Level 5 Computational Physics **Exercise 4**

The deadline for this exercise is Sunday 16th December 2012 at midnight. Your report and all program (*.c) files should be uploaded into Blackboard at the appropriate point in the Computational Physics (PHYS2COMP) course.

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Calculation of orbits

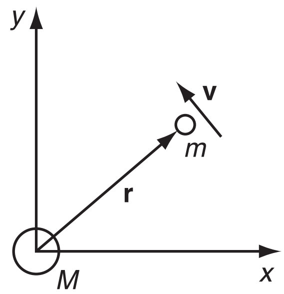
The aims of this exercise are:

- To use some of the computational tools and knowledge gained thus far in the Computational Physics course to solve real physical problems;
- To use the Runge-Kutta method to explore the motion of rockets and orbits.

Problem 1: Basic orbits (6/20 marks)

In this first attempt to simulate orbits, you will treat one body as being very massive and stationary at the origin. The other body will be in orbit around it. You will need to solve the equation of motion:

$$m\ddot{\mathbf{r}} = -\frac{mMG}{|\mathbf{r}|^2}\hat{\mathbf{r}} = -\frac{mMG}{|\mathbf{r}|^3}\mathbf{r} \tag{1}$$



where M is the stationary body, m the orbiting body and G the gravitational constant. You should use the 4th order Runge-Kutta approach, as discussed in lectures, to solve for $\mathbf{r}$ and $\dot{\mathbf{r}}$.

In your report you should simulate both circular orbits and eccentric, comet-like, orbits. Are the orbits stable and repeatable? Examine the energy of the moving body. Is it conserved?

Save this program and submit it with your report.

Problem 2: Moon shot (8/20 marks)

Now for a 'real world' problem that combines most of what you have learnt so far. Your goal is to launch a space probe from low Earth orbit (orbital radius 7000 km) such that it passes within 500 km of the Moon's surface to take photographs, 'slingshots' around the moon, and then passes close to Earth once more to send back the information by radio. You have sufficient fuel for only one rocket 'burn' to leave Earth orbit, and the rest of the flight is simply coasting.

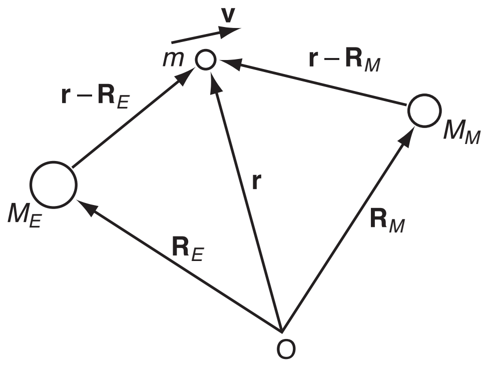
Adapt your previous program to simulate the motion of the probe and find the correct launch position (the point on the orbit where the rocket is fired) and velocity for the probe to fulfil the above requirements. How close can you get to the moon without crashing? How long does the flight to the moon and back take?

This is a non-trivial problem, and there are some important points to note if you want to avoid difficulties:

- This is another example of numerical integration of a differential equation; the equation of motion is now the combined influence of the Earth and Moon's gravitational field. To simplify the problem, treat the Earth and Moon as fixed in position for the duration of your flight. Then the equation of motion becomes (see figure):

$$m\ddot{\mathbf{r}} = -\frac{mM_E G}{|\mathbf{r}-\mathbf{R}_E|^3}(\mathbf{r}-\mathbf{R}_E) - \frac{mM_M G}{|\mathbf{r}-\mathbf{R}_M|^3}(\mathbf{r}-\mathbf{R}_M) \tag{2}$$

- Think about the simplifying symmetries in the problem. How many variables do you need? What is the best choice of coordinate system and origin? How many dimensions do you need to work in?
- The 4th order Runge-Kutta method is suitable, as before. However, unlike previous problems, you are not given the full boundary conditions at the start. Instead, you are given some boundary conditions which apply at the start, and some which apply at the end. You will need to use trial and error to establish the correct launch conditions to obtain the required ending.



- How will you know when to halt your calculation? How will you know if you've crashed?
- As always, check that your numerical solution has sufficient accuracy. Think about ways to check this, using boundary conditions that should give known behaviour. Remember that your solution should be independent of step size.
- Don't proceed blindly. Find ways to visualise your trial solutions, and make sure you are heading in the right direction.

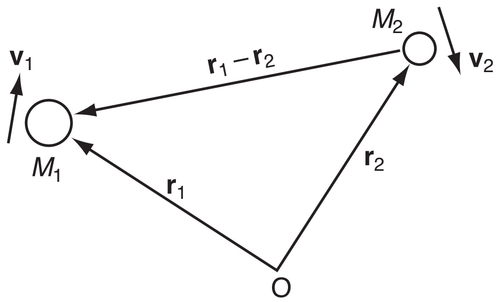
Save this program and submit it with your report.

Problem 3: Earth and moon (6/20 marks)

Finally, you are going to make the problem more challenging still, by simulating a 2-body problem, where both bodies are moving under their combined gravitational forces. The equations of motion will become:

$$M_1\ddot{\mathbf{r}}_1 = -\frac{M_1M_2G}{|\mathbf{r}_1-\mathbf{r}_2|^3}(\mathbf{r}_1-\mathbf{r}_2) \qquad (3a)$$

$$M_2\ddot{\mathbf{r}}_2 = -\frac{M_1M_2G}{|\mathbf{r}_2-\mathbf{r}_1|^3}(\mathbf{r}_2-\mathbf{r}_1) \qquad (3b)$$



You should generalise your program to handle two similar bodies, each moving under the influence of the other. Test your program by simulating the motion of two equal bodies, and then the moon and the earth. If you take too large a time step, the motion of the two bodies can become unstable and unpredictable – chaos can ensue. Can you find evidence for this? Discuss your findings in your report.

Submitting your work

You should submit the following to Blackboard:

1. A brief report, in MS Word or pdf format;
2. Your program solutions to problems 1 to 3.

As before, please note:

- Blackboard will not accept your compiled program—please only upload your "prog.c" files.
- Blackboard won't accept a ".c" file so please rename your file with a ".txt" extension.
- Please also give your programs sensible distinguishing names, including your name or userid e.g. "my_userid_ex4_prob_1.txt" or "myname_ex4_prob_2.txt".

If you have any problems submitting your work, please contact Dr. Hanna (s.hanna@bristol.ac.uk) or ask a demonstrator.