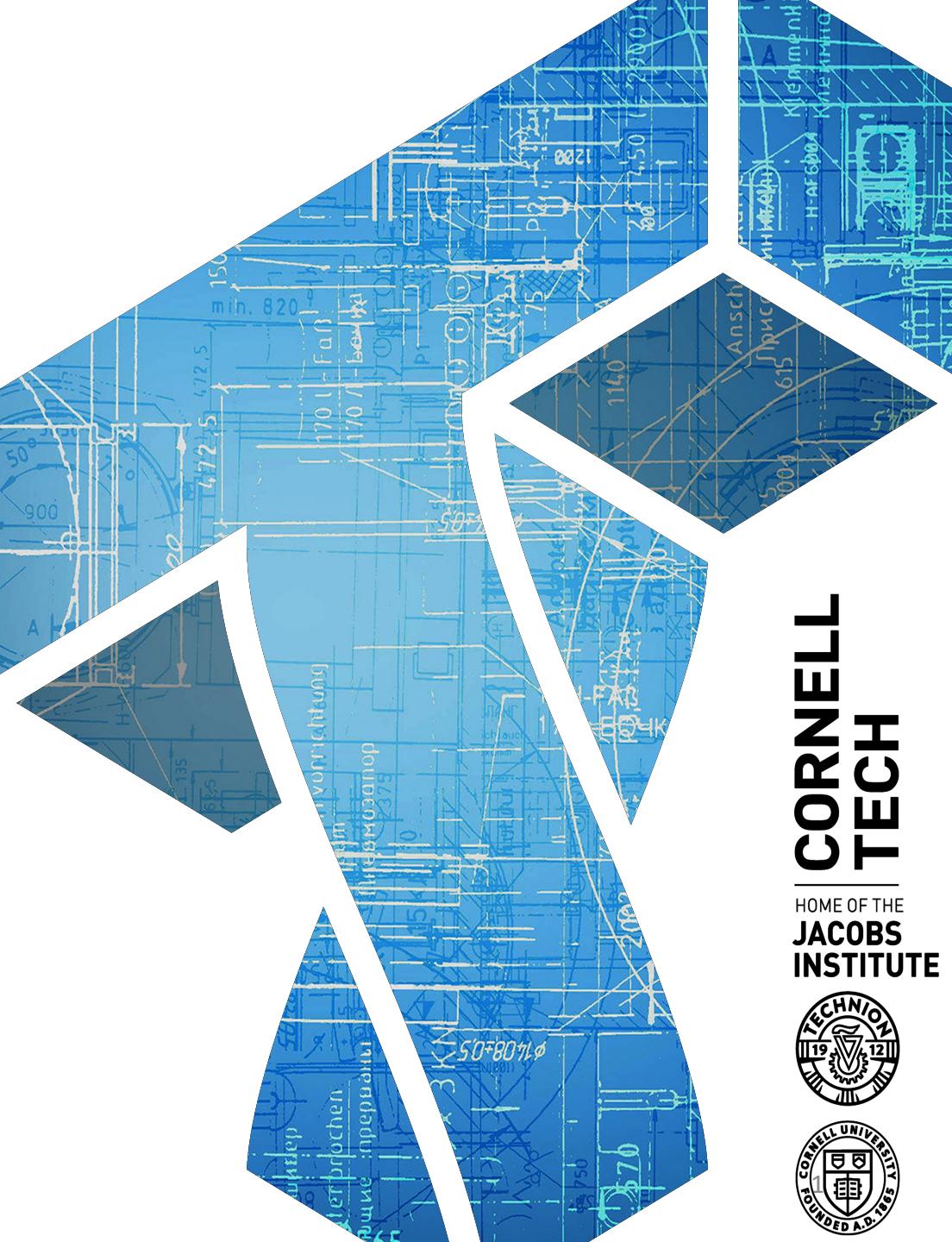


# CS 5830

# Cryptography



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# Recap and where we're at

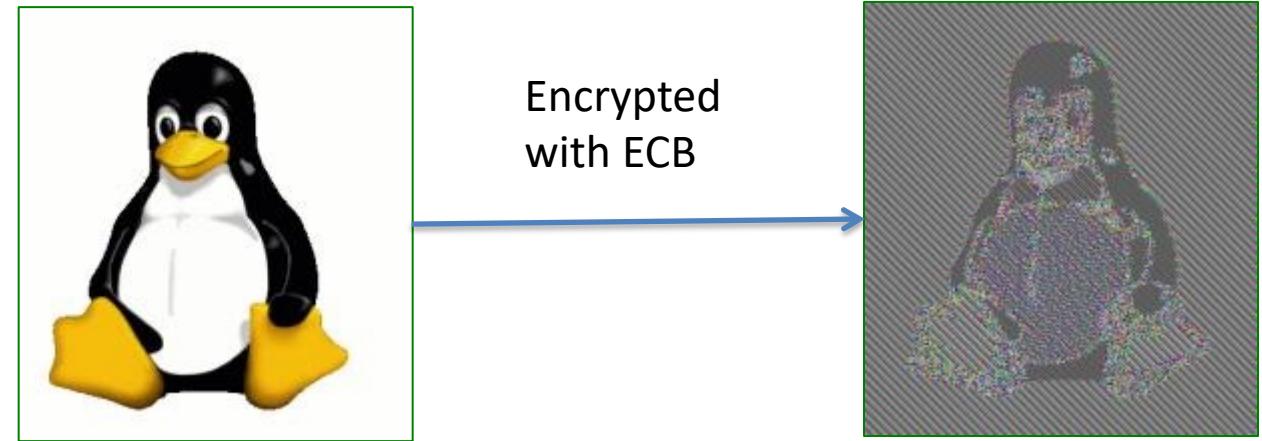
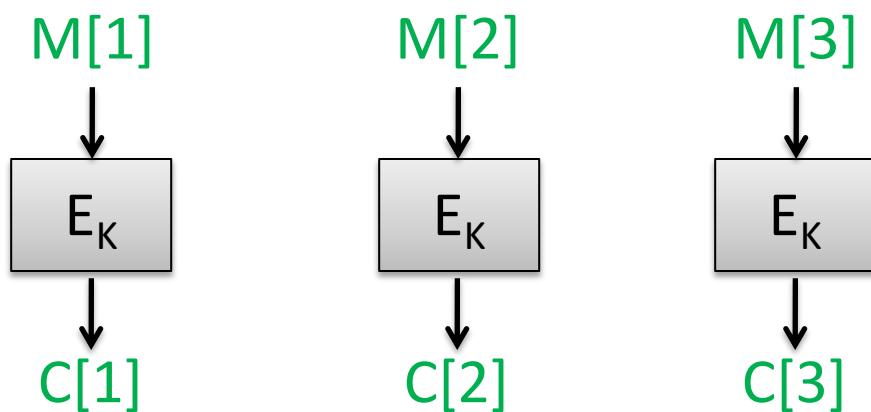
- Blockciphers and their security goals
  - Assume good blockciphers that achieve PRF security up to implications of best-known generic attacks
  - $\sim 2^k$  time (exhaustive key search)
  - $\sim 2^{n/2}$  time (birthday attacks)
- Today: modes of operation, IND-CPA security, & (time allowing) chosen-ciphertext attacks

# Block cipher modes of operation

Electronic codebook (ECB) mode

Pad message  $M$  to  $M[1], M[2], M[3], \dots$  where each block  $M[i]$  is  $n$  bits

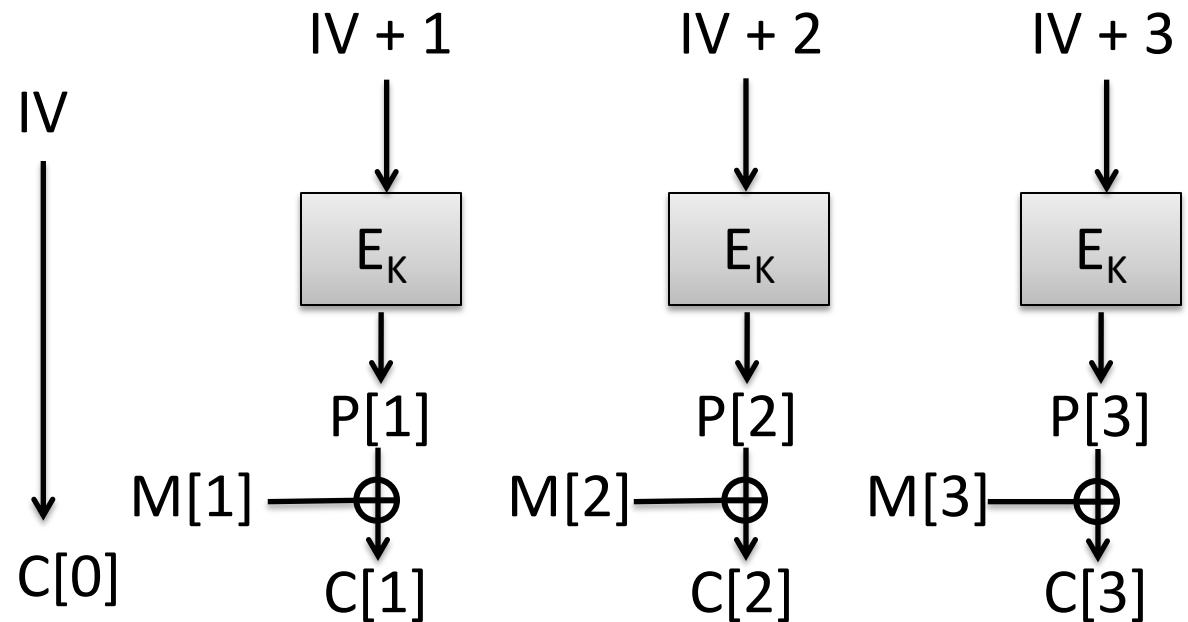
Then:



Images courtesy of  
[http://en.wikipedia.org/wiki/Block\\_cipher\\_modes\\_of\\_operation](http://en.wikipedia.org/wiki/Block_cipher_modes_of_operation)

# CTR mode

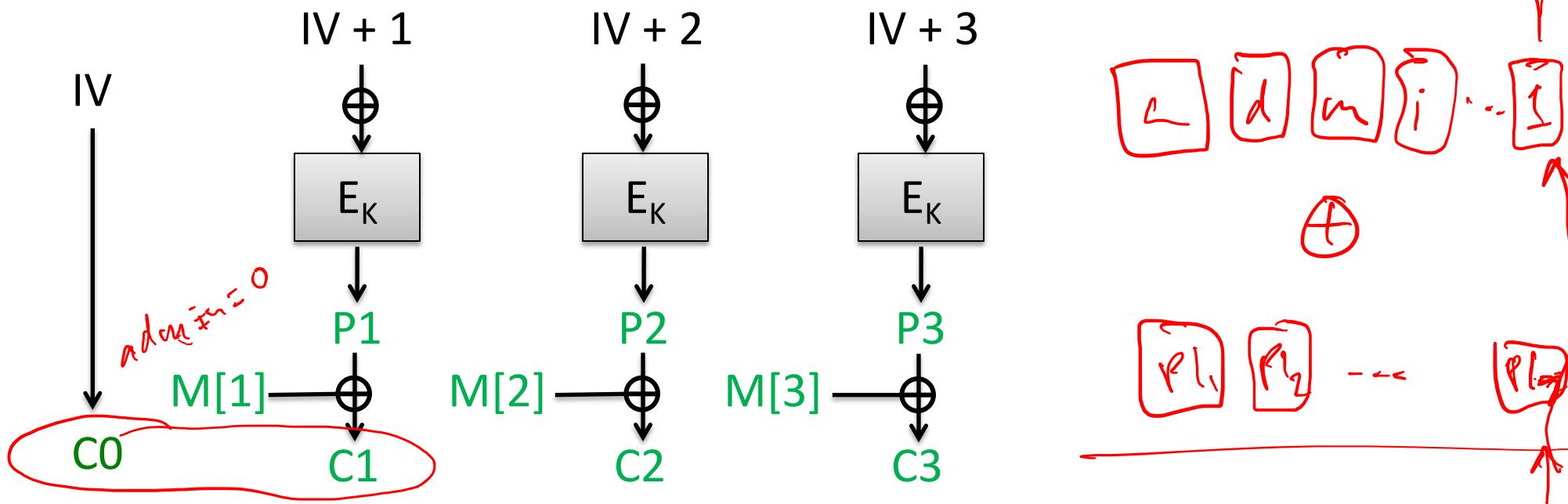
**Block cipher**  $E : \{0,1\}^k \times \{0,1\}^n \rightarrow \{0,1\}^n$  is a family of permutations  
Should be secure as a pseudorandom function (PRF)



CTR mode provides message confidentiality (nothing about message from ciphertext)  
assuming  $E$  is a PRF and number of message blocks encrypted  $<< 2^{n/2}$

# CTR is a blockcipher mode of operation

- How do we encrypt long messages with block cipher?
  - Modes of operation
- Long history: NIST standard
  - First published in 1980 specifically for DES
  - 2001 version:  
<https://nvlpubs.nist.gov/nistpubs/Legacy/SP/nistspecialpublication80-38a.pdf>
- Analyses starting in 1990s for chosen-plaintext attacks
- Analyses starting in 2000s for chosen-ciphertext attacks



Can attacker learn K from just  $C_0, C_1, C_2, C_3$ ?

Implies attacker can break E, i.e. recover block cipher key

Can attacker learn  $M = M[1], M[2], M[3]$  from  $C_0, C_1, C_2, C_3$ ?

Implies attacker can break PRF security of E

Can attacker learn one bit of M from  $C_0, C_1, C_2, C_3$ ?

Implies attacker can break PRF security of E

**Passive adversaries cannot learn anything about messages**

# Formalizing security for CPA attacks

- Indistinguishability under chosen-plaintext attacks (IND-CPA)
  - Multi-message, adaptive security
  - Query encryption oracle q times adaptively with pair of equal-length messages  $M_0, M_1$  ; get back  $\text{Enc}(K, M_b)$  for secret key K
  - Can adversary infer b?

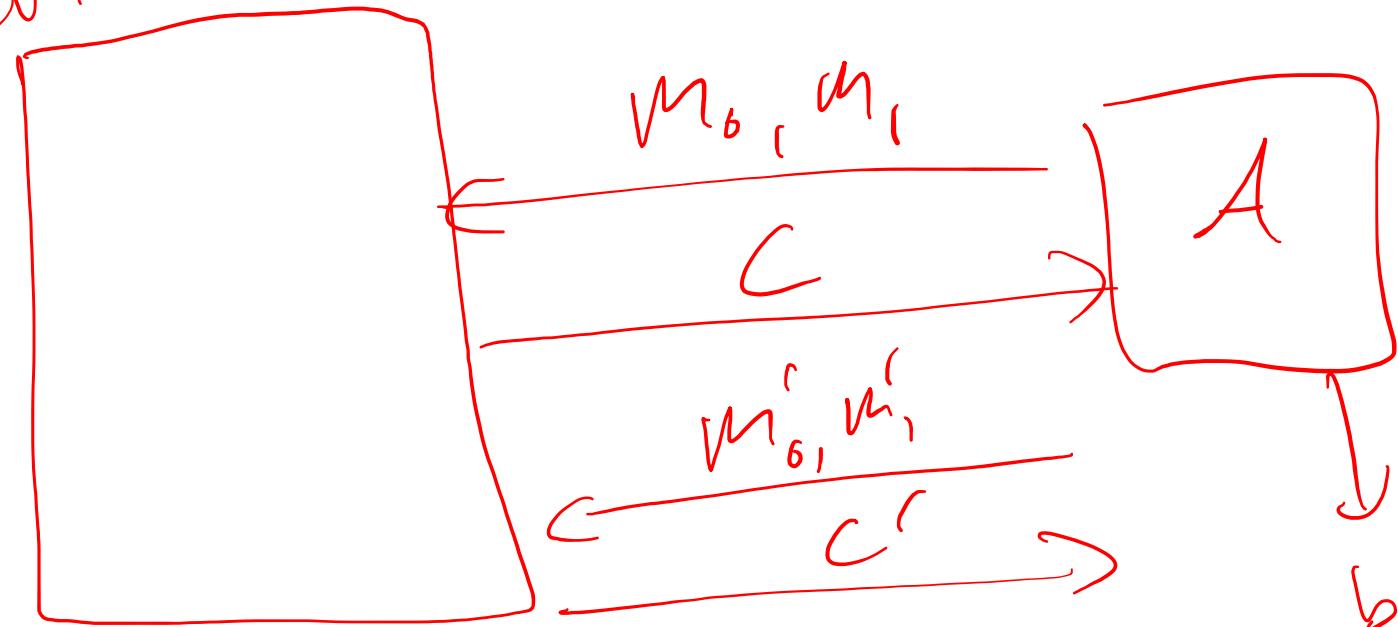
IND-CPA(SE,  $\mathcal{A}$ ):  
 $K \leftarrow \$ \text{Kg} ; b \leftarrow \$ \{0,1\}$   
 $b' \leftarrow \$ \mathcal{A}^{\text{LR}}$   
Return  $(b = b')$

LR( $M_0, M_1$ ):  
 $C \leftarrow \$ \text{Enc}(K, M_b)$   
Return C

Def. A symmetric encryption scheme is  $(t, q, L, \epsilon)$ -IND-CPA if for any adversary  $\mathcal{A}$  running in time at most t and making at most q queries of length at most L bits, it holds that

$$\Pr[\text{IND-CPA}(\text{SE}, \mathcal{A}) = 1] \leq 1/2 + \epsilon$$

IND-CPA



IND-CPA( $SE, \mathcal{A}$ ):

$K \leftarrow \$ Kg$  ;  $b \leftarrow \$ \{0,1\}$

$b' \leftarrow \$ \mathcal{A}^{LR}$

Return  $(b = b')$

LR( $M_0, M_1$ ):

$C \leftarrow \$ Enc(K, M_b)$

Return  $C$

Hybrid argument

$b=0$   
IND-CPA  
w/  $b=0$

Replace  $E$  with  
Random function

Don't  
call  $E$

Using  $b=1$

$b=1$   
IND-CPA  
w/  $b=1$

$\epsilon_{prf}$

$\frac{\epsilon^2}{2^n}$

0

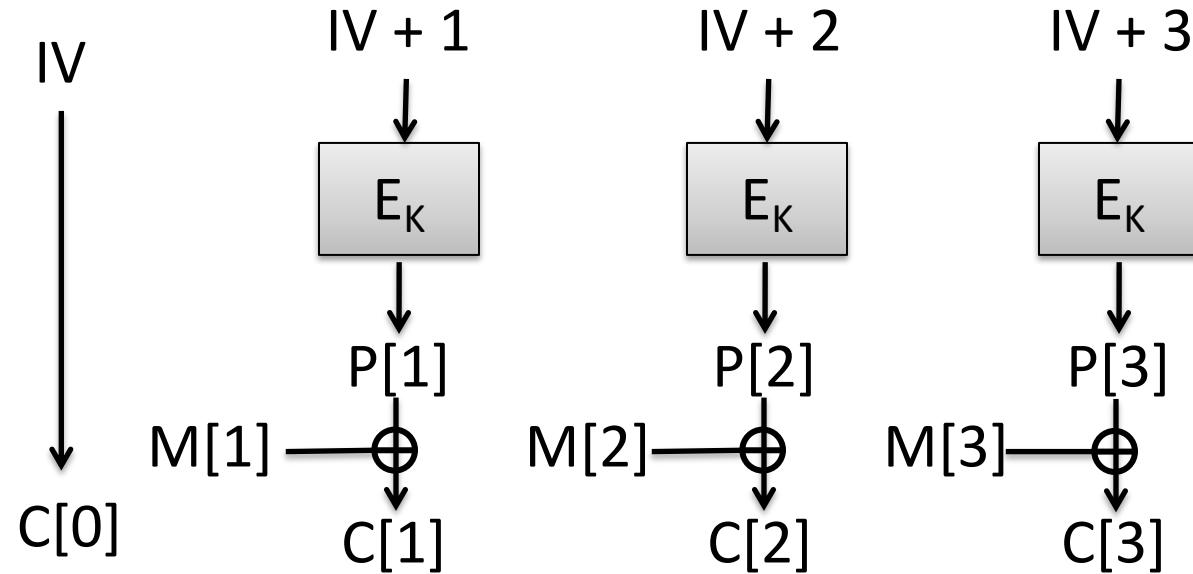
# CTR mode IND-CPA security

Thm. Let  $E$  be a blockcipher with blocksize  $n$ . Then CTR-mode using  $E$  is  $(t, q, L, \epsilon_{\text{cpa}})$ -IND-CPA with

$$\epsilon_{\text{cpa}} \leq \underbrace{\epsilon_{\text{prf}}}_{\text{close to } 0} + \underbrace{(\sigma q)^2 / 2^n}_{\text{large term}}$$

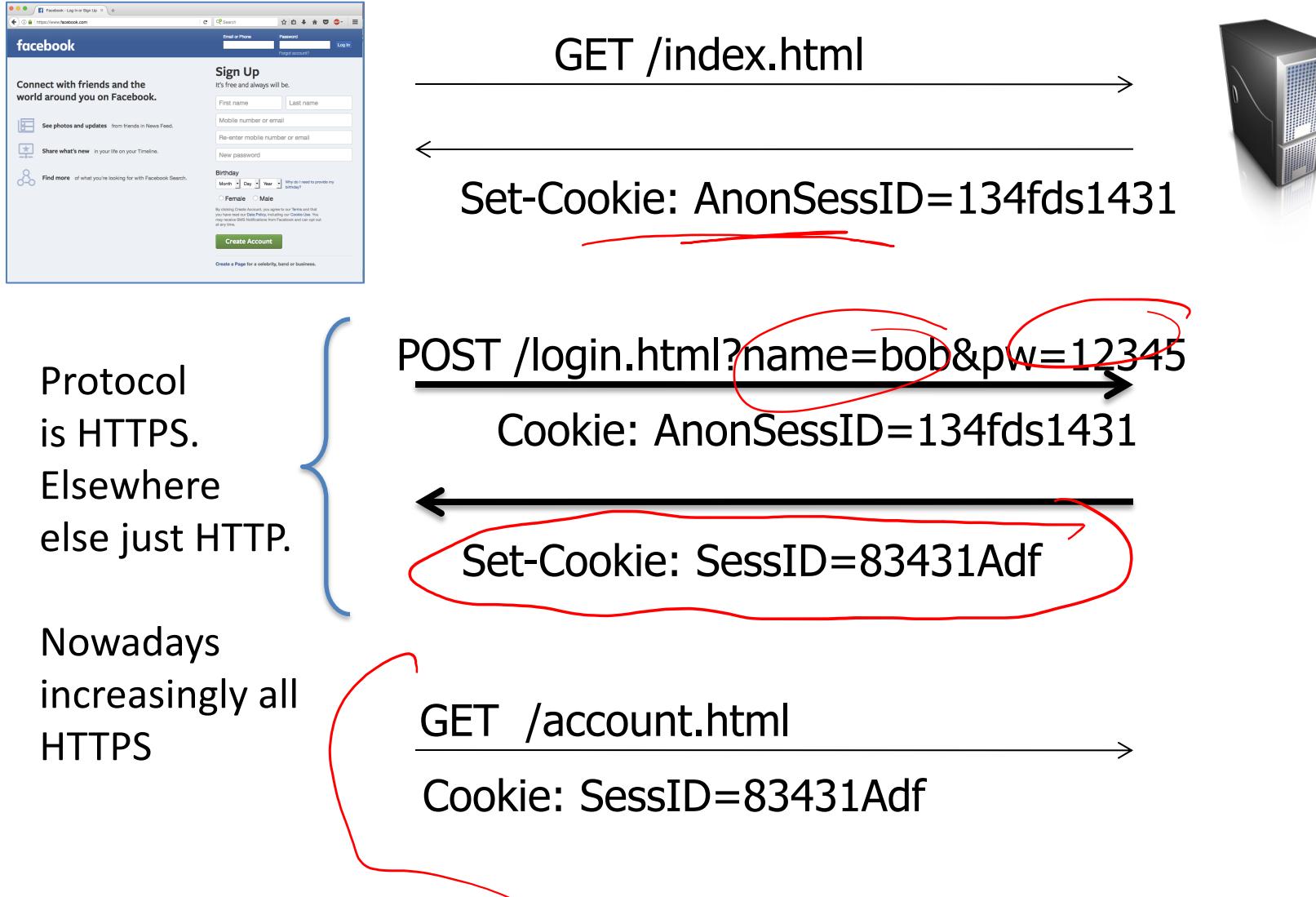
assuming  $E$  is  $(t, q\sigma, \epsilon_{\text{prf}})$ -PRF secure blockcipher for  $\sigma = \lceil L/n \rceil$ .

# CTR mode and different threat models

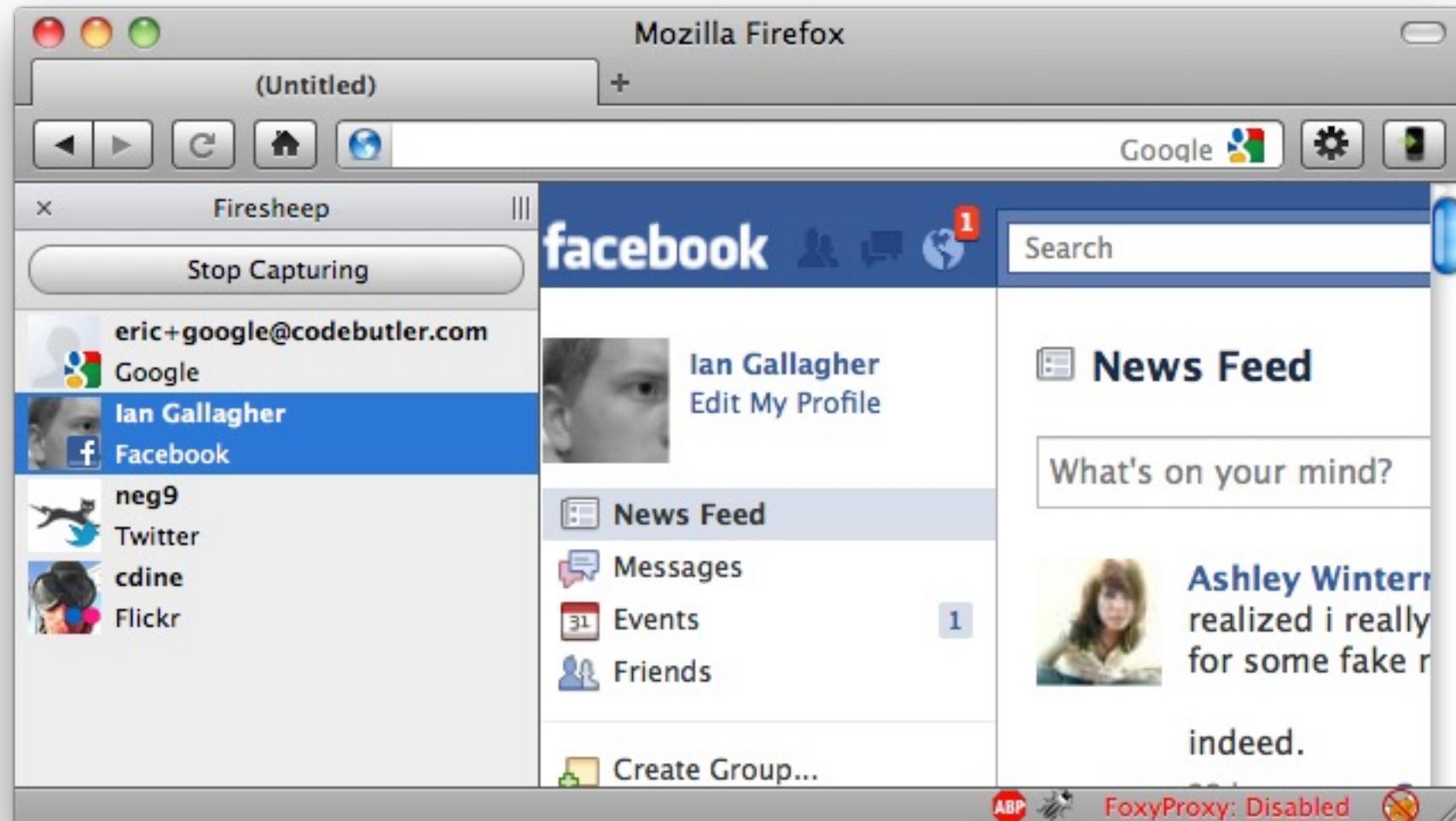


Attack type	Capability	Goal
Indistinguishability under chosen plaintext attack	Observe ciphertexts under secret key of chosen plaintexts with $\geq 1$ bit of uncertainty	Learn at least one bit of information about plaintext
Ciphertext integrity attack	Get example ciphertext(s); maul ciphertexts and submit to decryption oracle	Trick recipient into accepting forged plaintext
Indistinguishability under chosen-ciphertext attack	Get example ciphertext(s); maul ciphertexts and submit to partial decryption oracle	Learn information about plaintexts

# Session handling and login



# Without HTTPS: session hijacking often trivial



From <http://codebutler.com/firesheep>

# Security problems here?



Facebook.com

POST /login.html?name=bob&pw=12345



Cookie: AnonSessID=134fds1431



Set-Cookie: SessID=83431Adf

Secret key K only  
known to server

GET /account.html

Cookie: SessID=83431Adf

$$83431Adf = \text{CTR-Enc}(K, \text{"admin=0"})$$

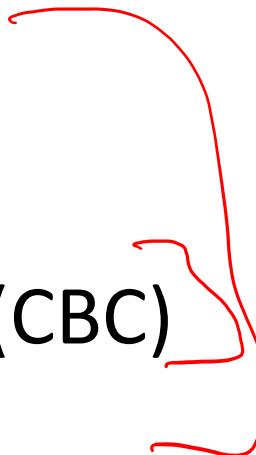
Malicious client can simply flip a few bits to change admin=1

Example of an ***integrity / authenticity violation***

Soon we will build authentication mechanisms to prevent this

# NIST Modes

- ~~Electronic codebook mode (ECB)~~
- Counter mode (CTR)
- Ciphertext feedback mode (CFB)
- Offset feedback mode (OFB)
- Ciphertext block chaining mode (CBC)



CTR, CBC found widespread use

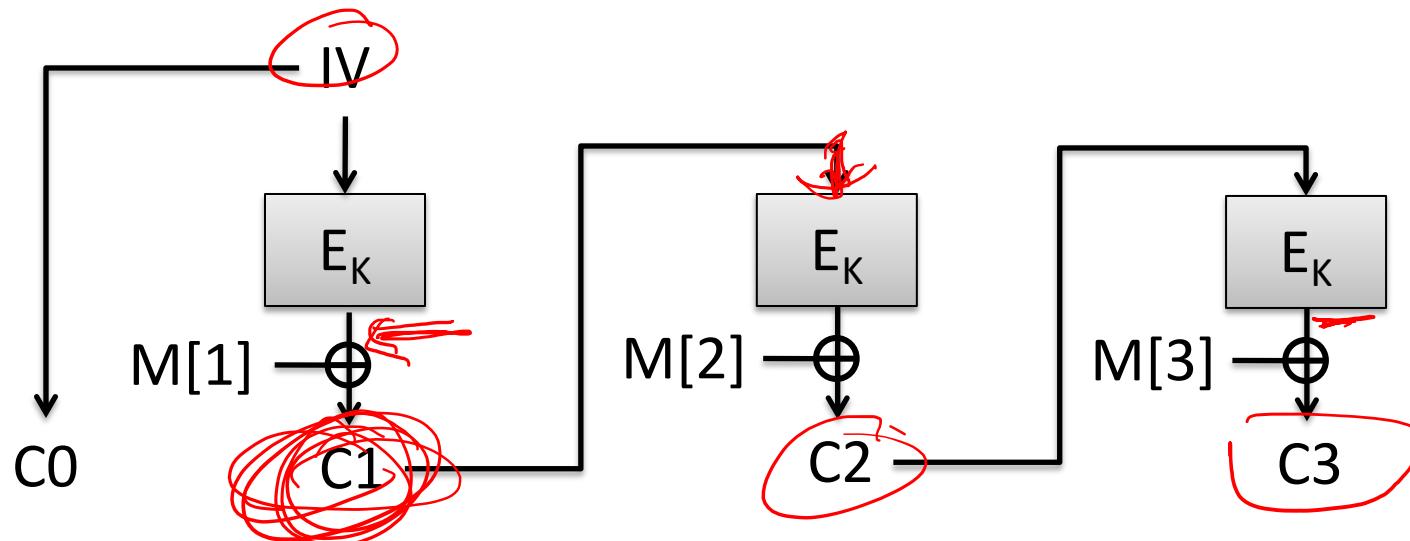
# CFB mode

Ciphertext feedback mode (CFB)

Pad message M to  $M[1], M[2], M[3], \dots$  where each block  $M[i]$  is n bits

Choose random n-bit string IV

Then:



How do we decrypt?

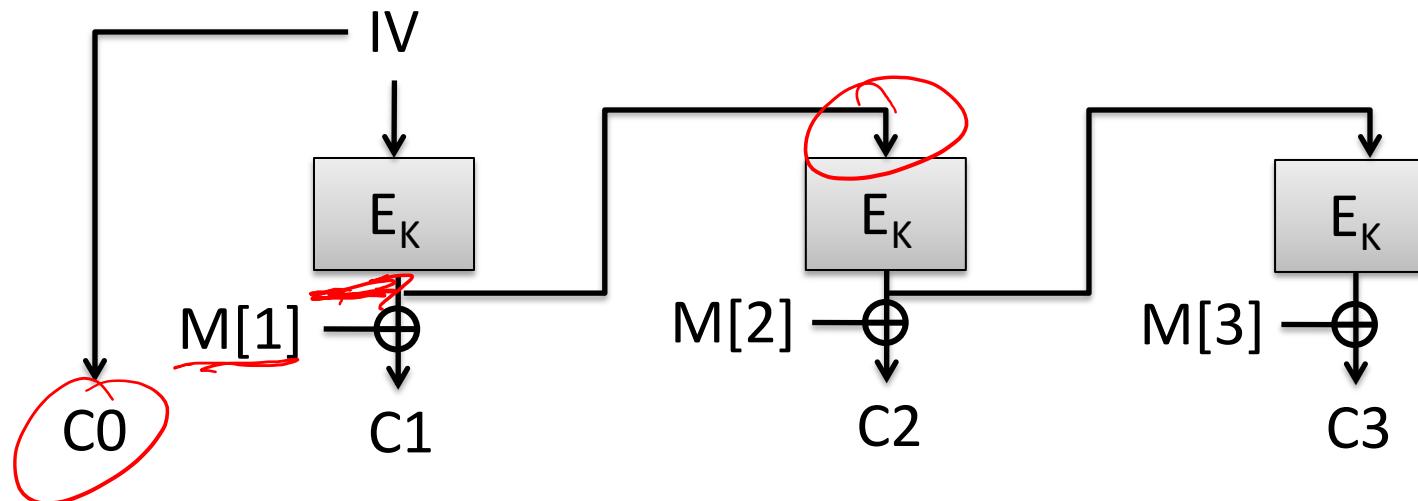
# OFB mode

Offset feedback mode (OFB)

Pad message  $M$  to  $M[1], M[2], M[3], \dots$  where each block  $M[i]$  is  $n$  bits

Choose random  $n$ -bit string  $IV$

Then:



How do we decrypt?

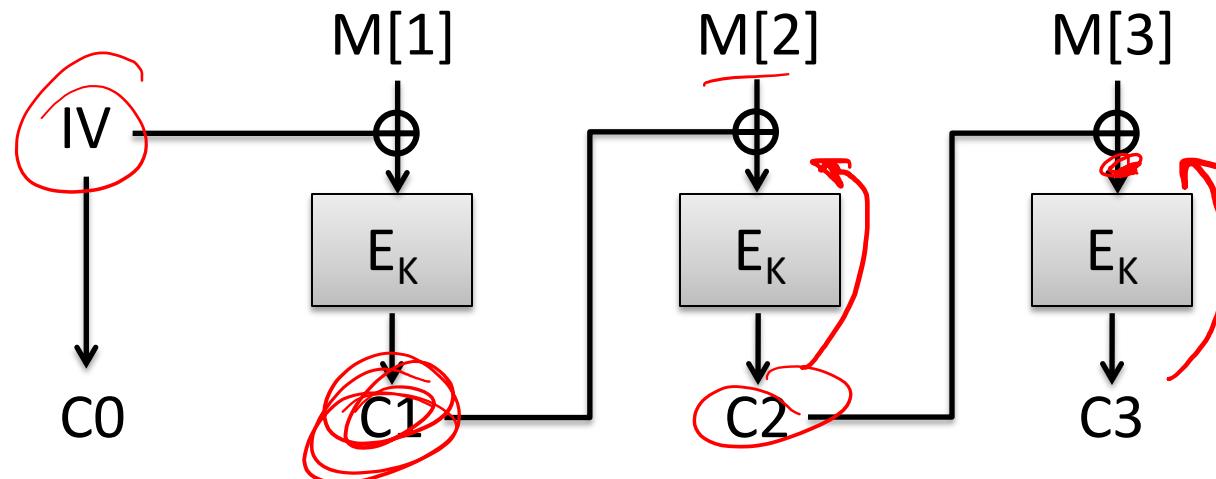
# CBC mode

Ciphertext block chaining (CBC)

Pad message  $M$  to  $M[1], M[2], M[3], \dots$  where each block  $M[i]$  is  $n$  bits

Choose random  $n$ -bit string  $IV$

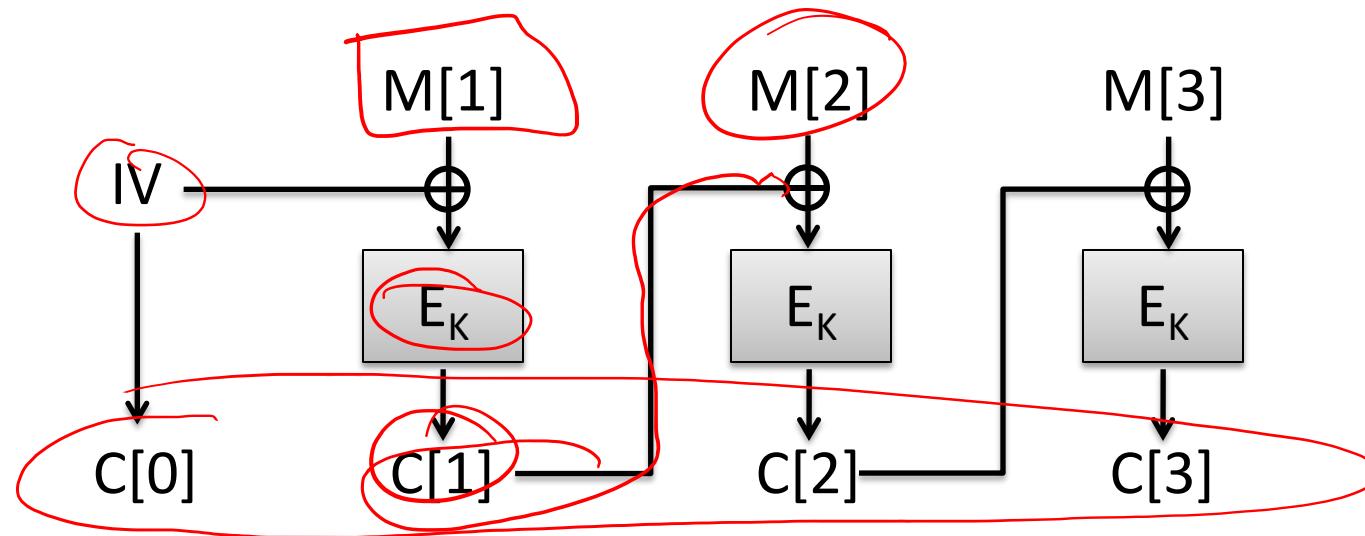
Then:



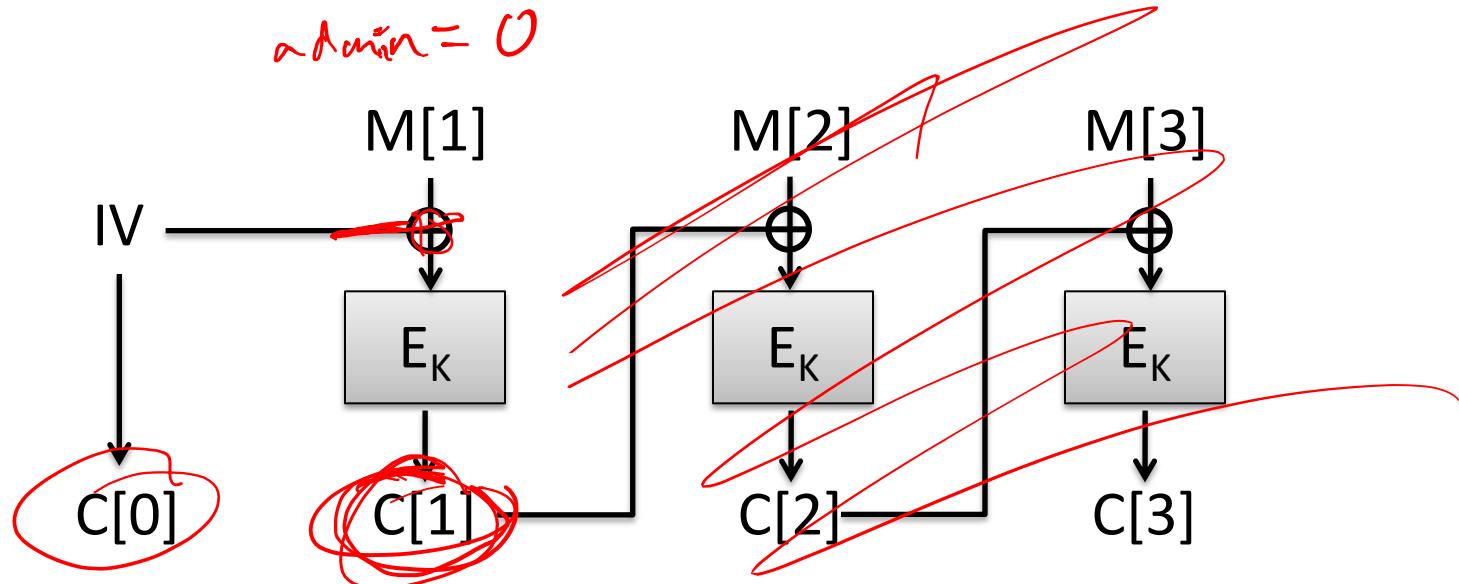
How do we decrypt?

# IND-CPA of CBC mode

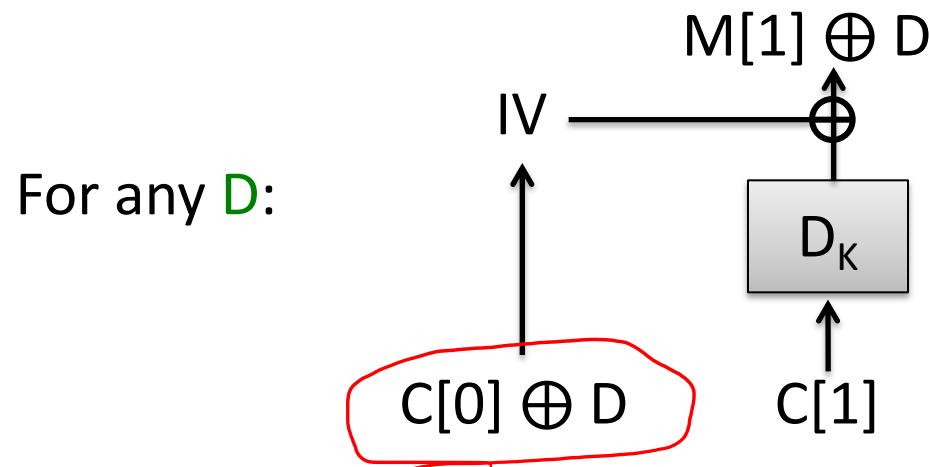
- CBC mode can also be shown to be IND-CPA secure, with similar security level as that of CTR mode
  - What if IV collides?
  - Must switch keys before number of encryptions  $q$  gets close to  $2^{n/2}$



# CBC mode has malleability issues, too



How do we change bits of  $M$  received by server?



# Padding for CBC mode

- CBC mode handles messages with length a multiple of n bits
- We use padding to make it work for arbitrary message lengths
  - PadCBC, UnpadCBC map to, from strings of length multiple of n
  - Will specify example padding schemes later

# Pseudocode for CBC mode with padding

Kg():

$K \leftarrow \{0,1\}^k$

CBC-Enc(K,M):

$L \leftarrow |M| ; m \leftarrow \text{ceil}(L/n)$

$C[0] \leftarrow \text{IV} \leftarrow \{0,1\}^n$

$M[1], \dots, M[m] \leftarrow \text{PadCBC}(M, n)$

For  $i = 1$  to  $m$  do

$C[i] \leftarrow E_K(C[i-1] \oplus M[i])$

Return  $C[0] \parallel C[1] \parallel \dots \parallel C[m]$

Pick a random key

CBC-Dec(K,C):

For  $i = 1$  to  $m$  do

$M[i] \leftarrow C[i-1] \oplus D_K(C[i])$

$M \leftarrow \text{UnpadCBC}(M[1], \dots, M[m], n)$

Return  $M$

PadCBC unambiguously pads  $M$  to a sequence of  $n$  bit message blocks

block cipher

inverse

UnpadCBC removes padding, returns appropriately long string

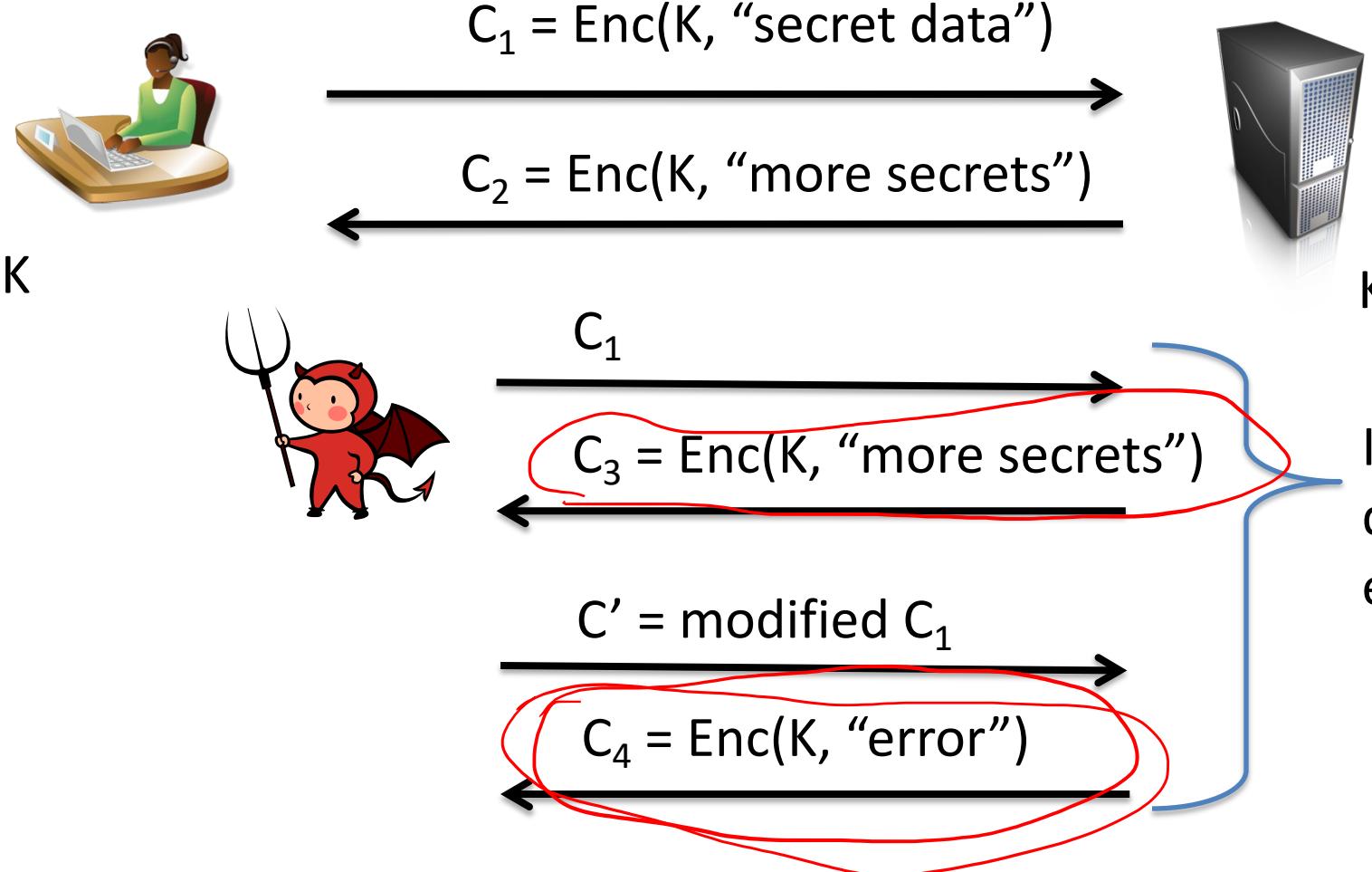
May output error if padding is wrong

In crypto, errors often denoted by  $\perp$

# Padding for CBC mode

- CBC mode handles messages with length a multiple of n bits
- We use padding to make it work for arbitrary message lengths
  - PadCBC, UnpadCBC map to, from strings of length multiple of n
  - Will specify example padding schemes later
- Padding checks often give rise to chosen-ciphertext attack called ***padding oracle attacks***
  - Given CBC mode encryption  $C = \text{Enc}(K, M)$  for unknown  $M$
  - Access to oracle that reveals just whether decryption succeeds
  - Recover  $M$

# Partial decryption oracles arise frequently in practice



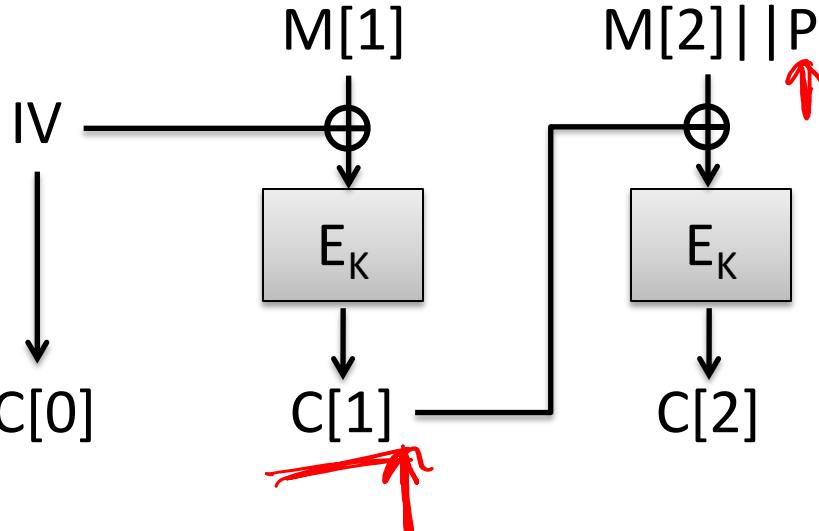
TLS/HTTPS canonical examples where decryption oracles arise

In practice usually easy to distinguish  $C_3$  from  $C_4$  even without  $K$

$$|C_4| \neq |C_3|$$

Timing differs for successful vs. unsuccessful decryption

# Simple situation: pad by 1 byte



Assume that  
 $M[1] \parallel M[2]$  has length  
2n-8 bits

P is one byte of padding  
that must equal 0x00



Adversary  
obtains  
ciphertext  
 $C[0], C[1], C[2]$

$C[0], C[1] \oplus 1, C[2]$

Dec( $K, C'$ )  
 $M'[1] \parallel M'[2] \parallel P' = \text{CBC-Dec}(K, C')$   
If  $P' \neq 0x00$  then  
Return error  
Else  
Return ok