

Laboratory 1: Entanglement and Bell's Inequalities

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Abstract

The purpose of this lab was to demonstrate the existence and nonlocal behavior of entangled systems by violating an altered version of Bell's inequalities created by Clauser, Horne, Shimony, and Holt (CHSH). This was achieved by utilizing avalanche photodiodes (APD) as detectors to detect coincident pairs of polarization-entangled photons, created via the process of spontaneous parametric down conversion (SPDC).

Background

In the early 20th century, pioneers like Bohr, Heisenberg, and Schrödinger uncovered and described a new, revolutionary kind of physical behavior that dramatically changed the way humanity conceptualizes nature. This behavior is called quantum mechanics. Perhaps the most counterintuitive characteristic of quantum behavior is the idea of a nonlocal reality, which is the property this experiment seeks to demonstrate. The idea of nonlocality was first proposed in a famous paper written in 1935 by Albert Einstein and his colleagues Boris Podolsky and Nathan Rosen (Einstein, Podolsky, & Rosen, 1935) wherein they sought to show that quantum theory is either incomplete or completely wrong by bringing to light a paradox between classical intuition and the predictions of quantum theory. They accomplished this by describing a *gedankenexperiment* (thought experiment) that showed an apparent exception to Heisenberg's foundational uncertainty principle

$$\Delta x \Delta p \geq \hbar/2,$$

where Δx denotes positional uncertainty, Δp represents the uncertainty in momentum, and \hbar represents Planck's constant. In the *gedankenexperiment*, two particles interact in a way such that they become entangled, which is to say that they become components of a single, inseparable state:

$$|\Psi_{12}\rangle \neq |\Psi_1\rangle \otimes |\Psi_2\rangle$$

After the particles interact, some arbitrarily large distance separates them, which is important. If a person measures some property of one of the particles, then he or she can deduce, based on how the particles interacted, information about the state of the other

particle without disturbing it. This leaves two possibilities: Either EPR has figured out how to extract information from the system about the state of one of the particles without disturbing it, which is a violation of the uncertainty principle, or measuring the state of one of the particles instantaneously affects the state of the other particle without dependence on the distance between the particles in order to preserve uncertainty in the system. Einstein famously called this latter possibility the “spooky action at a distance,” and he did not believe that it could be possible. Podolsky and Rosen rejected this idea too, so they assumed that they had uncovered a substantial flaw with quantum theory, thus making quantum theory an incomplete or incorrect description of reality. All three of them instead opted for a classical alternative to quantum mechanics, which posited that quantum behavior is the result of some yet undiscovered, classically behaving hidden variables.

Bell’s Inequalities

John Stewart Bell, in 1964, showed that an experiment could be performed that showed that it is not possible for quantum systems to be manifestations of hidden variables like EPR and David Bohm believed. He did this by setting up inequalities (Bell’s inequalities) that are, essentially, trivial mathematical relations between physical characteristics that hold true for all classical systems, but that are violated for some quantum systems. An example of such a trivial inequality is

$$|A| + |B| + |C| \geq |A + B + C|.$$

It would be impossible for a classically behaving system with parameters A, B, and C to violate this inequality. In quantum systems, however, trivial relations such as this are violated given the right conditions, which proves that classical notions about physics are inadequate for explaining nature.

For this experiment, a modified version of Bell’s inequalities, derived by Clauser, Horne, Shimony, and Holt (Clauser, Horne, Shimony, & Holt, 1969), was used to test the quantum mechanical behavior of the system. For this method, two measures of correlations must be made:

$$E(\alpha, \beta) = P_{VV}(\alpha, \beta) + P_{HH}(\alpha, \beta) - P_{VH}(\alpha, \beta) - P_{HV}(\alpha, \beta), \text{ and}$$

$$S = |E(a, b) - E(a, b')| + |E(a', b) + E(a', b')|, \text{ where}$$

$$E(\alpha, \beta) = \frac{N(\alpha, \beta) + N(\alpha_{\perp}, \beta_{\perp}) - N(\alpha, \beta_{\perp}) - N(\alpha_{\perp}, \beta)}{N(\alpha, \beta) + N(\alpha_{\perp}, \beta_{\perp}) + N(\alpha, \beta_{\perp}) + N(\alpha_{\perp}, \beta)}$$

Given these relations, we know that Bell’s inequality has been violated if we find $|S| > 2$, which shows that the system cannot be described by hidden variables.

Experimental Setup and Procedure

In order to test Bell's inequality, pairs of entangled particles must be generated. For this particular method of violating Bell's inequalities, we utilized polarization-entangled photons created via the spontaneous parametric down-conversion process. This was accomplished by sending a 100mW Argon ion pump beam of wavelength 363.8 nm into two type 1 beta barium borate (BBO) crystals. The cone of entangled pairs of photons that emerges from the crystals (demonstrated in figure 2) then travels to the APDs, which register incident photons and send corresponding TTL signals through fiber optic cables to the counter-card.

If the card registers a signal from both detectors within 26ns of each other, a coincidence count is recorded. However, because the detectors can receive two non-entangled photons within a 26ns timeframe, some of the coincidence counts are 'accidental' or 'random' coincidence counts. They do not significantly affect the experiment, though, because they only work to diminish the value of $|S|$. Therefore, if $|S| > 2$, it cannot be caused by accidental coincidences. A diagram of the experimental setup is shown in figure 1.

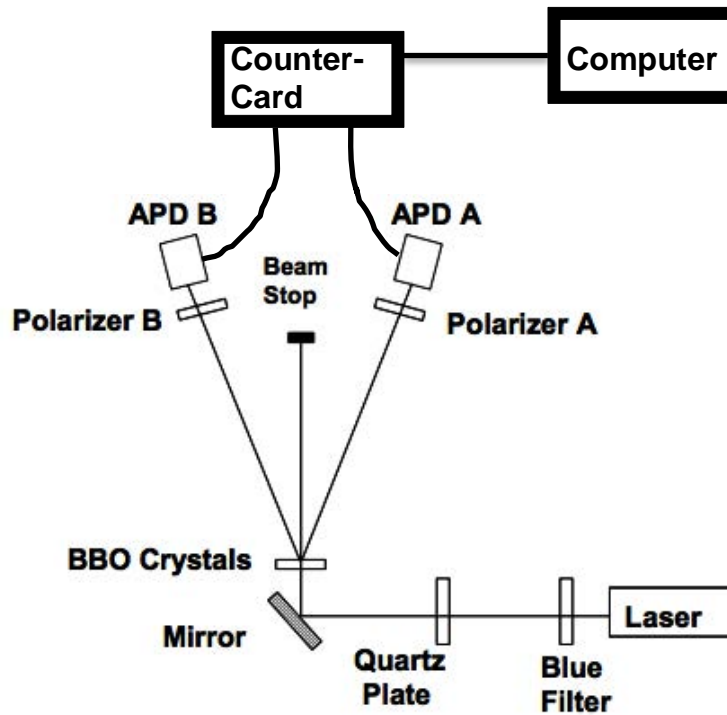


Figure 1: Setup for Violating Bell's Inequality.

BBO Crystals

The crystals that are used to create entangled pairs of photons are type 1 beta barium borate (BBO) crystals, which are nonlinear crystals that cause spontaneous parametric down-conversion (SPDC). As photons from the pump beam pass through the crystals, they either pass through unaffected or get down-converted into a pair of photons, which are usually called the signal (s) and idler (i) photons. Each of the signal and idler photons has wavelength $\lambda = 2\lambda_{\text{pump}}$, and a polarization state that is perpendicular to the parent photon's polarization. Each signal and idler pair is created in an entangled state.

The properties of the BBO crystals are dependent on the cut and orientation. Each crystal has a type 1 cut, and they are oriented with their cuts perpendicular to each other so that the down-conversion process is optimized to be able to affect vertically and horizontally polarized incident photons. Down-conversion of the vertically polarized component of the pump beam will happen in the first crystal. Similarly, down conversion of the horizontally polarized component of the pump beam happens in the second crystal (Kwiat, Waks, White, Appelbaum, & Eberhard, 1999).

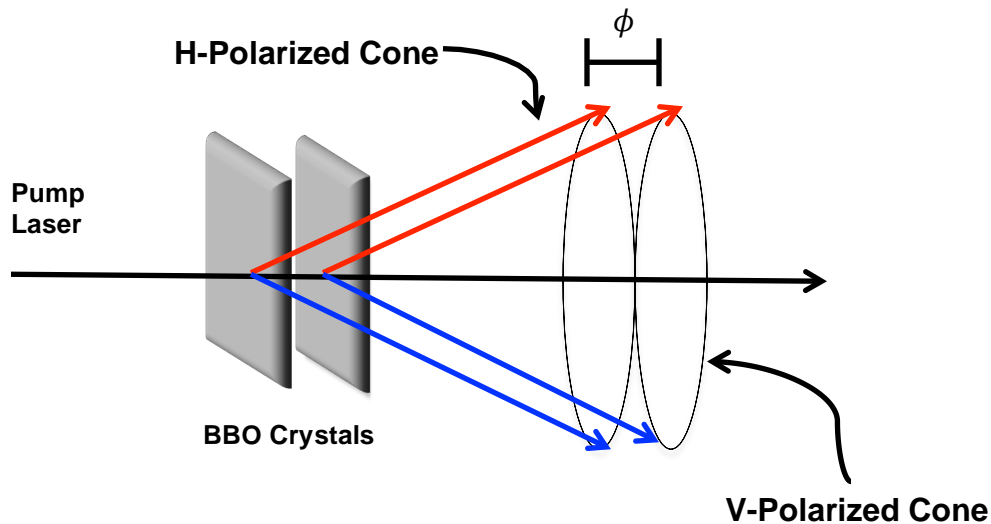


Figure 2: Spontaneous parametric down-conversion in type 1 beta barium boron crystals. The red lines represent the signal photons, and the blue lines describe the idler photons. Because the crystals have some distance between them, there is a difference in phase between the cone of H-Polarized photons and the V-Polarized photons.

Blue Filter & Quartz Plate

Before passing through the BBO crystals, the pump beam is passed through a simple blue filter and a quartz plate. The blue filter is used to remove 'dirty' fluorescence from the laser. The quartz plate is a birefringent material that creates a phase difference between the vertically and horizontally polarized components of the pump beam as it passes through.

This is done in order to compensate for the phase difference ϕ caused in the Horizontally and Vertically polarized cones of light that propagate from the BBO crystals, which can be seen in Figure 2.

In order to correct for the phase difference ϕ introduced by the BBO crystals, the quartz plate has to be calibrated. This is done experimentally by adjusting the plate's horizontal and vertical angles until an optimal configuration is found, which is defined as the configuration at which the plate produces the most consistent coincidence counts across different polarization angles. For our experiment, we only tested the vertical angle adjustment. Our results are displayed in Figure 3, which shows that the phase difference ϕ is most optimally corrected for when the quartz plate's vertical angle is adjusted to between 32.5° and 33° . When the quartz plate's vertical angle is set to some value in this range, the two cones of light should be superimposed¹.

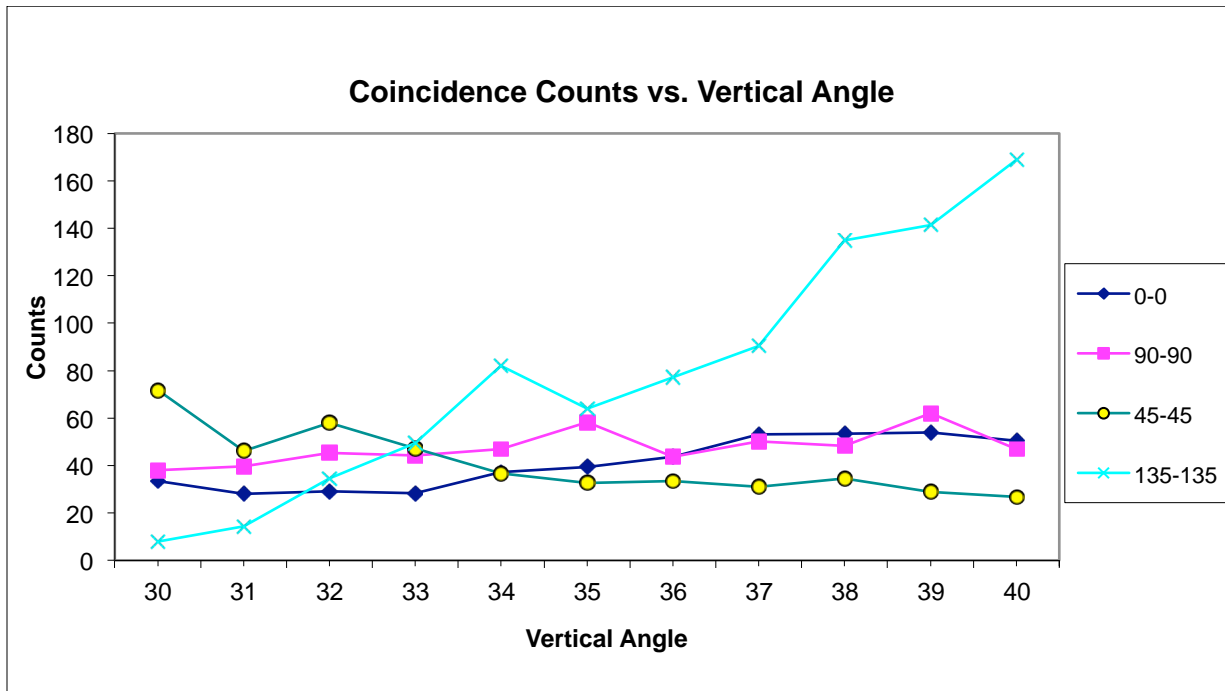


Figure 3: Data from vertical alignment of the quartz plate taken for four different sets of polarizer angles [0° - 0° , 45° - 45° , 90° - 90° , and 135° - 135°]. Laser power was measured at 77mW and acquisition time was set to 1 second.

Imaging the Down-Converted Light Cone

Assuming the quartz plate has corrected for the phase difference between the H-Polarized and V-Polarized light cones, there will be a single, observable cone with equal components of H-Polarized and V-Polarized light. In order to image this cone, we used an EM-CCD

¹ This is assuming that the horizontal angle of the quartz plate, which we did not test for, has already been optimized.

camera to record images of the cone at different polarization angles. This was accomplished by placing a polarizer in front of the camera that we rotated in increments of 20° from 0° to 360° , where we recorded an image of the cone at every increment.

Since the cone consists of equal amounts of H-Polarized and V-Polarized light, the intensity of light from the cone that is transmitted by the polarizer is not dependent on the angle of polarization. Example images are shown in figure 4, which demonstrates that changing the angle of the polarizer has little to no effect on the transmitted intensity

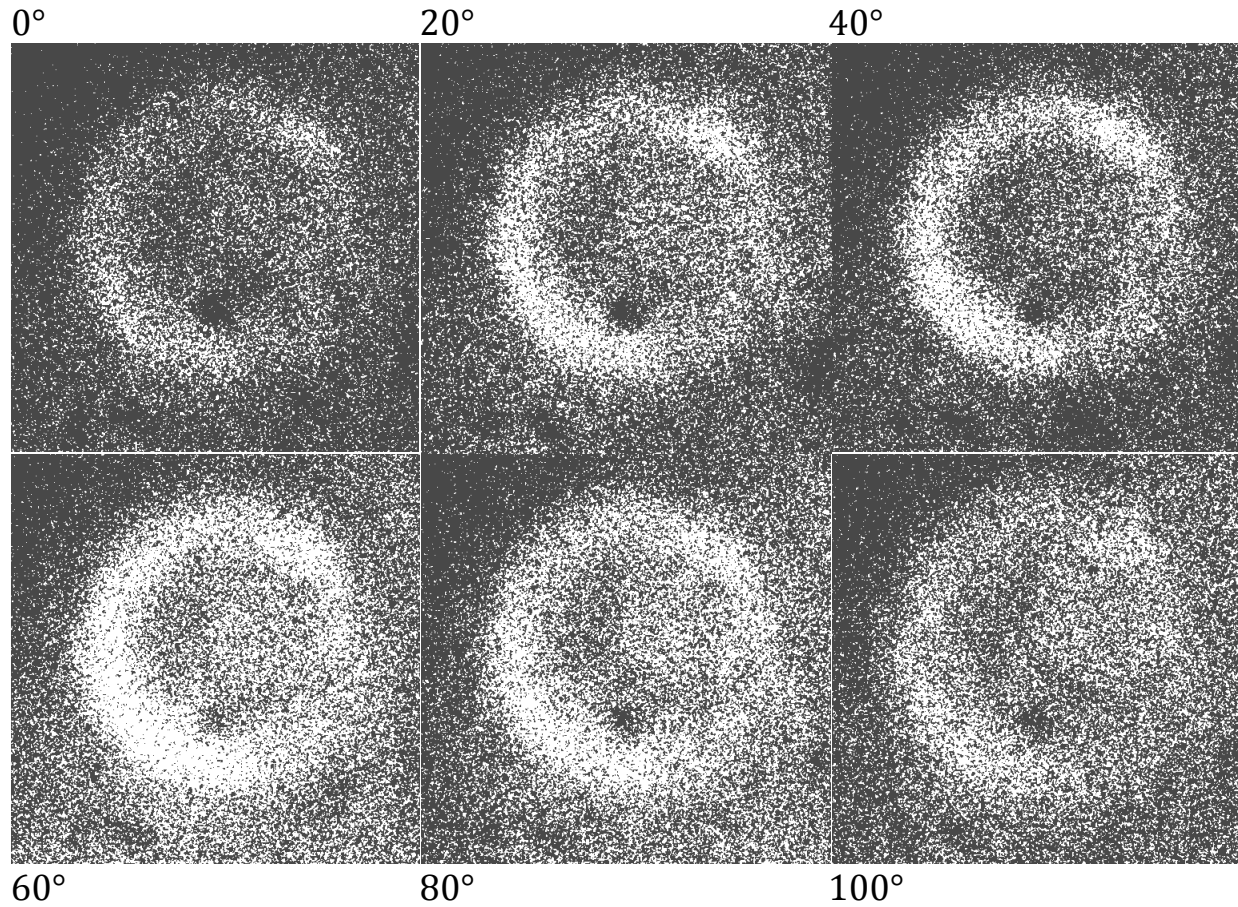


Figure 4: EM-CCD images of the Down-Converted light cone at different angles of polarization. The consistency of the intensity of the cone at different polarization angles indicates that the cone has equal components of Horizontally and Vertically polarized light, which is analogous to having no polarization. The dark circle near the bottom of the is an artifact that is probably fluorescent emission from the laser tube.

Checking the System Alignment

In order for Bell's inequality to be violated, the system must be properly aligned. We verify appropriate system alignment by making sure the coincidence counts recorded by each detector have a cosine-squared dependence on the difference of the polarizer angles. To do this, we set the polarizer in front of detector A to angle $\alpha = 45^\circ$, then incremented the polarizer in front of detector B by 10° , starting at $\beta = 0^\circ$ and ending at $\beta = 360^\circ$, measuring the coincidence counts on each detector at each increment. After finishing the process, we set polarizer A to $\alpha = 135^\circ$, and repeated the same steps. The cosine-squared dependence is clearly displayed in figure 5, indicating that the system is appropriately aligned for violating Bell's inequality.

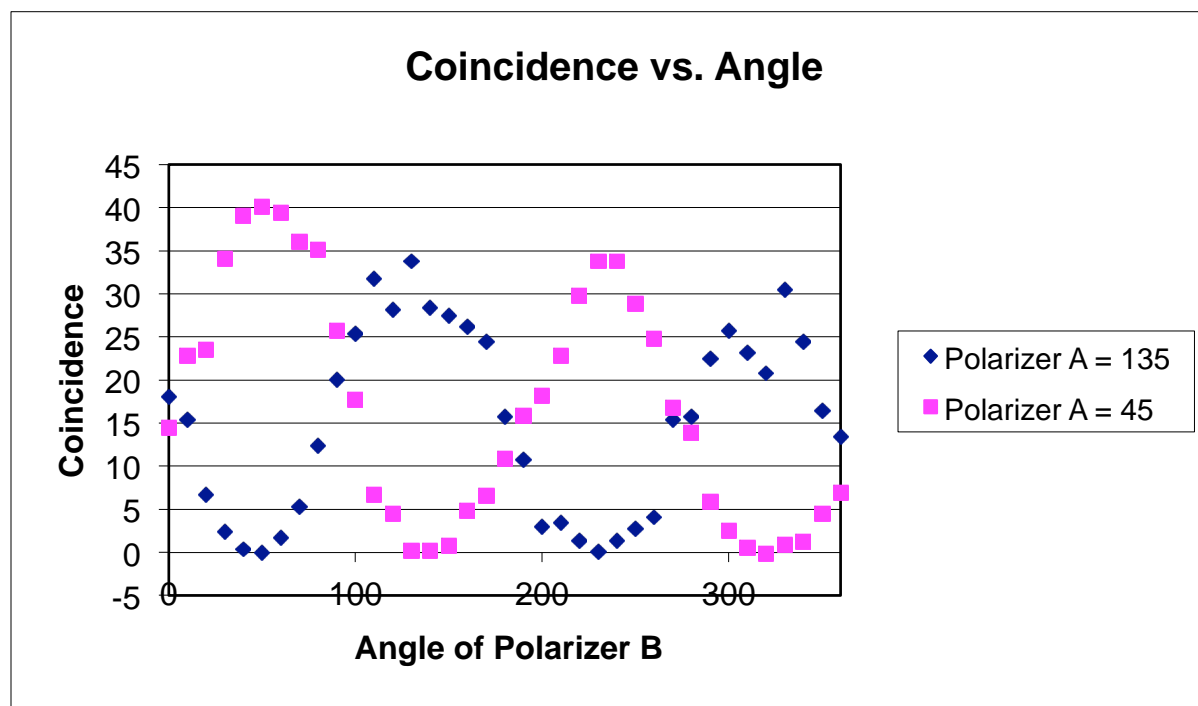


Figure 2: Plot of coincidence count dependence on the difference in polarizer angles. The plot shows cosine-squared dependence, and a phase shift of 90° between the data for $\alpha = 45^\circ$ and $\alpha = 135^\circ$

Results and Discussion

Following the procedure described by Kwiat *et. al.* we were able to violate the CHSH modification of Bell's inequality with an S value of 2.14. The data recorded to produce this result is shown in table 1. This violation of Bell's inequality indicates that we have successfully demonstrated the nonlocal behavior of particles in an entangled state, thereby dismissing the EPR paradox as a capable objection to quantum theory. It is apparent that classically behaving hidden variables are not capable of explaining the counterintuitive,

non-classical behavior of particles, thus verifying that the world is intrinsically quantum mechanical.

Polarizer A	Polarizer B	Singles A	Singles B	Average coincidence	Net coincidence
-45	-22.5	3000	2730	1902.5	1947.5
-45	22.5	3500	3000	2174.166667	2219.166667
-45	67.5	3000	3600	2222.5	2267.5
-45	112.5	3000	2500	1870.833333	1915.833333
0	-22.5	2800	2300	1692.5	1692.5
0	22.5	2800	2700	1840.833333	1840.833333
0	67.5	3000	3150	2072.5	2072.5
0	112.5	3000	2600	1904.166667	1904.166667
45	-22.5	3000	2200	1725.833333	1680.833333
45	22.5	2950	2796	1922.833333	1877.833333
45	67.5	3005	3068	2046.833333	2001.833333
45	112.5	3966	2381	2153.166667	2108.166667
90	-22.5	2967	2188	1710.833333	1620.833333
90	22.5	3078	3052	2050.833333	1960.833333
90	67.5	2716	2975	1919.5	1829.5
90	112.5	2813	2271	1732.166667	1642.166667

Table 1: Data from 1st attempt at observing entanglement. This trial was successful, producing an $|S|$ value of 2.14, clearly violating the CSH inequality.

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