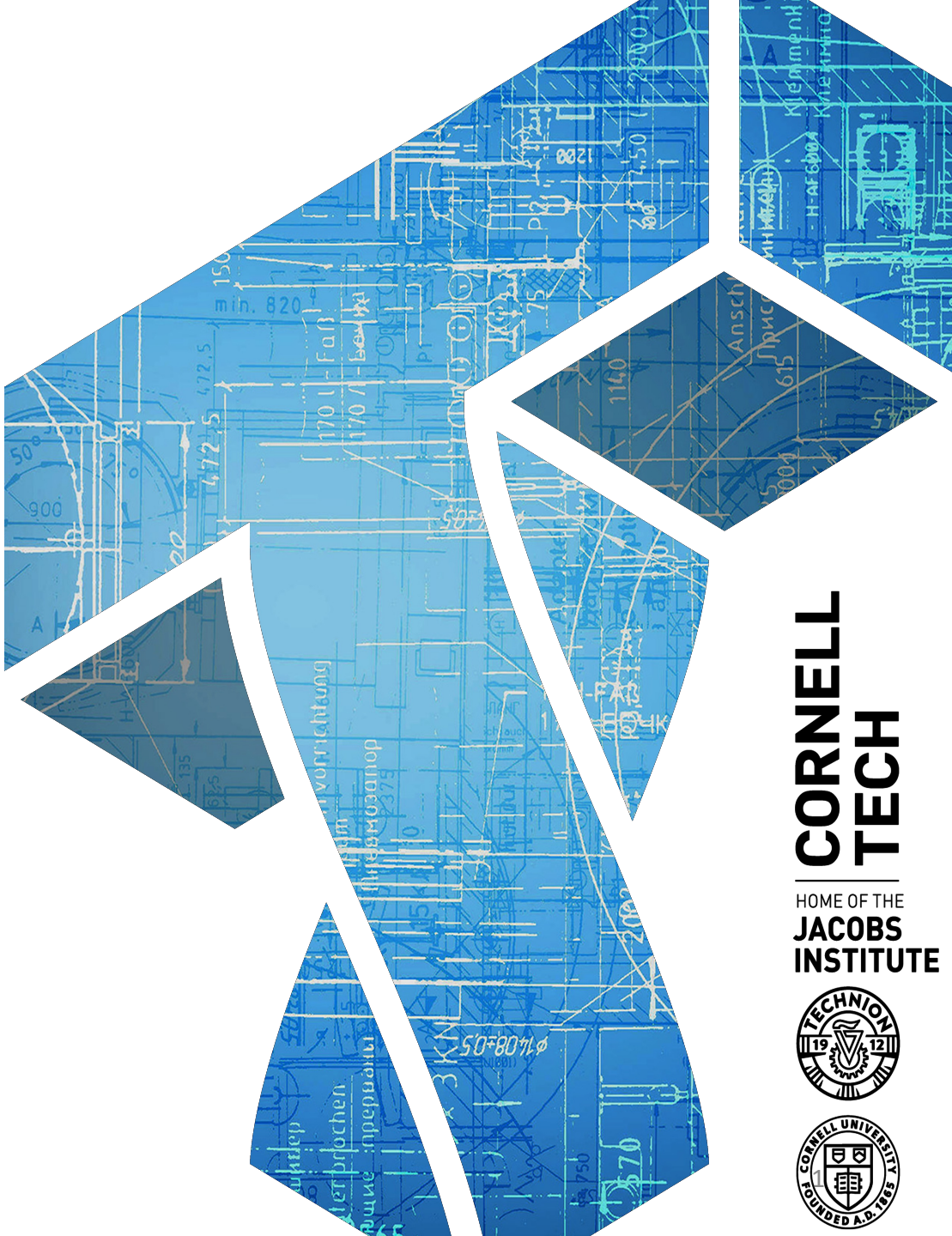
CS 5830 Cryptography

Instructor: Tom Ristenpart

TAs: ~~Yan Ji~~, Sanketh Menda

other TAs



CORNELL TECH

HOME OF THE JACOBS INSTITUTE

Administrivia

- Office hours are by demand – use slack first, and we can arrange in-person as requested

- Tentative homework schedule will be up on course schedule very soon
  - Will try to stick to announced schedule
  - Let us know if there are large conflicts

The many types of attackers & threat models

- Threat model describes adversarial capabilities and goals
- **Example capabilities:**
  - Passive attackers observe communications / data
  - Active attackers may additionally interact with systems
    - Chosen-plaintext attacks can see input-output examples for adversarial plaintexts
    - Chosen-ciphertext attacks allow submitting mauled ciphertexts to decryption box
- **Example goals:**
  - Confidentiality violation (recover keys, recover plaintexts)
  - Integrity violation (trick recipient into accepting modified plaintext)
- Modern cryptography tries to formalize threat models

Shannon's security notion (1949)

Def. A symmetric encryption scheme Enc is perfectly secure if for all messages M, M' and ciphertexts C

$$\Pr[\,Enc(K,M) = C\,] = \Pr[\,Enc(K,M') = C\,]$$

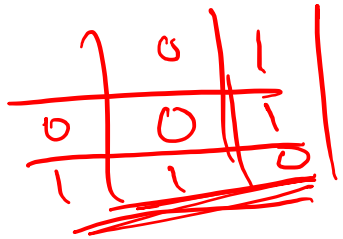
where probabilities are over choice of K

In words:
each message is equally likely to map to a given ciphertext

In other words:
seeing a ciphertext leaks nothing about what message was encrypted

Does a substitution cipher meet this definition? No!

One-time pad (OTP)



Kg():
K <-$ {0,1}^{L}

Pick a random bit string

Enc(K,M):
Return M ⊕ K

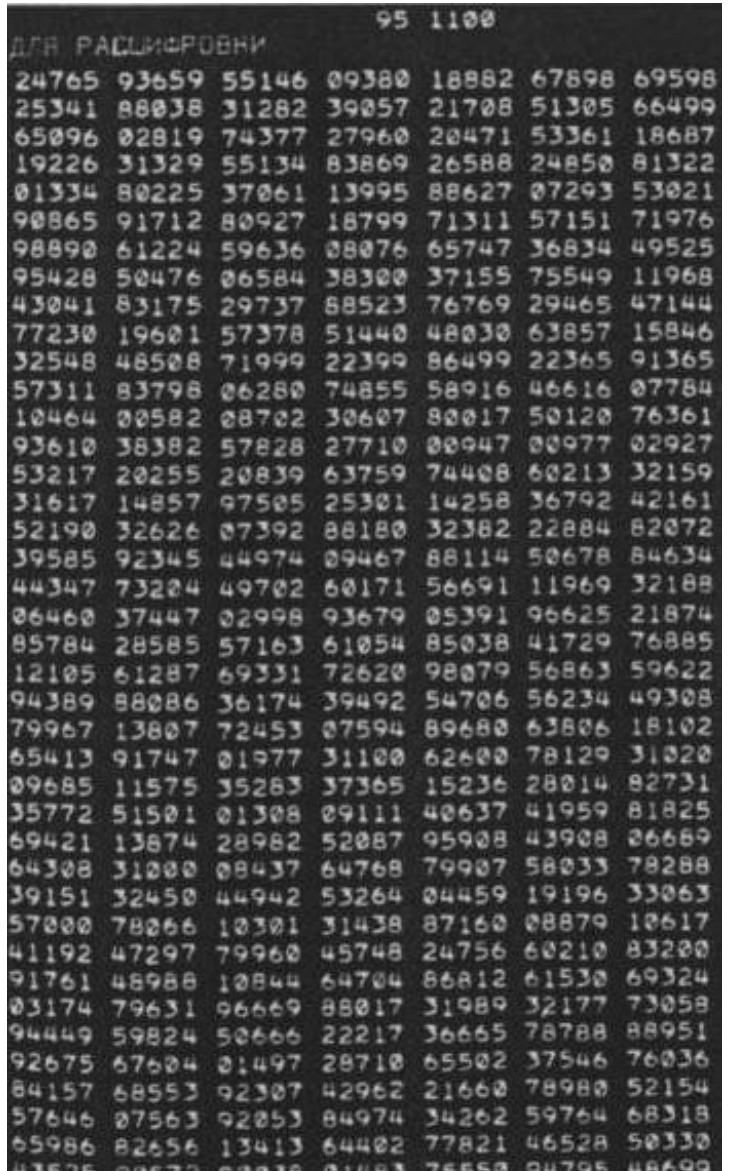
exclusive or

Assume M is L-bit string

Dec(K,C):
Return C ⊕ K

Assume C is L-bit string

Part of a CIA OTP used by Soviet diplomat spying for CIA



95 1100

ДЛЯ РАСШИФРОВКИ

24765	93659	55146	09380	18882	67898	69598
25341	88038	31282	39057	21708	51305	66499
65096	02819	74377	27960	20471	53361	18687
19226	31329	55134	83869	26588	24850	81322
01334	80225	37061	13995	88627	07293	53021
90865	91712	80927	18799	71311	57151	71976
98890	61224	59636	08076	65747	36834	49525
95428	50476	06584	38300	37155	75549	11968
43041	83175	29737	88523	76769	29465	47144
77230	19601	57378	51440	48030	63857	15846
32548	48508	71999	22399	86499	22365	91365
57311	83798	06280	74855	58916	46616	07784
10464	00582	08702	30607	80017	50120	76361
93610	38382	57828	27710	00947	00977	02927
53217	20255	20839	63759	74408	60213	32159
31617	14857	97505	25301	14258	36792	42161
52190	32626	07392	88180	32382	22884	82072
39585	92345	44974	09467	88114	50678	84634
44347	73204	49702	60171	56691	11969	32188
06460	37447	02998	93679	05391	90625	21874
85784	28585	57163	61054	85038	41729	76885
12105	61287	69331	72620	98079	56863	59622
94389	88086	36174	39492	54706	56234	49308
79967	13807	72453	07594	89680	63806	18102
65413	91747	01977	31100	62600	78129	31020
09685	11575	35283	37365	15236	28014	82731
35772	51501	01308	09111	40637	41959	81825
69421	13874	28982	52087	95908	43908	06669
64308	31000	08437	64768	79907	58033	78288
39151	32450	44942	53264	04459	19196	33063
57000	78066	10301	31438	87160	08879	10617
41192	47297	79960	45748	24756	60210	83200
91761	48988	10844	64704	86812	61530	69324
03174	79631	96669	88017	31989	32177	73058
94449	59824	50666	22217	36665	78788	88951
92675	67604	01497	28710	65502	37546	76036
84157	68553	92307	42962	21660	78980	52154
57646	07563	92053	84974	34262	59764	68318
65986	82656	13413	64402	77821	46528	50330

Shannon's security notion (1949)

Def. A symmetric encryption scheme Enc is perfectly secure if for all messages M, M' and ciphertexts C

$$\Pr[\,Enc(K,M) = C\,] = \Pr[\,Enc(K,M') = C\,]$$

where probabilities are over choice of K

Thm. OTP is perfectly secure

For any C and M of length L bits

$$\Pr[\,K \oplus M = C\,] = 1 / 2^L$$

$$\Pr[\,K \oplus M = C\,] = \Pr[\,K \oplus M' = C\,]$$

Shannon's security notion (1949)

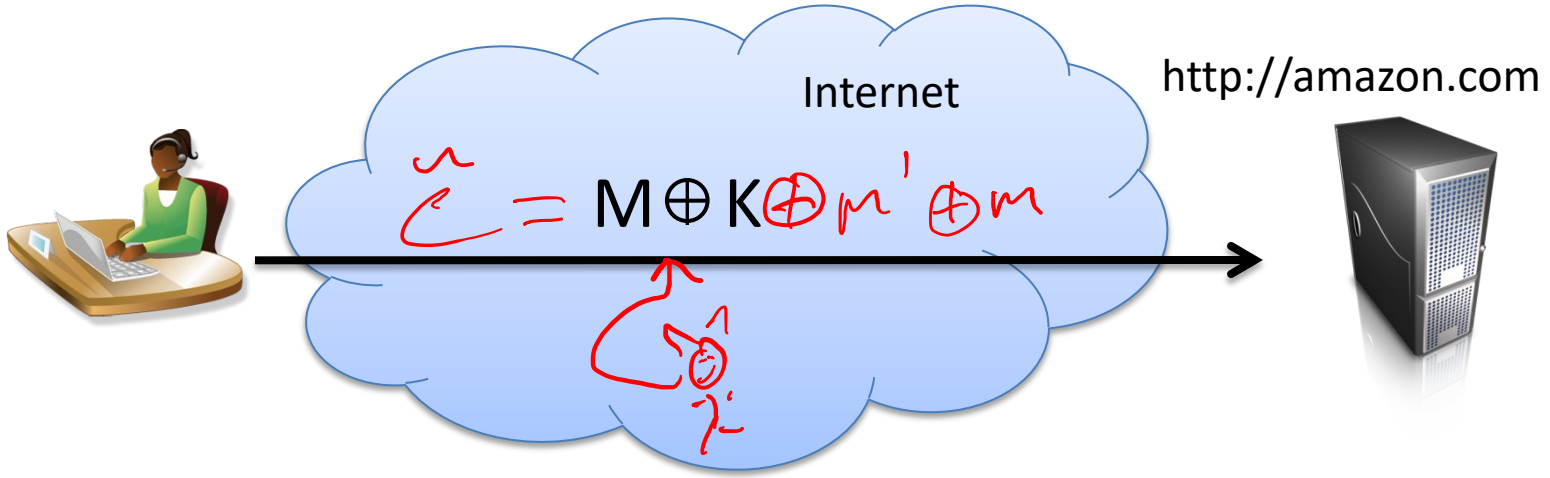
Def. A symmetric encryption scheme Enc is perfectly secure if for all messages M, M' and ciphertexts C

$$\Pr[\,Enc(K,M) = C\,] = \Pr[\,Enc(K,M') = C\,]$$

where probabilities are over choice of K

Thm. OTP is perfectly secure

Thm. Perfectly secure encryption requires $|K| \geq |M|$



Does OTP suffice for securing communications online?

Integrity easily violated

Reuse of K for messages M,M' leaks M ⊕ M'

Encrypting same message twice under K leaks the message equality

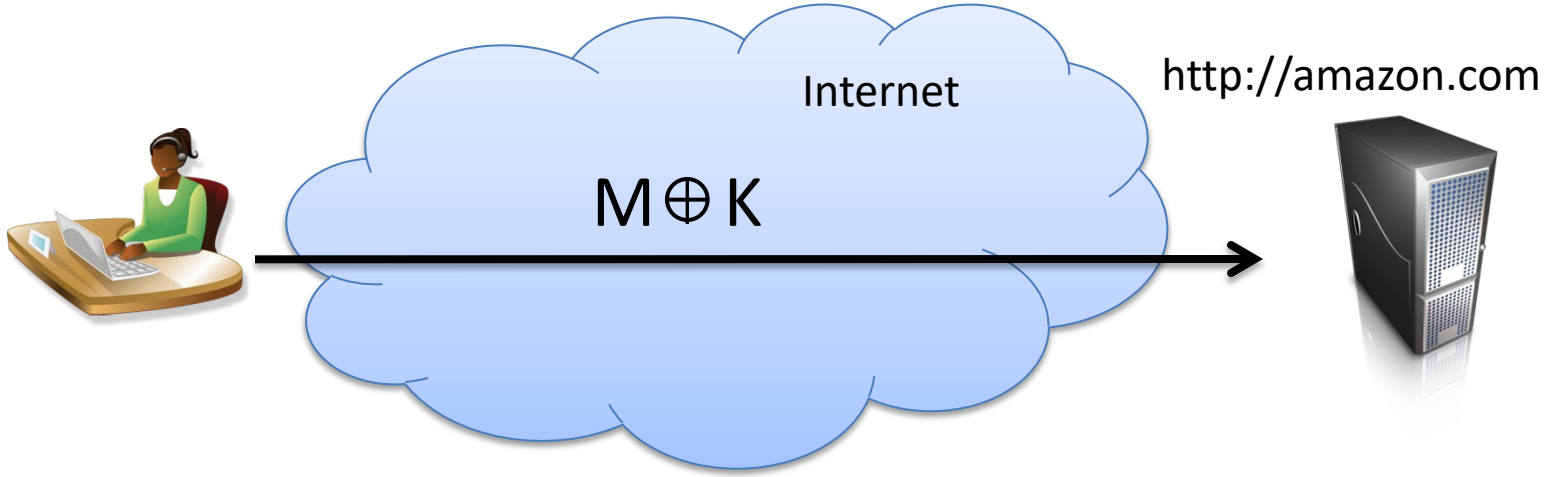
K must be as large as message

Message length revealed

Formal security notions are hard to get right

Simplifying abstraction to allow rigorous analysis

Amount of deployment details needed to capture attacks



Does OTP suffice for securing communications online?

Integrity easily violated

Reuse of K for messages M,M' leaks M ⊕ M'

Encrypting same message twice under K leaks the message equality

K must be as large as message

Message length revealed

Beyond Shannon

Thm. Perfectly secure encryption requires $|K| \geq |M|$

We will ***relax*** the definition of confidentiality:

1. Allow tiny adversarial success probability (often written as ϵ, e.g., $\epsilon = 2^{-80}$)
2. Focus on resource-efficient adversaries (e.g., only those given run time at most $t = 2^{80}$)

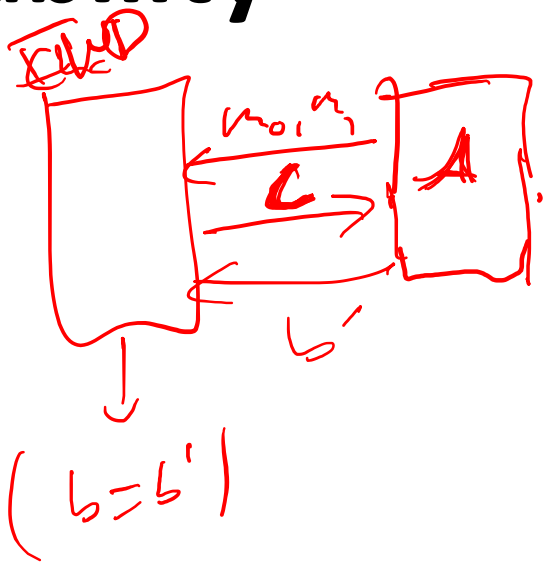
Intuition: we are ok if adversaries require prohibitive time to win, or only win with vanishingly small probability

Towards computational indistinguishability

Def. A symmetric encryption scheme is perfectly secure if for all messages M,M' and ciphertexts C

$$\Pr[\,Enc(K,M) = C\,] = \Pr[\,Enc(K,M') = C\,]$$

where probabilities are over choice of K



Let's give a game-based formulation of this using an adversary

Let SE = (Kg,Enc,Dec) be a symmetric encryption scheme

Let $\mathcal{A}$ Let be a randomized algorithm, called the adversary

IND(SE,$\mathcal{A}$):
(M_0,M_1) <-$ $\mathcal{A}$
K <-$ Kg ; b <-$ {0,1}
b' <-$ $\mathcal{A}$(Enc(K,M_b))
Return (b = b')

IND(SE,$\mathcal{A}$) output is 1 if (b = b').

We say then that the adversary succeeded

Thm. A scheme SE is perfectly secure if for every $\mathcal{A}$ it is the case that

$$\Pr[IND(SE,\mathcal{A}) = 1] = 1/2$$

Computational indistinguishability

Def. A symmetric encryption scheme is (t, ϵ)-indistinguishable if for any adversary $\mathcal{A}$ running in time at most t it holds that

$$\Pr[\text{IND}(SE, \mathcal{A}) = 1] < 1/2 + \epsilon$$

1) Tiny adversarial success

2) Computationally limited adversary

IND(SE, $\mathcal{A}$):
$(M_0, M_1) \leftarrow\$ \mathcal{A}$
$K \leftarrow\$ Kg$; $b \leftarrow\$ \{0,1\}$
$b' \leftarrow\$ \mathcal{A}(Enc(K, M_b))$
Return $(b = b')$

Later

Discussion questions:

1) Does (t, ϵ)-indistinguishability model known, chosen message attack? What about chosen ciphertext?
2) Is OTP (t, ϵ)-indistinguishable?
3) Is a substitution cipher (t, ϵ)-indistinguishable?

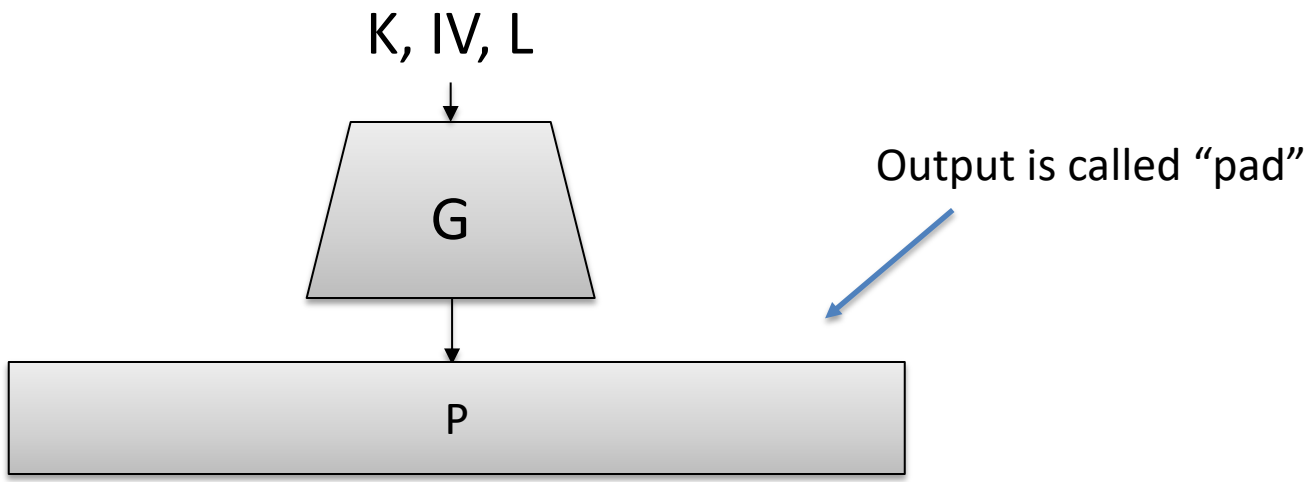
How do we build computationally-secure SE schemes?

- Game plan: need more tools
  - Stream ciphers
  - Block ciphers

Stream ciphers

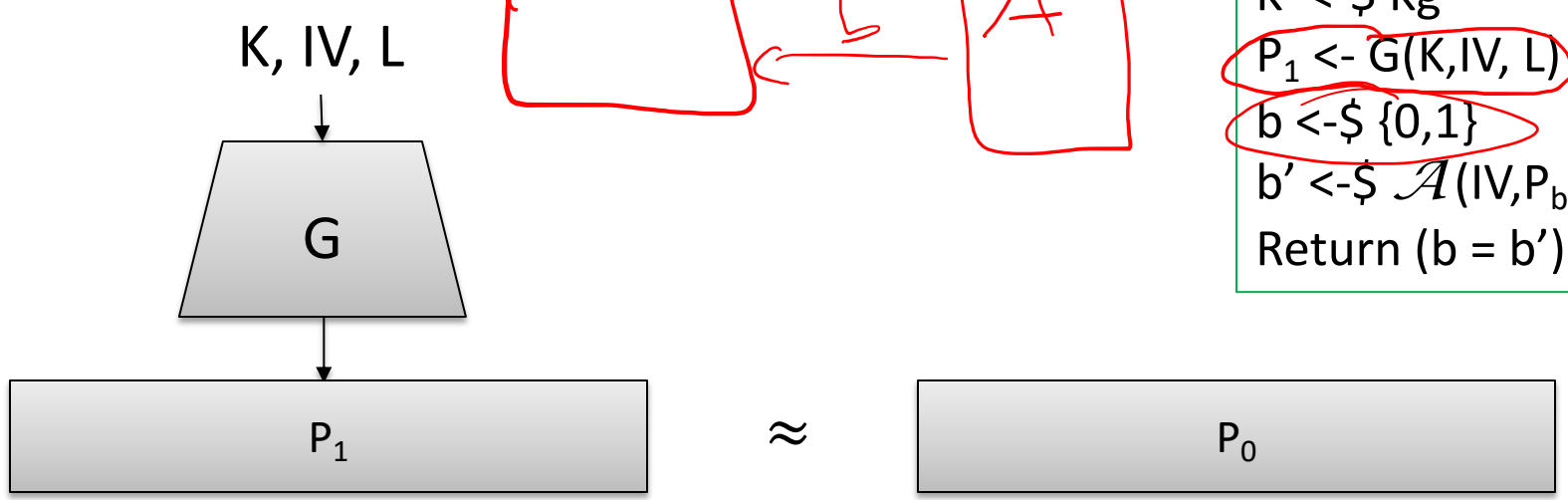
Stream cipher (aka pseudorandom generator, PRG) is pair of algorithms:

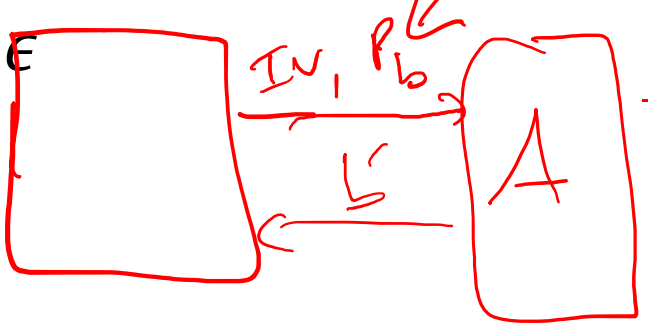
- Kg outputs a random key K
- G(K,IV,L) takes K, optionally an additional random value IV (initialization vector), desired length L
  - Outputs bit string P with |P| = L



Stream cipher security goal

(t, ϵ)-pseudorandom: no attacker limited to time t can distinguish between IV, G(K,IV,L) and IV, P_0 for P_0 random length-L bit string, with probability greater than ϵ





PRG(G,L,IV,$\mathcal{A}$):
K <-$ Kg
P_1 <- G(K,IV, L); P_0 <-$ $\{0,1\}^L$
b <-$ {0,1}
b' <-$ $\mathcal{A}$(IV,P_b))
Return (b = b')

Intuition: G output as good as one-time pad R (uniformly sampled)

Implies that no large ***biases*** exist, e.g. Pr[P[0] = 0x00] = $\frac{1}{256} + \delta$

SE from a stream cipher

Say we have a secure stream cipher. How do we build an SE scheme?

Kg():
K <-$ $\{0,1\}^k$

Pick a random key

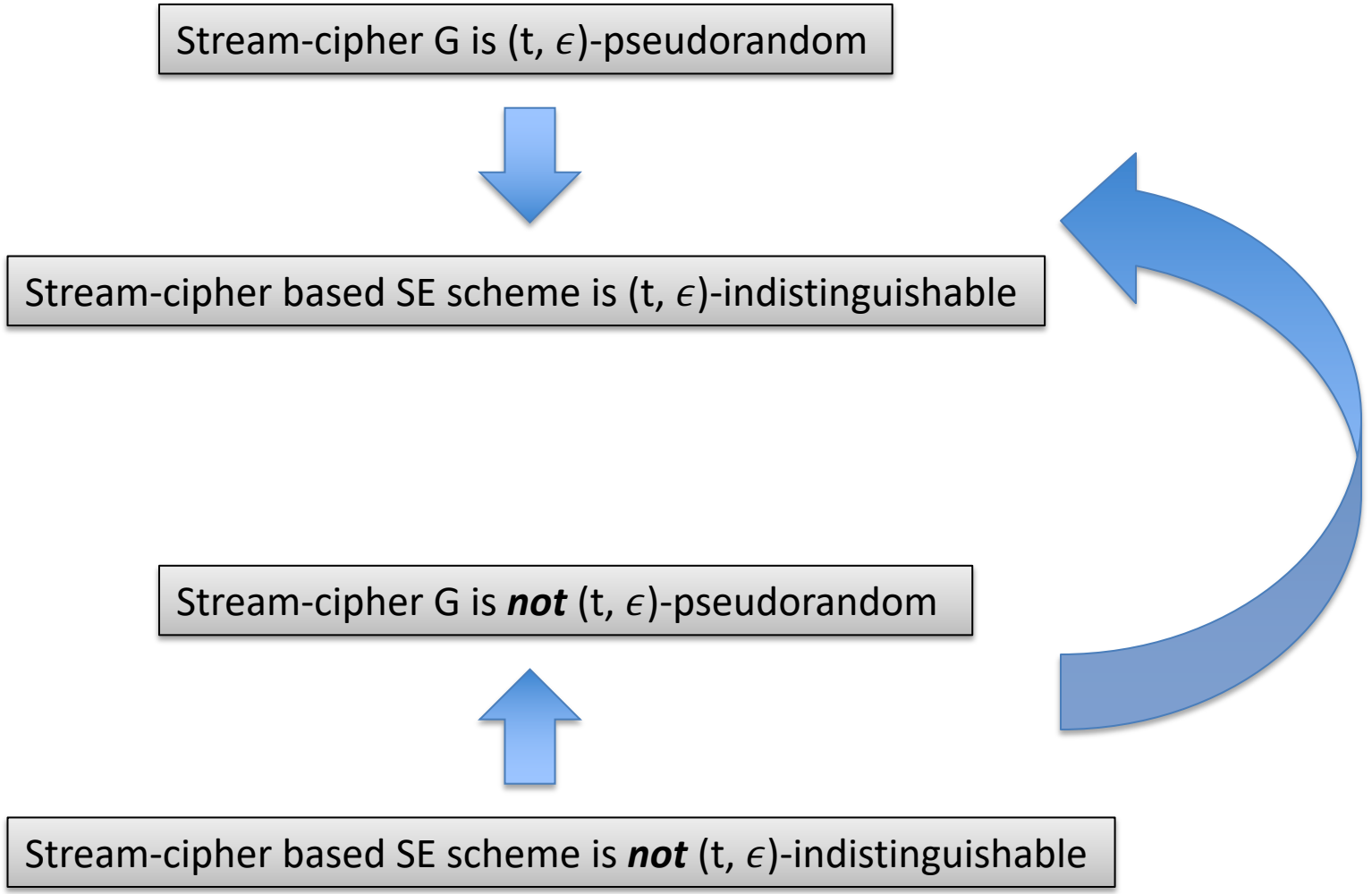
Enc(K,M):
L <- |M|
IV <-$ $\{0,1\}^n$
Return (IV, G(K,IV,L) $\oplus$ M)

Dec(K,(IV,C)):
L <- |C|
Return G(K,IV,L) $\oplus$ C

Assume ciphertext can be parsed into IV and remaining ciphertext bits

Reduction-based analysis: high level idea

What we would want to show:



What we would show:

Reduction-based analysis: high level idea

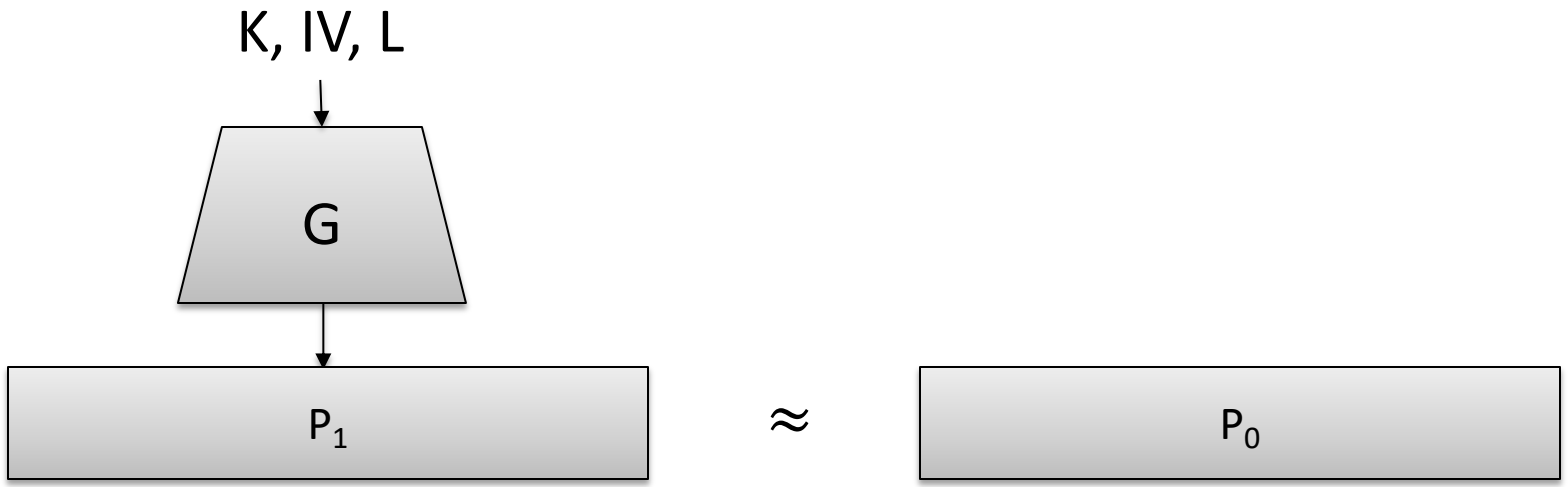
- **Reductionist analyses powerful tool**: rules out attacks that don't also break some (simpler to state) underlying hard computational problem
  - Often referred to as "provable security"
- Not a panacea:
  - Security definitions must capture relevant adversarial capabilities/goals
  - Proofs can sometimes be wrong
  - Must couple with other approaches such as direct cryptanalysis

Reduces task of building indistinguishable encryption to building pseudorandom stream cipher

Stream cipher security

- Someone gives you a stream cipher G
- How would you go about analyzing its security?
  1. **Key recovery attacks**: given IV, P (for some hidden K), recover K. How would you do it? What is run time of attack?
  2. **Bias-finding attacks**: can we find biases? How would you go about it?

E.g.: Pr[P[0] = 0x00] = $\frac{1}{256} + \delta$



Simplified view of bias-abusing attack

Assume stream cipher G with no IV and outputting single byte
Unknown byte M encrypted under q different keys K1,…,Kq
Adversary given C1,…,Cq

$C1 = G(K1) \oplus M$

$C2 = G(K2) \oplus M$

$C3 = G(K3) \oplus M$

$\vdots$

$Cq = G(Kq) \oplus M$

Suppose Pr[G(K) = 0x00] > Pr[G(K) = B] for any other B
Pad byte is most likely to be 0x00

Say Pr[G(K) = 0x00] = 1/3
How many ciphertexts do we expect to equal M? ~q/3

Simple attack:
Let N_r = number of ciphertexts that equal byte value r
Output r such that N_r is highest

Simplified view of bias-abusing attack

Assume stream cipher G with no IV and outputting single byte
Unknown byte M encrypted under q different keys K1,…,Kq
Adversary given C1,…,Cq

$C1 = G(K1) \oplus M$

$C2 = G(K2) \oplus M$

$C3 = G(K3) \oplus M$

$\vdots$

$Cq = G(Kq) \oplus M$

Alfardan et al. 2013 point out better approach

Build maximum likelihood estimator (MLE) that takes advantage of all known biases in G's output

In words:
Choose value M that most likely explains the C1,…,Cq seen given the known biases of G

http://www.isg.rhul.ac.uk/tls/RC4biases.pdf

RC4 stream cipher

- Rivest Cipher 4 (1987)
  - No IV: can't reuse K
  - L is implicit: call generate L times
  - All addition mod 256
- Originally proprietary secret
- Leaked to CypherPunks mail list (1994)
  - "Alleged RC4" or ARCFOUR
  - Rivest confirmed history later
- Very simple, fast to implement in software
- Used widely, only recently deprecated

Algorithm 1: RC4 key scheduling (KSA)

input : key K of l bytes
output : initial internal state st_0
begin
  for $i = 0$ **to** 255 **do**
    $\mathcal{S}[i] \leftarrow i$
  $j \leftarrow 0$
  for $i = 0$ **to** 255 **do**
    $j \leftarrow j + \mathcal{S}[i] + K[i \bmod l]$
    $\text{swap}(\mathcal{S}[i], \mathcal{S}[j])$
  $i, j \leftarrow 0$
  $st_0 \leftarrow (i, j, \mathcal{S})$
  return st_0

Algorithm 2: RC4 keystream generator (PRGA)

input : internal state st_r
output : keystream byte Z_{r+1} updated internal state st_{r+1}
begin
  parse $(i, j, \mathcal{S}) \leftarrow st_r$
  $i \leftarrow i + 1$
  $j \leftarrow j + \mathcal{S}[i]$
  $\text{swap}(\mathcal{S}[i], \mathcal{S}[j])$
  $Z_{r+1} \leftarrow \mathcal{S}[\mathcal{S}[i] + \mathcal{S}[j]]$
  $st_{r+1} \leftarrow (i, j, \mathcal{S})$
  return (Z_{r+1}, st_{r+1})

From:
http://www.isg.rhul.ac.uk/tls/RC4passwords.pdf

RC4 stream cipher

- Key scheduling algorithm uses K to build permutation *S* on {0,…,255} and initialize internal state (i,j,S)

  Interesting discussion of permutation generation:
  https://blog.codinghorror.com/the-danger-of-naivete/

- Keystream generator outputs byte Z_{r+1} and updated internal state
  - *S* remains permutation throughout
  - i is a counter
  - j varies as function of i, *S*, previous j

Algorithm 1: RC4 key scheduling (KSA)

input : key K of l bytes
output : initial internal state st_0
begin
  for $i = 0$ **to** 255 **do**
    $\mathcal{S}[i] \leftarrow i$
  $j \leftarrow 0$
  for $i = 0$ **to** 255 **do**
    $j \leftarrow j + \mathcal{S}[i] + K[i \bmod l]$
    $\text{swap}(\mathcal{S}[i], \mathcal{S}[j])$
  $i, j \leftarrow 0$
  $st_0 \leftarrow (i, j, \mathcal{S})$
  return st_0

Algorithm 2: RC4 keystream generator (PRGA)

input : internal state st_r
output : keystream byte Z_{r+1} updated internal state st_{r+1}
begin
  parse $(i, j, \mathcal{S}) \leftarrow st_r$
  $i \leftarrow i + 1$
  $j \leftarrow j + \mathcal{S}[i]$
  $\text{swap}(\mathcal{S}[i], \mathcal{S}[j])$
  $Z_{r+1} \leftarrow \mathcal{S}[\mathcal{S}[i] + \mathcal{S}[j]]$
  $st_{r+1} \leftarrow (i, j, \mathcal{S})$
  return (Z_{r+1}, st_{r+1})

From:
http://www.isg.rhul.ac.uk/tls/RC4passwords.pdf

RC4 stream cipher

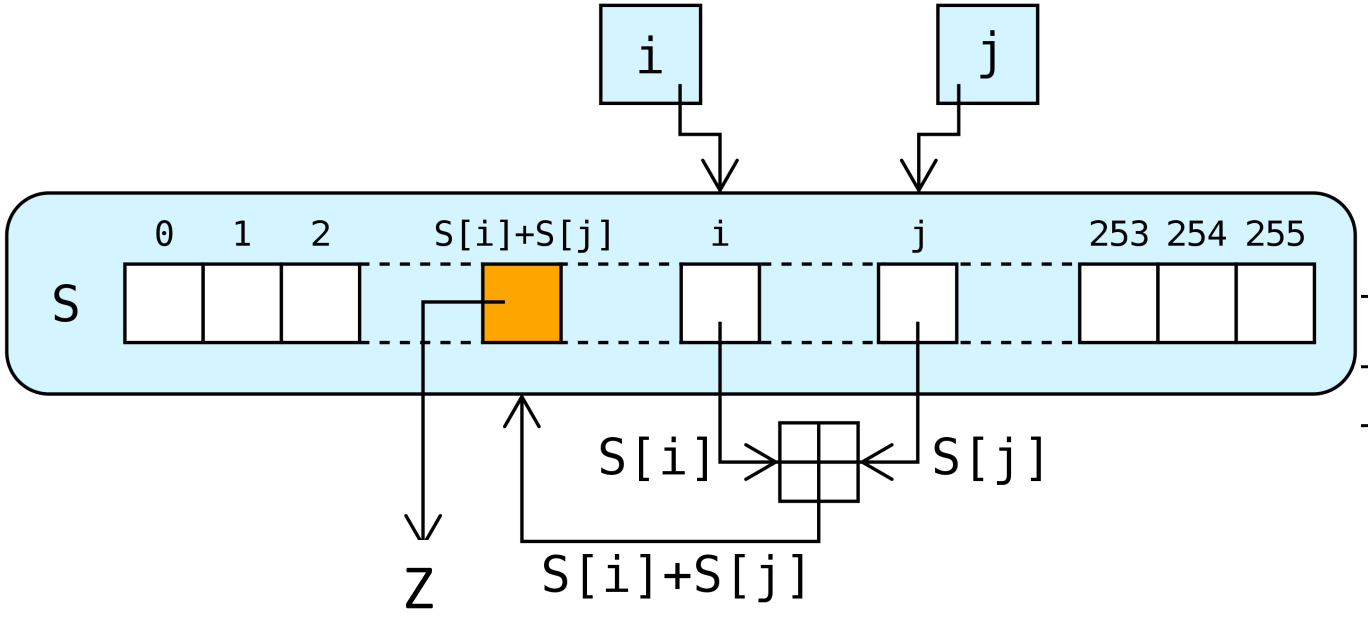


Diagram courtesy of https://en.wikipedia.org/wiki/RC4

Algorithm 1: RC4 key scheduling (KSA)

input : key K of l bytes
output : initial internal state st_0
begin
  for $i = 0$ **to** 255 **do**
    $\mathcal{S}[i] \leftarrow i$
  $j \leftarrow 0$
  for $i = 0$ **to** 255 **do**
    $j \leftarrow j + \mathcal{S}[i] + K[i \bmod l]$
    $\text{swap}(\mathcal{S}[i], \mathcal{S}[j])$
  $i, j \leftarrow 0$
  $st_0 \leftarrow (i, j, \mathcal{S})$
  return st_0

Algorithm 2: RC4 keystream generator (PRGA)

input : internal state st_r
output : keystream byte Z_{r+1} updated internal state st_{r+1}
begin
  parse $(i, j, \mathcal{S}) \leftarrow st_r$
  $i \leftarrow i + 1$
  $j \leftarrow j + \mathcal{S}[i]$
  $\text{swap}(\mathcal{S}[i], \mathcal{S}[j])$
  $Z_{r+1} \leftarrow \mathcal{S}[\mathcal{S}[i] + \mathcal{S}[j]]$
  $st_{r+1} \leftarrow (i, j, \mathcal{S})$
  return (Z_{r+1}, st_{r+1})

From:
http://www.isg.rhul.ac.uk/tls/RC4passwords.pdf

RC4 stream cipher

- How do evaluate RC4 security?
  - Manual analysis:

    walk through code and analyze probabilities

    [Mantin-Shamir 2001]: $\Pr[Z_2 = 0x00] \approx 1/128$

  - Statistical tests:

    for a random key, do you get expected distribution of output bytes?

    for many random keys, do you get expected distribution for each byte?

    [AlFardan et al. 2013] calculated outputs for 2^{44} keys and reported on empirical distributions

Algorithm 1: RC4 key scheduling (KSA)

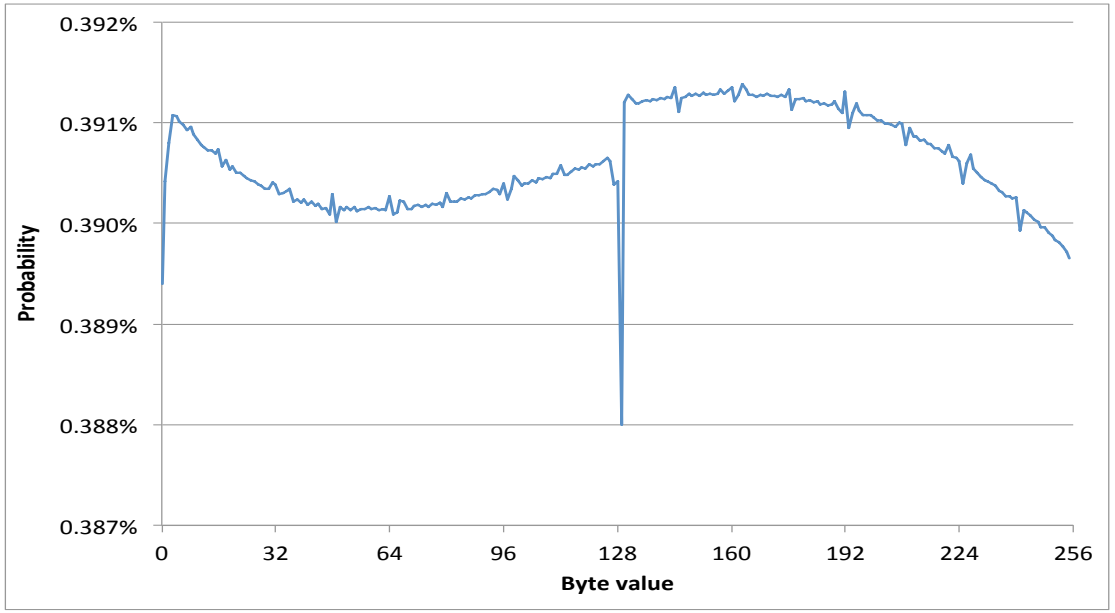
input : key K of l bytes
output : initial internal state st_0
begin
  for $i = 0$ **to** 255 **do**
    $\mathcal{S}[i] \leftarrow i$
  $j \leftarrow 0$
  for $i = 0$ **to** 255 **do**
    $j \leftarrow j + \mathcal{S}[i] + K[i \bmod l]$
    $\text{swap}(\mathcal{S}[i], \mathcal{S}[j])$
  $i, j \leftarrow 0$
  $st_0 \leftarrow (i, j, \mathcal{S})$
  return st_0

Algorithm 2: RC4 keystream generator (PRGA)

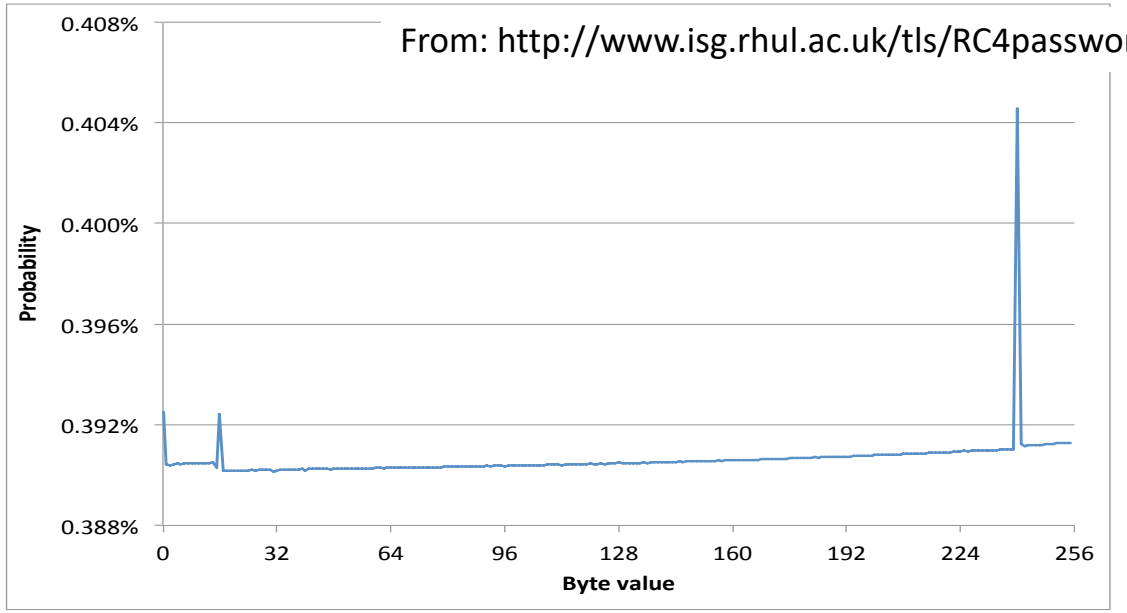
input : internal state st_r
output : keystream byte Z_{r+1} updated internal state st_{r+1}
begin
  parse $(i, j, \mathcal{S}) \leftarrow st_r$
  $i \leftarrow i + 1$
  $j \leftarrow j + \mathcal{S}[i]$
  $\text{swap}(\mathcal{S}[i], \mathcal{S}[j])$
  $Z_{r+1} \leftarrow \mathcal{S}[\mathcal{S}[i] + \mathcal{S}[j]]$
  $st_{r+1} \leftarrow (i, j, \mathcal{S})$
  return (Z_{r+1}, st_{r+1})

From:
http://www.isg.rhul.ac.uk/tls/RC4passwords.pdf

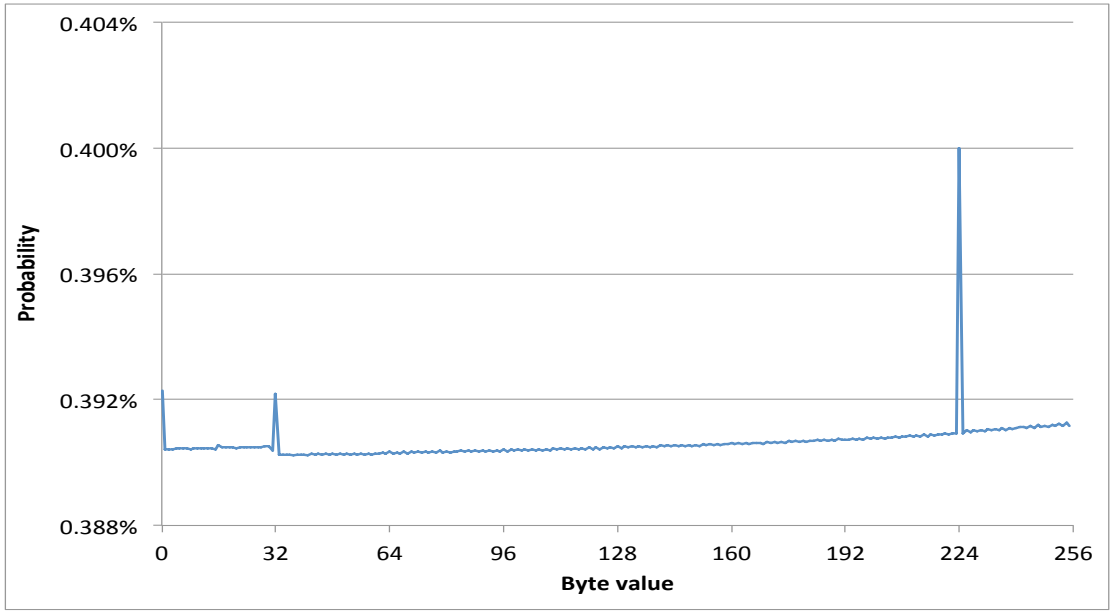
From: http://www.isg.rhul.ac.uk/tls/RC4passwords.pdf



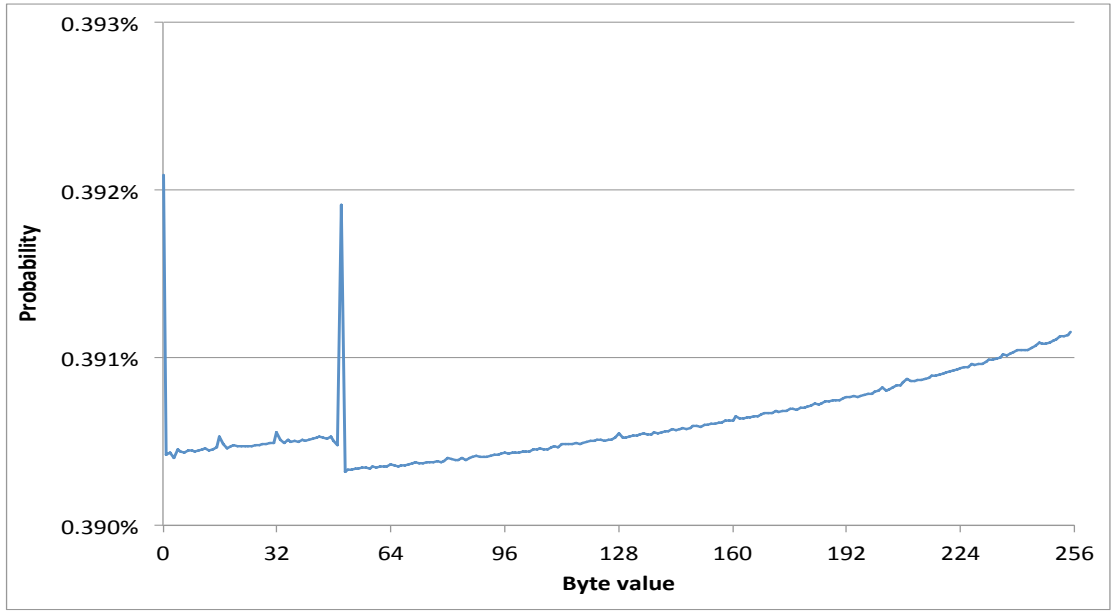
(a) Byte Z_1



(b) Byte Z_{16}



(c) Byte Z_{32}



(d) Byte Z_{50}

Fallout from 2013 RC4 attacks

- RC4 was the most widely used encryption method for TLS at the time (circa 2013)
- Attack required about q = 2^{26} (67 million) to start recovering plaintexts
  - Not quite practical, but within realm of feasibility
  - Enough to be considered ***significant problem***, and potentially within reach of intelligence agencies
- Needed to move on from RC4, but all other TLS encryption methods had even more severe security problems (stay tuned)
- Have replaced now with encryption based on ***block ciphers***

Block ciphers

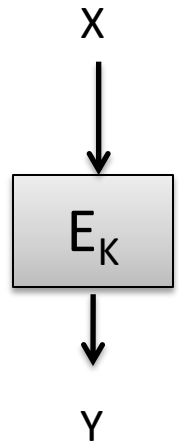
Family of permutations, one permutation for each key

$$E : \{0,1\}^k \times \{0,1\}^n \rightarrow \{0,1\}^n$$

Use notation E(K,X) = Y

Define inverse D(K,Y) = X such that D(K,E(K,X)) = X

E,D must be efficiently computable



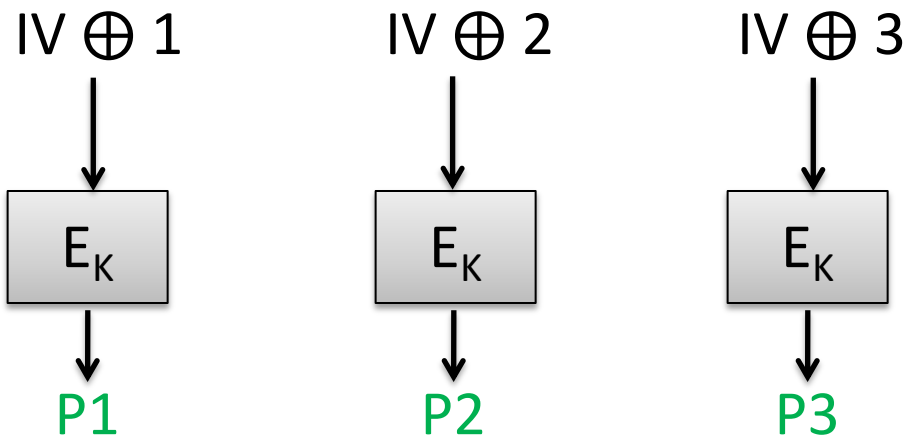
Pick K uniformly at random from $\{0,1\}^k$

CTR mode stream cipher

Counter mode stream cipher:

- Kg outputs random k-bit key for block cipher
- $G(K,IV,L) = E_K(IV \oplus 1) \| E_K(IV \oplus 1) \| \ \ldots \ \| \text{trunc}(E_K(IV \oplus m))$

  where $m = \text{ceil}(L / n)$



Truncate P3
to get L total bits

CTR-mode SE scheme

Kg():
K <-$ $\{0,1\}^k$

Pick a random key

Enc(K,M):
L <- |M| ; m <= ceil(L/n)
IV <-$ $\{0,1\}^n$
P <- $E_K(IV \oplus 1) \| \cdots \| \text{trunc}(E_K(IV \oplus m))$
Return (IV, P $\oplus$ M)

What security properties do we need from the block cipher?

Dec(K,(IV,C)):
L <- |C| ; m <= ceil(L/n)
P <- $E_K(IV \oplus 1) \| \cdots \| \text{trunc}(E_K(IV \oplus m))$
Return (IV, P $\oplus$ C)

Assume ciphertext can be parsed into IV and remaining ciphertext bits

Summary

- Target security against computational attackers
- Stream ciphers are computational equivalent of OTP
  - Keys must be large enough to avoid brute force
  - Even moderate biases in output stream must be avoided
- Stream ciphers give efficient symmetric encryption secure against passive adversaries
- None achieve security against active adversaries (stay tuned)

Next up

- Block cipher security goals
  - Pseudorandom functions and permutations
- Building & analyzing block ciphers
  - Feistel networks and DES
  - AES cipher