CS 5830
Cryptography

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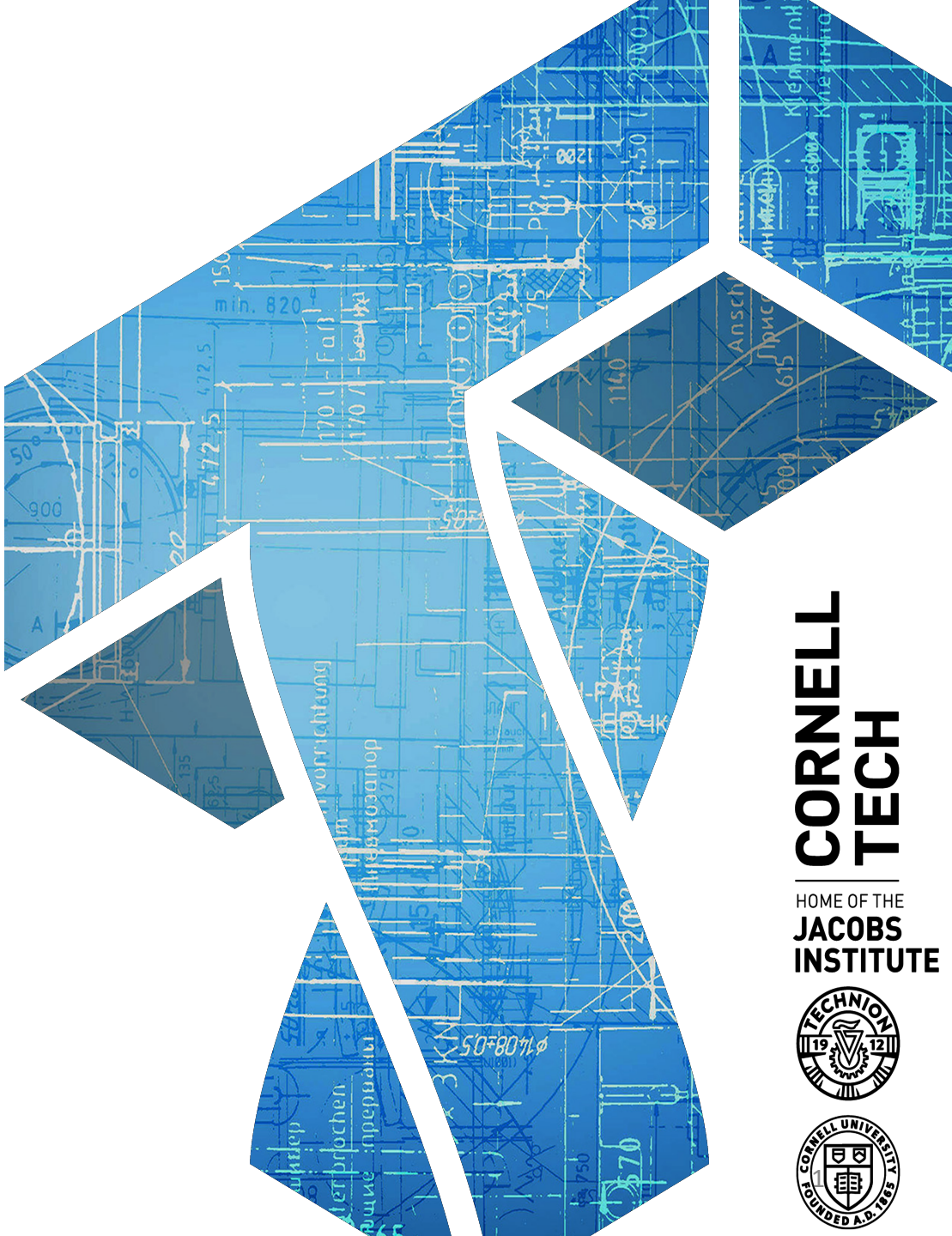

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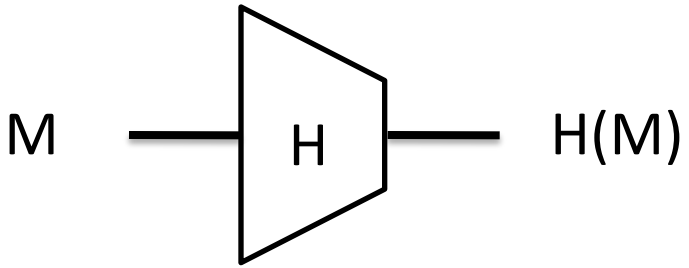
Where we're at

- **Message authentication schemes (MACs)**
  - Secret key + message => authentication tag
  - Examples: CBC-MAC
- **Authenticated encryption (with associated data)**
  - Confidentiality + ciphertext integrity
- **Today: cryptographic hash functions**

Cryptographic hash functions

A function H that maps (essentially) arbitrary bit string to fixed length string of size n



MD5: n = 128 bits
SHA-1: n = 160 bits
SHA-256: n = 256 bits

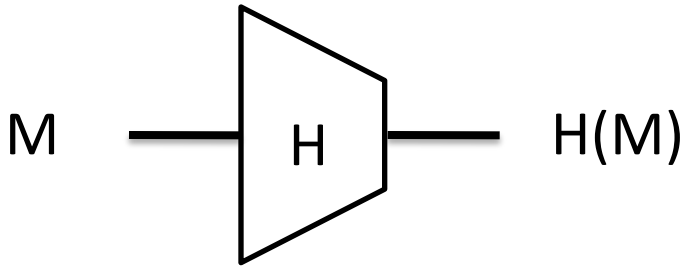
Many security goals asked of hash functions
Ideally, they behave as if they were a (public) random function

Applications of hashing

- File comparison
- Digital signatures (coming up later)
- Password hashing and password-based encryption
- Message authentication codes
- Key derivation
- Randomness extraction (in random number generators)

Cryptographic hash functions

A function H that maps (essentially) arbitrary bit string to fixed length string of size n



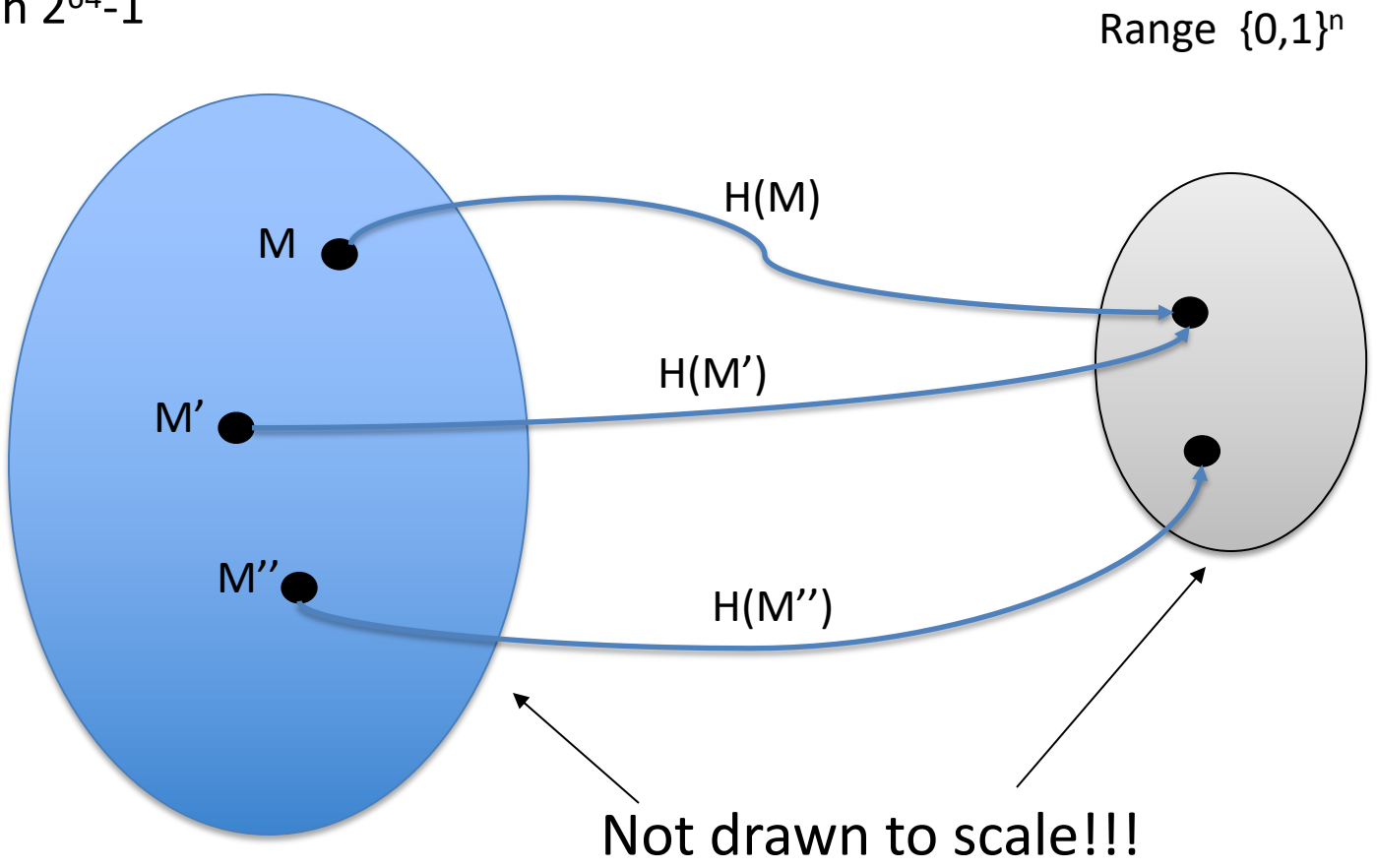
MD5: n = 128 bits
SHA-1: n = 160 bits
SHA-256: n = 256 bits

Collision resistance:
No computationally efficient adversary can find M ≠ M' such that H(M) = H(M')

CR(H,$\mathcal{A}$):
(M,M') <-$ $\mathcal{A}$
If M ≠ M' and H(M) = H(M')
    Return 1
Return 0

Collisions always exist

Domain (usually all strings up to some length)
SHA-1: up to length 2^{64}-1



Pigeonhole principle:
size of domain larger than size of range implies there must be collisions

Hash functions: Keyed or unkeyed?

- Hash functions traditionally are ***unkeyed***
  - Function H: $\mathcal{M}$ -> {0,1}^{n} mapping a message to fixed-length string
  - $\mathcal{M}$ is message space (e.g., all bit strings up to length 2^{64})
  - Examples: MD5, SHA1, SHA256
- Some hash functions also defined ***with keys***
  - Function H: {0,1}^{k} x $\mathcal{M}$ -> {0,1}^{n} mapping a key, message pair to fixed-length bit string
  - Most often used in cases where we need message authentication, not just CR
  - Examples: HMAC (we will see later), Envelope MAC

Hash functions: Keyed or unkeyed?

Transforming keyed to unkeyed

- Pick a key K and make it fixed, public parameter, hardcode it
- Pick random key K per application, or per use of hash
  - Rarely done

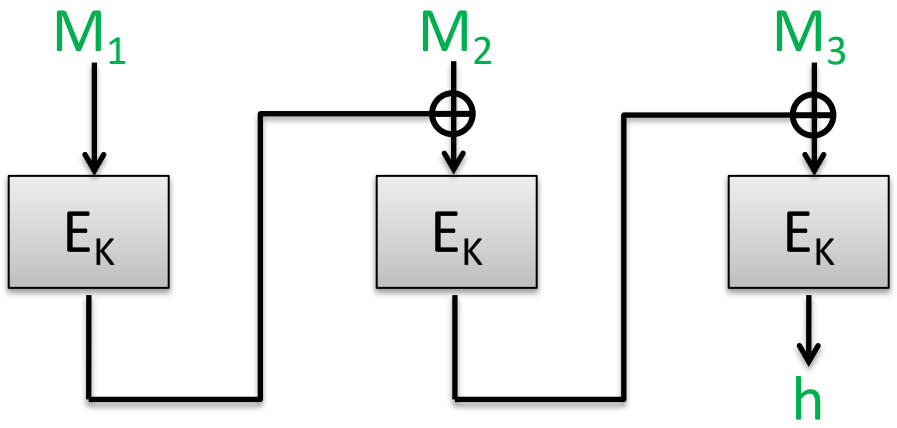
Transforming unkeyed to keyed

- Need a keying strategy. We will see some soon (e.g., HMAC)

CBC-MAC as CR hash?

Consider CBC-MAC as keyed hash
Key was secret in message authentication setting

If key is public is CBC-MAC collision resistant?



CR(H,$\mathcal{A}$):
K <-$ $\{0,1\}^k$
(M,M') <-$ $\mathcal{A}$ (K)
If M ≠ M' and H(M) = H(M')
    Return 1
Return 0

Adversary $\mathcal{A}$(K):
$h \leftarrow E_K(0^n)$
$M_2 \leftarrow E_K(1^n)$
Return $(0^n, 1^n || M_2)$

Universal hash functions as CR hashes?

A keyed function $H : \{0,1\}^k \times \mathcal{M} \rightarrow \{0,1\}^n$ is an ϵ-almost universal (AU) hash function if for all $M \neq M'$

$$\Pr\left[\, H_K(M) = H_K(M') \,\right] \leq \epsilon$$

where the probability is over choice of K.

What is difference between this and CR notion?

CR: Adversary ***can*** choose message as function of K

Universal: Adversary ***cannot*** choose message as function of K

Universal hash functions as CR hashes?

Example universal hash from Carter-Wegman:

$$H_{a,b}(M) = ((aM+b) \bmod p) \bmod 2^n$$

$p > 2^n$ is a prime

Universality: for any $M \neq M'$, $\Pr[H_{a,b}(M) = H_{a,b}(M')] = 1/2^n$

What is a CR adversary?

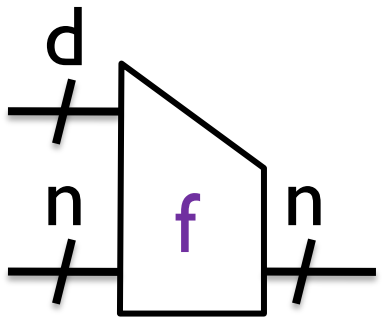
Pick any M, compute $y = ((aM+b) \bmod p$

Solve linear equation: $y = aM' + b \bmod p$

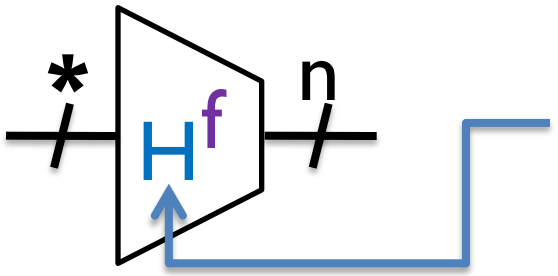
Universal hash functions don't provide collision-resistance

Two-step design approach for hash functions

Compression Function

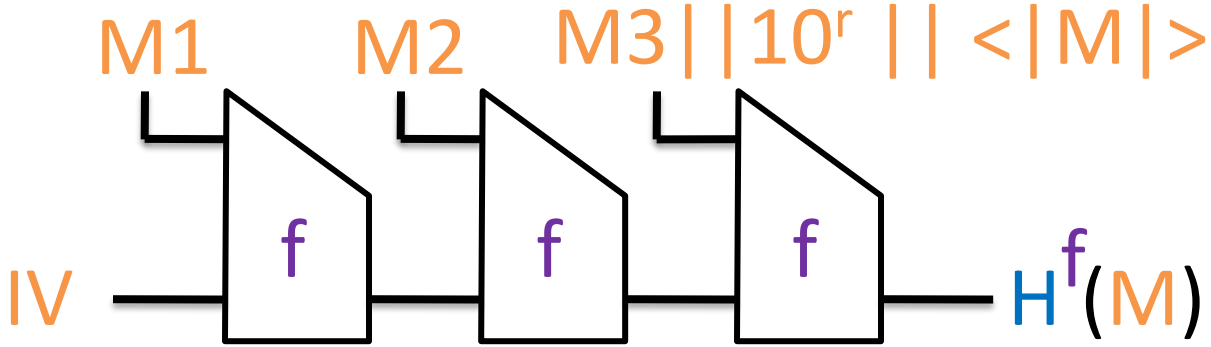


Hash Function



Domain extension transform

Domain extension called “Merkle-Damgard with strengthening”



Used in MD-x, SHA-1, SHA-256, …

IV is a fixed constant. Not randomly chosen!

Building compression functions

- Can build compression functions from suitable block ciphers

$$f(z,m) = E(m,z) \oplus z$$

  Called Davies-Meyer construction

- Can use AES, but security too low. Why?

- SHA-1 uses custom E with k = d = 512 and n = 160
  - Message block length of SHA-1 is 512 bits

SHA-1 compression function

Expand 512-bit message m into

$W_1,\ldots,W_{80}$ strings of length 32 bit values

(Think of this as "key schedule")

Chaining variable is 160 bits, 5 32-bit values

A , B , C , D , E

F(B,C,D) function that changes over rounds:

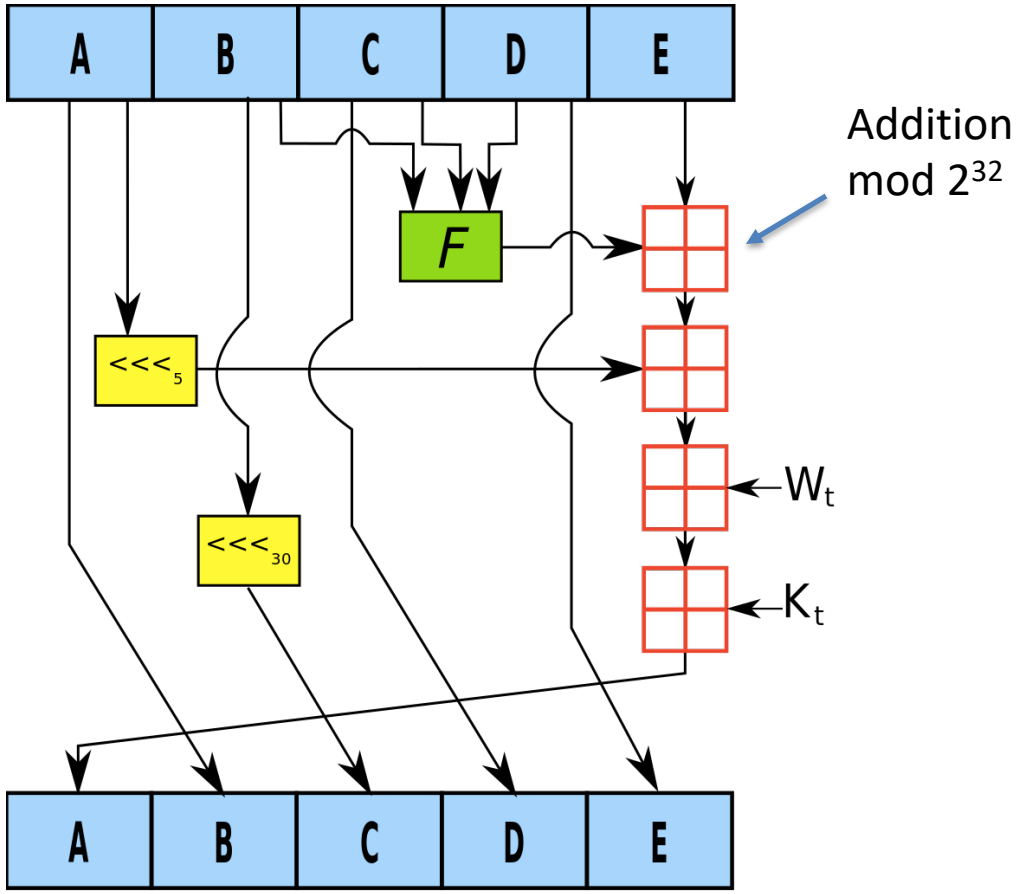
0-19: (B **and** C) **or** ((**not** B) **and** D)

20-39: B **xor** C **xor** D

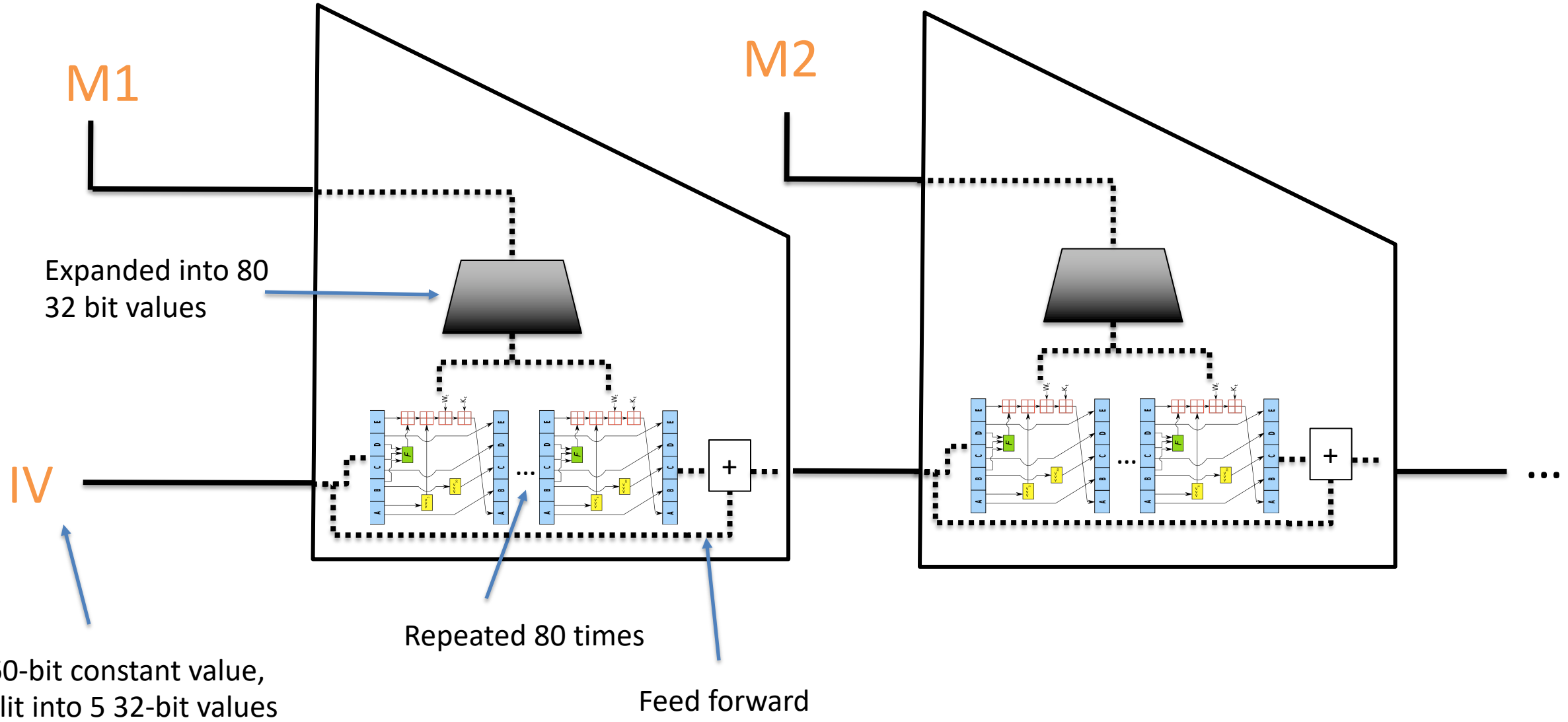
40-59: (B **and** C) **or** (B **and** C) **or** (C **and** D)

60-79: B **xor** C **xor** D

Constants K_1 , … , K_{80} differ across rounds



SHA1 hash function



Birthday attacks

- What is best possible security achievable by hash function with output length n bits?

- Answer: security is only achievable up to $2^{n/2}$ hash computations

The birthday problem

Choose q values $Y_1,\ldots,Y_q$ from $\{0,1\}^n$ at random. What is probability that two are the same?

Let $Coll_i$ be event that $Y_i = Y_j$ for some $j < i$

$$\Pr[Coll] = \Pr[Coll_1 \vee \ldots \vee Coll_q]$$
$$\leq \Pr[Coll_1] + \ldots + \Pr[Coll_q]$$
$$= \frac{0}{2^n} + \frac{1}{2^n} + \frac{2}{2^n} + \ldots + \frac{q}{2^n}$$
$$= \frac{q(q-1)}{2^n}$$

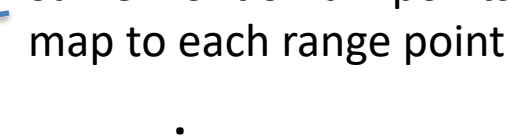
Another proof shows that if $q \leq 2^{(n+1)/2}$

$$\Pr[Coll] \geq \frac{0.3 \cdot q(q-1)}{2^n}$$

The birthday attack

Let m be some length in domain of hash function H

Adversary A:
For i = 1 to q do:
X_i <-$ $\{0,1\}^m$
h_i <- $H(X_i)$
If exists i,j s.t. $X_i \neq X_j$ and $h_i = h_j$ then
Return (X_i , X_j)
Return fail



If H is *regular* then probability of success is exactly birthday probability

$$\Pr[\text{A finds collision}] \geq \frac{0.3 \cdot q(q-1)}{2^n}$$

Birthday attack run times

MD5:	n = 128 bits	2^{64}	MD5 computations
SHA-1:	n = 160 bits	2^{80}	SHA-1 computations
SHA-256:	n = 256 bits	2^{128}	SHA-256 computations

2^{64} too small by today's standards!

Bitcoin network computes about ~2^{67} SHA-256 hashes ***per second***

https://blockchain.info/charts/hash-rate

Faster attacks than birthday?

- 2004: Full break of MD5 announced by Xiaoyun Wang and co-authors
  - MD5 is easy to find collisions against
  - You can download programs to do it on your laptop
- 2005: Announced faster than 2^{80} attack against SHA-1 by Wang et al.
  - Not practical to run (2^{69} estimated cost)
- 2017: CWI and Google announce first demonstrated collision

SHAttered attack

Attacker-chosen prefix P. Find two pairs of message blocks M_1 , M_2 and M_1' , M_2' such that for any suffix S:

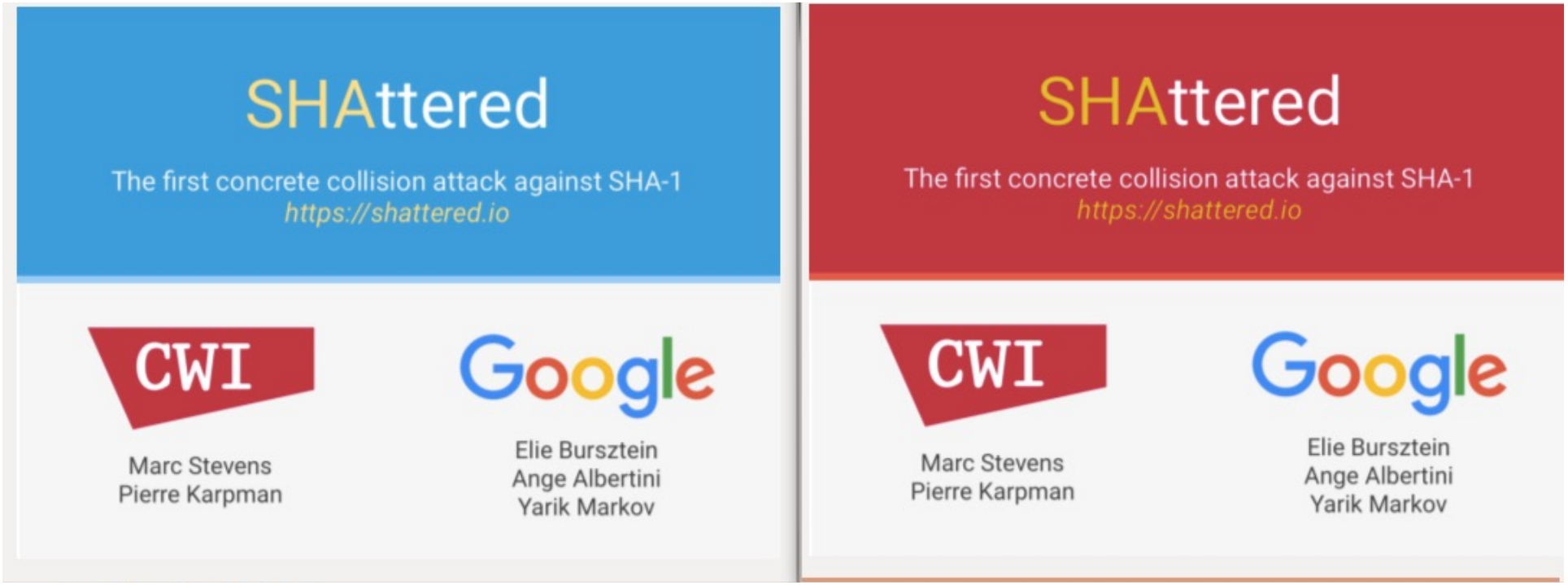
$$\text{SHA-1}(P \,||\, M_1 \,||\, M_2 \,||\, S) = \text{SHA-1}(P \,||\, M_1' \,||\, M_2' \,||\, S)$$

Referred to as an *identical-prefix collision attack*

How? Pick P, find M_1 and M_1' that form *near-collision* on chaining variable. Then complete collision by finding M_2 and M_2'

They show how to extend to build colliding PDF files

SHAttered attack



Required $2^{63.1}$ SHA-1 compression function applications

100,000x faster than birthday attack (2^{80})

Fallout of attack

SVN repositories can be broken (DoS attack)

- Checking in the two SHAttered PDFs corrupts repo

Marc Stevens & Dan Shumow (Microsoft) developed *counter-cryptanalysis*

Way to detect if a particular file is one half of colliding pair

Deployed at several large companies

Ongoing migration away from SHA-1 to SHA-256 / SHA-3

Applications of hashing

- File comparison
- Digital signatures (coming up later)
- Password hashing and password-based encryption
- For message authentication codes
- Key derivation