The very first idea is to make this document a learning tool for git, latex and, of course, of Electronics. A primary goal is to maintain English as its unique language.

Introduction to Electronics: theory, formulas and utilities

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and

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Contents

1 Semiconductors

Semiconductors are material with an electrical conductivity value falling between that of a conductor and an insulator. An important property of semiconductors is that, during a temperature increment, their resistance decreases. Common semiconductors material are:

- Silicon, Si. Z (i.e. atomic number) = 14. IV (14^{th}) group.
- Germanium, Ge. Atomic number = 32. IV (14^{th}) group
- Gallium Arsenide, GaAs, a compound of Gallium and Arsenic.

Another important characteristic of semiconductors is that there are two charge carriers available, holes and electrons.

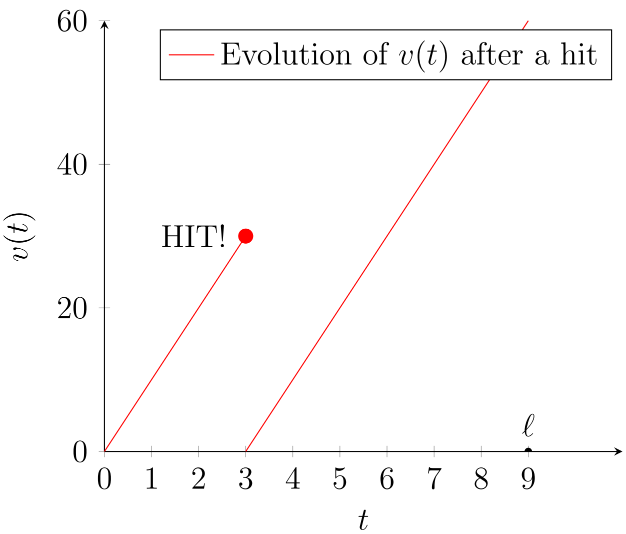
1.1 Movement analysis of electric charge in a semiconductor bar

Given a bar of length ℓ, and an Electric field E, every electron in valence band will be affected by a force $F = q \cdot E$, and thus its acceleration a will be, until the electron is subjected to the Electric field, $a(t) = \frac{F}{m} = \frac{q \cdot E}{m}$; being this quantity a constant respect to time, the speed will be a straight line with angular coefficient $m = a = \frac{q \cdot E}{m}$:

$$v(t) = \frac{q \cdot E}{m} \cdot t = a \cdot t \tag{1}$$

However, scientists have observed that during its path to freedom (i.e. the end of the bar), the electrons hit some nucleus, this resetting their speed (and, by consequence, kinetic energy) to zero.

`NTtA [Note To the author]: Look up the axis measures and ticks to make it realistic.`



In a difficult way, the average speed of the electrons can be expressed by the area underlying the red line divided by the total amount of time: $\bar{v}$ of function $v(t)$ is $\frac{1}{t_1}\int_0^{t_1} v(t) \cdot dt$.
In an easier way we can observe that between the $(i-1)^{th}$ and the i^{th} hit $\bar{v}_i = \frac{h}{2} = \frac{a \cdot \Delta t}{2}$, and so $\bar{v} = \frac{\bar{h}}{n} = a \cdot \frac{\sum \Delta t}{n} = a \cdot \bar{t}$, where n is the number of hits and $\bar{t}$ is the *average flying time*. By consequence

$$\bar{v} = a \cdot \bar{t} = \frac{\bar{t} \cdot q}{m} \cdot E = \mu_p \cdot E \tag{2}$$

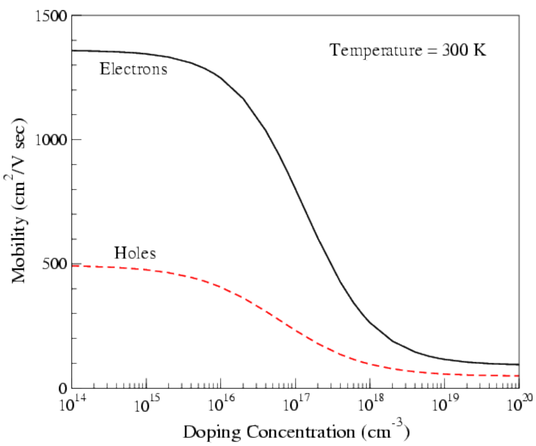
Where we introduce the ***Electron Mobility*** μ_p, that depends on $\bar{t}$, which is a material constant.

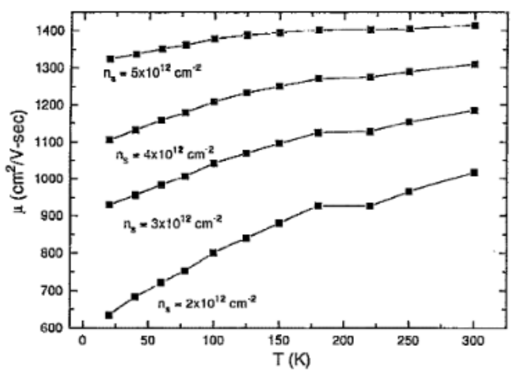
$$\mu_p = [\frac{cm^2}{V \cdot s}] \tag{3}$$

Mobility is a constant even for **holes** and usually it is related to electron mobility by the law

$$\mu_p \sim 3\mu_n \tag{4}$$

where μ_p is the *electron mobility* and μ_n is the *holes mobility* . The fact that $\mu_p > \mu_n$ doesn't surprise ourself because **holes** move in *valence band*, which is normally fulfilled of carriers, while **electrons** move in *conductance band*, where the movement is less influenced by other carriers and by nuclei.





In the figure we can observe that mobility is negatively influenced by doping concentration, but positively by temperature. NOT TRUE, HAD TO BE REVIEWED

1.1.1 Numerical analysis

Given a bar with:

- left-side potential = 0 V
- right-side potential = $+V$ (given by a generator)
- bar lenght ℓ and area A

Check for signs in equations Being $V = -\int_0^\ell E d\ell$ and assuming the electric field E being constant in the bar, $V = -E\int_0^\ell d\ell = -E \cdot \ell$, so we can say $|E| = \frac{V}{\ell}$.
By definition,

$$I = \frac{\Delta Q}{t} = I_n + I_p \tag{5}$$

where t is the transit-time and I_n, I_p being relatively the current generated by negative carriers going to the right side of the bar (electrons) and by positive carriers going to the left side of the bar (holes).

1.1.2 I_n

$I_n = \frac{q \cdot number\ of\ electrons}{t_{tr}}$. Being $number of electrons = n \cdot v$, where n is the electrons concentration in the semiconductor bar $[\frac{electrons}{cm^3}]$ and $v = \ell \cdot A$ is the volume of the bar, we can write the following equation: $I_n = \frac{q \cdot n \cdot \ell A}{t_{tr}}$. Writing $t_{tr} = \frac{\ell}{v_n} = \frac{\ell}{\mu_p \cdot E}$ we finally can say that

$$\boxed{I_n = \frac{qn\ell A}{\frac{\ell}{\mu_p \cdot E}} = \frac{qn\ell A}{\frac{\ell}{\mu_p \cdot \frac{V}{\ell}}} = \mu_n qn \frac{A}{\ell} \cdot V} \tag{6}$$

We observe that 6 is the famous **Ohm's law**

$$R = \frac{1}{\mu_n qn} \frac{\ell}{A} \tag{7}$$

where all the variables are related to the physical state of the bar.

1.1.3 Resistivity and conductivity

Doing a reasoning analogue to that for I_n, we can observe that $I_p = \mu_p qn \frac{A}{\ell} \cdot V$. The general expression of the current intensity in the bar is so given by:

$$I = (n\mu_n + p\mu_p)q \cdot \frac{\ell}{A} \cdot V \tag{8}$$

where the first term, $(n\mu_n + p\mu_p)q = \sigma$ is called *conductivity* of the material.
$\rho = \frac{1}{\sigma}$ is the resistivity of the material. Resistivity is a special characteristic of the material.

$$\rho = [cm \cdot \Omega] \tag{9}$$

1.1.4 Diffusion Current

The phenomenon of Diffusion Current is a characteristic of semiconductor materials.
When there is a nonuniform concentration of charge carriers (e.g. *holes*) we can 'sit' in a point (the point 0 in the figure) and observe that there is a *gradient* $\frac{dp}{dx}$ for the function p to the left direction. The existence of this gradient implies that on the left of it the holes concentration is higher and on the right of it it will be lower. Imaging this point as a surface, it will be reasonably to think that there are more holes that will come from right to left than otherwise. So, we can assert that there is a statistical current (only due to this statistical observation) over the semiconductor called **Diffusion Current**.
The *Diffusion hole-current density* $J_p[\frac{A}{cm^2}]$ is proportional to the concentration gradient $\frac{dp}{dx}$ and is given by

$$J_p = -qD_p \frac{dp}{dx} \tag{10}$$

where D_p is called the *diffusion constant* for holes. The minus sign is necessary because the gradient is directed to the direction of maximum increment, so that J_p will be positive for increments of x.

1.1.5 Total current

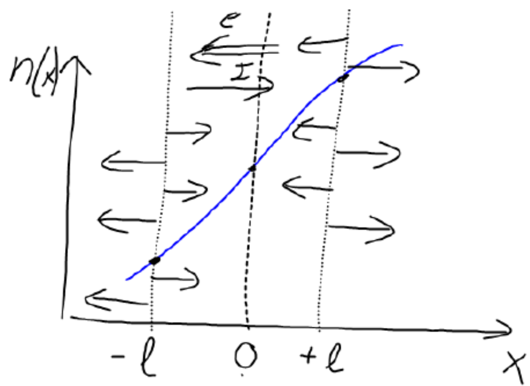
Hence we can state that the total current over a doped semiconductor bar is given by:

$$I_{TOT} = I_p + I_n \tag{11}$$

where

1. $I_n = q\mu_n n \frac{A}{L} V + qD_n \frac{dn}{dx}$
2. $I_p = q\mu_p p \frac{A}{L} - qD_p \frac{dp}{dx}$

Diffusion current is a current in a semiconductor caused by the change of concentration of carriers in a semiconductor.



We can say, by an act of semi-religious acceptance, that the flux at point 0 is subjected to the law:

$$\Phi_e(0) = \frac{1}{2}n(+\ell)v_{TH} - \frac{1}{2}n(-\ell)v_{TH} \tag{12}$$

And, at the end of some points that I can't get at this moment, we conclude that

$$I_{DIFF} = -qA\Phi_e = (qv_{TH}\ell)A\frac{dn}{dx} = qD_n\frac{dn}{dx}A \tag{13}$$

Where V_{TH} is the *thermal speed*, and D_n is the *diffusion constant* of the electrons.

1.2 Semiconductors: a nuclear approach

In 1913 Niehls Bohr proposed a model of atom in which *quantization of the angular momentum* explained some interesting results regarding the light emitted by Hydrogen atoms heated. This model is very useful to explain the internal structure of atoms and can tell us something even about Silicon. A neutral Silicon atom possesses 14 protons and many electrons.

- the first 10 of them lies in the first two energy level: they fit perfectly around the nucleus and are generally not involved in chemical reactions (because of the very high amount of energy necessary to "interact" with them).

- the remaining 4 electrons are called *valence electrons* and are at higher energy levels. They are involved in the *covalent bond*.

When temperature is near at the absolute zero (-273^oC) the Silicon atoms assume a crystal form in which every *valence electron* is shared with the nearest 4 atoms of Silicon with a covalent bond: in this circumstance Silicon act as an **insulator**. At higher temperatures, (i. e. furnishing energy to the atom) covalent bounds can break.

1.2.1 Electrons and Holes

When the break happens, an electron *jumps* from valence band to conduction band and a hole is generated. The conduction-band electron is now a **charge carrier** and the atom it left is a charged positively (cation).

1.2.2 Valence and conduction bands

Without the necessary energy, electrons remain under the *Fermi level*, in a zone called *Valence Band*. when the *jump* occurs, electrons step over the *Fermi level* and stay in the *conduction band*. In the conduction band charge carrier have less constraints and can easily move in the crystalline lattice, depending on the intensity of the electro-magnetic field.

1.2.3 Silicon properties

- The amount of charge carriers in a cm^3 (i.e. electrons in the conduction band) at ambient temperature $n = 10^{10}cm^{-3}$. It is reasonably a little number compared to the global amount of atoms in a Silicon cm^3 ($5 \cdot 10^{22}$). [Think that if atoms were people on earth, only 7 people would result a cation with the loss of their electron].
- It is compatible with SiO_2 and Al

1.3 Doping

The doping process consists in significantly change the number of *electrons* or *holes* in a semiconductor. This result is achieved inserting atoms of the doping element in the crystalline arrangement of the doped.

1.3.1 Utilities and numbers

- $N_d \approxeq 10^{12} : 10^{20} cm^{-3}$ number of donors
- $p[\frac{electrons}{cm^3}]$ number of holes
- $n[\frac{electrons}{cm^3}]$ number of electrons

Remember that:

- n-doping leaves free electron in conduction band and is obtained by inserting DONORS atoms of *Phosphorus*, or other elements from the Vth group.
- p-doping creates holes in the lattice structure and is obtained by inserting ACCEPTORS atoms of *Boron*, or other elements from the IIIth group.

1.3.2 The intrinsic Concentration

The Mass action Law states that, in doped semiconductor, the electrons and holes density are related by the following expression:

$$n \cdot p = n_i^2 \tag{14}$$

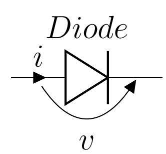
Where n_i is constant respect to doping but varies on temperature following the law:

$$n_i^2 = A_0 T^3 \epsilon^{\frac{-E_{GO}}{kT}} \tag{15}$$

where E_{GO} is the energy gap at $0^o K$ in electron volts, k is the Boltzmann constant (in $eV/^oK$) and A_0 is a constant independent from T.

- Germanium at 300^o K: $n_i \simeq 10^{13} cm^{-3}$
- Silicon at 300^o K: $n_i \simeq 10^{10} cm^{-3}$

2 Diodes



Diodes are defined as a p-n junction. Is is made by two parts: the left part (according to the figure) is a p-doped semiconductor in which there are holes left. The other part, on the contrary, is n-doped in which there are electrons left.

3 FORMULAS

3.1 Electric field and semiconductors

Electrostatic force between two charges $q_1 q_2$ at a distance r:

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} \tag{16}$$

The Electric field (as the force which a charge q is subjected to in a point of the space divided by the charge q)

$$E = \frac{F}{q} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} \tag{17}$$

Note that the Electric field is a **CONSERVATIVE FIELD**
A conservative field is a field in which the *Work* done for placing an element from a point a to a point b depends only from the different location of a and b and not by the route done.

The *Work* in a conservative field is defined as follows:

$$W_{a\to b} = \int_a^b \overrightarrow{F}(x)dx = q(V_b - V_a)d \tag{18}$$

The *Potential energy* of a charge subjected to an Electric field is

$$U = qV[eV] \tag{19}$$

and the measure unit is the electronVolt eV, where $1eV = 1,6\ 10^{-19}J$, the energy necessary to move an electron from a point a to a point b with $V_a - V_b = 1V$.

The *Electric Potential* at a point r in an Electric Field E is given by

$$V_E = -\int_C E \cdot \mathrm{d}\boldsymbol{\ell} \tag{20}$$

Where C is an arbitrary path from the point r to the point with zero potential.

The *Electric Current* is defined as i (from the french *intensité du courant*):

$$I = \frac{Q}{t} = \frac{V}{R} \qquad [Amp] \tag{21}$$

The *Law of mass action* is a relation about the concentration of free electrons and free holes in a semiconductor under thermal equilibrium. It states that the product of the concentration of free electrons and free holes is constant

$$p \cdot n = n_i^2 \tag{22}$$

Where n_i, *intrinsic concentration*, is the concentration of electrons (or hole, that is the same) of a *pure* semiconductor. Question: i_n isn't a properly constant value: it depends on temperature and material (?)

4 Engineer's corner

Question 1: How much is $1C$?

At first attempt we can say that 1 $Coulomb$ is *very* approximately 10^{19} times the fundamental charge unit (the electron). However it is a bit difficult rendering the idea of 10^{19} electrons.
Thinking about batteries, a common AAA battery has a capacity of 540 mAh, which are equivalent to 1944 C.
A common AA battery has a charge of 9 kC.
In everyday terms, a Coulomb is $\frac{1}{2000}$ times the charge occurring to empty a normal AAA battery.

5 Millman's questions

2.2 **Define mobility and give its dimension**
Mobility is defined as the coefficient between the Electric field applied to a bar and the speed of charge carriers moving into the bar. Hence we can say $\mu = \frac{\bar{v}}{E} = [\frac{cm^2}{E}]$, it is usually described with non SI dimension $\frac{cm^2}{E}$.

2.3 **Define conductivity and give its dimension**
Conductivity is a material constant used to determine the resistance of a semiconductor. It is defined as $\sigma = (n\mu_n + p\mu_p)q$ and depends on doping. Its dimension are $[\frac{1}{\Omega \cdot m} = \frac{S}{m}]$ because it can also be expressed as $\sigma = \frac{1}{\rho} = \frac{\ell}{R \cdot A}$

2.4 **Define a hole (in a semiconductor)**
A hole is the positive charge generated by the migration of an electron from an atom to another, leaving the first atom positively charged and neutralizing the second. In a general approach we can consider the holes mobility as a proper current.

2.6 **Define intrinsic concentration of holes**
intrinsic concentration of holes is a coefficient n_i temperature dependent that indicates the amount of free electrons (or holes, that is the same) in a pure semiconductor. The relation between the intrinsic concentration of holes and the holes density is given by the following $n_i^2 = n \cdot p$. At $0^oK, n_i = 0$. This fact has two explanation: the first is that at the absolute zero no energy is given to the electrons from the outside, hence it is impossible to break the covalent bonds between atoms. The second is based on the expression of n_i: $n_i \propto \epsilon^{-\frac{1}{T}}$, so the limit for this function for $T \to 0 = 0$.

2.9 **Define donor and acceptors impurities**
A semiconductor can be doped by donors (acceptors) atoms from the IV (III) group, like Phosphorus (Boron). These atoms, called impurities, fit in the lattice structure and tend to donate (accept) an electron, due to their atomic structure. In fact, Phosphorus (Boron) atoms reach their stability with 3 (5) electrons.

2.2 **A semiconductor is doped with both donors and acceptors of concentration N_D and N_A respectively. Write the equation from which to determine the electron and hole concentrations (n and p)**
The equation ruling these relations are

$$n \cdot p = n_i^2, N_p + p = N_n + n$$

Problem 6. a) Suppose an entire function f is bounded by M along $|z| = R$. Show that the coefficients C_k in its power series expansion about 0 satisfy

$$|C_k| \leq \frac{M}{R^k}.$$

b) Suppose a polynomial is bounded by 1 in the unit disc. Show that all its coefficients are bounded by 1.

Solution Part a): Since f is an entire function it can be expressed as an infinite power series, i.e.

$$f(z) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} z^k = \sum_{k=0}^{\infty} C_k z^k.$$

If we recall Cauchy's Integral we have

$$f(z) = \frac{1}{2\pi i} \int_\gamma \frac{f(w)}{w - z} \, dw,$$

carefully notice that $\frac{1}{w-z} = \frac{1}{w} \cdot \frac{1}{1-\frac{z}{w}}$ can be written as a geometric series. We have

$$\begin{aligned}
\frac{1}{2\pi i} \int_\gamma \frac{f(w)}{w - z} \, dw &= \frac{1}{2\pi i} \int_\gamma \left\{ \frac{f(w)}{w} \cdot \left(\frac{1}{1 - \frac{z}{w}} \right) \right\} \, dw \\
&= \frac{1}{2\pi i} \int_\gamma \left\{ \frac{f(w)}{w} \cdot \left(1 + \frac{z}{w} + \frac{z^2}{w^2} + \frac{z^3}{w^3} + \cdots \right) \right\} \, dw \\
&= \left(\frac{1}{2\pi i} \int_\gamma \frac{f(w)}{w} \, dw \right) z^0 + \left(\frac{1}{2\pi i} \int_\gamma \frac{f(w)}{w^2} \, dw \right) z^1 + \left(\frac{1}{2\pi i} \int_\gamma \frac{f(w)}{w^3} \, dw \right) z^2 \cdots
\end{aligned}$$

Now take the modulus of C_k to get

$$|C_k| = \left| \frac{1}{2\pi i} \int_\gamma \frac{f(w)}{w^{k+1}} \, dw \right| \leq \frac{1}{2\pi} \int_\gamma \frac{|f(w)|}{|w^{k+1}|} \, |dw| \leq \frac{M}{2\pi} \int_\gamma \frac{|dw|}{|w^{k+1}|}$$

Then integrate along $\gamma(\theta) = Re^{i\theta}$ for $\theta \in [0, 2\pi]$ to get

$$|C_k| \leq \frac{M}{2\pi} \int_0^{2\pi} \frac{|iRe^{i\theta} \, d\theta|}{|R^{k+1} e^{ik\theta}|} = \frac{M}{2\pi \cdot R^k} \int_0^{2\pi} \, d\theta = \frac{M}{R^k}.$$

Hence, $|C_k| \leq \frac{M}{R^k}$.

Index