

## **Need to know about the qubit-experiments (v0\_6–v0\_6.1)**

**Theme:** Systematic signature detection + evidence layer for physics-native subspaces

**Scope:** v0\_6 (discover signature families) and v0\_6.1 (robustness + enrichment + hold-out replication)

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## 0. What changed vs. v0\_3–v0\_5

v0\_3–v0\_5 established a pipeline that (i) finds near-degenerate eigenpairs, (ii) inspects eigenvector structure, and (iii) tests dynamic stability through leakage under time evolution (locally and, where possible, on backend).

**v0\_6 adds population-level structure:** it asks whether “good” candidates cluster into **repeatable classes** (signatures) across many random Hamiltonians.

**v0\_6.1 adds an evidence layer:** robustness sweeps, enrichment tests, and hold-out replication to quantify repeatability and stability-correlation without over-claiming.

## 1. Mathematical objects you scan

### 1.1 Hilbert space and computational basis

An  $n$ -qubit system lives in a complex Hilbert space of dimension  $2^n$ , with computational basis  $\{|x\rangle: x \in \{0,1\}^n\}$ .

**Equation (Word/LaTeX input):**

$$\dim(\mathcal{H}_n) = 2^n$$

**Equation (Word/LaTeX input):**

$$|\psi\rangle = \sum_{x \in \{0,1\}^n} \alpha_x |x\rangle, \sum_x |\alpha_x|^2 = 1$$

### 1.2 Pauli-sum Hamiltonians

All v0\_6–v0\_6.1 experiments use Hamiltonians built from Pauli strings.

**Equation (Word/LaTeX input):**

$$H = \sum_k c_k P_k, P_k \in \{I, X, Y, Z\}^{\otimes n}$$

This matters because:

- The **spectrum** (eigenvalues) is used to detect candidate manifolds (near-degeneracy).
- The **eigenvectors** encode structure (dominant basis patterns, entropy).
- Pauli structure supports implementable approximations for time evolution (link to v0\_5 lineage).

## 2. Spectral decomposition: why eigenpairs are central

Diagonalizing  $H$  yields eigenpairs  $\{(E_j, |\psi_j\rangle)\}$ .

Equation (Word/LaTeX input):

$$H |\psi_j\rangle = E_j |\psi_j\rangle$$

Equation (Word/LaTeX input):

$$H = \sum_j E_j |\psi_j\rangle\langle\psi_j|$$

Equation (Word/LaTeX input):

$$\langle\psi_i|\psi_j\rangle = \delta_{ij}$$

Operationally, v0\_6 uses this to extract:

- **Eigenvalue spacings** (where candidate pairs exist)
- **Eigenvector structure** (how “low-dimensional” the state looks in the computational basis)
- **Dominant basis states** (which bitstrings carry most weight)

## 3. Near-degenerate pairs and candidate subspaces

A near-degenerate eigenpair  $(i, j)$  is detected via a threshold  $\varepsilon$ .

Equation (Word/LaTeX input):

$$|E_i - E_j| < \varepsilon$$

This defines a candidate 2D invariant subspace:

Equation (Word/LaTeX input):

$$\mathcal{S}_{ij} = \text{span}\{|\psi_i\rangle, |\psi_j\rangle\}$$

In v0\_6, each detected  $\mathcal{S}_{ij}$  is treated as a **candidate physics-native information unit** (provisionally), then characterized by structure + stability descriptors.

## 4. The heart of v0\_6: from “found pairs” to “signature families”

### 4.1 Structural descriptors per candidate

For each candidate subspace  $\mathcal{S}_{ij}$ , v0\_6 extracts a compact descriptor set:

1. **Dominant basis count**  
How many basis states  $|x\rangle$  account for most probability mass in each eigenvector (e.g., top components capturing  $> 90\%$ – $95\%$  weight).
2. **Amplitude entropy** (structure proxy)  
Let  $p_x = |\alpha_x|^2$ . Define:

**Equation (Word/LaTeX input):**

$$H_{\text{amp}}(|\psi\rangle) = - \sum_x p_x \log p_x$$

Low  $H_{\text{amp}}$  indicates concentrated, compressible structure; high entropy indicates spread-out eigenvectors.

3. **Leakage proxy** (lightweight dynamic stability)  
Prepare  $|\psi(0)\rangle$  supported on the candidate structure and evolve under  $H$ , then measure how much remains “inside” the candidate manifold.

**Equation (Word/LaTeX input):**

$$|\psi(t)\rangle = U(t) |\psi(0)\rangle, U(t) = e^{-iHt}$$

A conceptual leakage definition using an orthonormal basis  $\{|\phi_a\rangle\}$  for the target subspace:

**Equation (Word/LaTeX input):**

$$L(t) = 1 - \sum_{a \in \mathcal{S}} |\langle \phi_a | \psi(t) \rangle|^2$$

(Implementation-wise, you often approximate this using a practical “inside-set” defined by dominant bitstrings or a projector surrogate, to keep runs scalable.)

### 4.2 Integer-binned signature keys

Raw floats produce “false uniqueness” (tiny numerical differences fragment classes).

So v0\_6 defines discrete **signature keys** via integer binning:

- $d$ = dominant basis count (integer)
- $b_i, b_j$ = entropy bins (integers)
- $\ell$ = leakage bin (integer)

Conceptually you can view a key as:

**Equation (Word/LaTeX input):**

$$\kappa = (d, b_i, b_j, \ell) \in \mathbb{Z}^4$$

v0\_6 output is essentially a **frequency distribution** over keys  $\kappa$ : which families recur, and how often.

## 5. One-sentence summary of what v0\_6 adds

**v0\_6 converts individual degeneracy findings into repeatable signature families, using discrete (integer-binned) structural and stability descriptors so that recurrence can be quantified across large seed scans.**

## 6. v0\_6.1: the evidence layer (why it is more persuasive)

### 6.1 Robustness sweep (quick)

Re-run with 2–3 values of parameters (e.g.,  $\varepsilon$ , entropy bin width, leakage bin width, dominance threshold). Then compare the top- $K$  signature sets across settings.

A standard overlap metric is the Jaccard index:

**Equation (Word/LaTeX input):**

$$J(A, B) = \frac{|A \cap B|}{|A \cup B|}$$

Interpretation: high overlap means “top families are not artifacts of one tuning.”

### 6.2 Enrichment test (high-value, easy to explain)

Define a stable set (e.g., top 1% or 5% by stability/interestingness score).

For a signature family  $S$  (i.e., a particular  $\kappa$ ), compare its frequency inside the stable set vs. overall.

**Equation (Word/LaTeX input):**

$$\text{Enrichment}(S) = \frac{\Pr(S \mid \text{stable})}{\Pr(S \mid \text{all})}$$

Enrichment  $> 1$  indicates  $S$  is over-represented among stable candidates. A bootstrap CI makes the claim statistically readable.

### 6.3 Hold-out replication (clean replication logic)

Split seeds into two halves:

## Need to know about the qubit-experiments (v0\_6–v0\_6.1)

- learn “top families” in half A,
- test whether they also rank high in half B.

Replication can be summarized with overlap of top- $K$  families (again Jaccard, or rank correlation).

## 7. “Interestingness” (kept minimal and method-first)

In v0\_6.1, “interestingness” is a ranking device to define the stable set. A typical conservative direction is:

- reward low leakage (stability),
- reward small dominant basis count (structured).

A minimal form:

**Equation (Word/LaTeX input):**

$$\text{score} = w_1 (1 - \bar{L}) + w_2 \frac{1}{d}$$

where  $\bar{L}$  is a leakage summary (e.g., average over sampled times) and  $d$  is dominant basis count. The key is that the *evidence tests* remain the same even if the exact score is adjusted.

## 8. How to read your outputs (what counts as “the result”)

When you review v0\_6/v0\_6.1 logs, focus on four outputs:

1. **Top signature families by occurrence** (recurrence structure)
2. **Top families by enrichment** (stability association)
3. **Robustness overlaps** across parameter settings (fragile vs stable conclusion)
4. **Hold-out replication overlap** (reproducibility on unseen seeds)

A careful, defensible claim is:

**“We observe repeatable signature families; some are enriched among low-leakage candidates; robustness and hold-out checks quantify how stable that conclusion is.”**

## 9. Limitations (credibility section)

- Still controlled at  $n = 3$  qubits (by design).
- Signature keys depend on discretization (bin width trade-offs).

### **Need to know about the qubit-experiments (v0\_6–v0\_6.1)**

- Leakage is a proxy (lightweight dynamics), not full open-system decoherence.
- Random Pauli-sum Hamiltonians are a test bed; physics-motivated families are the next step.