Observations on Factoring Using the GNFS

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Session ID: xxx-xxxx
Session Classification: xxxxxxxxxxxx

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How Do I Factor - GNFS

1. Polynomial Selection
2. Sieving
3. Combine

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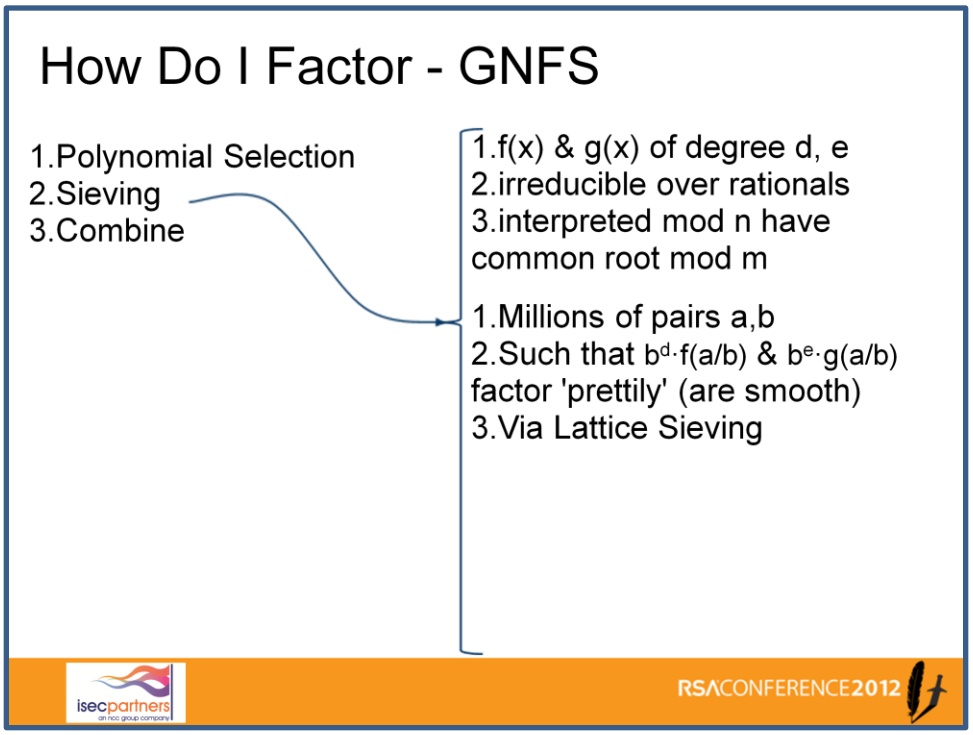
1. f(x) & g(x) of degree d, e
2. irreducible over rationals
3. interpreted mod n have common root mod m



Some more on this:

http://mersenneforum.org/showthread.php?t=15796

How Do I Factor - GNFS

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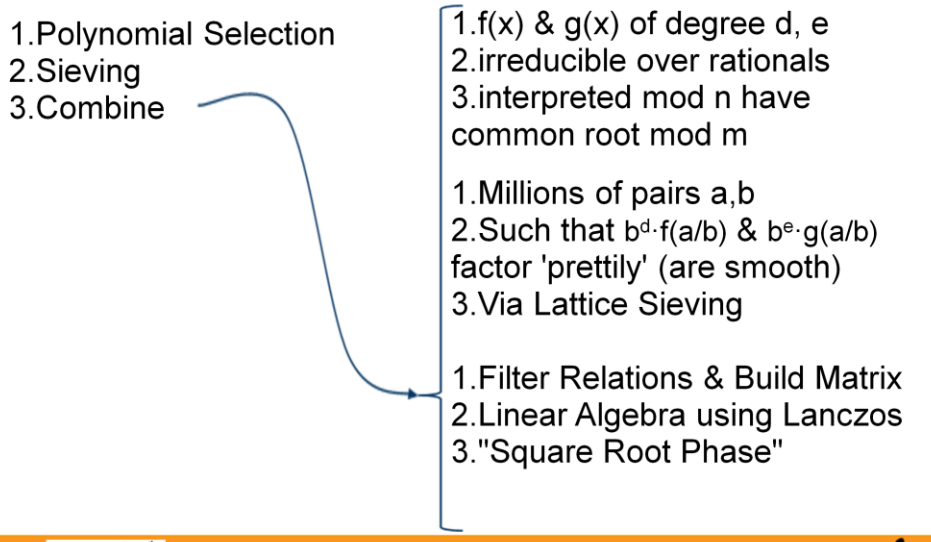
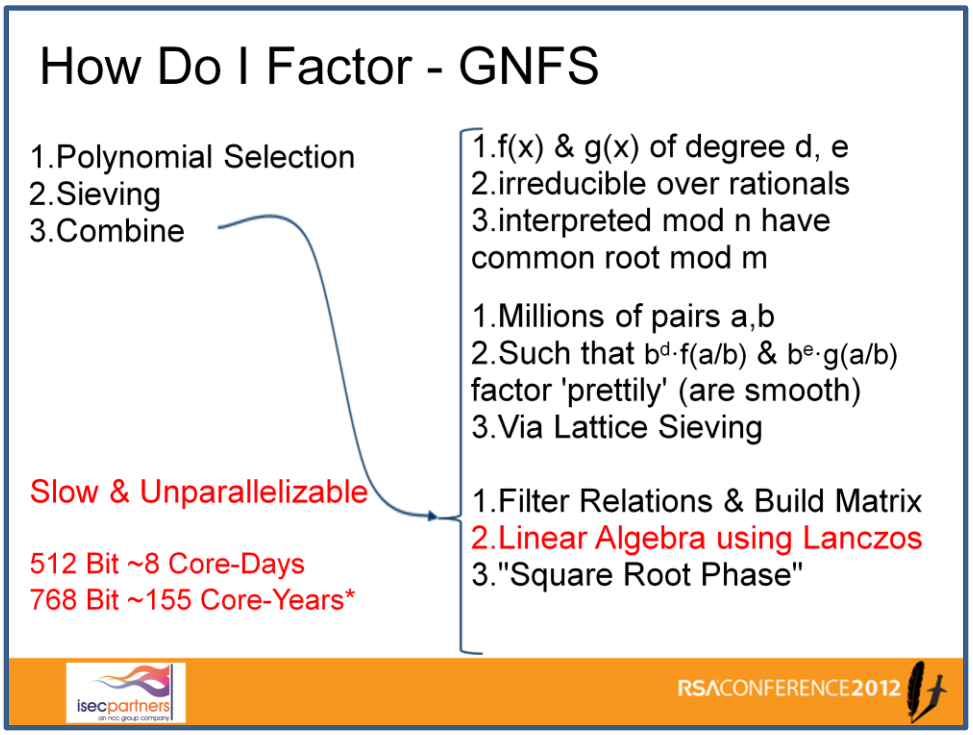
1. Millions of pairs a,b
2. Such that $b^d \cdot f(a/b)$ & $b^e \cdot g(a/b)$ factor 'prettily' (are smooth)
3. Via Lattice Sieving

How Do I Factor - GNFS





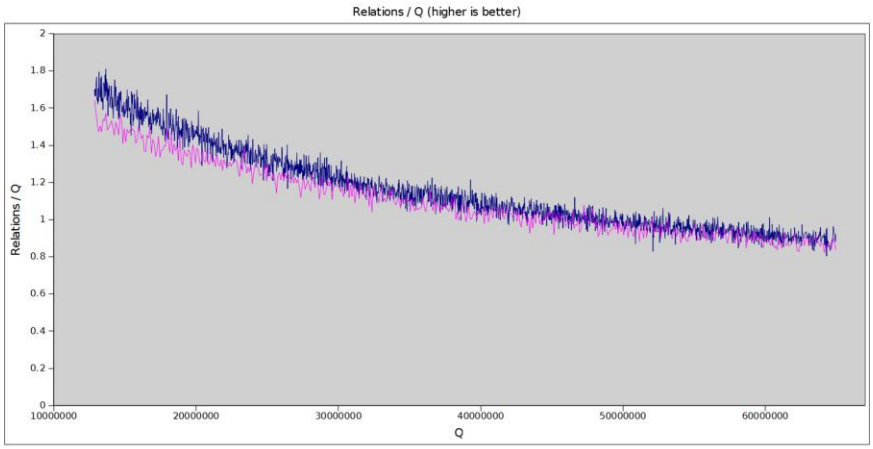
Why is it unparrellizable?
http://www.mersenneforum.org/showthread.php?t=15361

* is because the 768 bit semiprime used Block Weildmann as opposed to msieve's block lanczos algorithm.
http://www.mersenneforum.org/showthread.php?t=12958

Some Details on Factoring

- Polynomial Selection
- Siever Comparisons
- Oversieving

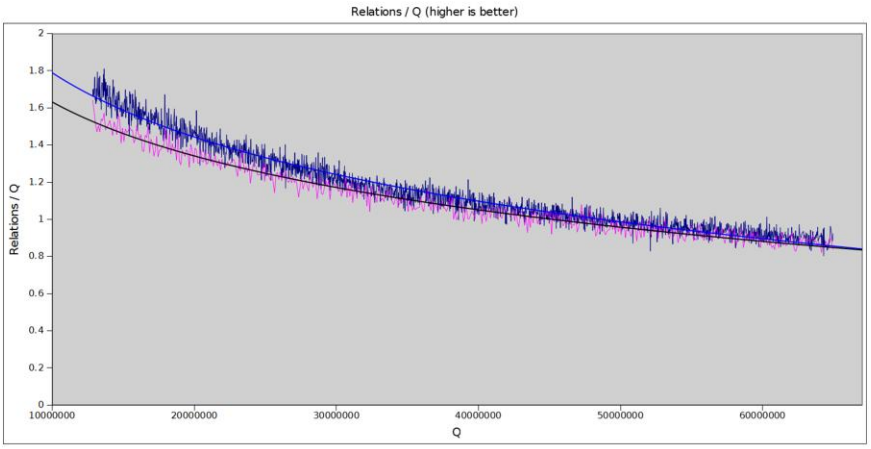
Misconceptions about Polynomials



Comparison of sieve results for two polynomials

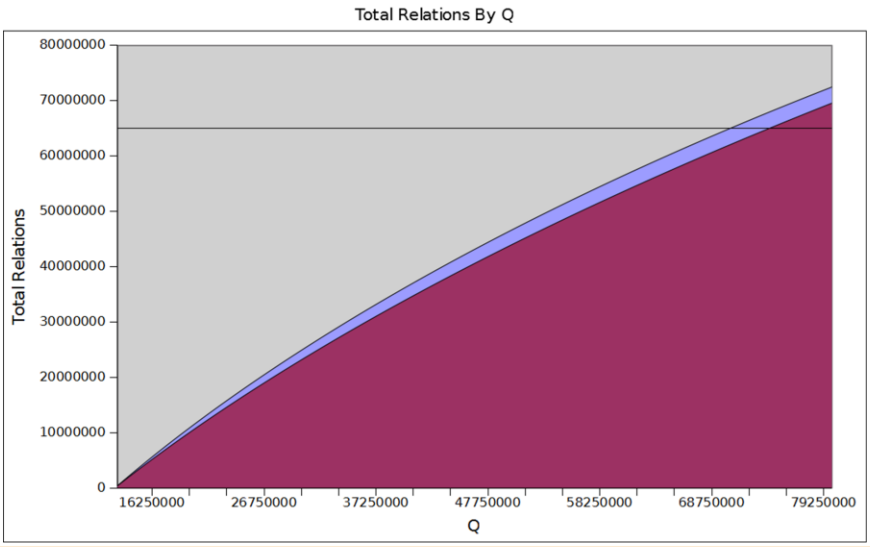
- Murphy 2.615e-12
- Murphy 3.023e-12

Misconceptions about Polynomials



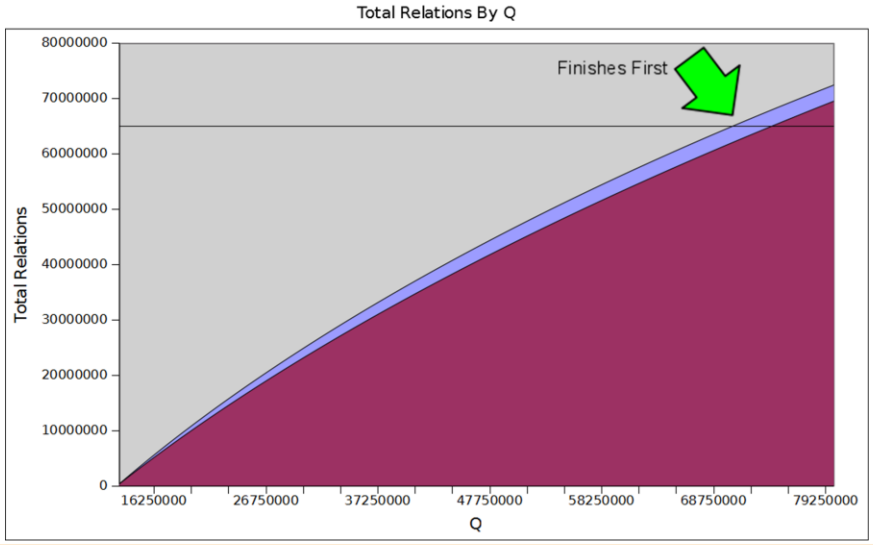
Trend Lines. We can integrate under these curves to get...

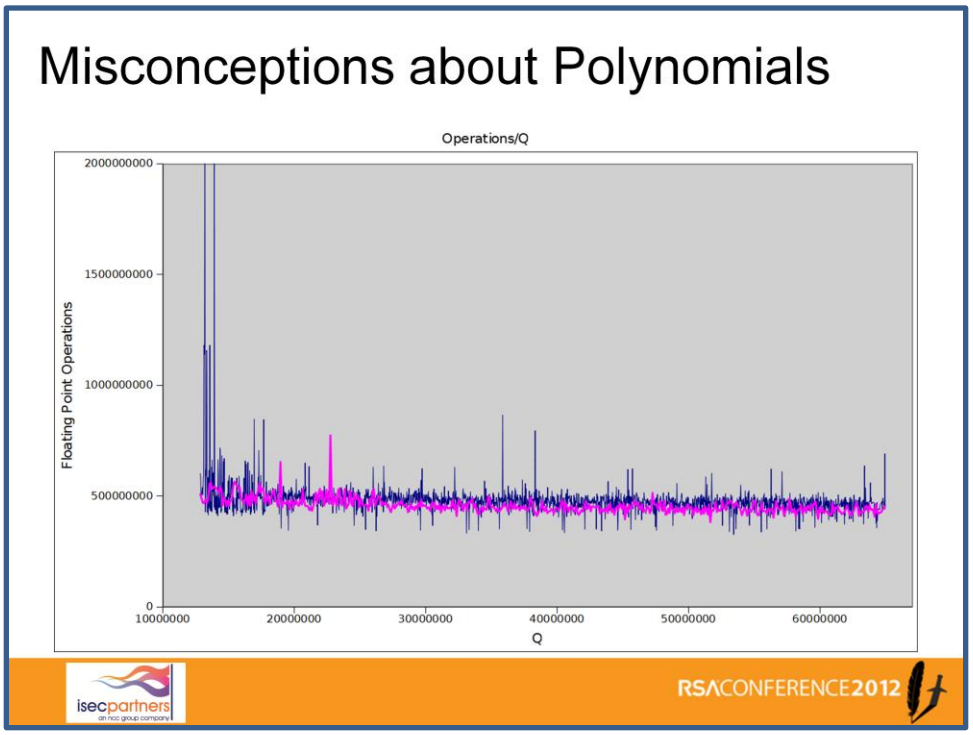
Misconceptions about Polynomials



The total sieve pairs as a function of Q

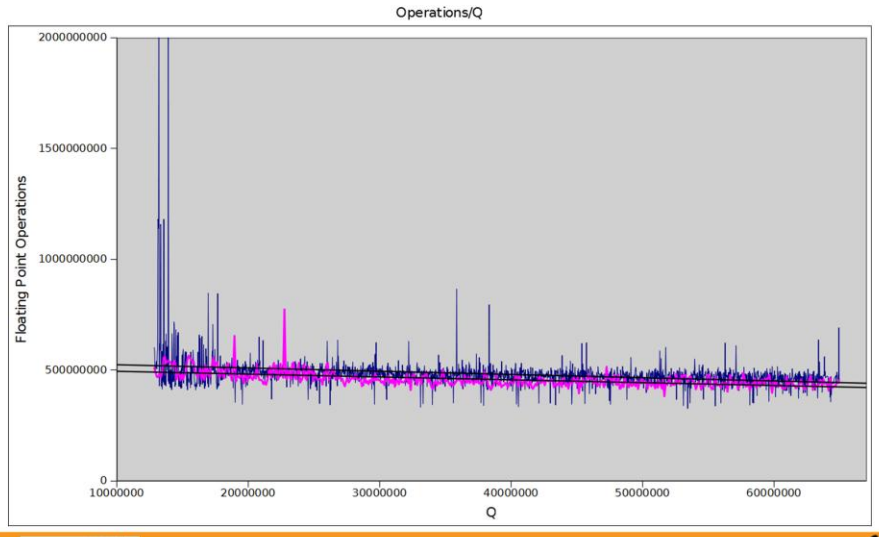
Misconceptions about Polynomials





Floating Point Operations per polynomial

Misconceptions about Polynomials



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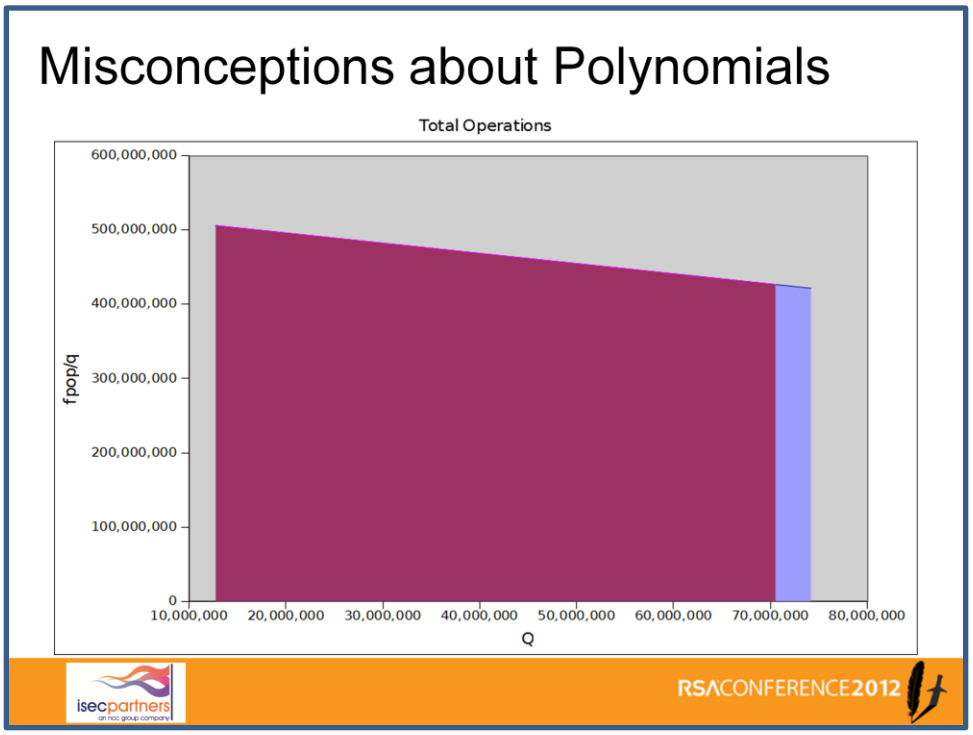
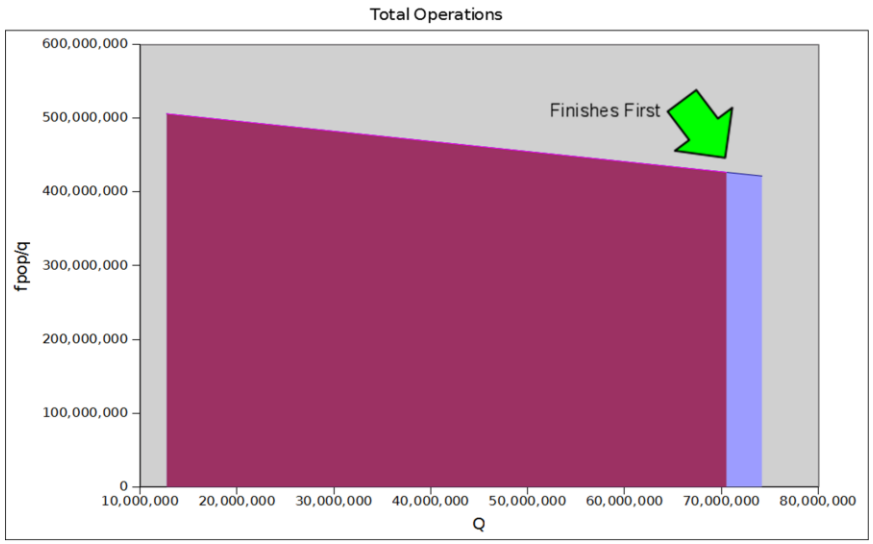
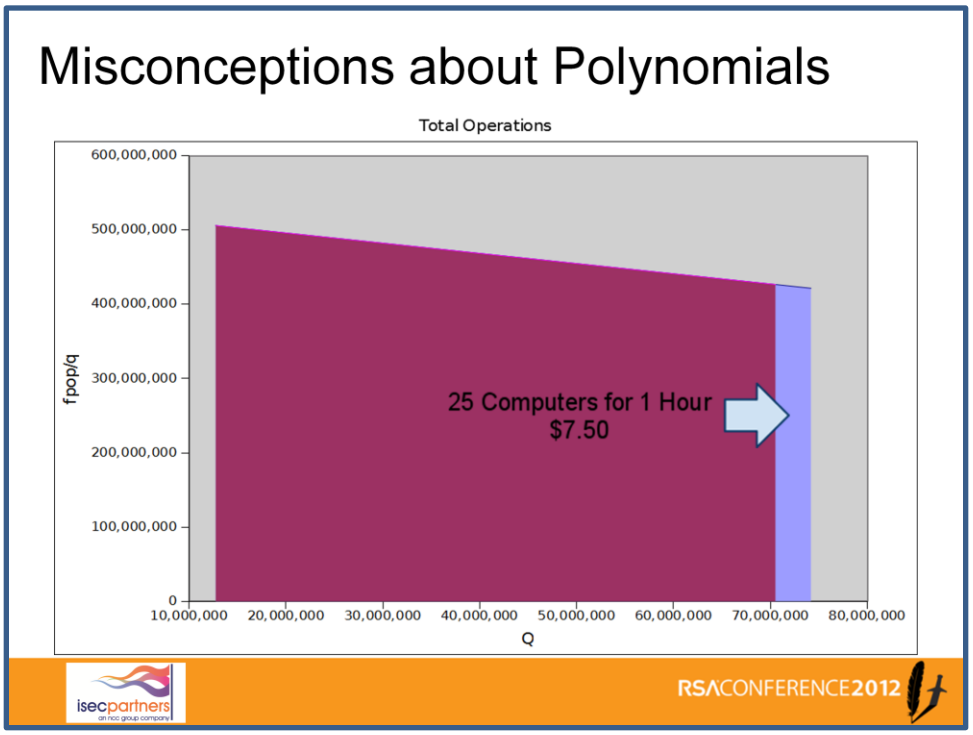
They're pretty much the same.



Integrate under that line (the average actually) and cut it off when the polynomial finishes gathering enough relations, and we have the total amount of work done for each polynomial to achieve the requisite number of sieve pairs.

Misconceptions about Polynomials

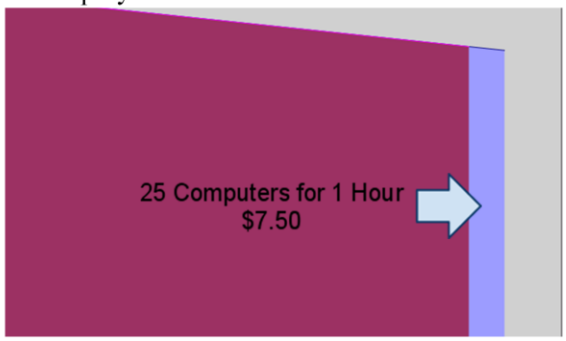




Now, because sieving scales horizontally perfectly, and I was working in EC2, that extra bit of work has a real dollar amount on it. And it's not very much.

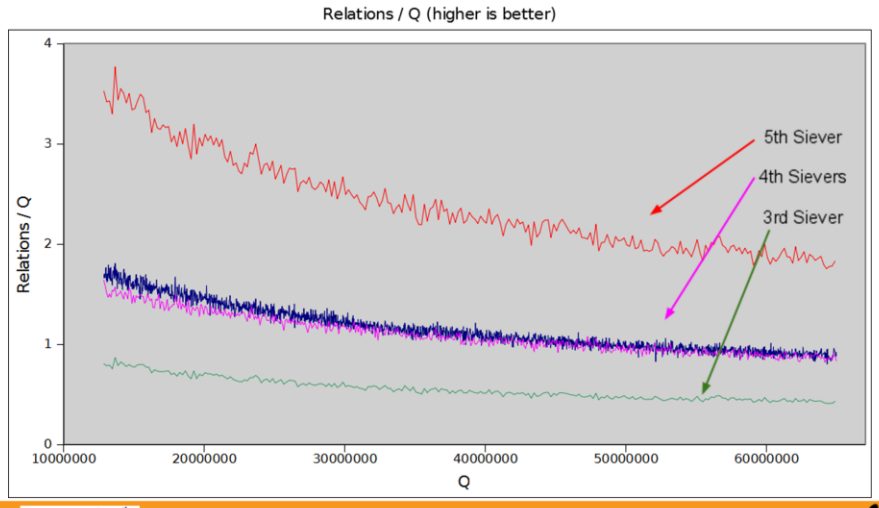
Misconceptions about Polynomials

If time is more valuable to you than (not much) money it is in your best interest to take the first polynomial you get and sieve with that, rather than doing another poly-selection run.



(this advice is only for 512-bit semiprimes.)

Siever Comparisons



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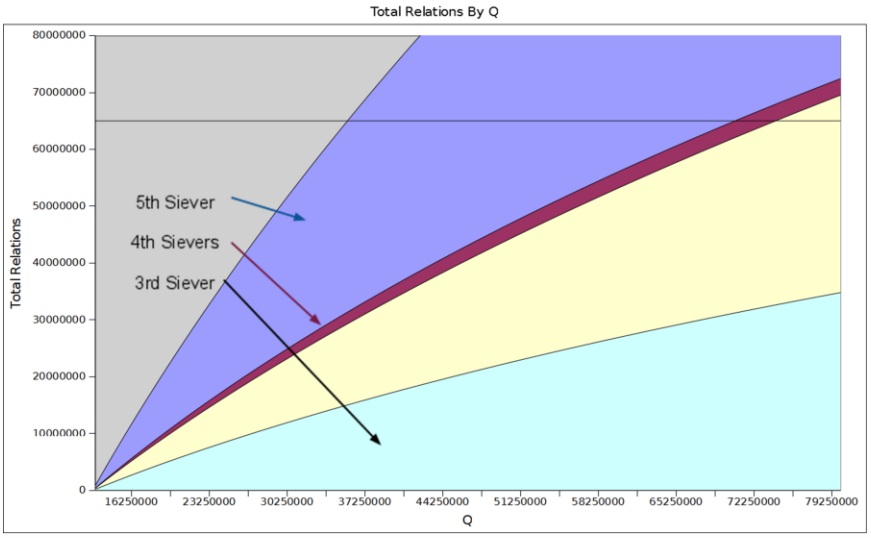
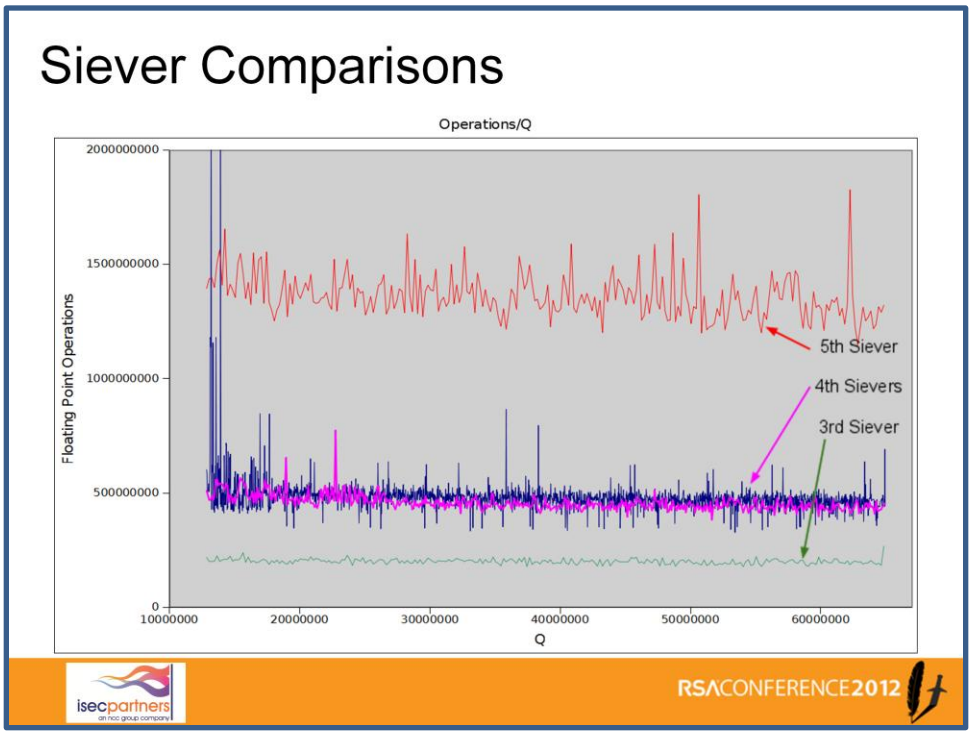
Comparison of
- gnfslasieveI13e
- gnfslasieveI14e for two polynomials
- gnfslasieveI15e

Siever Comparisons

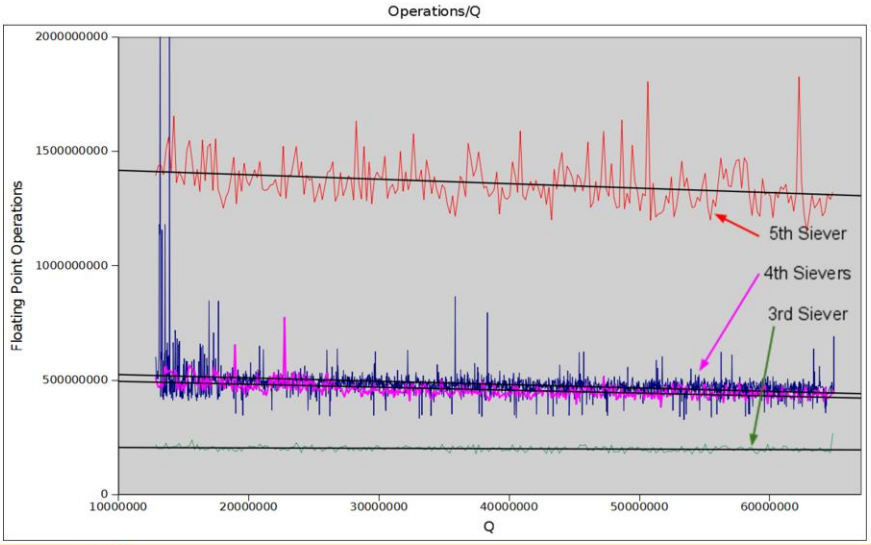


We can again fit trend lines and see where they each finish.



But we need to do an apples-to-apples comparison. While the 5th siever gathers relations much earlier in Q, it also takes much more CPU time to gather those relations.

Siever Comparisons



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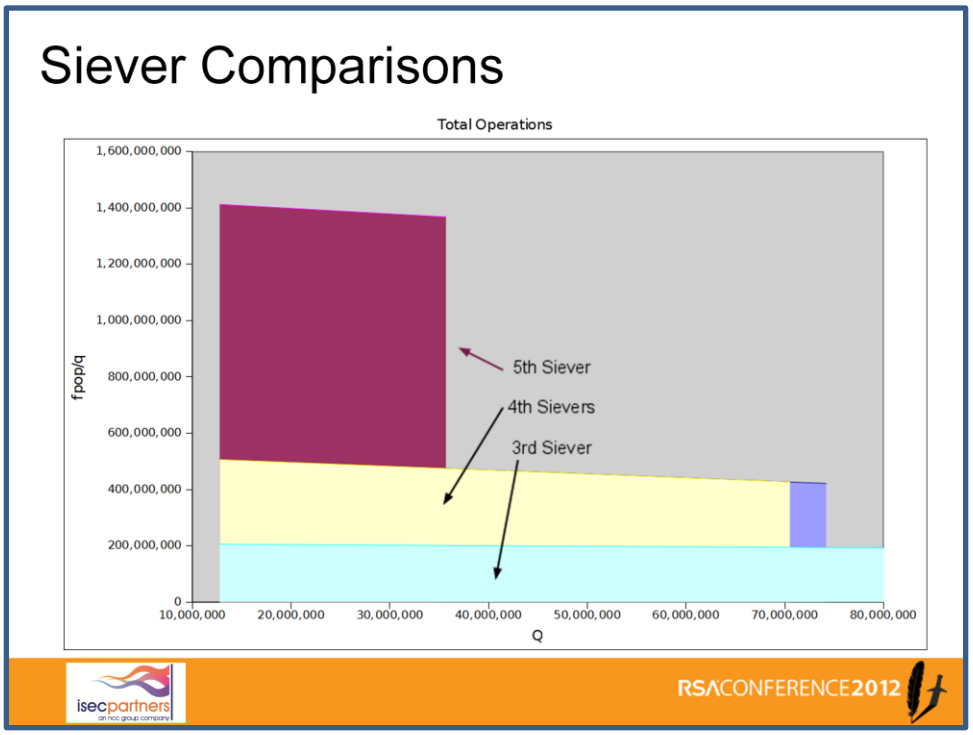
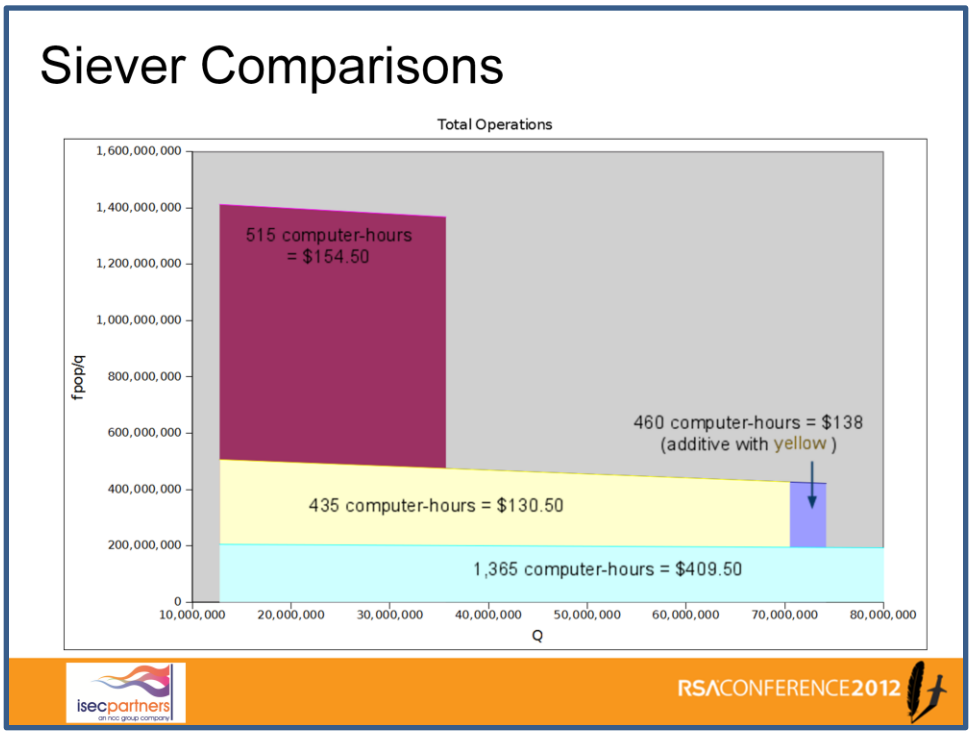
Trend Lines

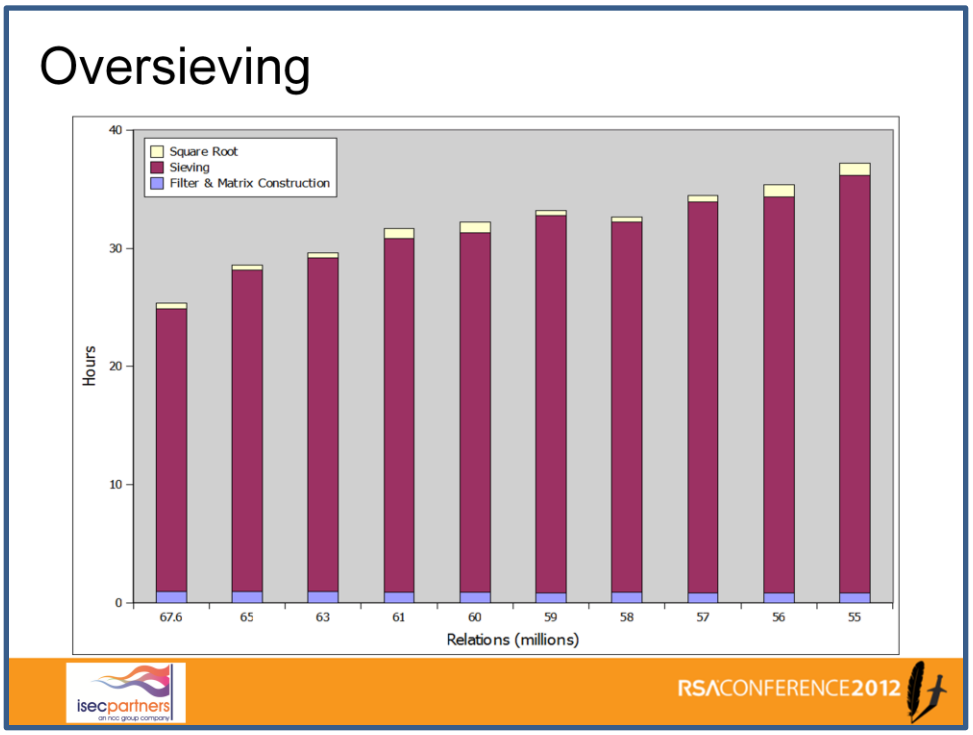


By integrating under the floating point operations trend lines, and stopping when they achieve enough relations we can compare the sievers total work done.



And again, because this is in EC2, we can put this into dollar figures.

This matches up reasonably well with:
http://eprint.iacr.org/2011/254



This is a comparison of the Combine done with various numbers of relations. It is clearly in your best interest to oversieve. Sieving scales perfectly out to more machines, and can save you 10 hours in the last step, which is not parrellelizable.

Obligatory Ending Slide

Fin

Thanks:
- iSEC Partners
- Gotham Digital Science
- NYSec
- MersenneForum & jasonp

Tom Ritter

Big Ups To:
- jasonp

http://www.isecpartners.com/
https://github.com/tomrittervg/cloud-and-control