

Observations on Factoring Using the GNFS

Tom Ritter
iSEC Partners



Session ID: xxx-xxxx

Session Classification: xxxxxxxxxxxx

RSACONFERENCE2012

How Do I Factor - GNFS

1. Polynomial Selection
2. Sieving
3. Combine



How Do I Factor - GNFS

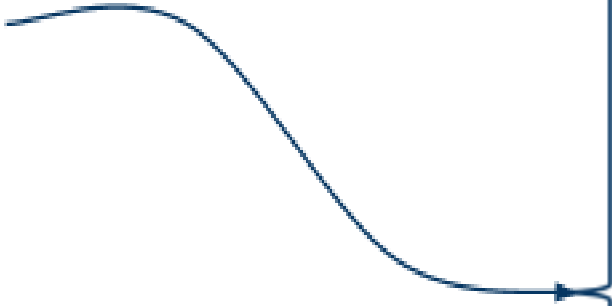
- 1. Polynomial Selection
- 2. Sieving
- 3. Combine

- 1. $f(x)$ & $g(x)$ of degree d, e
- 2. irreducible over rationals
- 3. interpreted mod n have common root mod m



How Do I Factor - GNFS

- 1. Polynomial Selection
- 2. Sieving
- 3. Combine

- 
- 1. $f(x)$ & $g(x)$ of degree d, e
 - 2. irreducible over rationals
 - 3. interpreted mod n have common root mod m
-
- 1. Millions of pairs a, b
 - 2. Such that $b^d \cdot f(a/b)$ & $b^e \cdot g(a/b)$ factor 'prettily' (are smooth)
 - 3. Via Lattice Sieving



How Do I Factor - GNFS

- 1. Polynomial Selection
- 2. Sieving
- 3. Combine

- 1. $f(x)$ & $g(x)$ of degree d, e
- 2. irreducible over rationals
- 3. interpreted mod n have common root mod m

- 1. Millions of pairs a, b
- 2. Such that $b^d \cdot f(a/b)$ & $b^e \cdot g(a/b)$ factor 'prettily' (are smooth)
- 3. Via Lattice Sieving



How Do I Factor - GNFS

- 1. Polynomial Selection
- 2. Sieving
- 3. Combine

- 1. $f(x)$ & $g(x)$ of degree d, e
- 2. irreducible over rationals
- 3. interpreted mod n have common root mod m

- 1. Millions of pairs a, b
- 2. Such that $b^d \cdot f(a/b)$ & $b^e \cdot g(a/b)$ factor 'prettily' (are smooth)
- 3. Via Lattice Sieving

- 1. Filter Relations & Build Matrix
- 2. Linear Algebra using Lanczos
- 3. "Square Root Phase"



How Do I Factor - GNFS

1. Polynomial Selection
2. Sieving
3. Combine

1. $f(x)$ & $g(x)$ of degree d, e
2. irreducible over rationals
3. interpreted mod n have common root mod m

1. Millions of pairs a, b
2. Such that $b^d \cdot f(a/b)$ & $b^e \cdot g(a/b)$ factor 'prettily' (are smooth)
3. Via Lattice Sieving

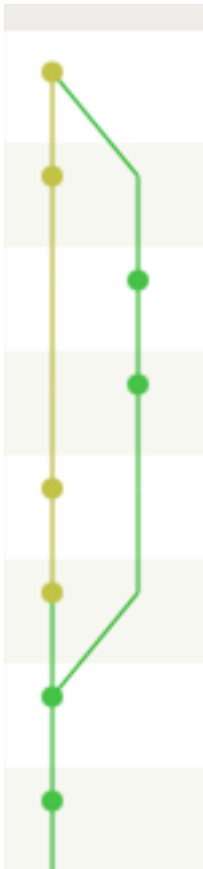
Slow & Unparallelizable

512 Bit ~8 Core-Days

768 Bit ~155 Core-Years*

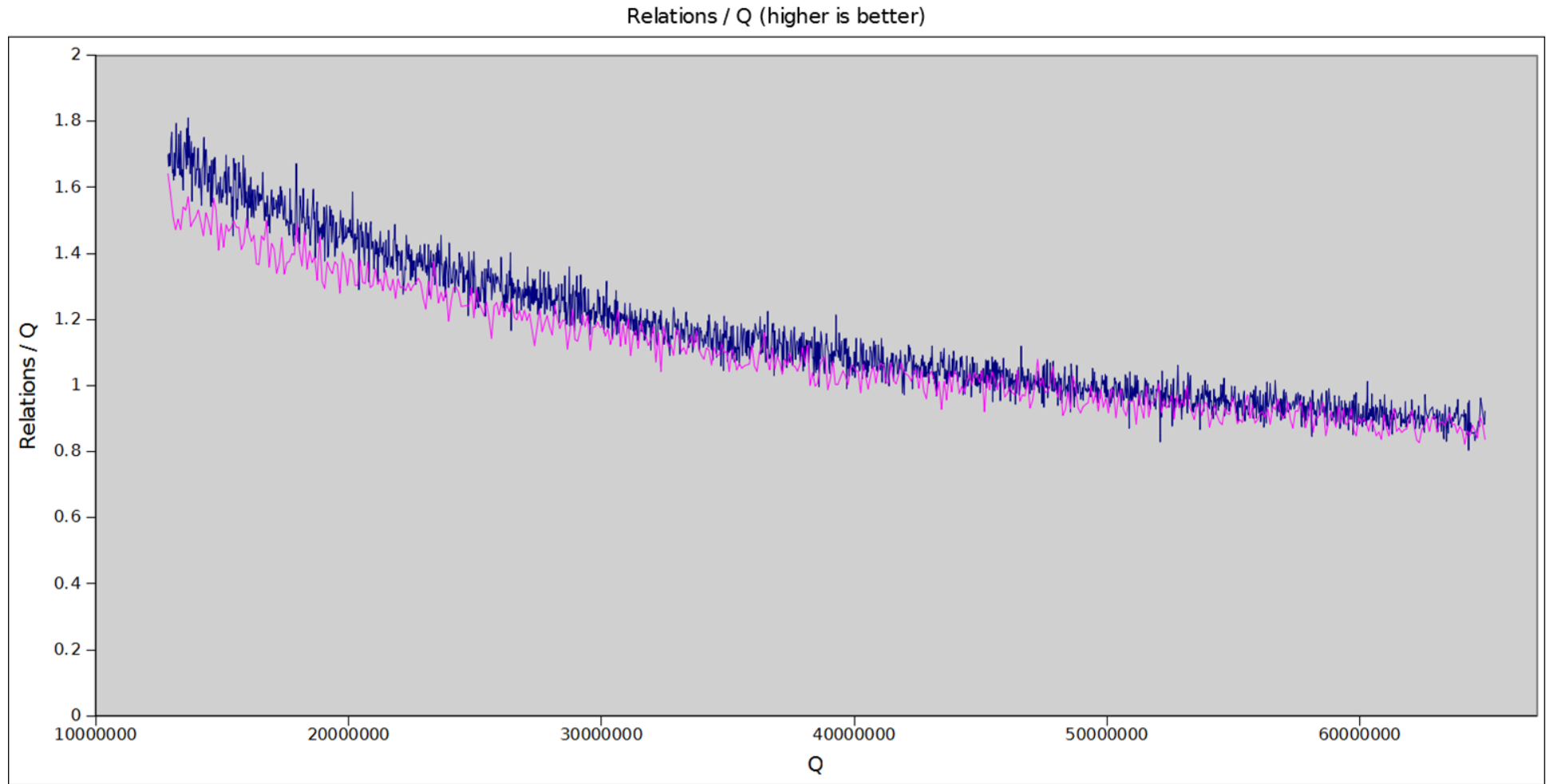
1. Filter Relations & Build Matrix
2. Linear Algebra using Lanczos
3. "Square Root Phase"

Some Details on Factoring

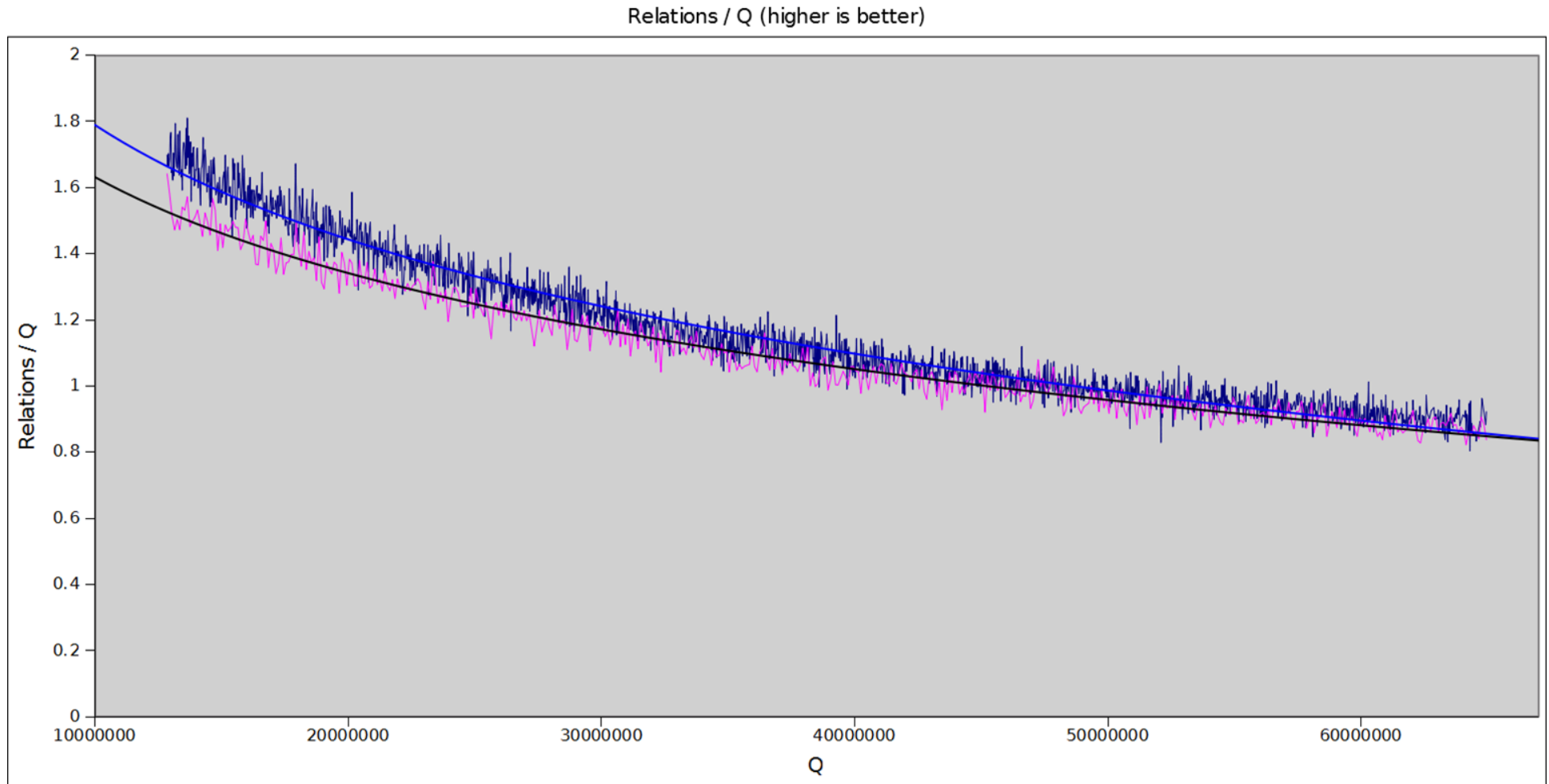


- Polynomial Selection
- Siever Comparisons
- Oversieving

Misconceptions about Polynomials

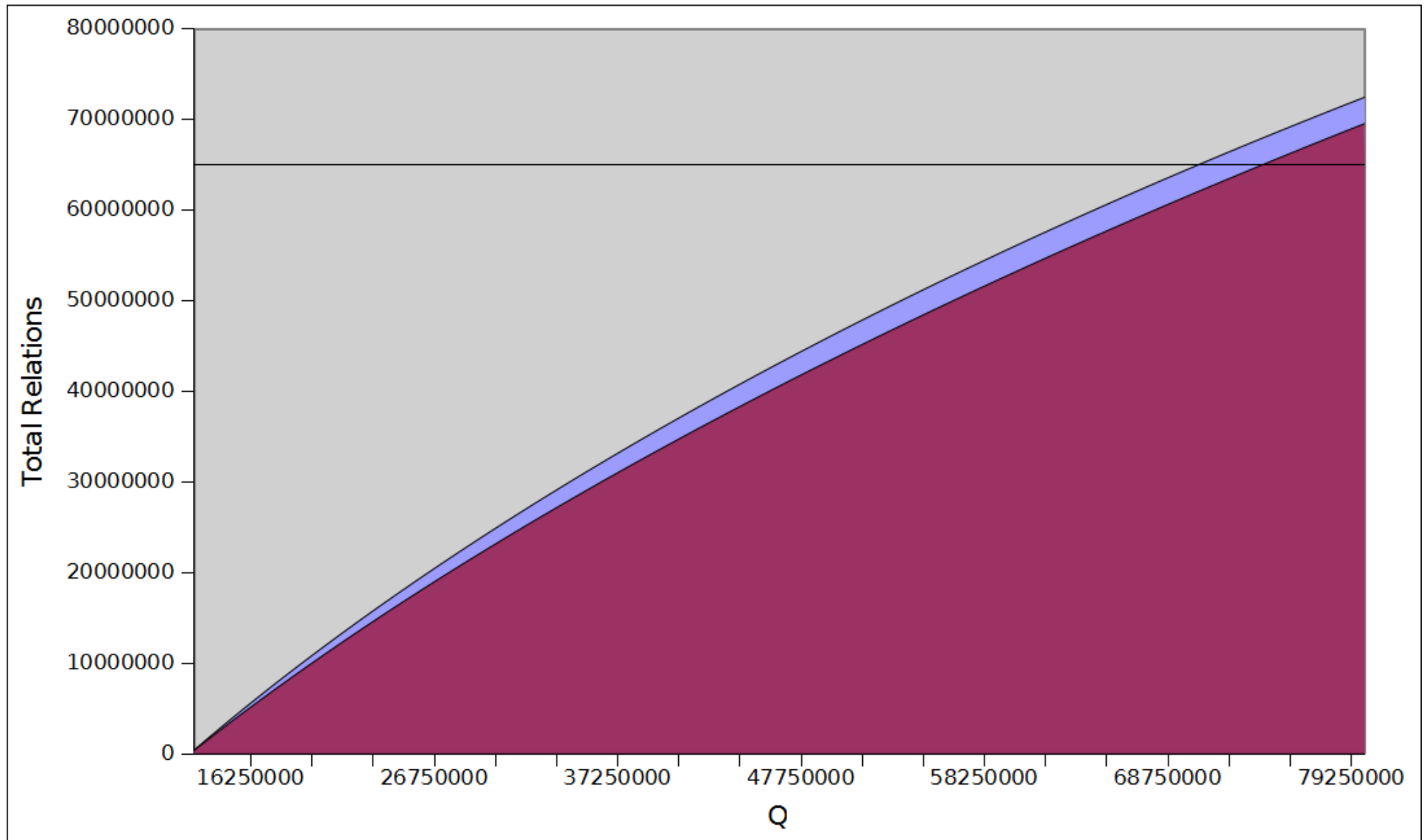


Misconceptions about Polynomials



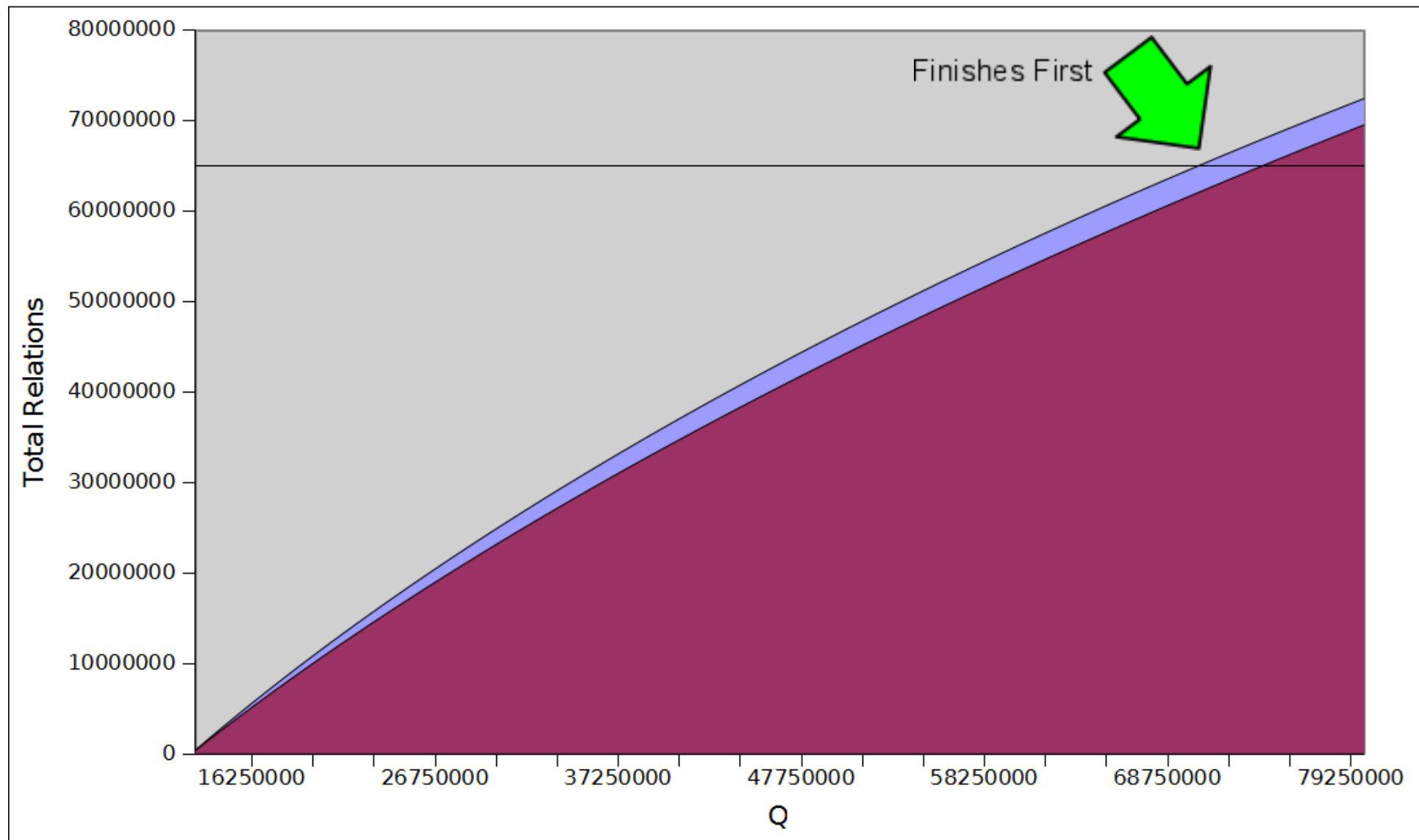
Misconceptions about Polynomials

Total Relations By Q

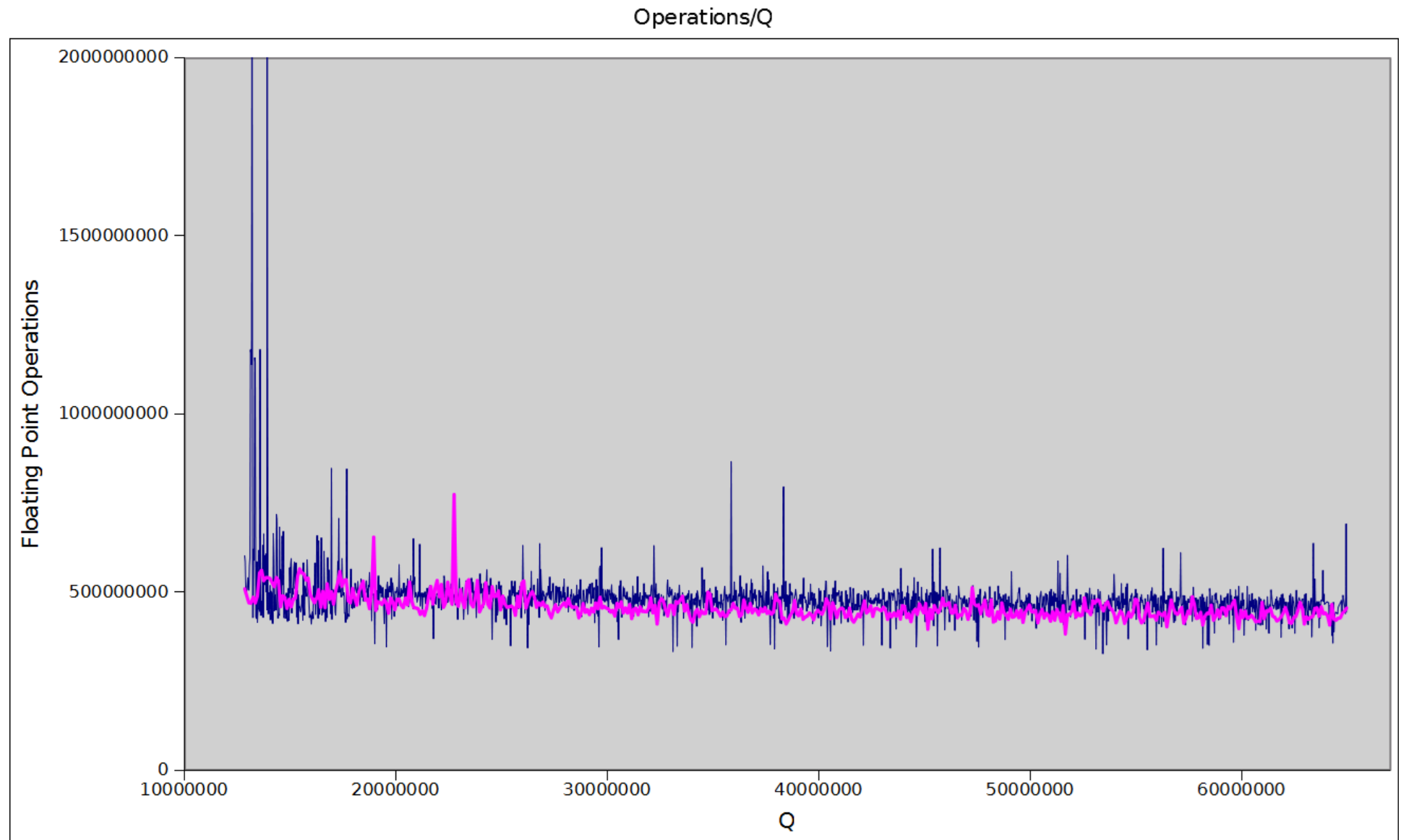


Misconceptions about Polynomials

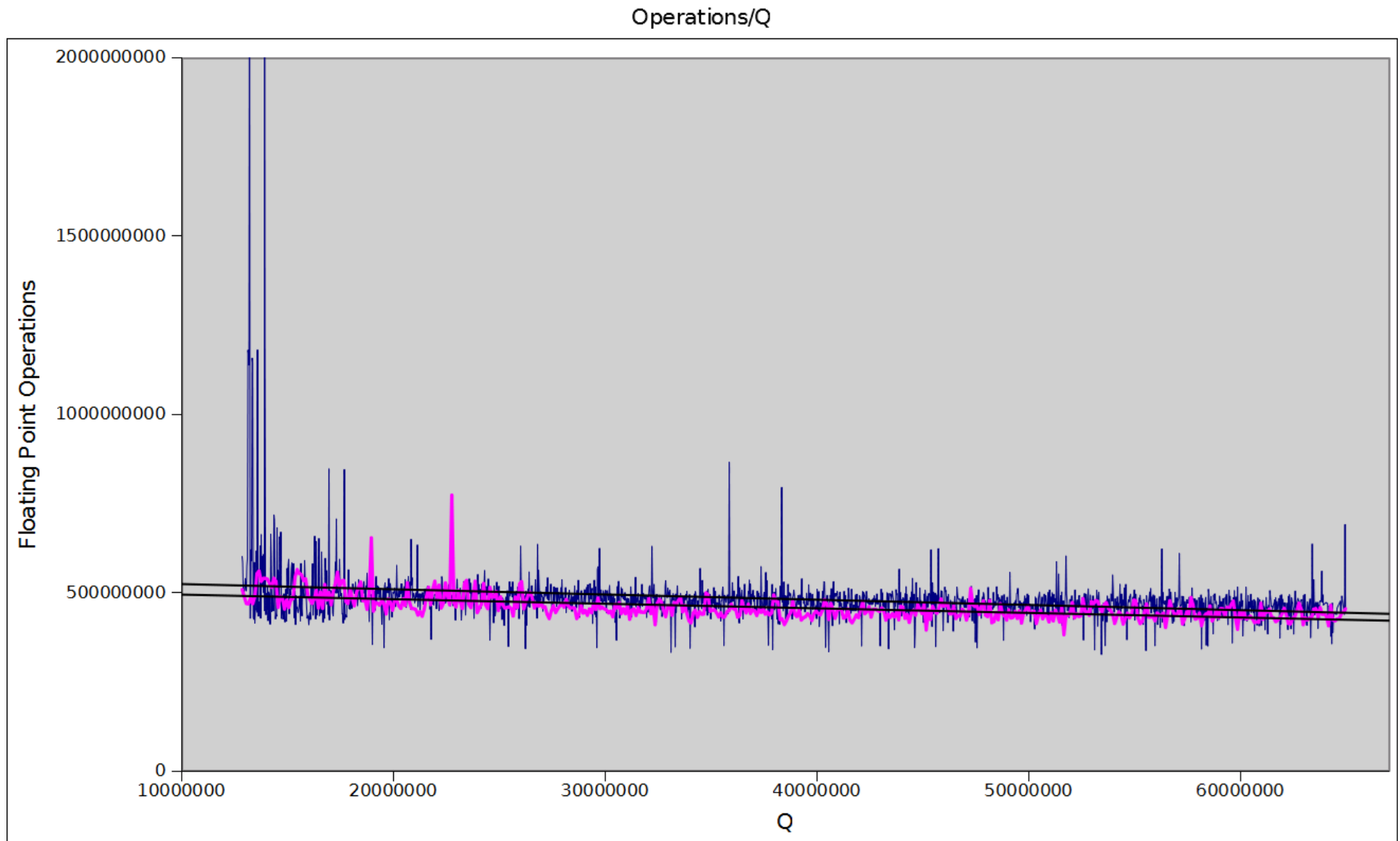
Total Relations By Q



Misconceptions about Polynomials

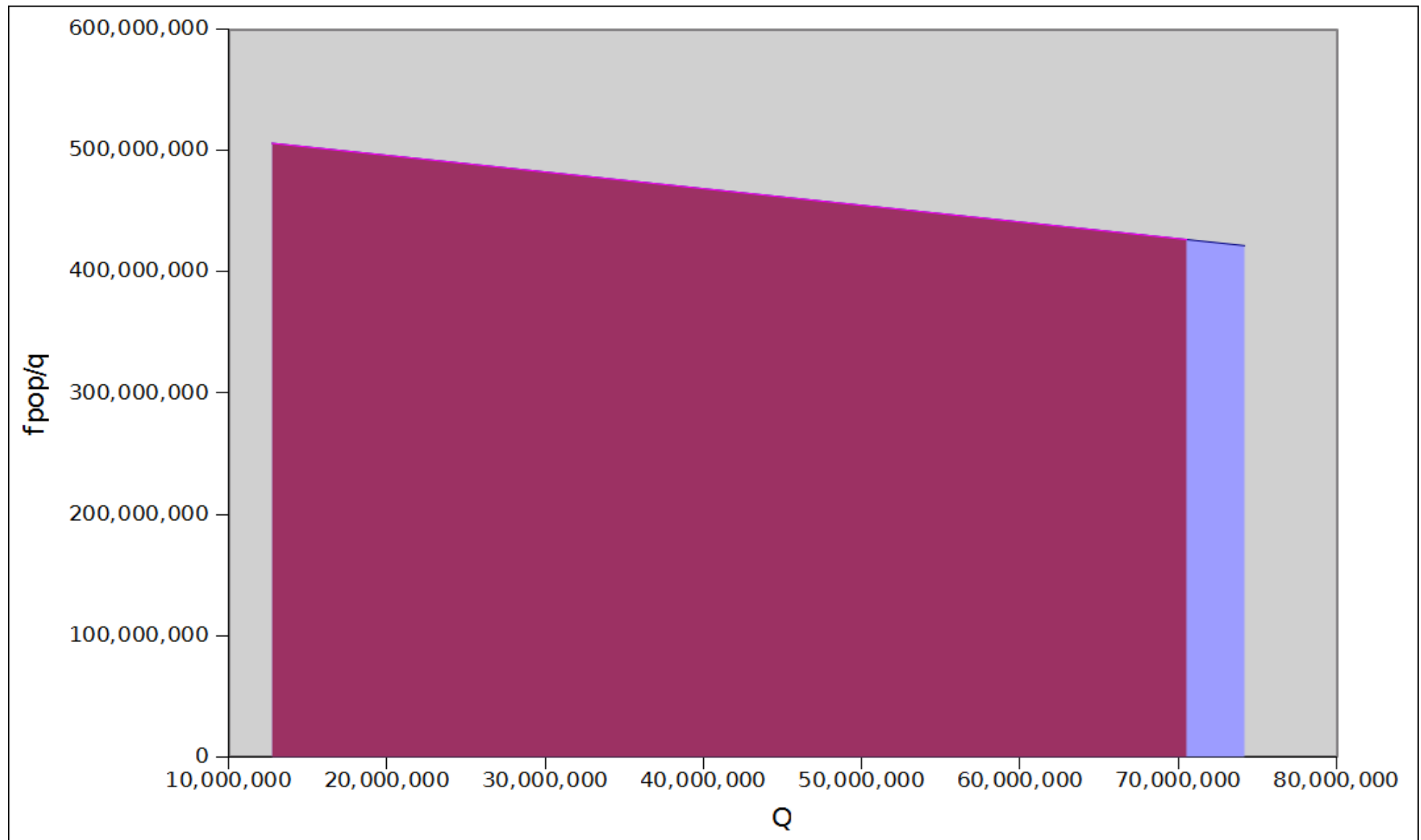


Misconceptions about Polynomials



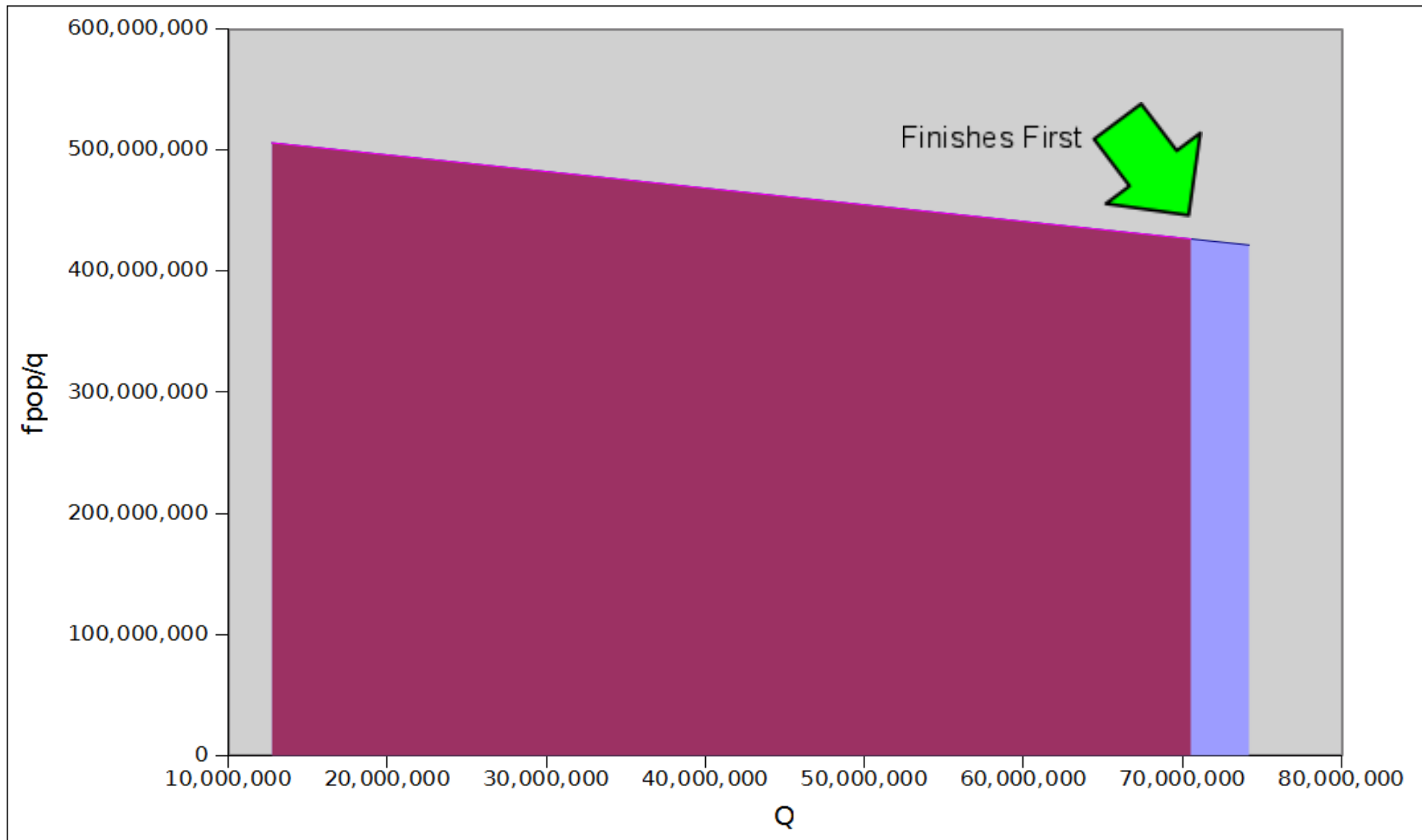
Misconceptions about Polynomials

Total Operations



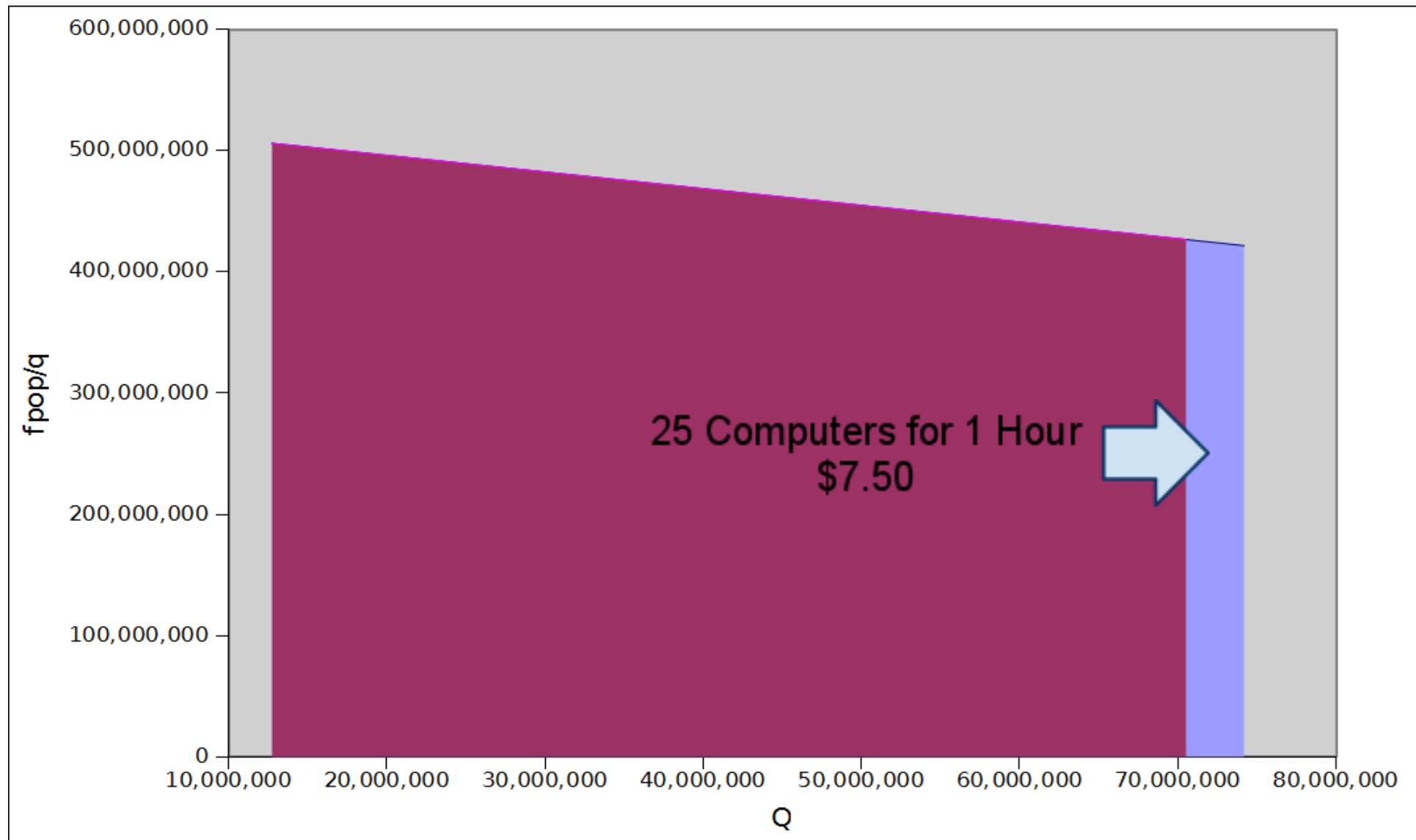
Misconceptions about Polynomials

Total Operations



Misconceptions about Polynomials

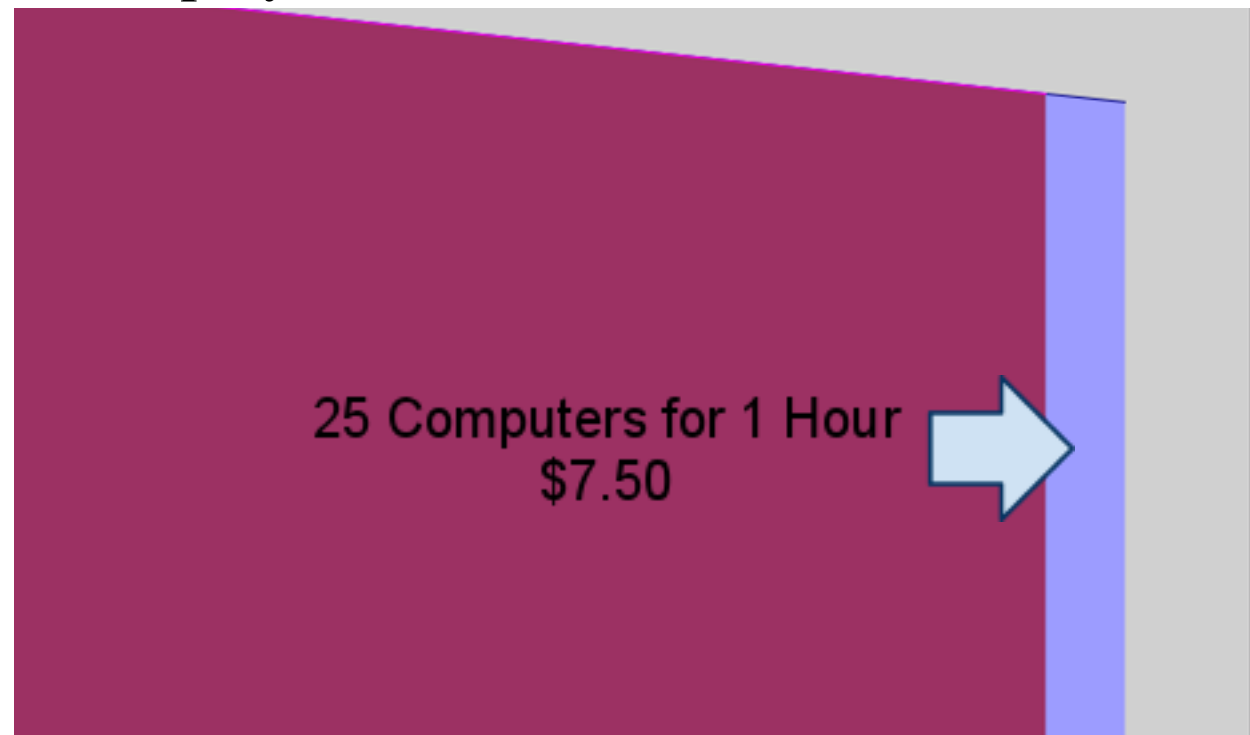
Total Operations



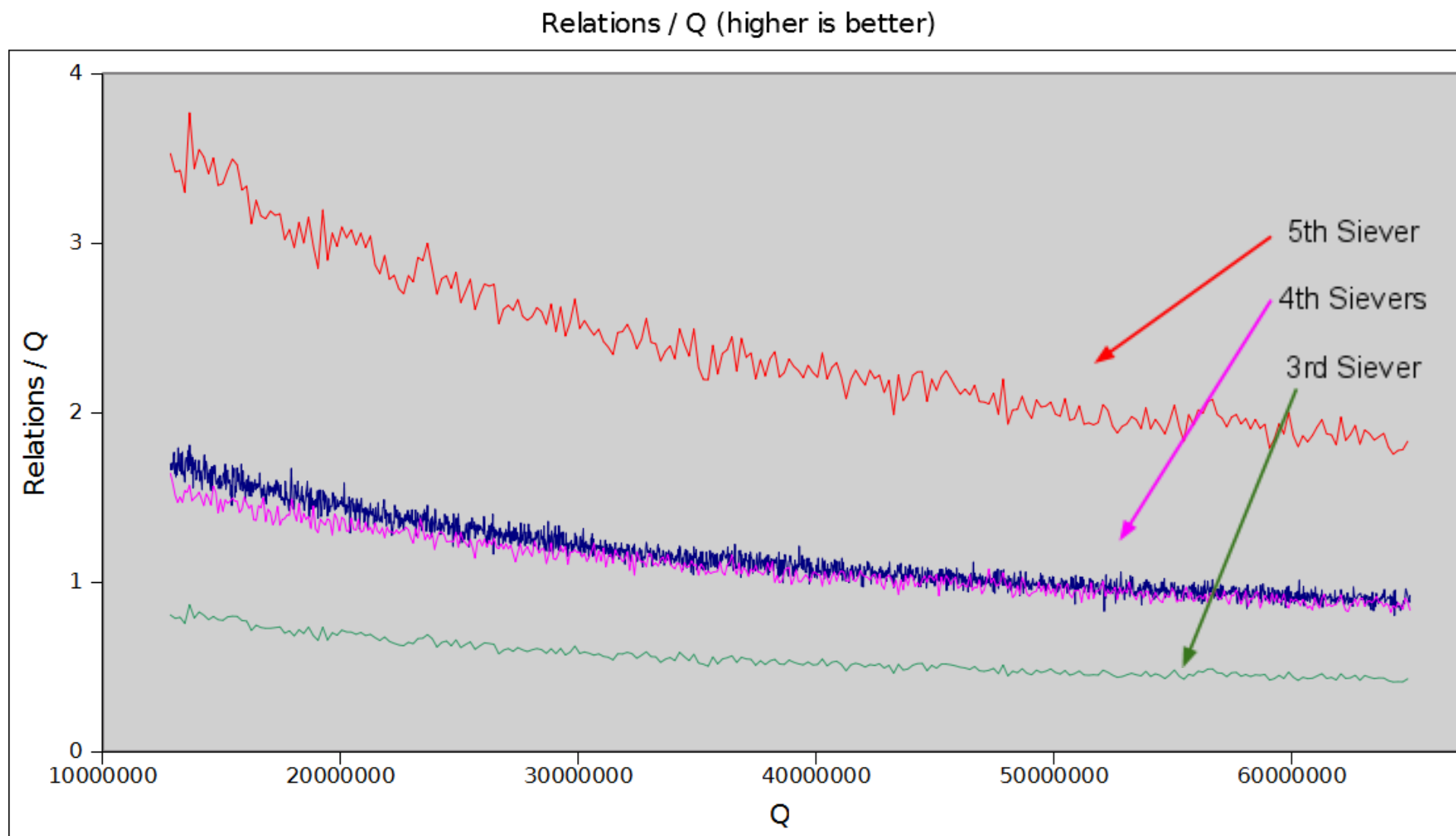
Misconceptions about Polynomials

If time is more valuable to you than (not much) money it is in your best interest to take the first polynomial you get and sieve with that, rather than doing another poly-selection run.

(this advice is only
for 512-bit semiprimes.)

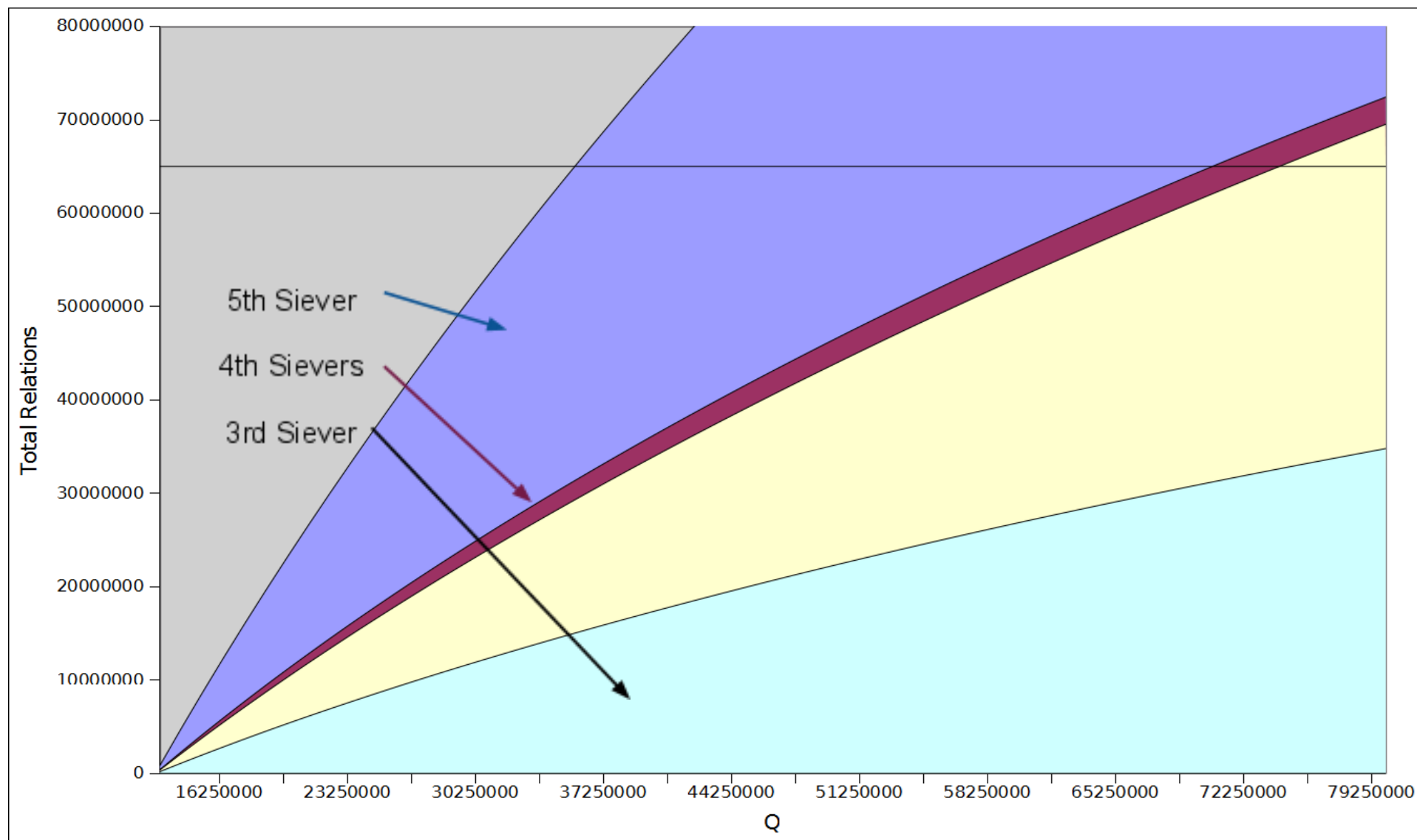


Siever Comparisons



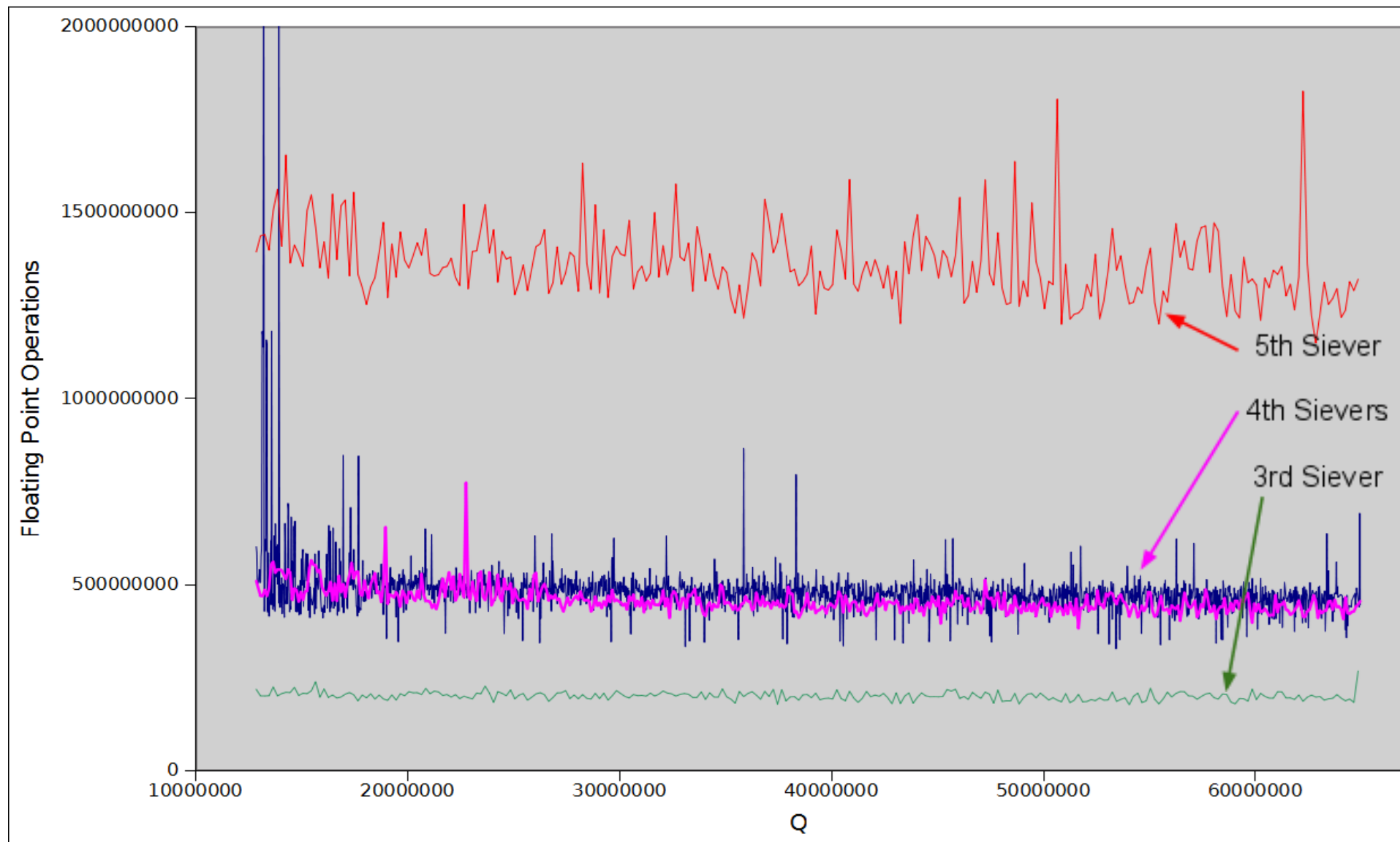
Siever Comparisons

Total Relations By Q



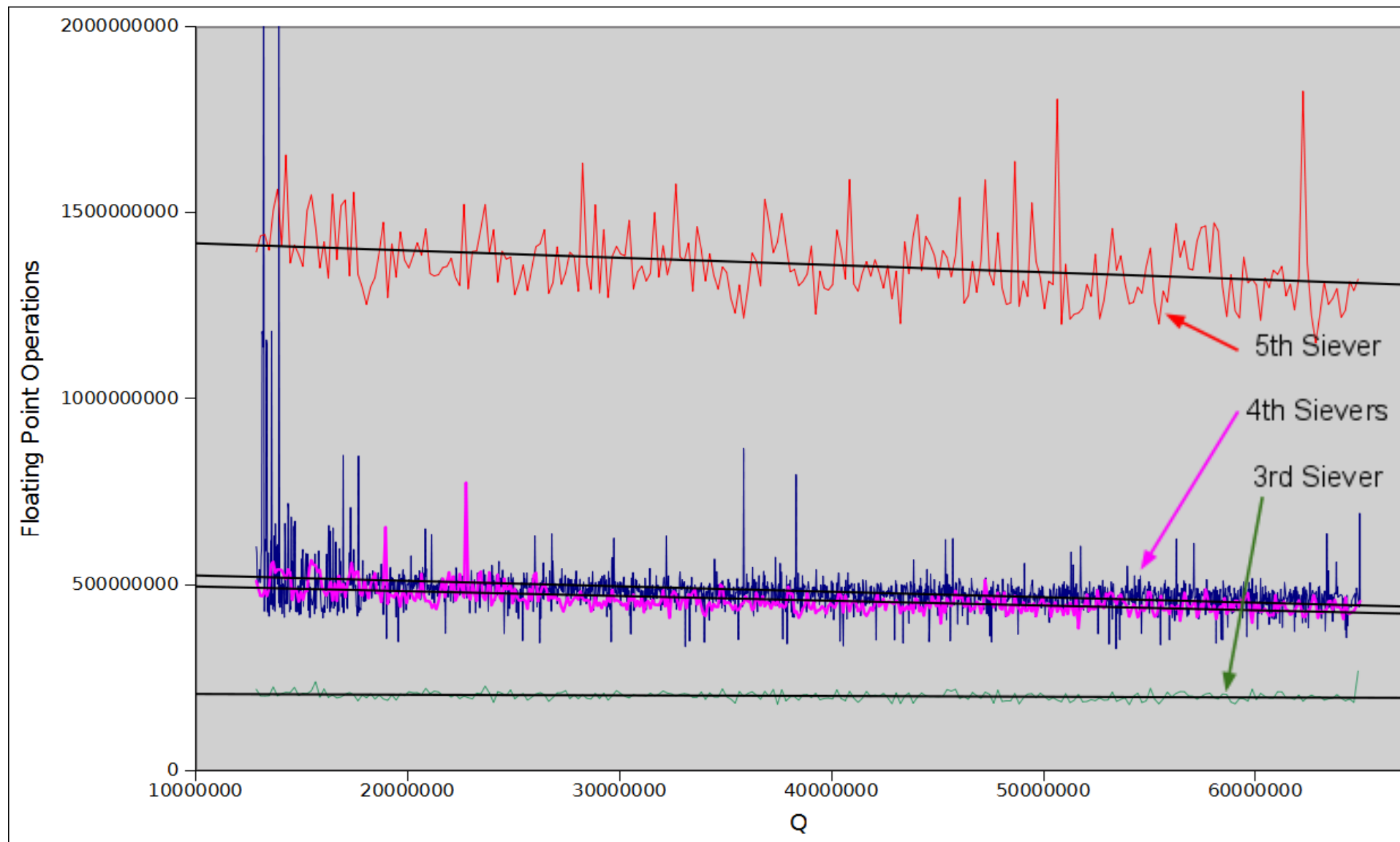
Siever Comparisons

Operations/Q

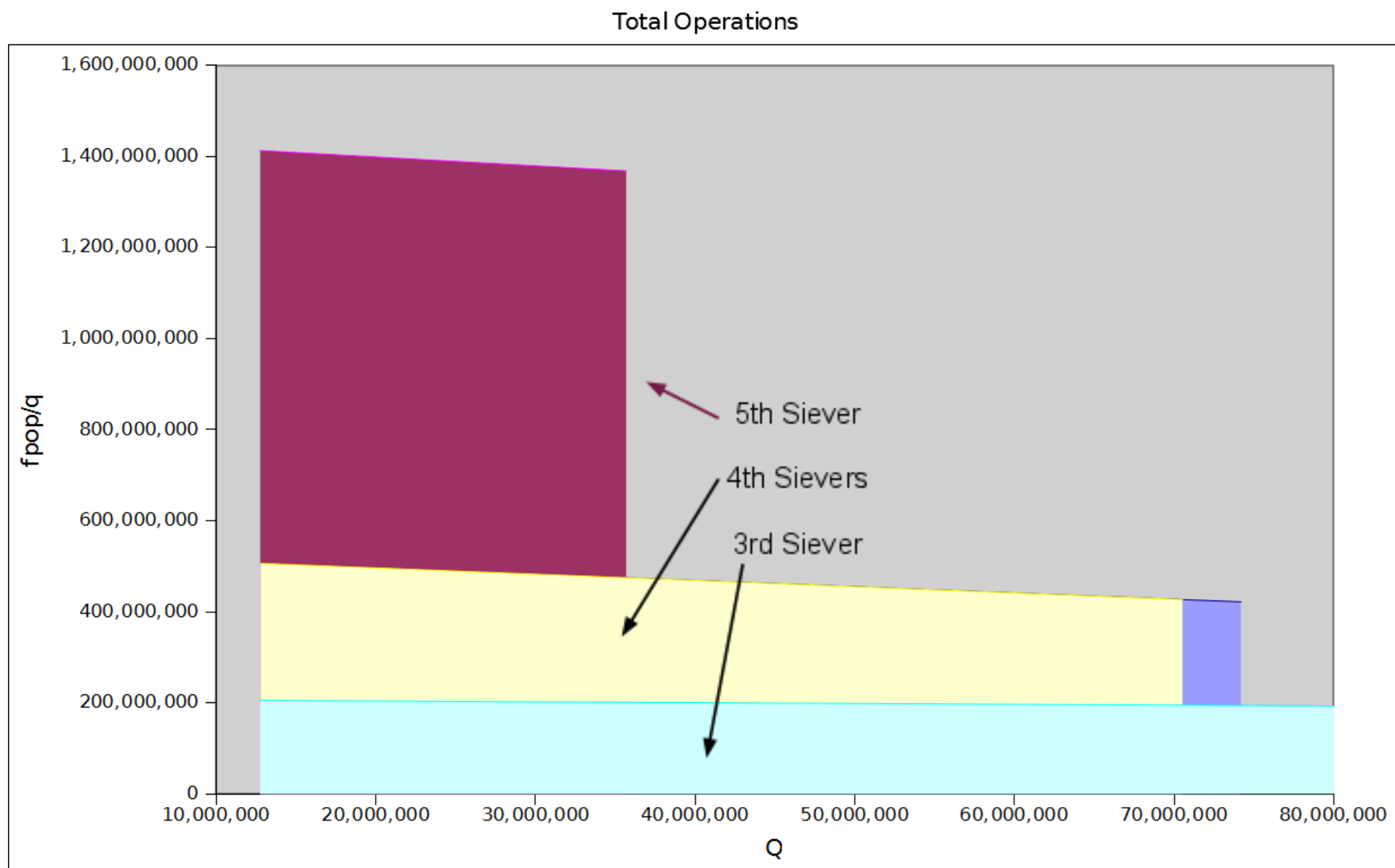


Siever Comparisons

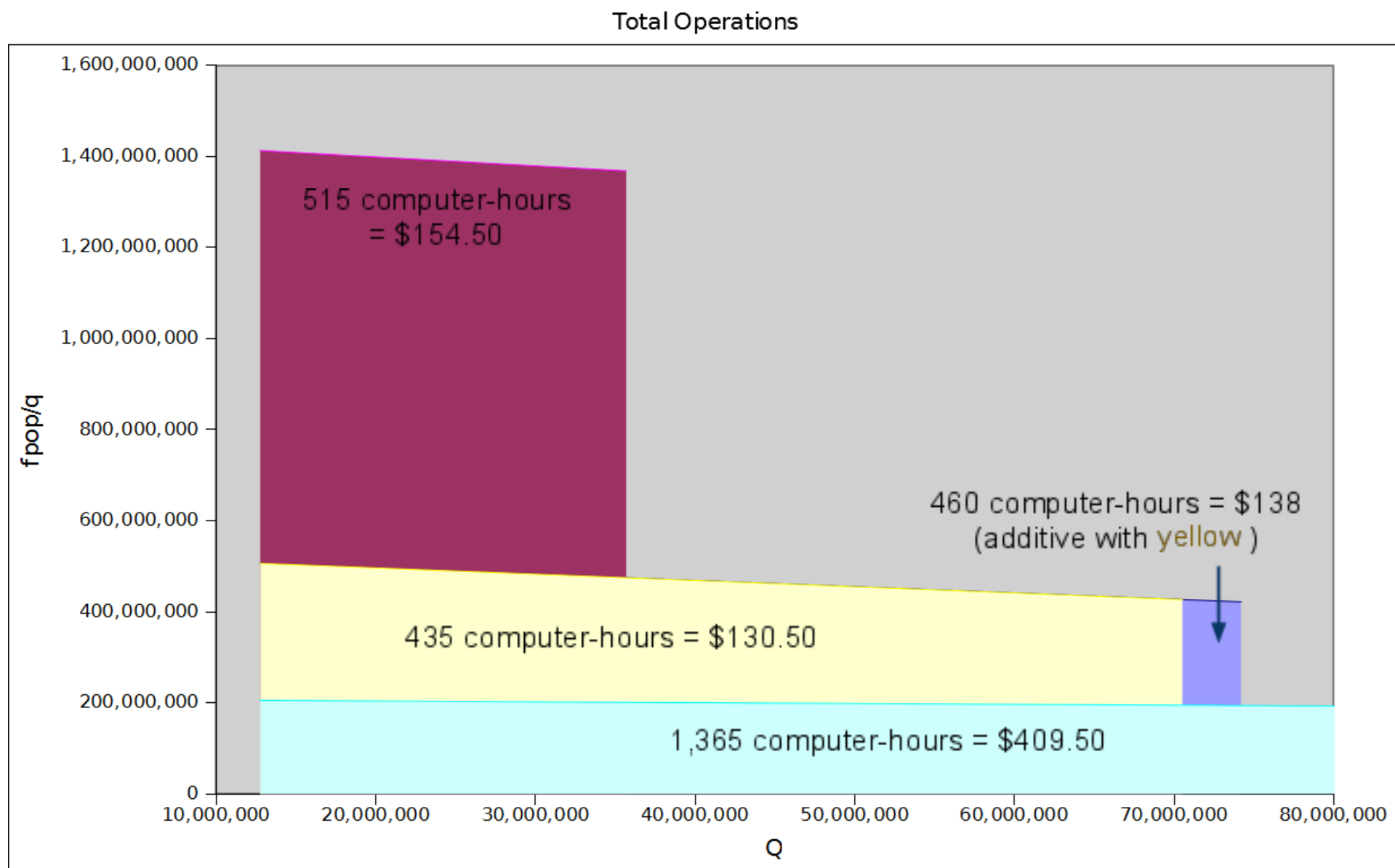
Operations/Q



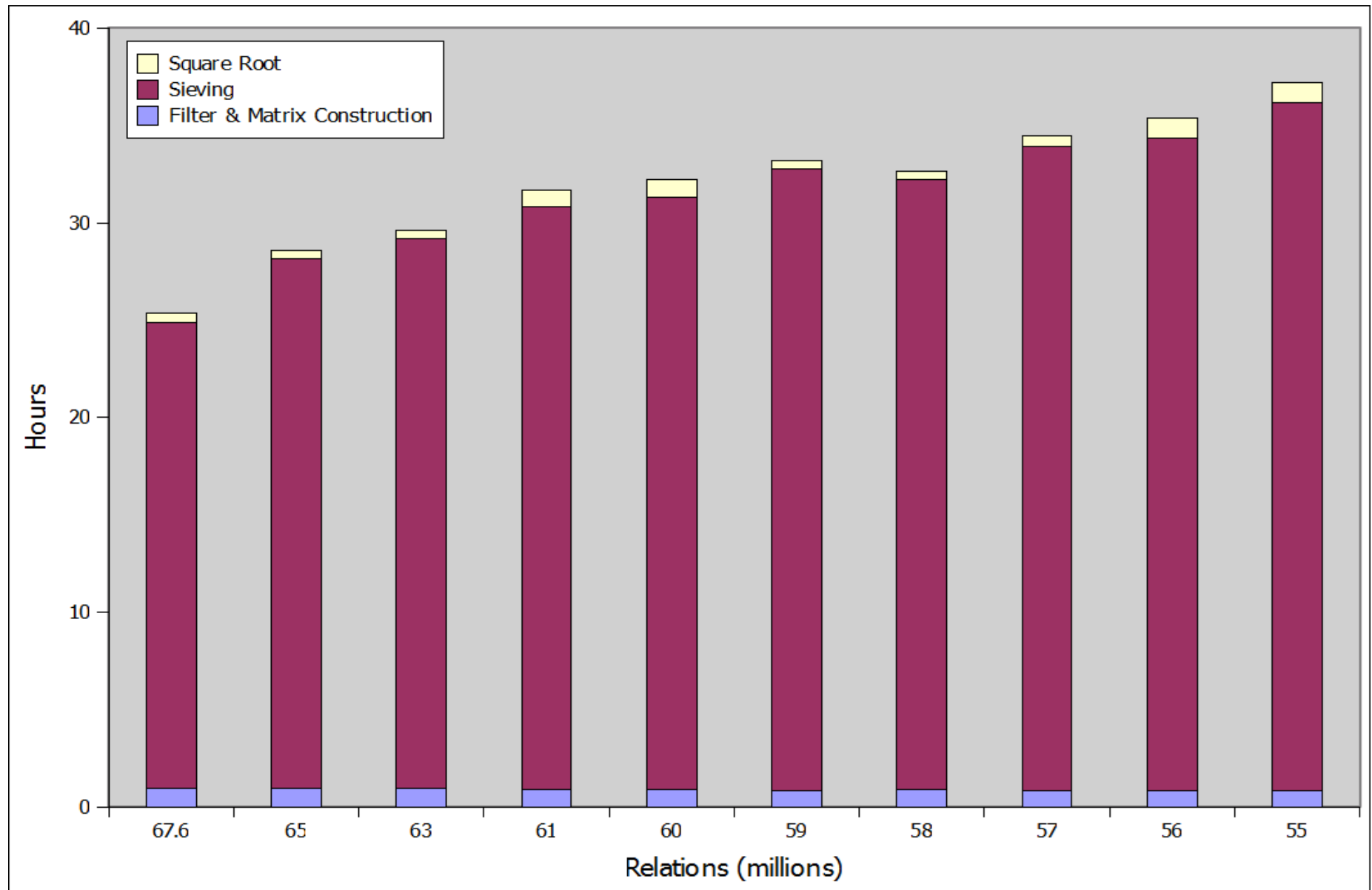
Siever Comparisons



Siever Comparisons



Oversieving



Obligatory Ending Slide

Fin

Thanks:

- iSEC Partners
- Gotham Digital Science
- NYSec
- MersenneForum & jasonp

Tom Ritter

Big Ups To:

- jasonp

<http://www.isecpartners.com/>

<https://github.com/tomrittervg/cloud-and-control>

