

CS 5830

Cryptography

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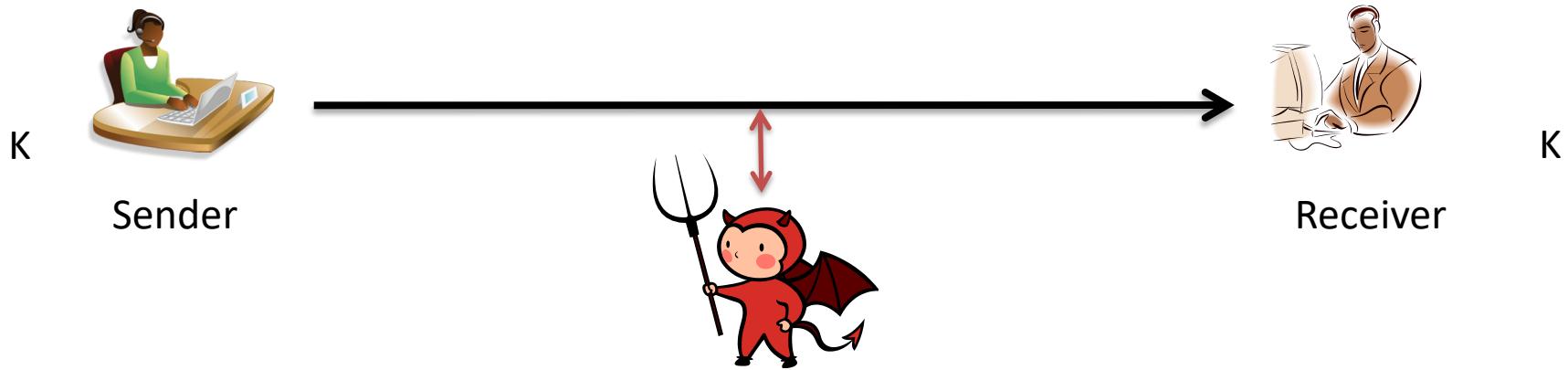
Recap and where we're at

- What we have so far:
 - Block cipher modes of operation (CBC, CTR)
 - Computationally secure against passive adversaries (IND-CPA)
 - Insecure against active attacks
 - Integrity problems: bit flip “mauling” attacks against CTR, CBC
 - Confidentiality problems: Padding oracle attacks against CBC
- We need another tool:
authenticity mechanisms

Roadmap towards authenticated encryption

- Message authentication schemes
 - Do not concern themselves with confidentiality, just authenticity
- Apply message authentication to IND-CPA ciphertexts
 - Yields ***authenticated encryption***
 - Message confidentiality (under both CPAs & CCAs)
 - Message integrity

Message authentication

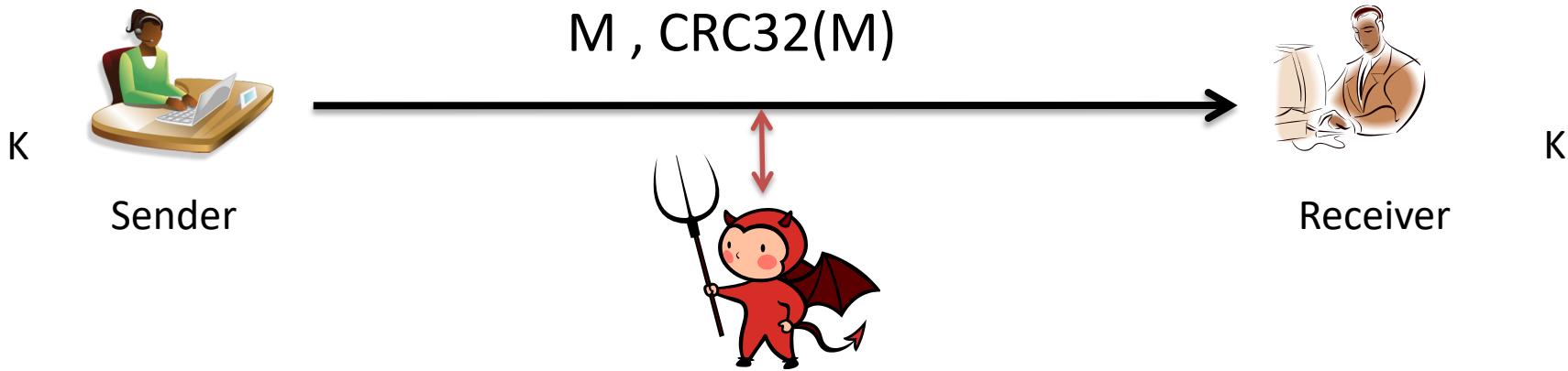


Attacker has read/write access to communications channel

The strategy:

Arrange so that all bits received can be validated as having come **only** from sender (the entity with key K)

Message authentication

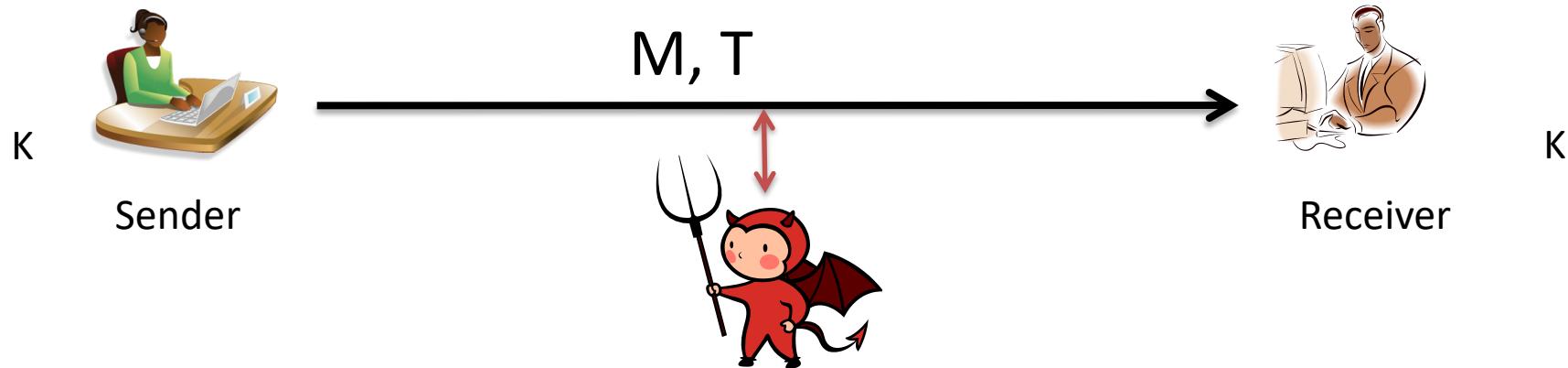


Recall error correction codes (ECCs):

- Don't use a cryptographic key
- Used to help detect & fix **random errors**
- CRC is just one example:
 - Reed-Solomon codes, credit card last digit (Luhn check), etc.

Don't use secret key. Adversary can easily forge an ECC!

Message authentication schemes



Scheme is triple of algorithms $MA = (Kg, Tag, Verify)$:

- (1) Kg outputs secret key K
- (2) $\text{Tag}(K, M)$ outputs a tag T
- (3) $\text{Verify}(K, M, T)$ outputs 0/1 (invalid / valid)

Message authentication code (MAC):
Tag deterministic +
Verify checks by recomputing Tag

Correctness: $\text{Verify}(K, M, \text{Tag}(K, M)) = 1$ always

Security: No computationally efficient attacker can forge tags for a new message even when attacker gets

$$(M_1, T_1), (M_2, T_2), \dots, (M_q, T_q)$$

for messages of their choosing and reasonably large q .

Unforgeability under chosen-message attack (UF-CMA)

Security: No computationally efficient attacker can forge tags for a new message, even given chosen message attack

UF-CMA(MA, \mathcal{A}):

```
K <- $ Kg  
(M*, T*) <- $  $\mathcal{A}^{\text{Tag}}$   
If M* ∈ MsgSet then  
    Return 0  
Return Verify(K, M*, T*)
```

Tag(M)

```
MsgSet <- MsgSet U {M}  
Return Tag(K, M)
```

Measure advantage of adversary via probability wins game:

$$\Pr[\text{UF-CMA}(\text{MA}, \mathcal{A}) \Rightarrow 1]$$

Insecure:

Close to 1 for \mathcal{A} using reasonable time and Tag queries

Secure:

Negligibly far from 0 for any \mathcal{A} using reasonable time and queries

Unforgeability under chosen-message attack (UF-CMA)

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Warm-up: tag length is important

Assume M is n-bit message. Let E be a blockcipher on n bits
Trunc_t truncates input string to first t bits

Let's say we set Tag(K, M) = Trunc_t(E_K(M))

Easy attack strategy?

Unforgeability under chosen-message attack (UF-CMA)

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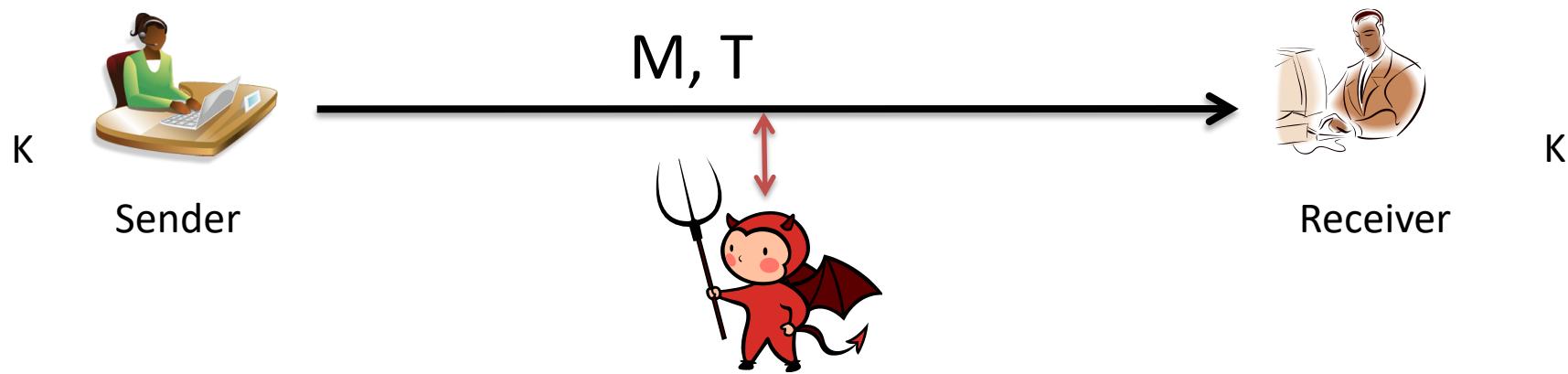
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Let's say we set Tag(K, M) = Trunc_t(E_K(M))

Any ideas for attack strategy?

Message authentication using pseudorandom functions (PRFs)



Let $F : \{0,1\}^k \times \{0,1\}^* \rightarrow \{0,1\}^t$ be a secure PRF

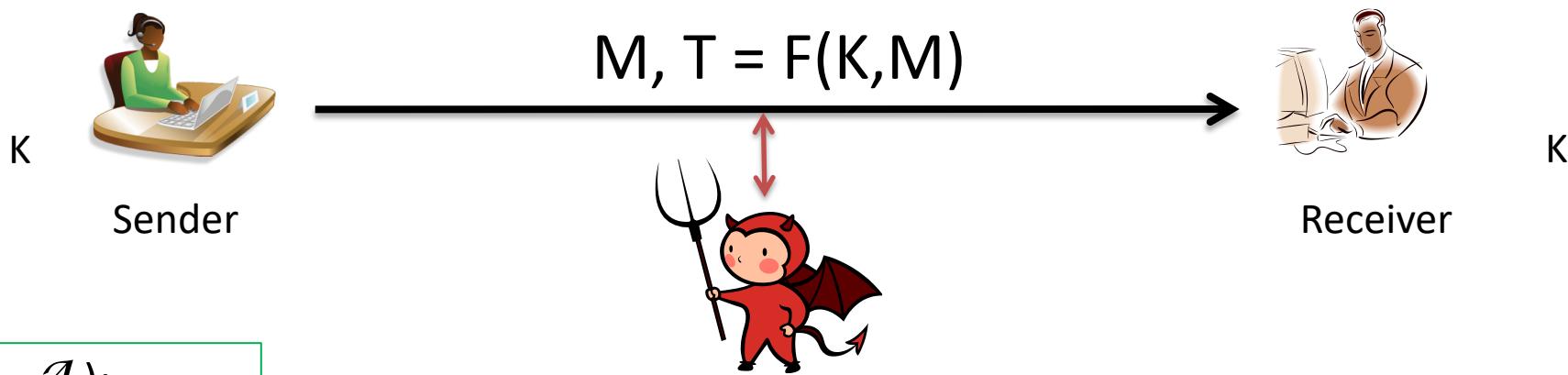
Tag(K,M)
Return $F(K,M)$

Verify(K,M,T):
If $F(K,M) = T$ then Return 1
Return 0

When is this secure?

Can you sketch an argument for why?

Message authentication using pseudorandom functions (PRFs)



UF-CMA(MA, \mathcal{A}):

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K <- $ Kg  
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```

PRFs for variable-length messages

- Blockciphers are good PRFs, but only accept n-bit input
- We want to construct PRFs for longer and variable lengths
- How do we do this? We can again build a PRF from a blockcipher, using it in a mode of operation
 - Goal is different from building encryption
 - $F : \{0,1\}^k \times \{0,1\}^* \rightarrow \{0,1\}^t$

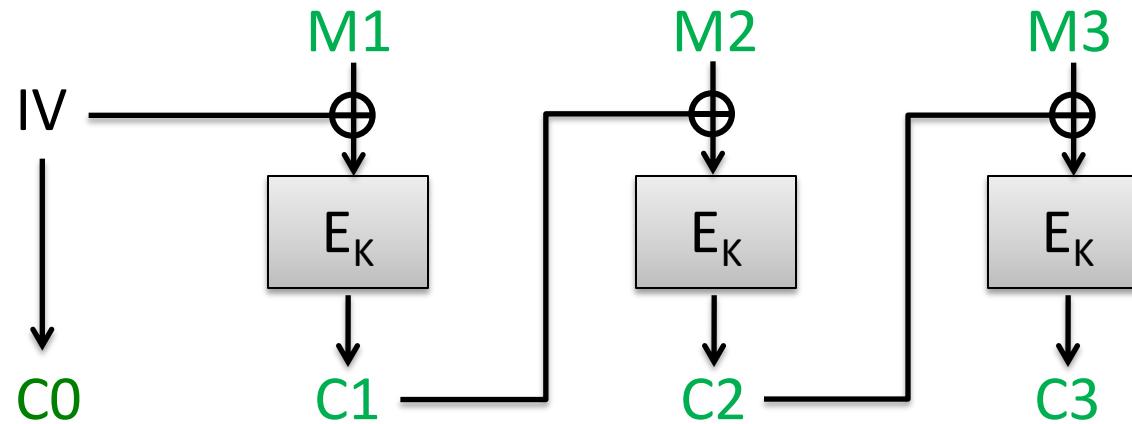
Recall CBC mode

Ciphertext block chaining (CBC)

Pad message M to M_1, M_2, M_3, \dots where each block M_i is n bits

Choose random n-bit string IV

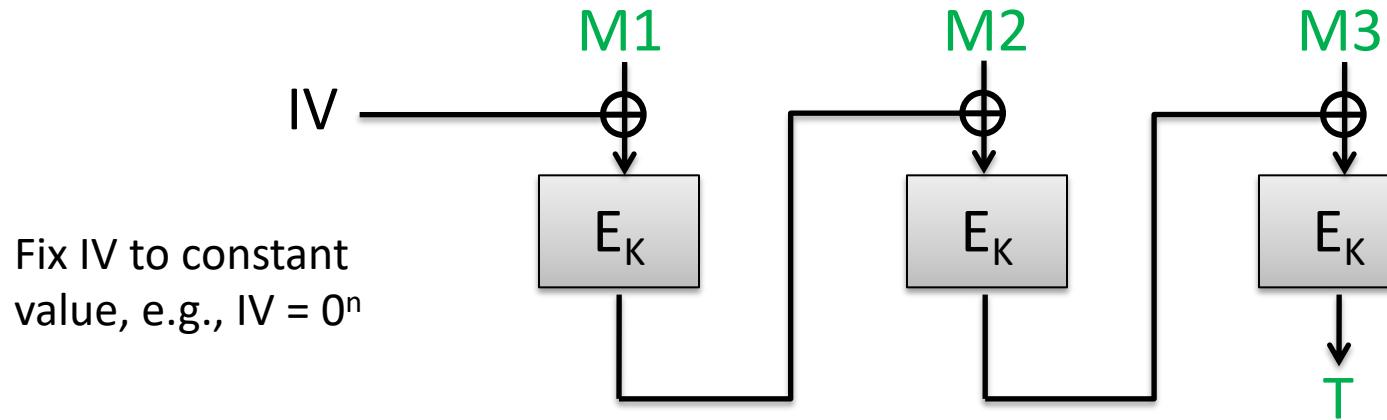
Then:



Can we convert this into variable-message-length PRF?

CBC-MAC

Message authentication code (MAC)



Turns out this is (provably) a good PRF
if K used only on same-length messages

Breaking variable-input-length CBC-MAC

Consider CBC-MAC but allow any messages of length a multiple of n.

$$\text{E.g.: } \text{Tag}(K, M_1) = E_K(M_1) \quad \text{Tag}(K, M_1 || M_2) = E_K(M_2 \oplus E_K(M_1))$$

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If M* ∈ MsgSet then  
    Return 0  
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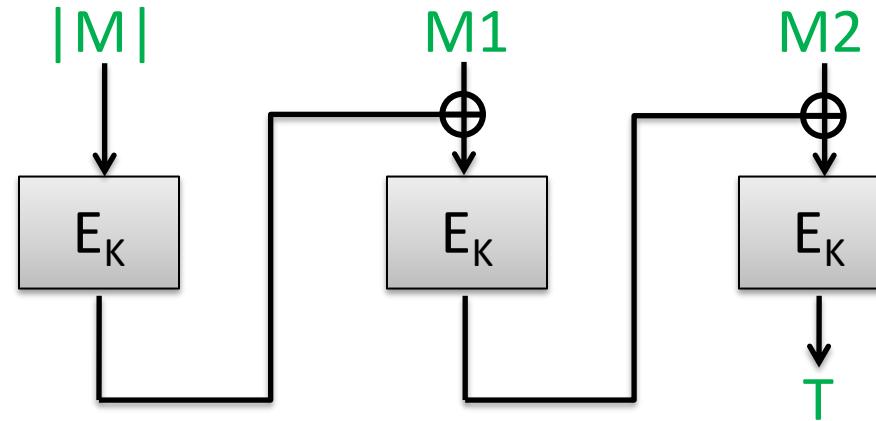
Tag(M)

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MsgSet <- MsgSet U {M}  
Return Tag(K, M)
```

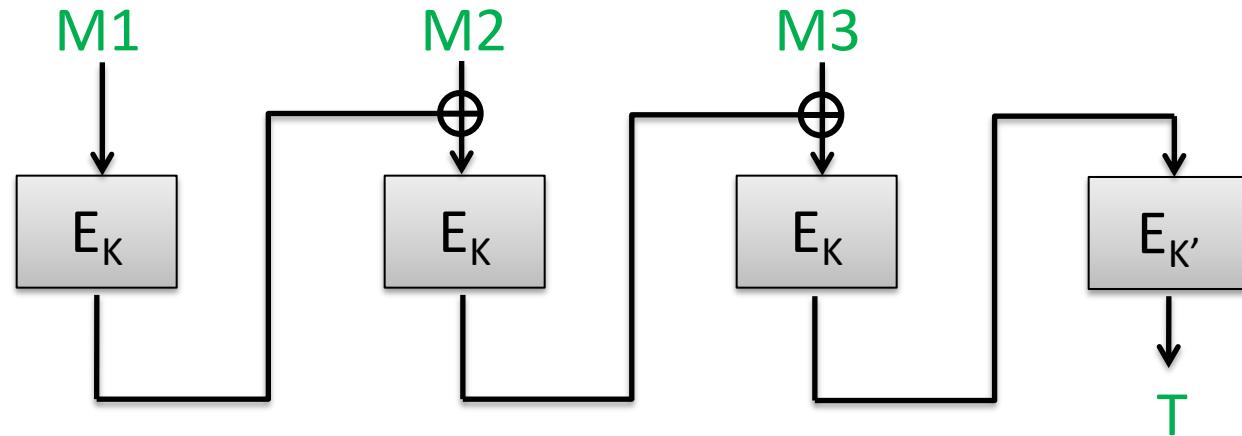
Give UF-CMA adversary \mathcal{A} that forges successfully using few queries

Secure variable-message-length CBC-MAC

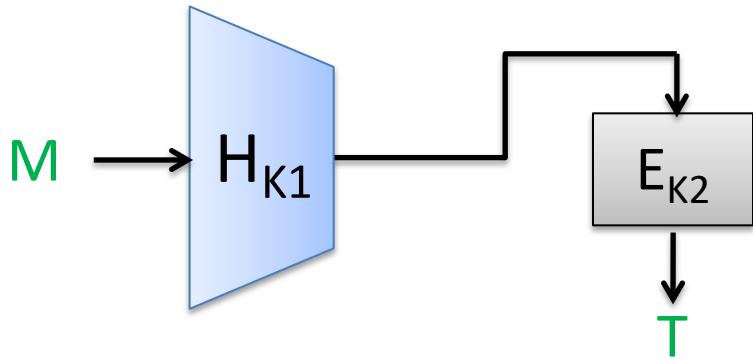
- Prepend message length



- Encrypted CBC-MAC



Universal hash then PRF approach



Turns out this is a good construction for any H that is a (computational) ***universal hash function***

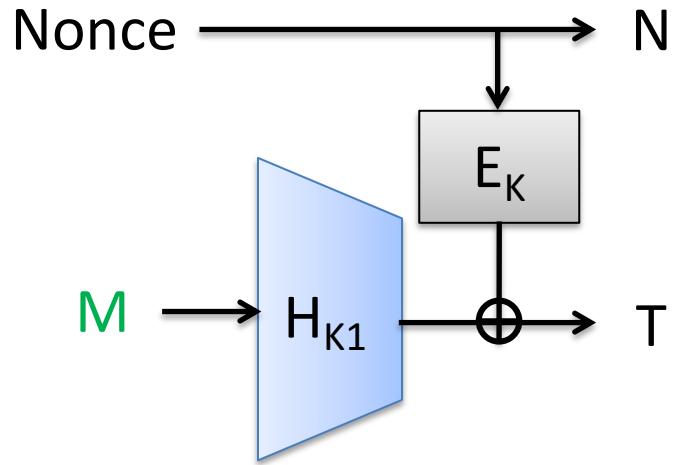
A keyed function $H: \{0, 1\}^k \times \mathcal{M} \rightarrow \{0, 1\}^n$ is an ϵ -almost universal (AU) hash function if for all $M \neq M'$

$$\Pr [H_K(M) = H_K(M')] \leq \epsilon$$

where the probability is over choice of K .

Intuition: E_{K2} hides info about $K1$ unless collision. For q queries, chance of collision at most $q^2 \epsilon / 2$

Carter-Wegman MA Schemes



Nonce must be fresh for each M
Random value or counter
If repeated, security fails

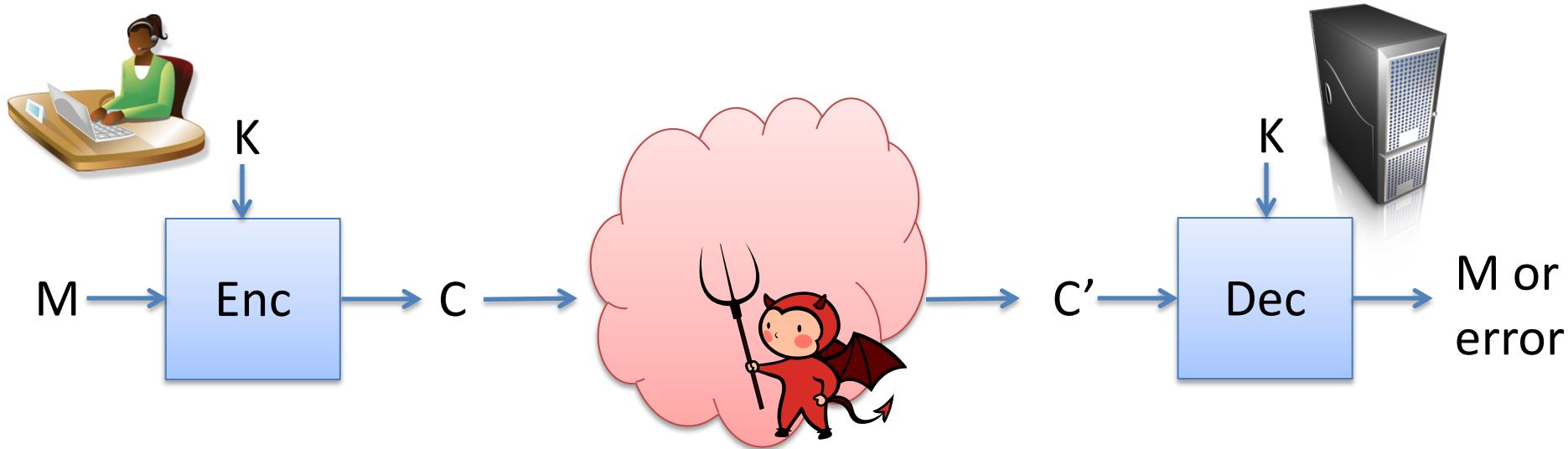
Intuition:
Hide universal hash by encrypting it

A keyed function $H: \{0, 1\}^k \times \mathcal{M} \rightarrow \{0, 1\}^n$ is an ϵ -almost universal (AU) hash function if for all $M \neq M'$

$$\Pr [H_K(M) = H_K(M')] \leq \epsilon$$

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Authenticated encryption (AE)

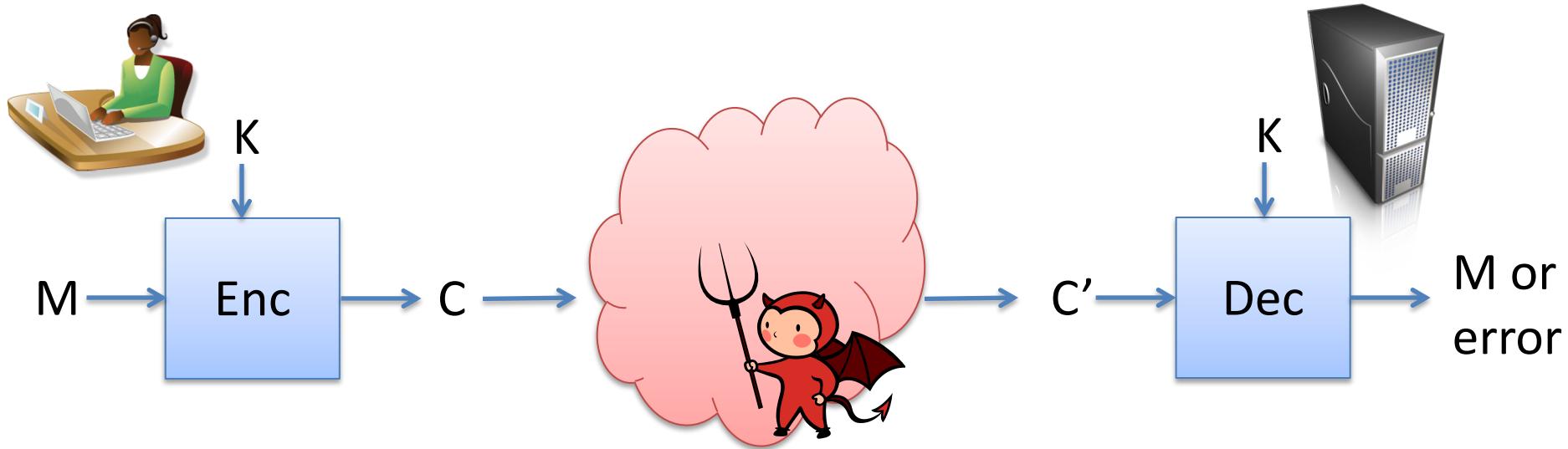


What security properties do we need from symmetric encryption?

- 1) Confidentiality: should not learn any information about M
- 2) Authenticity: should not be able to forge ciphertexts

Often referred to as Authenticated Encryption security

Authenticated encryption (AE)

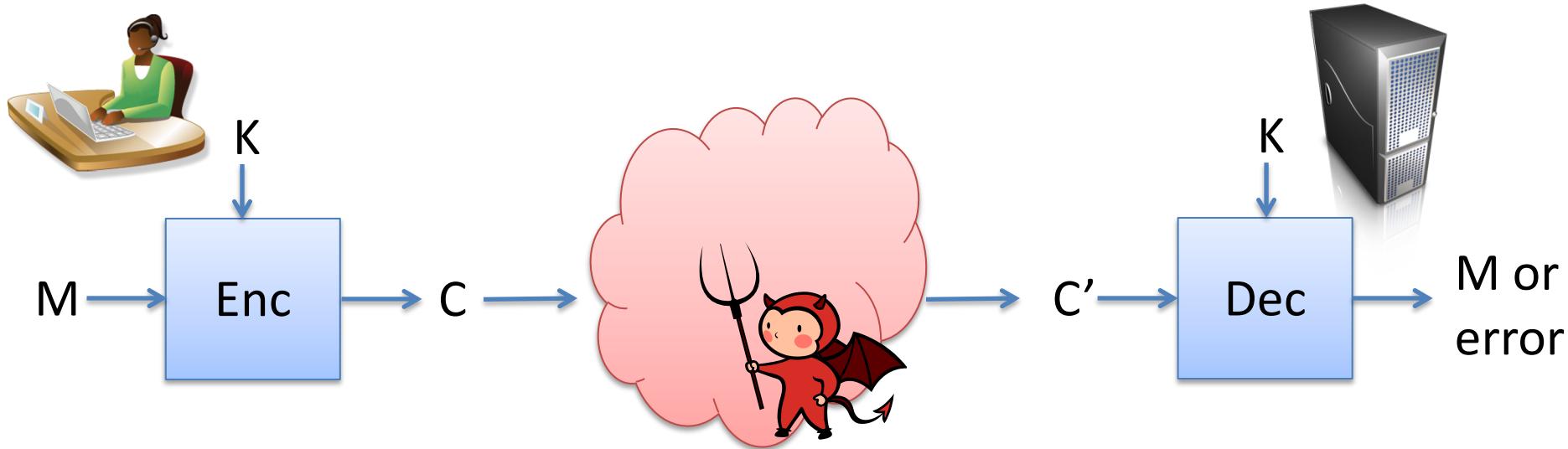


What security properties do we need from symmetric encryption?

- 1) Confidentiality: computational indistinguishability
- 2) Authenticity: ???

Often referred to as Authenticated Encryption security

Authenticated encryption (AE)



Ciphertext unforgeability: Let K be honestly generated secret key. No computationally efficient attacker can construct ciphertext C^* that decrypts correctly under K , even when given

$$(M_1, C_1), (M_2, C_2), \dots, (M_q, C_q)$$

for messages of his choosing and ciphertexts generated under K . It must be that $C^* \neq C_i$ for $1 \leq i \leq q$

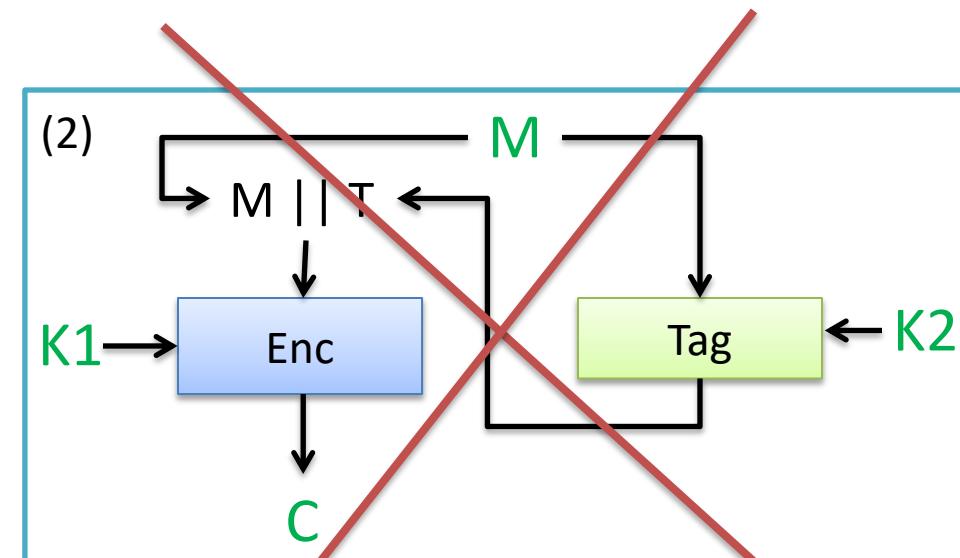
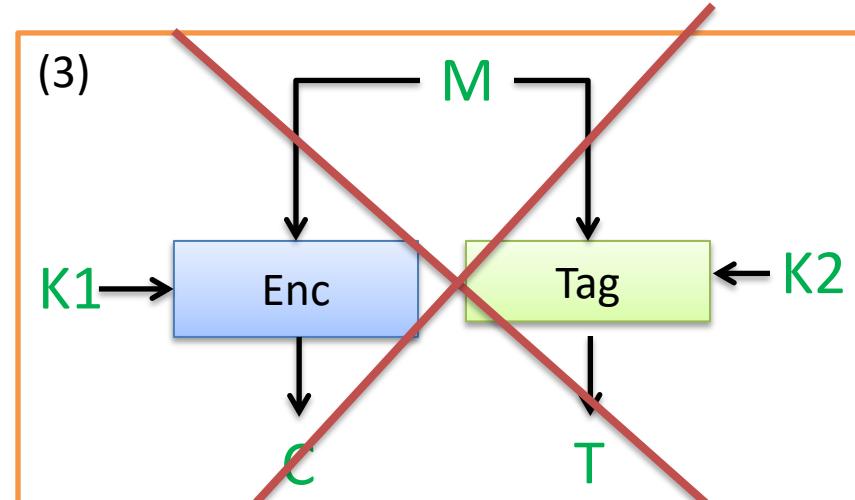
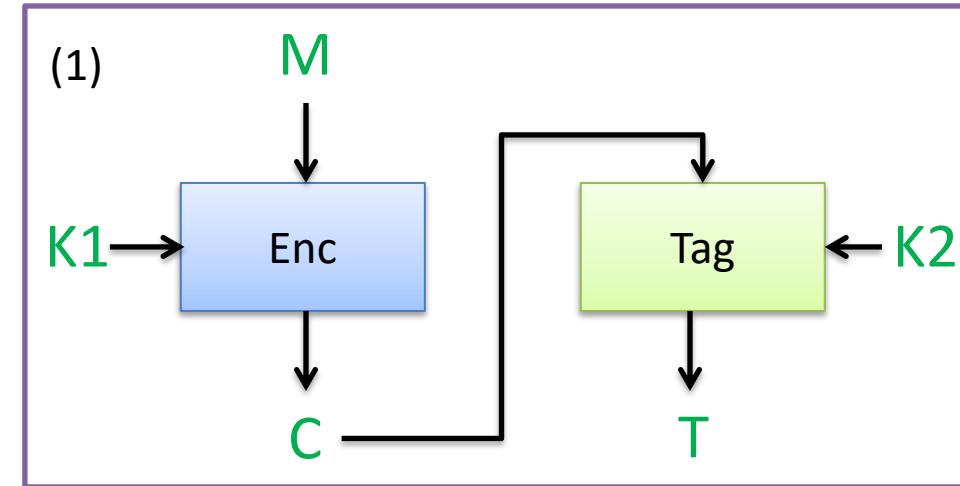
How do we do it?

Build a new scheme from Enc mode (CBC, CTR) and MAC

Kg outputs Enc key K1 and MAC key K2

Several ways to combine:

- (1) encrypt-then-mac
- (2) mac-then-encrypt
- (3) encrypt-and-mac

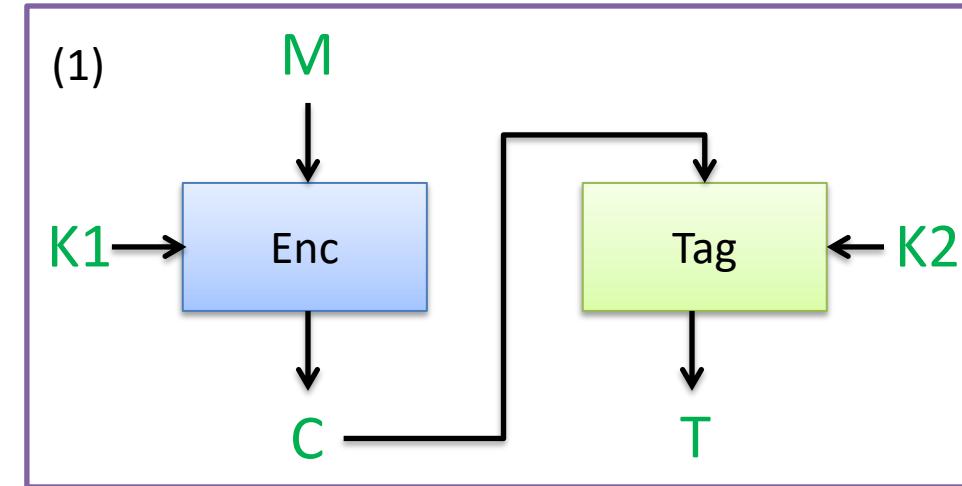


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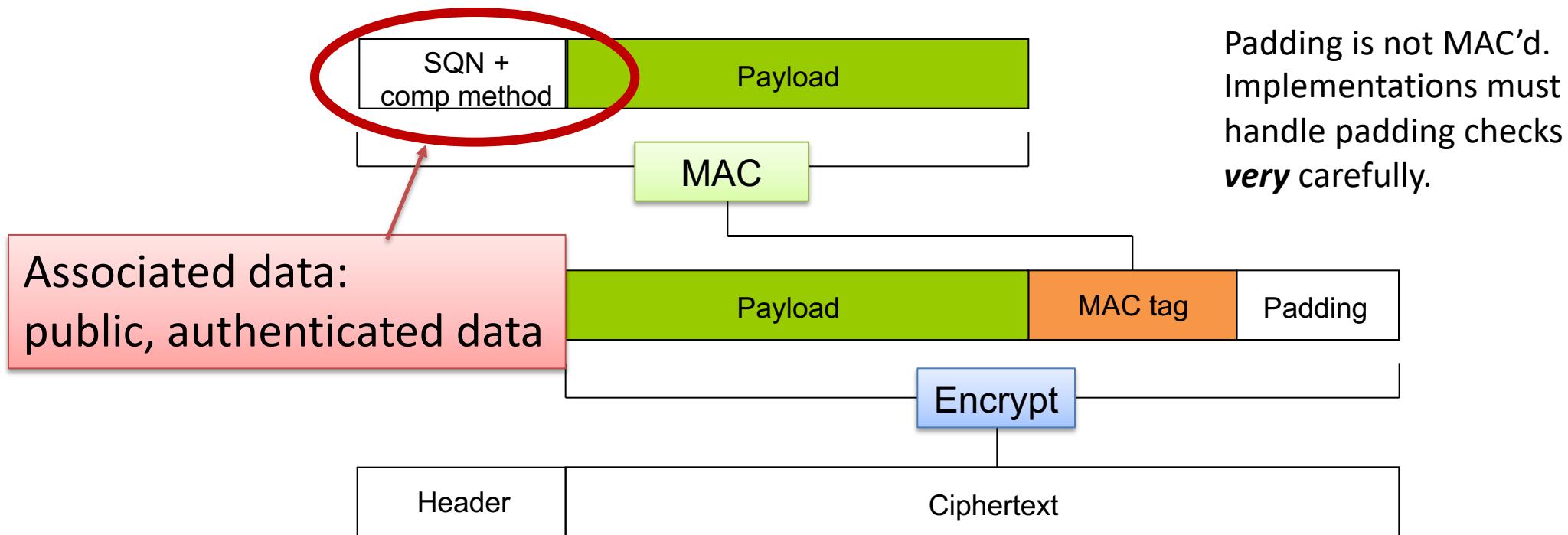
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- (1) encrypt-then-mac
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Thm. If encryption scheme provides confidentiality against passive attackers and MAC provides unforgeability, then Encrypt-then-MAC provides secure authenticated encryption

TLS 1.2 record protocol: MAC-Encode-Encrypt (MEE)



MAC

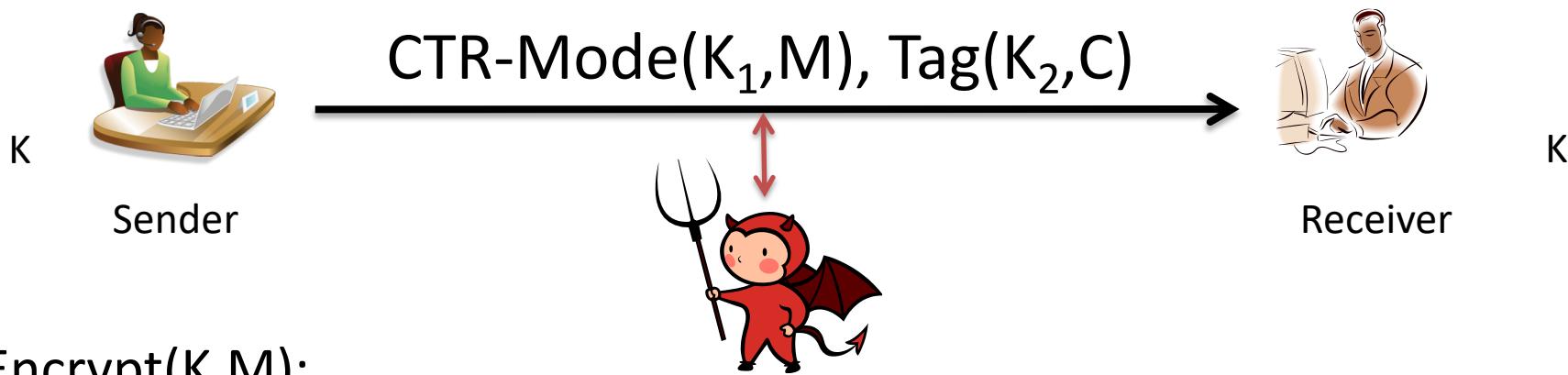
HMAC-MD5, HMAC-SHA1, HMAC-SHA256

Encrypt

CBC-AES128, CBC-AES256, CBC-3DES, RC4-128

Now TLS uses better authenticated-encryption schemes, this is deprecated

Composing encryption and authentication



Encrypt(K, M):

Use secret keys K_1 and K_2 . These can be derived from K if needed

$$K_1 \leftarrow \text{AES}(K, 0^n)$$

$$K_2 \leftarrow \text{AES}(K, 1^n)$$

$C \leftarrow \$ \text{CTR-Mode}(K_1, M)$

$T \leftarrow \text{Tag}(K_2, C)$

Output $C || T$

Decrypt($K, C || T$)

If $\text{Verify}(K_2, C, T) \neq 1$ then Return error

Return CTR-Mode-Decrypt(K_1, C)

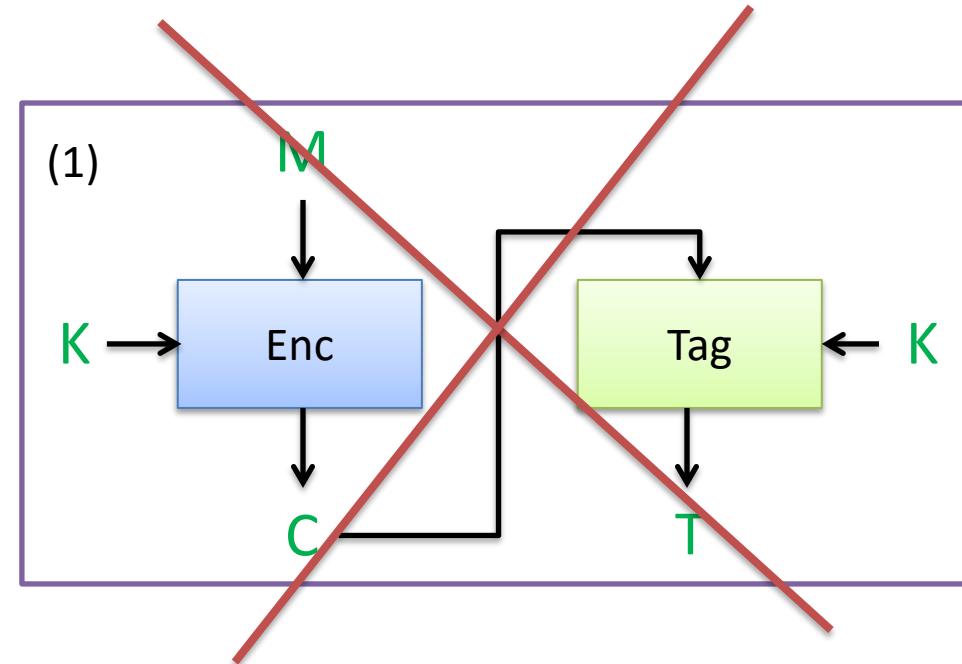
Key derivation function (KDF):
Way to use one key to generate several

HKDF is widely used general-purpose KDF

Key separation is essential

If one uses same key for both encryption and MAC, attacks can arise

Consider CBC-MAC plus CBC-mode encryption



General rule: different crypto primitives or different applications of same primitive, need independent keys

Dedicated authenticated encryption schemes

Not a generic composition of Enc, MAC.

Directly construct from blockcipher, stream cipher, etc.

Attack	Inventors	Notes
OCB (Offset Codebook)	Rogaway	One-pass (one blockcipher call per block of message)
GCM (Galois Counter Mode)	McGrew, Viega	CTR mode plus specialized MAC
Salsa20/Poly1305	Bernstein	Stream cipher plus Carter-Wegman MAC
CWC	Kohno, Viega, Whiting	CTR mode plus Carter-Wegman MAC
CCM	Housley, Ferguson, Whiting	CTR mode plus CBC-MAC
EAX	Wagner, Bellare, Rogaway	CTR mode plus OMAC (variant of CBC-MAC)

Symmetric Encryption Advice

Never use CTR mode or CBC mode by themselves

Passive security is almost never good enough!!

Encrypt-then-MAC best way to do generic composition

Dedicated modes that have been analyzed thoroughly
are way to go in practice