DISCRETE STRUCTURES
Lab 4
Proof techniques

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Abstract

In this Laboratory, we will practice using python and proving statements using the proof techniques introduced in Week5_6_Proof_Techniques.pdf.

1 Exercises

1. Prove/Disprove the conclusion C using the given data:

   (a) Given:
   $P =$"Phong has Visa"
   S1="If Phong can fly, Phong will go to America"
   S2="If Phong has Visa, Phong will go to America"
   S3="If Phong can speak English, Phong will go to America"
   $Conclusion : C =$"Phong goes to America"

   (b) Given:
   $P =$"It's hot yesterday"
   S1="If it's hot, it will rain the next day"
   S2="If and only if it's not rain, Nam goes outside"
   S3="If it's not hot, Nam will go outside"
   $Conclusion : C =$"Nam goes outside"

   (c) Given:
   $Q =$"An wake up late"; $R =$"The traffic is flowing smooth"
   S1="The traffic is always heavy on school day"
   S2="If An wake up late, he will be late for school on school day"
   S3="An only have to go to school on school day"
   S4="If An don't have to go to school, An can't be late for school"
   $Conclusion : C =$"An is late for school"

   (d) Given:
   $P =$"$\exists x, y \in \mathbb{R}, (x + y)^2 \leq x^2 + y^2$";
   $Q =$"$\forall x, y \in \mathbb{Z}^+, x + y \geq x + y$"
   $R =$"$\forall x, y \in \mathbb{Z}^+, x + y + 2\sqrt{xy} \geq x + y$"

$T =$ "$\forall x, y \in \mathbb{Z}^+, \sqrt{x+y} \geq 0$"
$S1 =$ "$\forall x, y \in \mathbb{Z}^+, x^2 \geq y^2 \rightarrow x \geq y$"
$S2 =$ "$\forall x, y \in \mathbb{Z}^+, x \geq y \rightarrow x^2 \geq y^2$"
$S3 =$ "$\forall x, y \in \mathbb{R}^+, x \geq y \rightarrow x^2 \geq y^2$"
$S4 =$ "$\forall x, y \in \mathbb{R}^+, x^2 \geq y^2 \rightarrow x \geq y$"
$Conclusion : C =$ "$\forall x, y \in \mathbb{Z}^+, \sqrt{x} + \sqrt{y} \geq \sqrt{x+y}$"

Guide: Student should print which given data used to Prove/Disprove the conclusion in order: For example:

```
print("P and S2 -> C")
print("P:%s"%(P))
print("S2:%s"%(S2))
print("C:%s"%(C))
```

2. Prove/Disprove the following:

   (a) $\exists x \in \mathbb{Z}, 0 \leq x \leq 100, x^2 = 15^2 + 16^2$

   (b) $\exists x \in \mathbb{Z}, 0 \leq x \leq 100, x^2 = 12^2 + 16^2$

   (c) $\exists x \in \mathbb{Z}, -50 \leq x \leq 50, x^2 \geq 99x$

   (d) $\exists x \in \mathbb{Z}, 50 \leq x \leq 100, x(x+1)(x+2)\%6 \neq 0$

   (e) $\exists x, y \in \mathbb{Z}, 0 \leq x \leq 100, \sqrt{x+y} = \sqrt{x} + \sqrt{y}$

   Then print the proof that the statements are valid or invalid such as:
   "the given statement is correct when x equal to ..."or
   "the given statement is incorrect for all values x within the given domain."
   Hint: Using Loop to test every possible cases in the given domains.

3. Print the negated statements from the following statement then prove/disprove them:

   (a) $\forall x \in \mathbb{Z}, 0 \leq x \leq 100, x^3 >= x$

   (b) $\forall x \in \mathbb{Z}, 0 \leq x \leq 100$, and x is even, $x * 3 + 1$ is a prime number

   (c) $\forall x \in \mathbb{Z}, 1 \leq x, y \leq 100$, and x is even, $x * 5 + 3$ is a prime number

   (d) $\forall c \in \mathbb{Z}, 0 < c \leq 100, c\%4 = 0, \exists a, b \in \mathbb{Z}^+, c^2 = a^2 + b^2$

   (e) $\forall c \in \mathbb{Z}, 0 < c \leq 100, c\%5 = 0, \exists a, b \in \mathbb{Z}^+, c^2 = a^2 + b^2$

   (f) $\exists c \in \mathbb{Z}, 0 < c \leq 100, c^2 \leq c$

   Note: the negated statements is logical in equivalent to the original statements.

4. Prove/disprove that:

   (a) $\sum_{x=0}^{10} \sum_{y=0}^{10} (x+y)^2 > \sum_{x=0}^{10} \sum_{y=0}^{10} (x+2y)^2$

   (b) $20! < \sum_{x=0}^{10} x!$

(c) $\sum_{x=0}^{10} 3x^2 >= 10^3$

(d) $\sum_{x=5}^{10} (4x^3 + 6x^2 + 2x) > 10^4 + 2 * 10^3 + 10^2 - 5^4 - 2 * 5^3 - 5^2$

5. Prove/Disprove the following arguments manually without using truth table.

(a) $p \to r$
$\neg p \to q$
$q \to s$
$\therefore \neg r \to s$

(b) $p \to (q \to r)$
$p \lor s$
$t \to q$
$\neg s$
$\therefore \neg r \to \neg t$

(c) $p \to q$
$\neg r \lor s$
$p \lor r$
$\therefore \neg q \to s$

(d) p
$p \to r$
$p \to (q \lor \neg r)$
$\neg q \lor \neg s$
$\therefore s$

Example Prove:
p
$\neg r \to \neg p$
$\therefore r$

```
G1="p"
G2="~r->~p"
G3="G2->'p->r',contrapositive"
C="G1 + G3 -> C='r'"
print("G1=%s"%(G1))
print("G2=%s"%(G2))
print("G3=%s"%(G3))
print("%s"%(C))
```