Coimisiún na Scrúduithe Stáit
State Examinations Commission

Junior Certificate Examination 2018

Mathematics

Paper 1
Higher Level

Friday 8 June
Afternoon 2:00 to 4:30

300 marks

Examination Number

Centre Stamp

Running Total	

For Examiner					
Q.	Ex.	Adv. Ex.	Q.	Ex.	Adv. Ex.
1			11		
2			12		
3			13		
4			14		
5					
6					
7					
8					
9					
10			Total		

Grade

Instructions

There are 14 questions on this examination paper. Answer **all** questions.

Questions do not necessarily carry equal marks. To help you manage your time during this examination, a maximum time for each question is suggested. If you remain within these times you should have about 10 minutes left to review your work.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You may lose marks if your solutions do not include supporting work.

You may lose marks if you do not include the appropriate units of measurement, where relevant.

You may lose marks if you do not give your answers in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Question 1   **(Suggested maximum time: 5 minutes)**

(a) Only one linear pattern begins with "1, 7".

Fill in the three boxes below so that the numbers form this **linear** pattern.

Linear pattern: 1,   7,   ☐ ,   ☐ ,   ☐

(b) Many different quadratic patterns begin with "1, 7".

Fill in the three boxes below so that the numbers form a **quadratic** pattern.
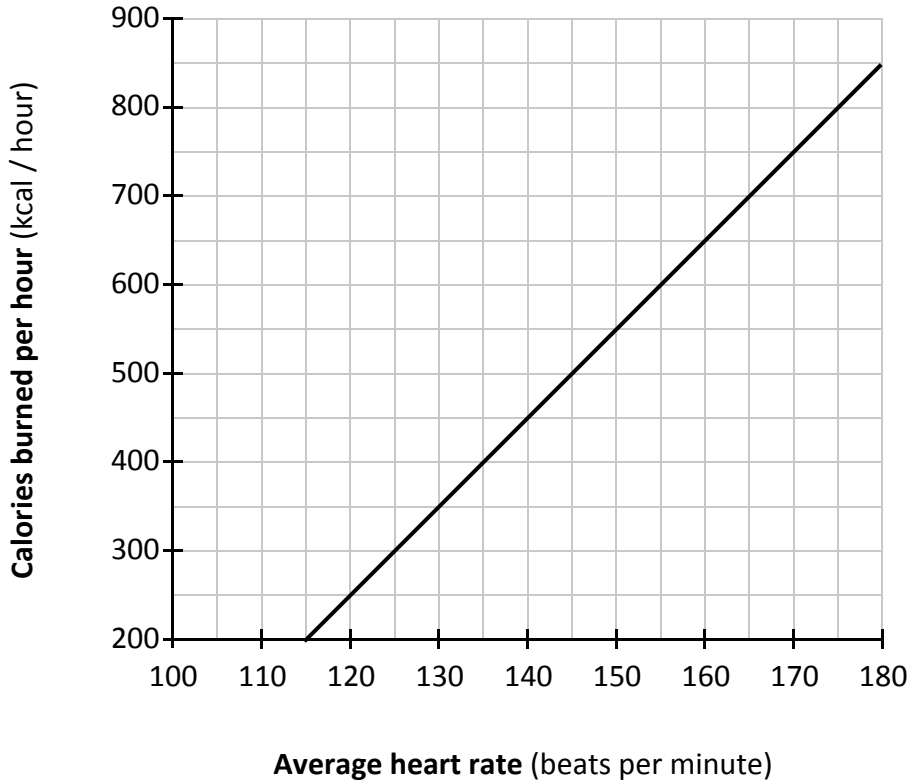
Quadratic pattern: 1,   7,   ☐ ,   ☐ ,   ☐

Question 2   **(Suggested maximum time: 15 minutes)**

(a) Phil is an athlete. The graph below shows the number of calories (in kcal) she burns per hour, depending on her average heart rate.
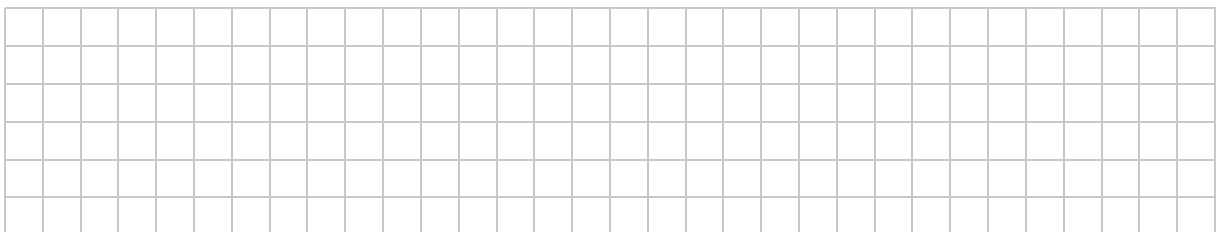
Note: the axes do not start at $(0,0)$.

Average heart rate (beats per minute)

(i) Use the graph to estimate how many calories Phil would burn in one hour if she had an average heart rate of **170 beats per minute**.

Calories burnt in one hour = ☐ kcal

(ii) Use the graph to work out Phil's average heart rate when she burns **300 kcal** in **30 minutes**.

Average heart rate = ☐ beats per minute

(b) Phil runs a series of stages. In each stage she runs a slow run of 60 m, and then a sprint. In each stage after stage 1, she sprints 50 m more than she did in the previous stage.

Complete the table below, showing the distance that she runs slowly and the distance that she sprints in each stage, as well as the total distance per stage.
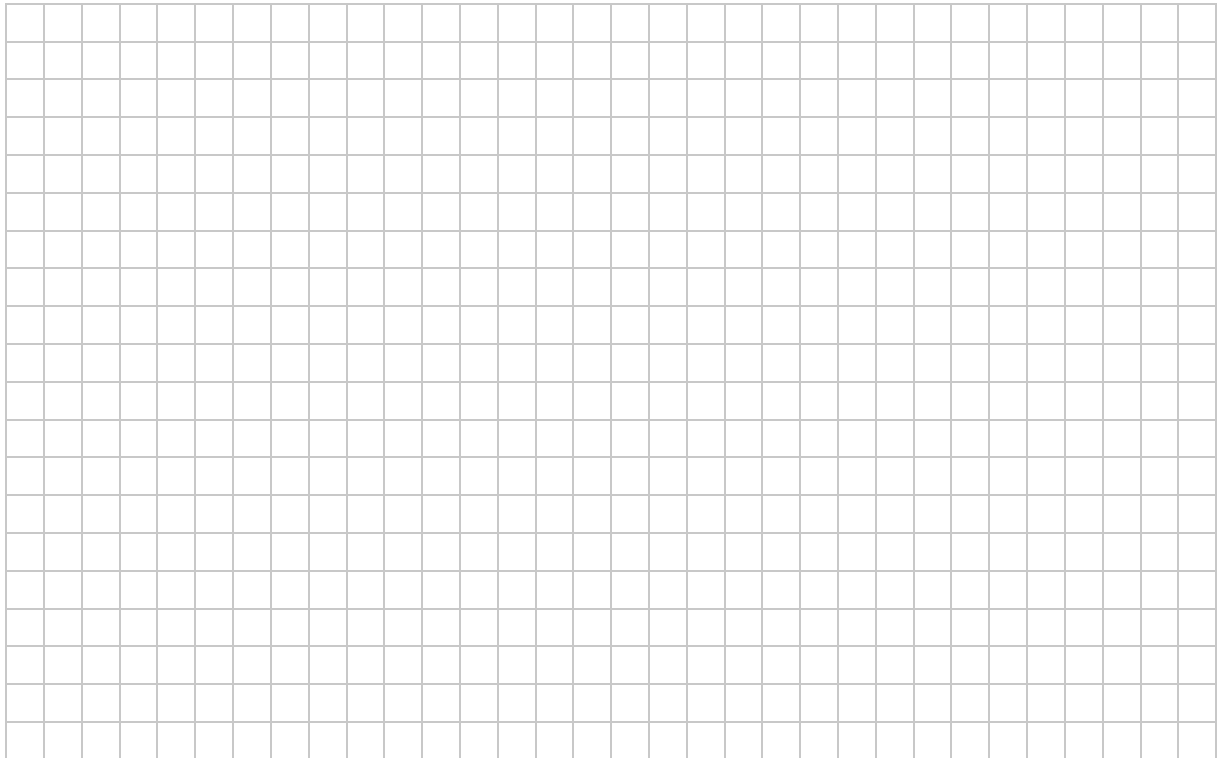Give the values in the last row in terms of n, where appropriate.

Stage	Slow run (metres)	Sprint (metres)	Total distance (metres)
1	60	50	110
2	60	100	160
3	60	150	
4			
5			
n			
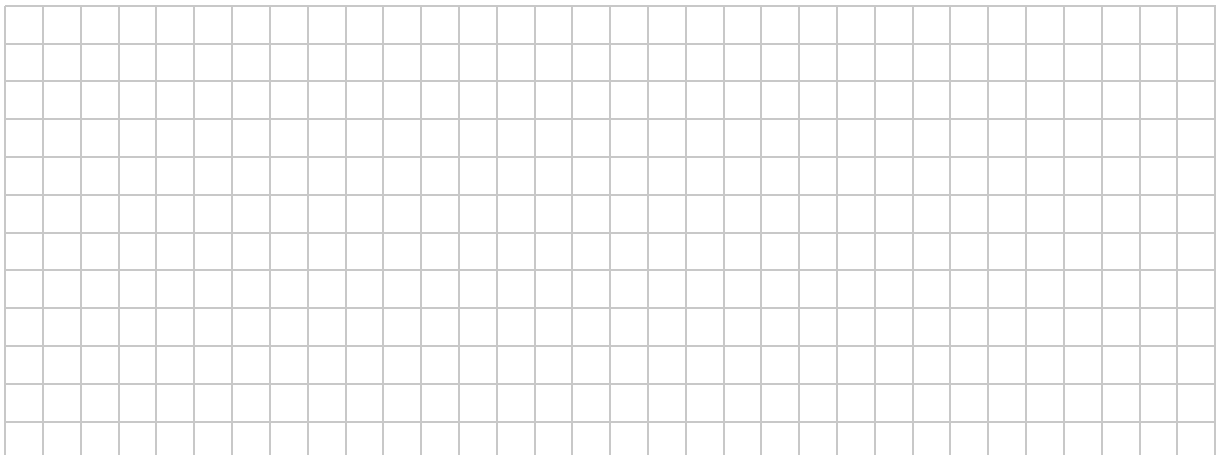
This question continues on the next page.

(c)   In one of the stages, Phil:

runs slowly for 60 metres at 2 metres per second, and then
sprints 150 metres at 7·5 metres per second.

Work out Phil's **average speed** for this stage, in metres per second.

Question 3   **(Suggested maximum time: 5 minutes)**

Patrick worked for a weekend washing cars in a garage.

(a) In total, 35 cars were washed in the garage that weekend. Patrick washed 14 of them.
Work out the **percentage** of the cars that Patrick washed.

(b) Patrick was paid £200 Sterling for the weekend. He converted this money to US Dollars ($).
The exchange rates are shown below.

$$€1 = £0 \cdot 88 \text{ Sterling}$$

$$€1 = \$1 \cdot 18 \text{ US Dollars}$$

Work out how many US Dollars ($) Patrick got for his £200 Sterling.
Give your answer correct to the nearest cent.

Question 4 (Suggested maximum time: 5 minutes)

F is the set of **factors** of **12**.

(a) List the six elements of the set F.

$$F = \{ \quad , \quad , \quad , \quad , \quad , \quad \}$$
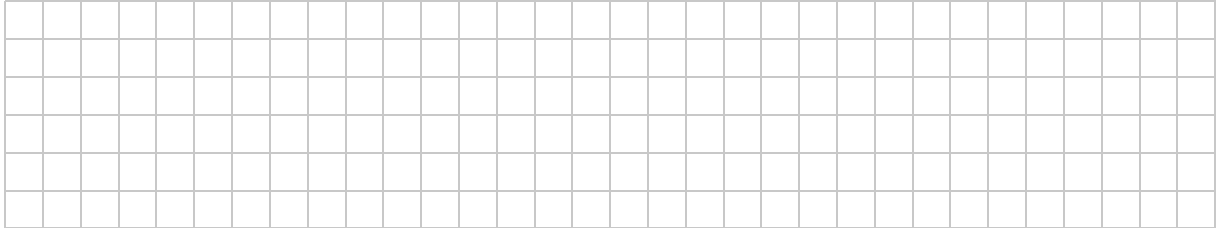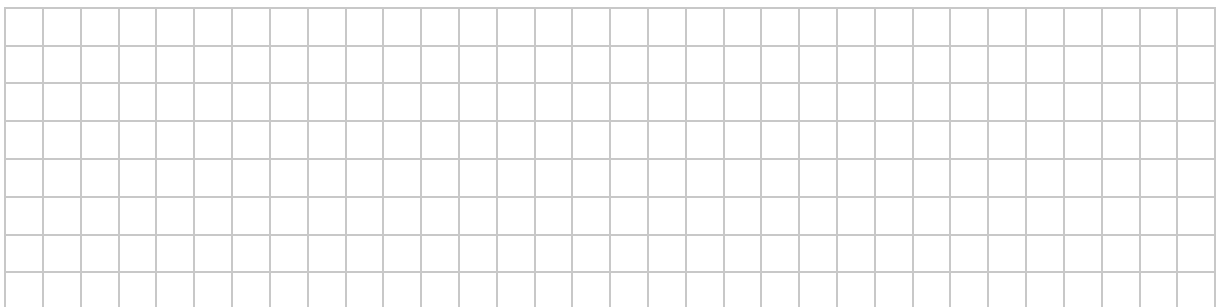
The sets P and S are as follows:

P is the set of **all prime** numbers
S is the set of **all square** numbers (i.e. all numbers n^2, where $n \in \mathbb{N}$).

The Venn diagram below shows the sets F, P, and S.
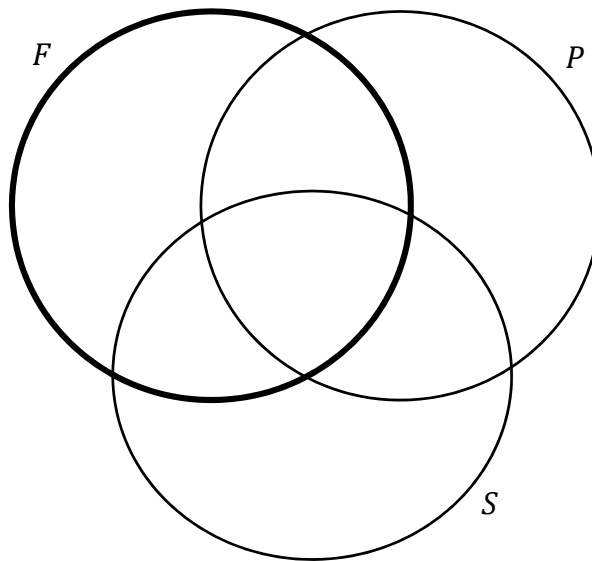
(b) Write each of the factors of 12 from part **(a)** into the correct region in the set F
in the Venn diagram.
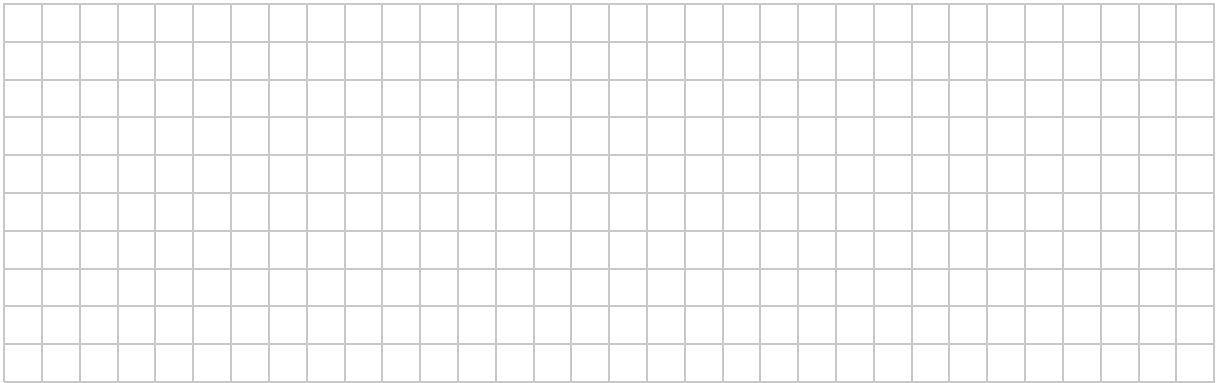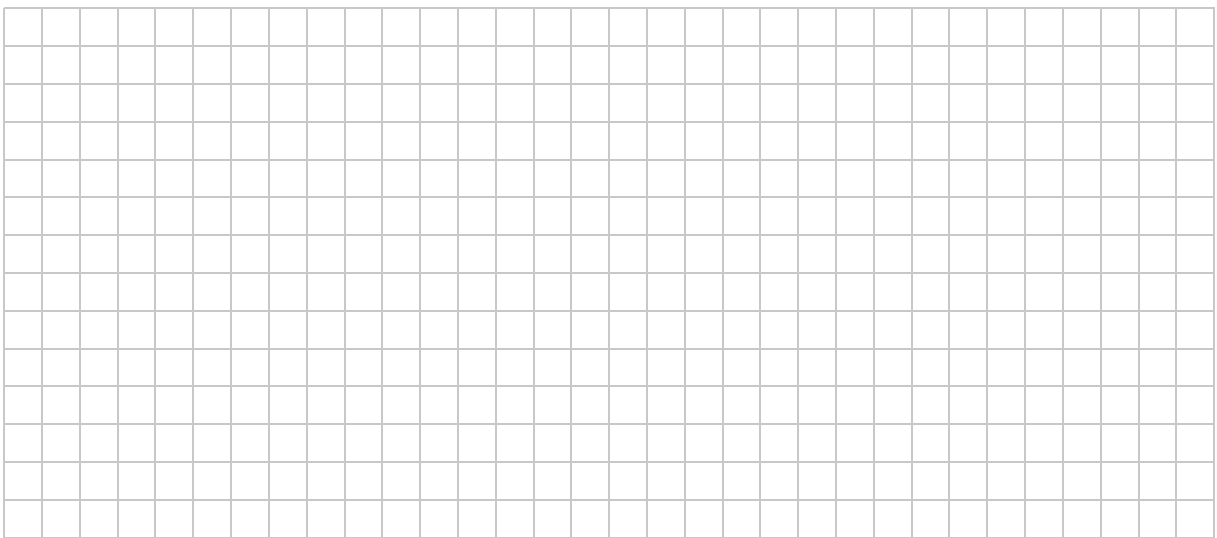
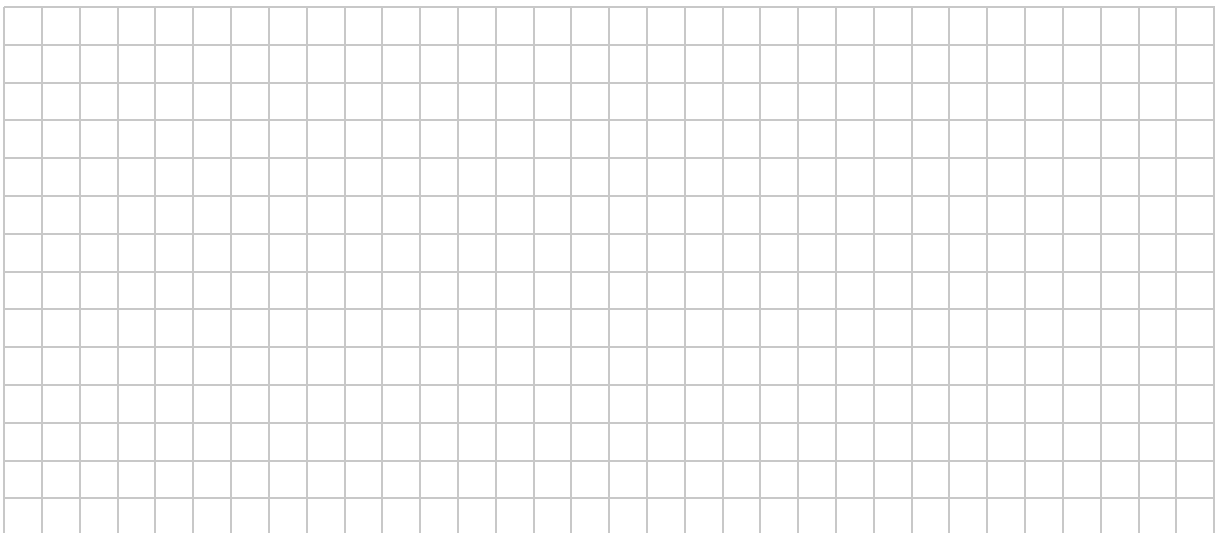**Question 5** (Suggested maximum time: 10 minutes)

(a) Work out the value of $3p - 4t^2$, when $p = 6$ and $t = 5$.

(b) Multiply out and simplify $(2x - 3)(4 - 5x + x^2)$.

(c) Factorise fully $10de - df - 5ef + 2d^2$.

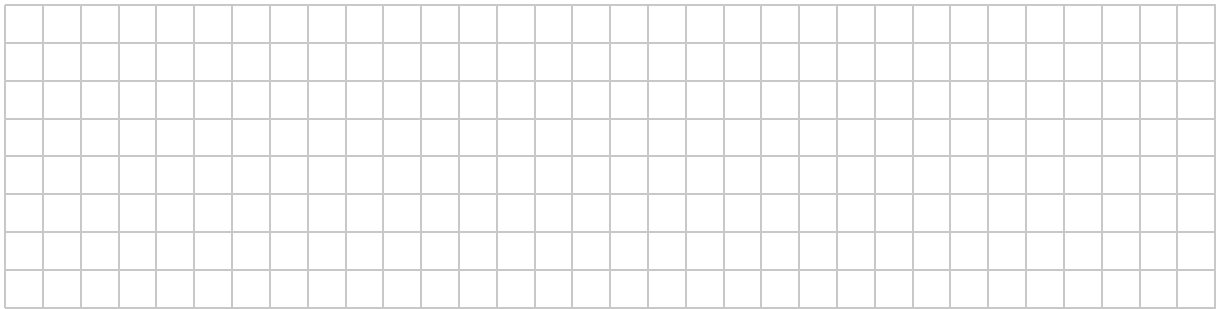| previous	page	running

Question 6 (Suggested maximum time: 15 minutes)

A sheet is folded in half a number of times.

$h(x) = 2^x$ is the number of layers after x folds.

(a) Fill in the table to show the number of layers after each of the first 6 folds.

Number of folds, x	1	2	3	4	5	6
Number of layers, $h(x)$	2	4	8			

(b) List the elements of the domain and the range of $h(x)$ that are shown in the table above.

Domain = { , , , , , }

Range = { , , , , , }

(c) Work out the least number of folds that would be needed to have **more than 500** layers.

(d) Work out the number of layers after 12 folds.
Give your answer in the form $a \times 10^n$, where $a \in \mathbb{R}$, $1 \le a < 10$, and $n \in \mathbb{N}$.

(e) Explain what the following statement means, in terms of folds and layers.

$$h(14) > 10\,000$$

(f) Put a tick in the correct box to show what kind of pattern is made by the number of layers.
Tick (✓) **one** box only. **Justify** your answer.

linear	quadratic	exponential	none of these
☐	☐	☐	☐

Justification:

This question continues on the next page.

(g) After a certain number of folds, there are k layers.

How many layers will there be after 3 **more** folds?
Give your answer in terms of k.

(h) After a certain number of folds, there are 2^p layers.

How many layers will there be after 3 **more** folds?
Give your answer in the form 2^m, where m is in terms of p.

Question 7 **(Suggested maximum time: 10 minutes)**

Joonas has an unlimited supply of €5 notes and €2 coins.

(a) Fill in the table to show three different ways in which he can use these to make exactly €27. One way is already done.

	Number of €5 notes	Number of €2 coins	Total amount of money
Way 1	1	11	€27
Way 2			€27
Way 3			€27

(b) Explain how he could use his supply of €5 notes and €2 coins to make every whole number value of money greater than €3 (i.e. to make € n for every $n \in \mathbb{N}$, where $n > 3$).

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Question 8   **(Suggested maximum time: 10 minutes)**

(a)   Solve the following equation. Give each answer correct to two decimal places.

$$x^2 - 4x - 7 = 0$$
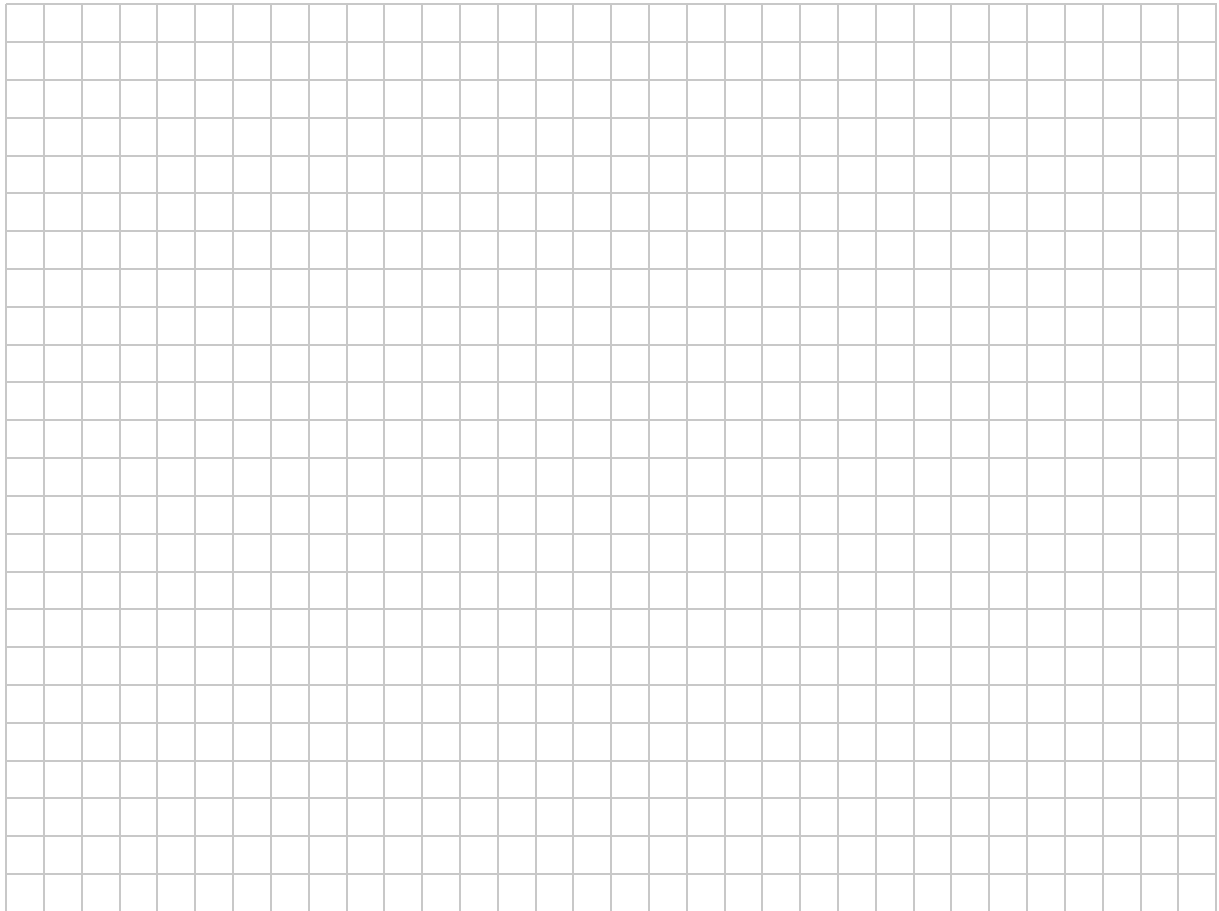
(b) The solutions of the equation $x^2 - x - 2 = 0$ are $x = -1$ and $x = 2$.

The table below shows three graphs, **A**, **B**, and **C**.
One of these is the graph of the function $f(x) = x^2 - x - 2$.

Graph **A**	Graph **B**	Graph **C**

Put a tick in the correct box to show which of these is the graph of $f(x) = x^2 - x - 2$.
Tick (✓) **one** box only. **Justify** your answer.

      A   **B**   **C**

☐   ☐   ☐

Justification:

Question 9   **(Suggested maximum time: 15 minutes)**

A group of friends makes a video and posts it on YouTube.
They know that they will be paid approximately €**15 for each 10 000 views** of the video.
Use this information to answer the following questions.

(a) Work out how many views of the video they would need in order to be paid €**45**.
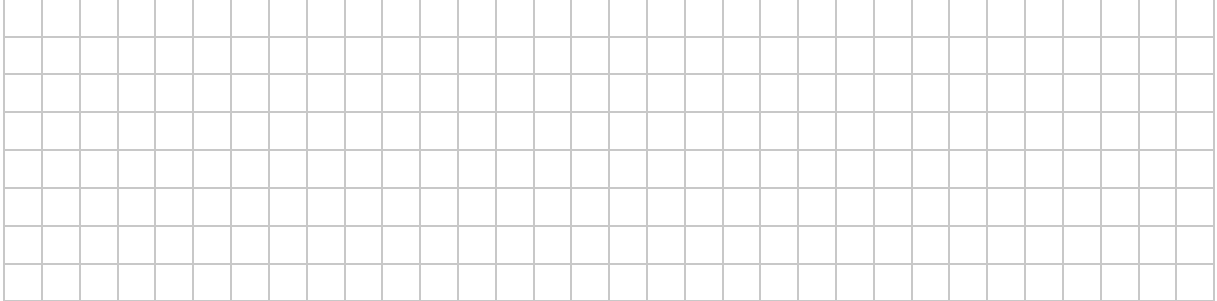
(b) Work out how much (in euro) they will be paid for **80 000 views** of the video.
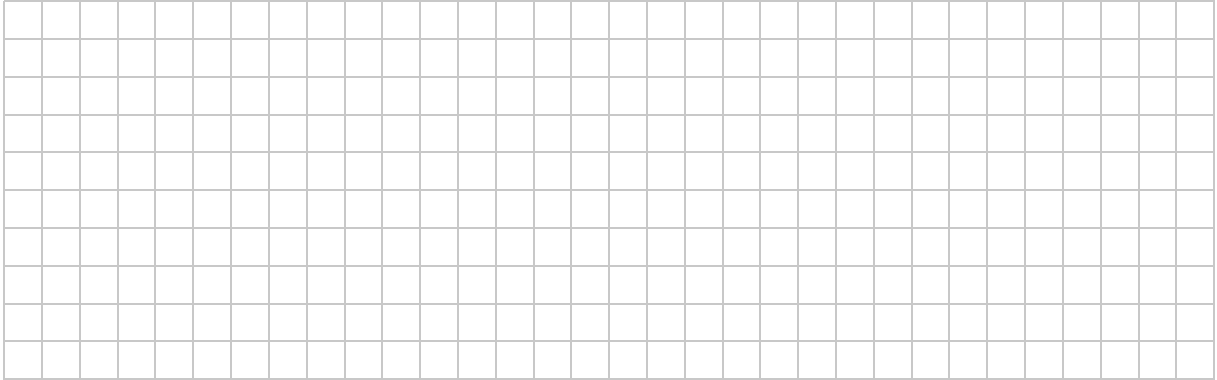
(c) Work out how much (in euro) they will be paid, on average, for **1 view** of the video.

The video costs them €70 to make. It does not cost them anything to post the video on YouTube.

(d) Write an expression in x for the **profit** (in euro) they will get for x views of the video.

There are **7 people** in the group.
They divide the **profit** from the video evenly between them. They **each** get €200 in profit.

(e) **(i)** Write an equation in x to represent this information, where x is the total number of views of the video.

(ii) Work out the value of x, the total number of views of the video.

Question 10   **(Suggested maximum time: 15 minutes)**

The rates and bands for income tax are as follows:
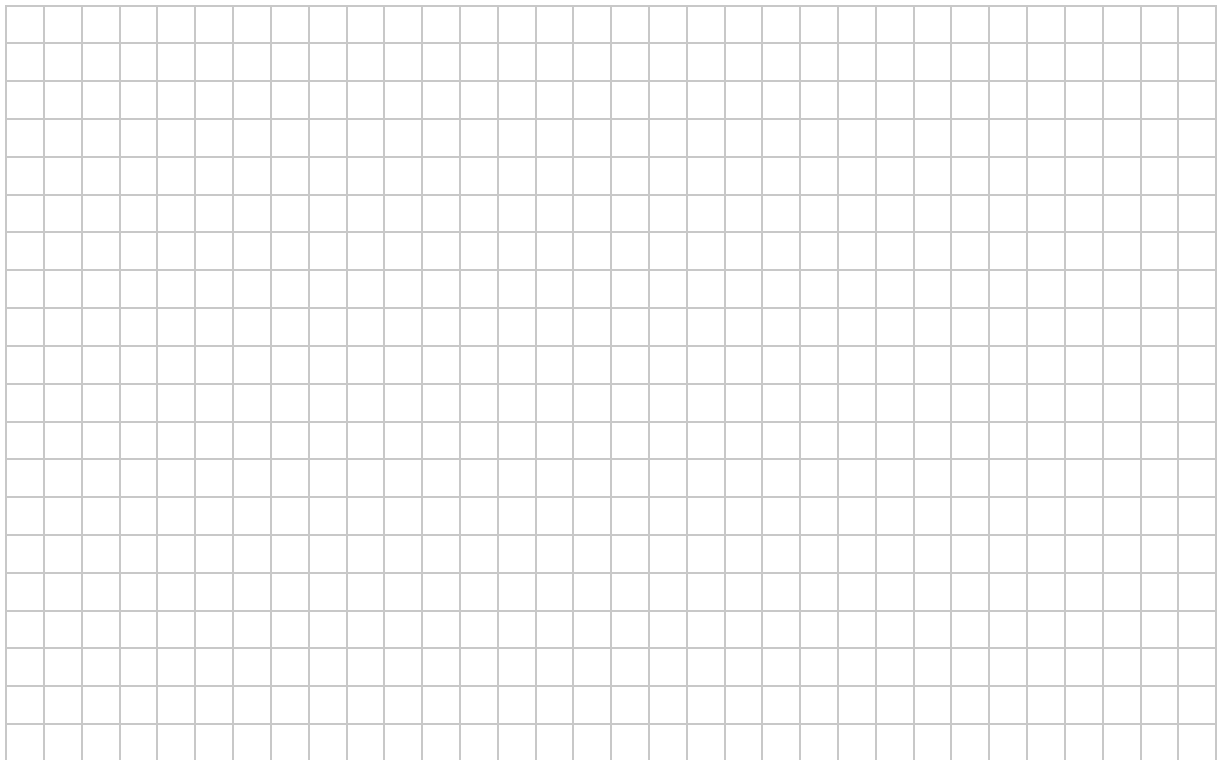
> €35 000 @ 20%,
>
> balance @ 40%.

(a)   Lorna has a gross income of €50 000 for the year.
Using the rates and bands above, work out her **total** income tax for the year.

(b)   Complete the **graph** on the next page to show the amount of income tax for every gross income up to €60 000 per year. Show all your working out.

The amount of income tax is shown for gross incomes up to €10 000.
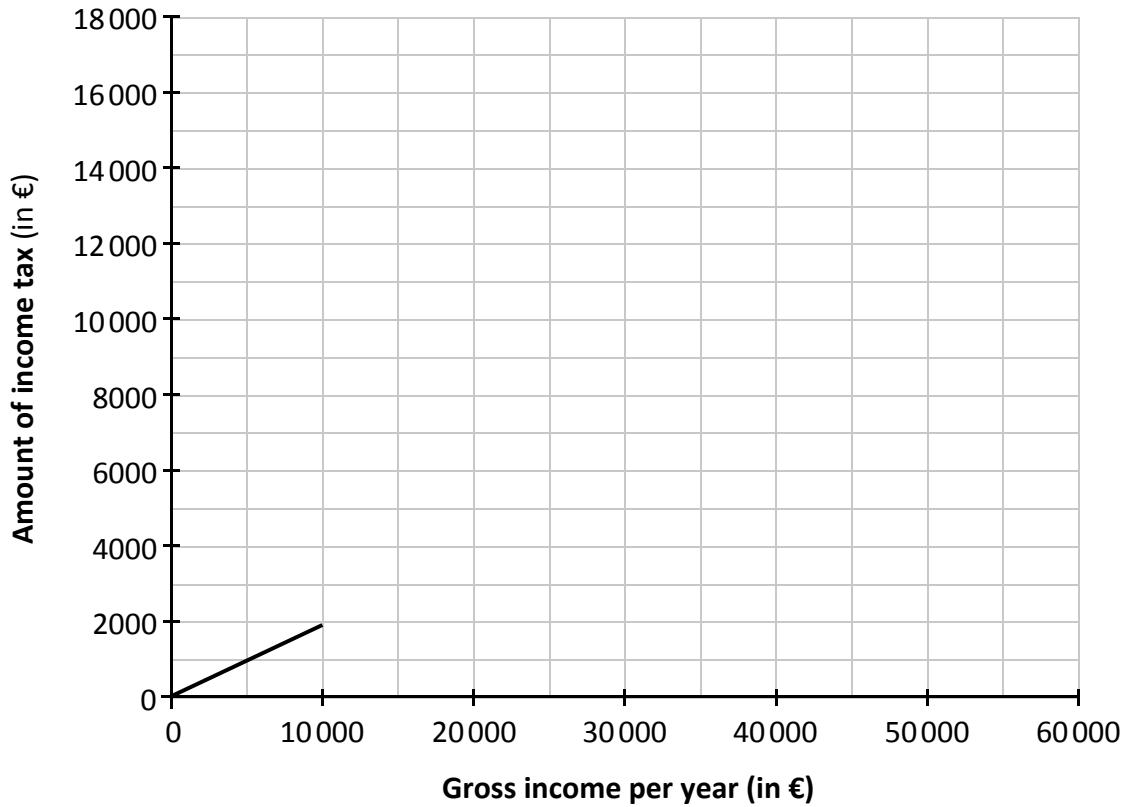
(c) This graph doesn't take tax credits into account.
Assume that everyone gets a **tax credit** of €**3000**.
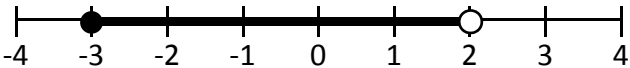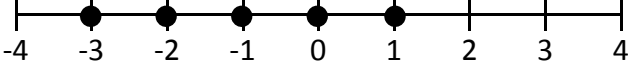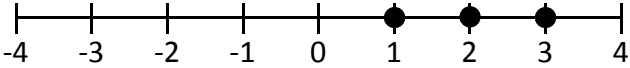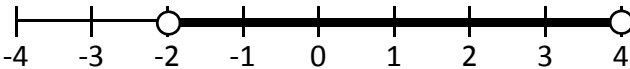
On the diagram above, **draw** a graph to show the amount of income tax to be paid for every gross income from €15 000 to €60 000 per year, taking this tax credit into account.
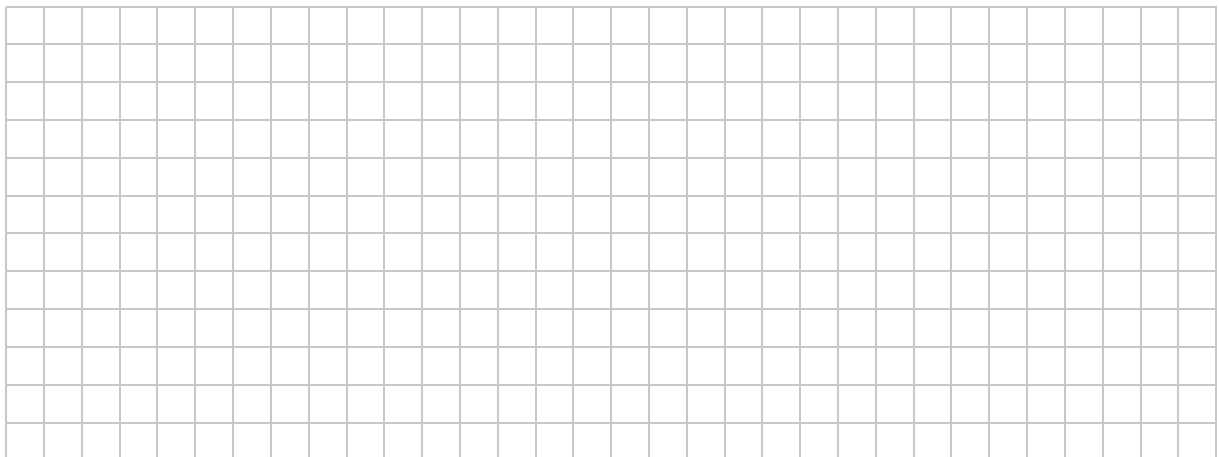
Question 11   **(Suggested maximum time: 5 minutes)**

Write down an inequality in x represented by each of the number lines shown below.
Put a tick (✓) in the correct box in each case to show whether $x \in \mathbb{N}$, $x \in \mathbb{Z}$, or $x \in \mathbb{R}$.
The first one is done.

Number line	Inequality in x	Domain (Tick **one** box only in each case)
(number line from −3 filled to 2 open)	$-3 \leq x < 2$	$\mathbb{N}$ ☐ $\mathbb{Z}$ ☐ $\mathbb{R}$ ✓
(points at −3, −2, −1, 0, 1)		$\mathbb{N}$ ☐ $\mathbb{Z}$ ☐ $\mathbb{R}$ ☐
(points at 1, 2, 3)		$\mathbb{N}$ ☐ $\mathbb{Z}$ ☐ $\mathbb{R}$ ☐
(open at −2 to open at 4)		$\mathbb{N}$ ☐ $\mathbb{Z}$ ☐ $\mathbb{R}$ ☐

Question 12 **(Suggested maximum time: 10 minutes)**

(a) Write the following as a single fraction in its simplest form.

$$\frac{2}{n-3} - \frac{5}{2n+5}$$

(b) Show that $(4x-3)^2 + 24x$ is **positive** for all values of $x \in \mathbb{R}$.

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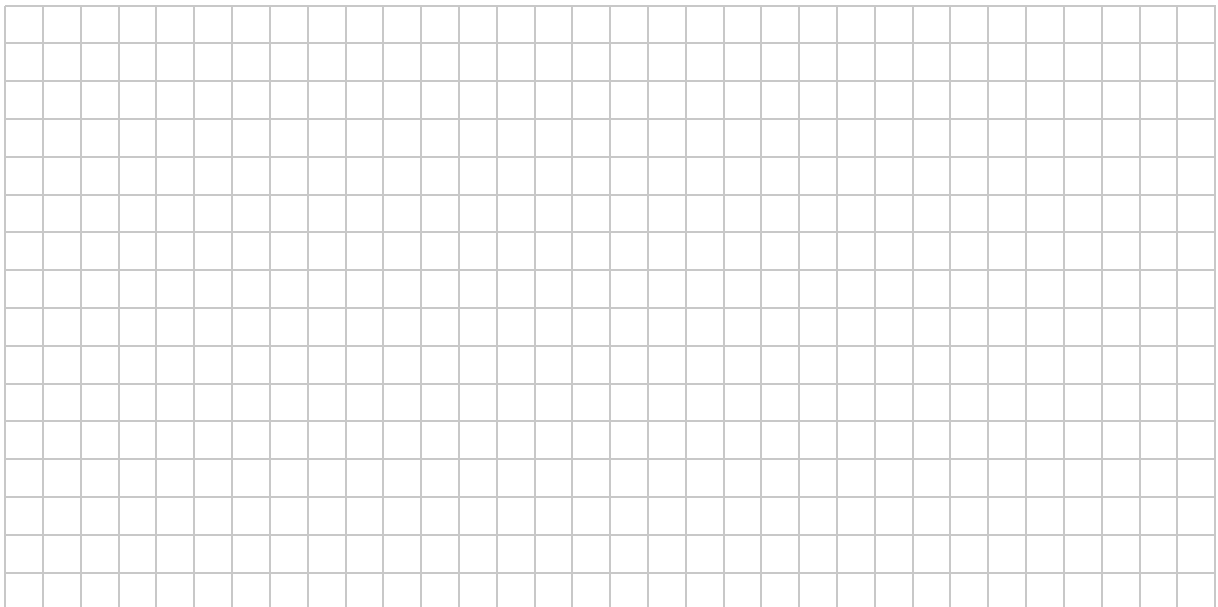**Question 13**   (Suggested maximum time: 10 minutes)

Below are four statements about three **non-empty** sets P, Q, and R in a universal set U.

(a) Put a tick ($\checkmark$) in the correct box in each case to show whether each statement is **always true**, **sometimes true**, or **never true**.

Note: P' is the complement of the set P.

	Statement	Always true	Sometimes true	Never true
		Tick **one** box only in each case		
1	$\#(P \cup Q) = \#(P) + \#(Q)$	☐	☐	☐
2	$P \backslash Q = P \cap Q$	☐	☐	☐
3	$(P \cap Q)' = P' \cup Q'$	☐	☐	☐
4	$P \cup (Q \cap R) = (P \cup Q) \cap (P \cup R)$	☐	☐	☐

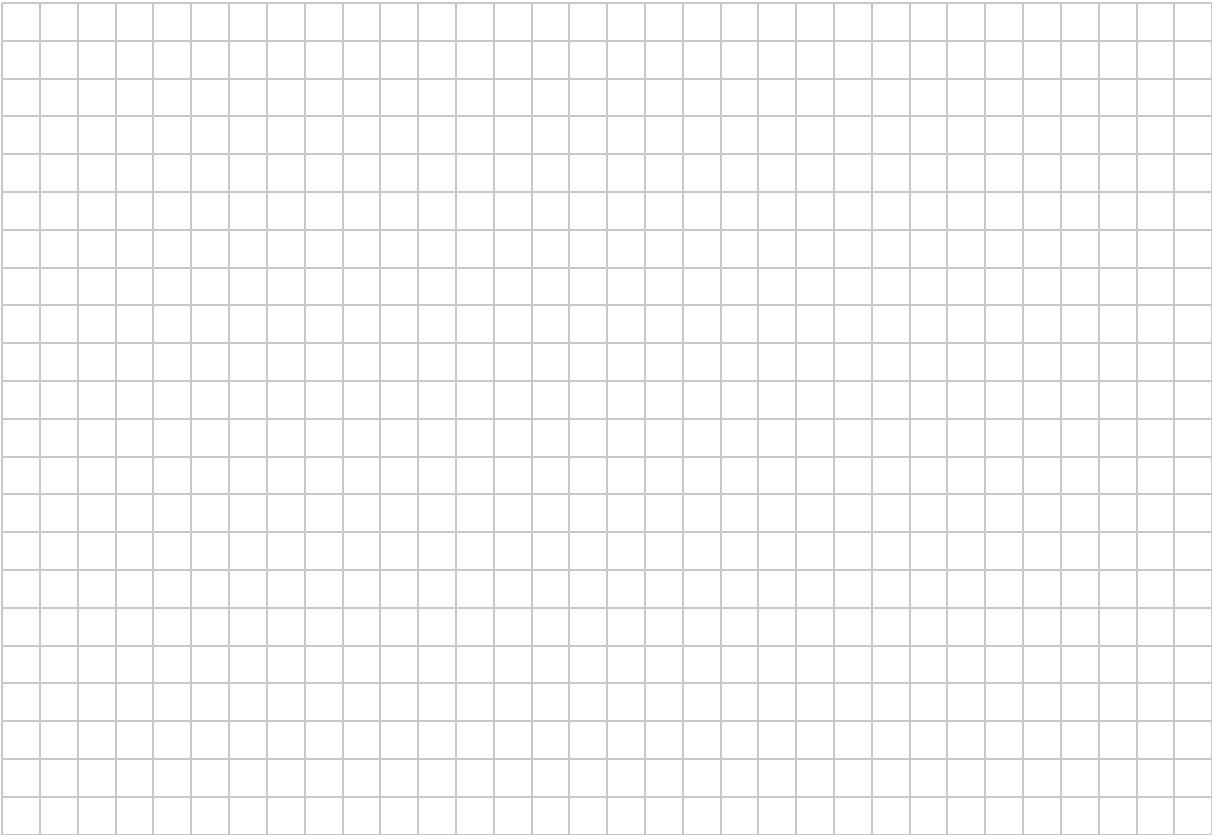**(b)** In the case of **Statement 1** in the table, justify your answer.

Justification:

Question 14 (Suggested maximum time: 10 minutes)

(a) Use factors to simplify $\dfrac{2n^2+n-15}{n^2-9}$.

(b) For all values of a, b, and $x \in \mathbb{R}$:

$$(x + a)(x + b) = x^2 + (a + b)\,x + ab.$$

Using this fact, or otherwise, answer the following two questions.

(i) Solve the following equation in x, where a and b are constants. Give each answer in terms of a or b.

$$x^2 + (a + b)\,x + ab = 0$$

Answer: $x = $ ☐   or $x = $ ☐

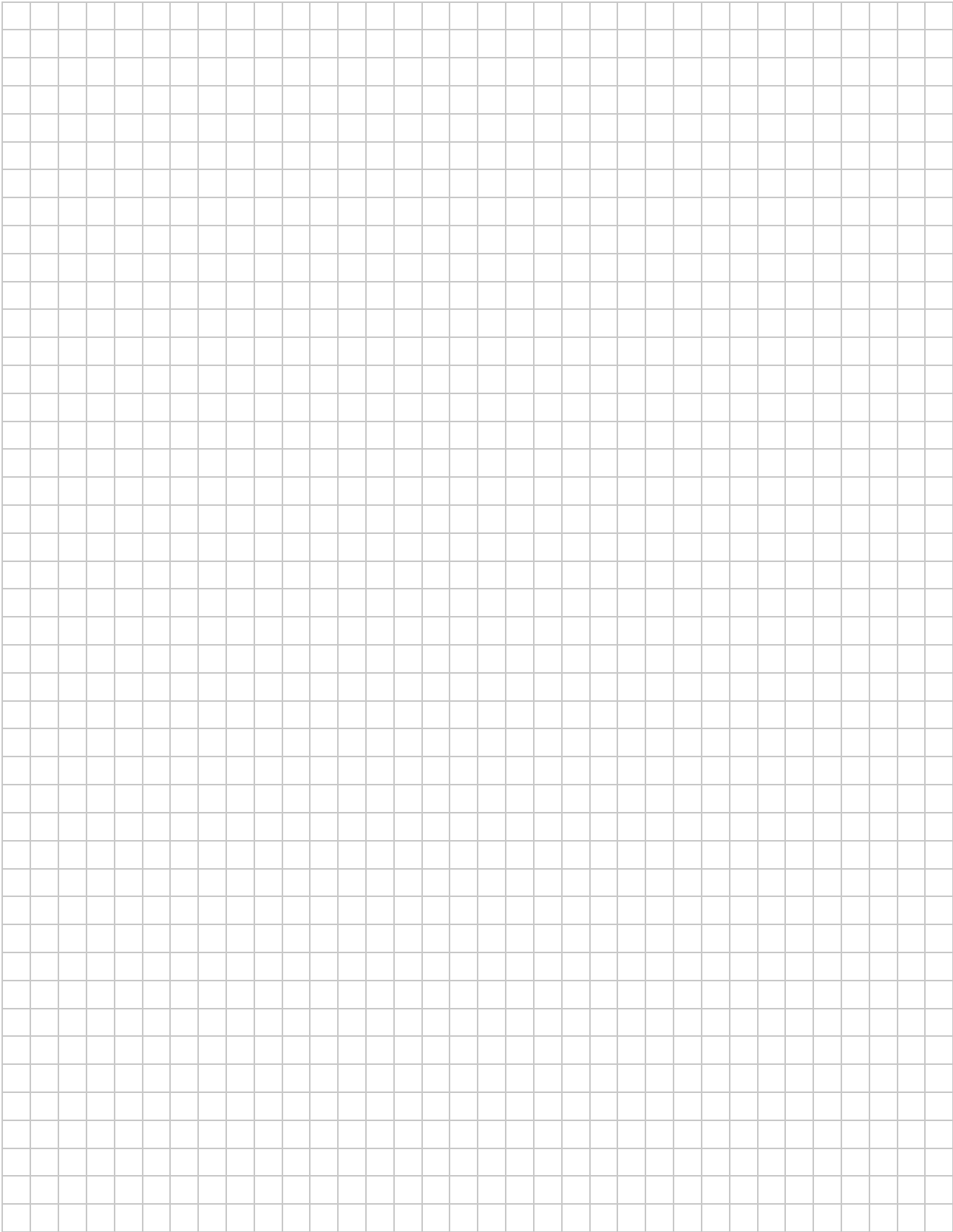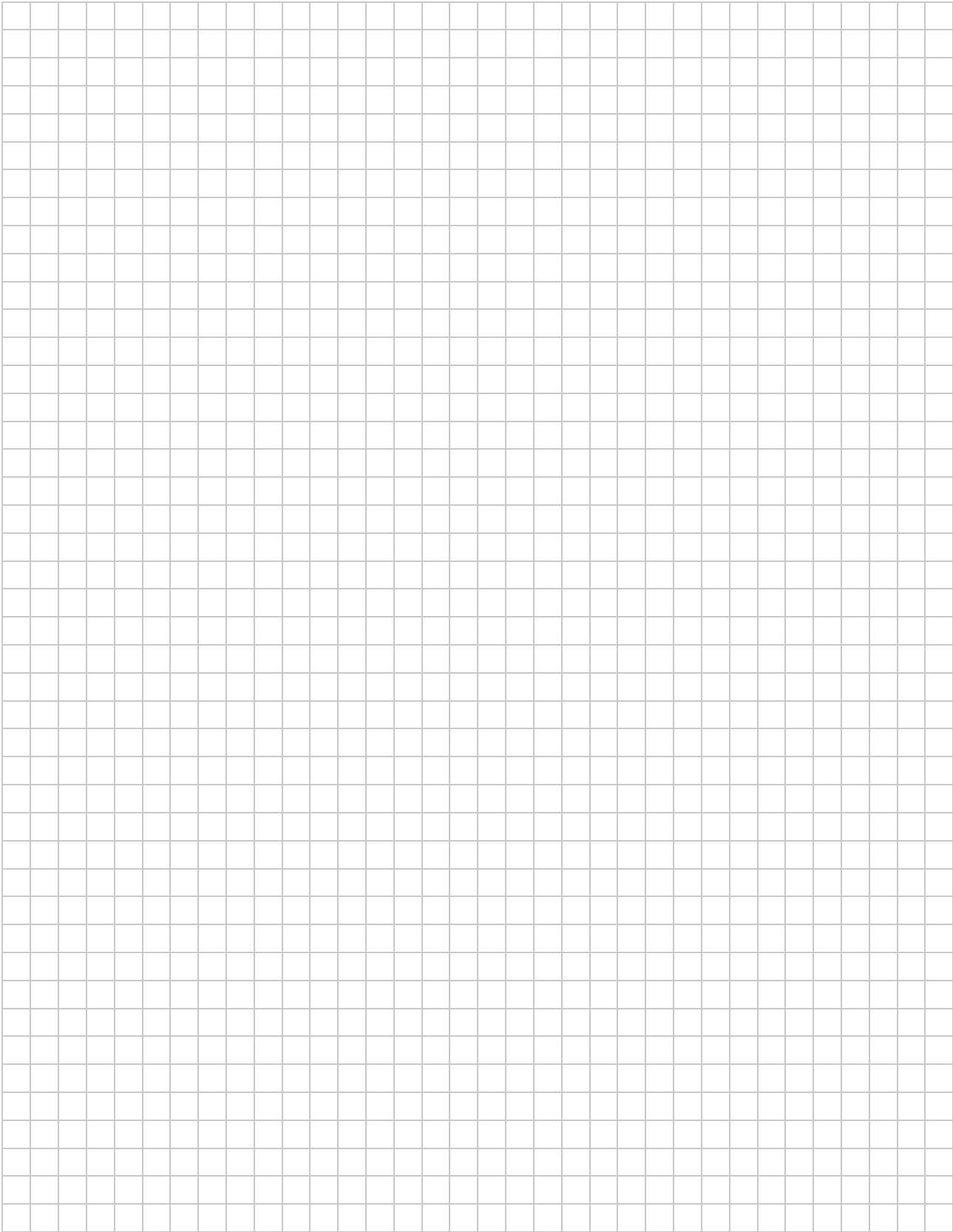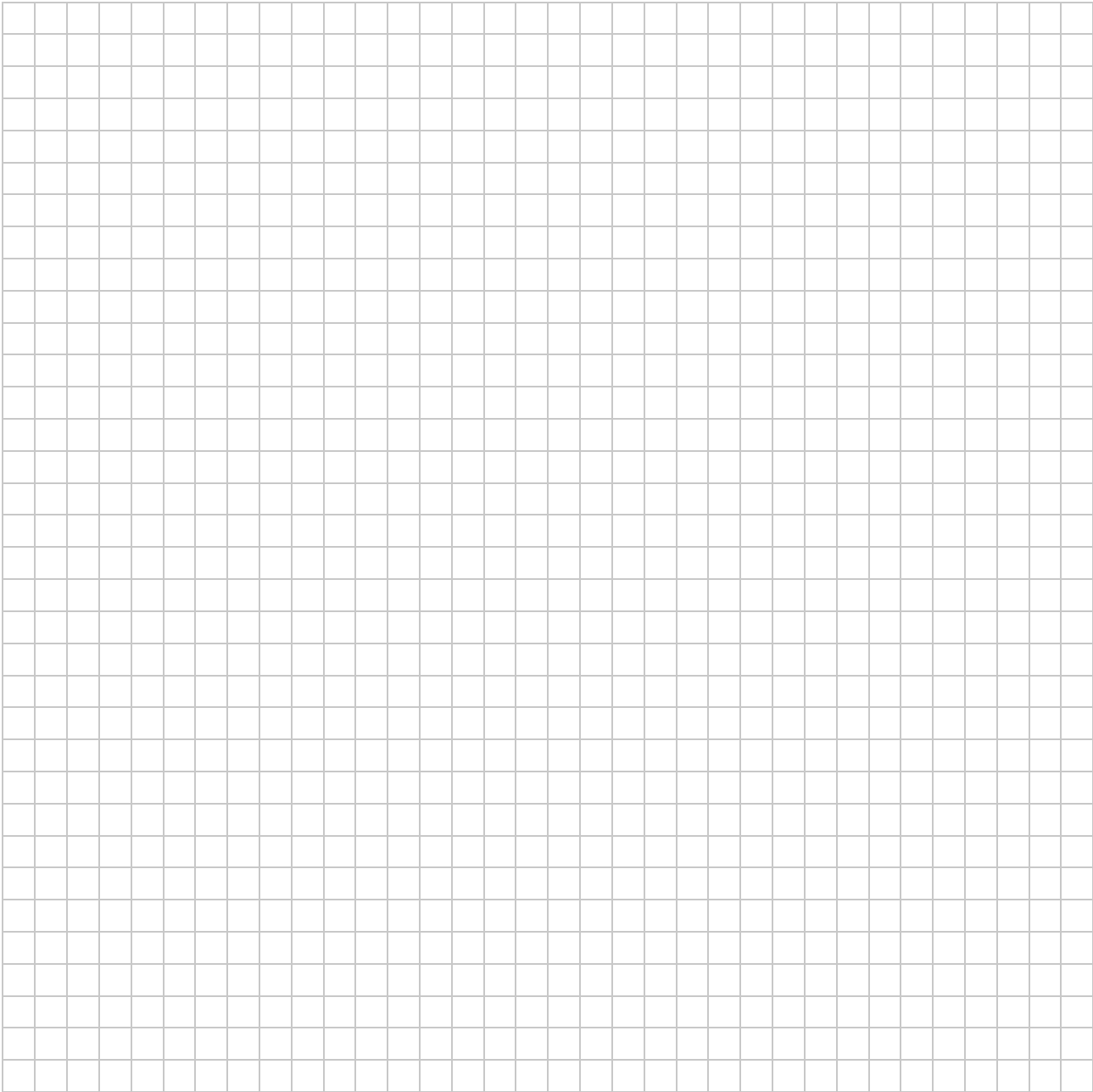**(ii)** Simplify $[x^2 + (a + b)\,x + ab] \div (x + a)$.

Page for extra work.
Label any extra work clearly with the question number and part.

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Label any extra work clearly with the question number and part.

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