Experiment v0.4 – Physics-Native Subspace Discovery

This document reports experiment v0.4 in the project "AI-Driven Discovery of Physics-Native Quantum Information Units". The aim of v0.4 is to scan random 3-qubit Hamiltonians for near-degenerate eigenpairs and examine whether the associated eigenstates form structured, low-dimensional subspaces that could function as physics-native information units.

Indhold

1. Introduction

Earlier prototypes (v0.1–v0.3) demonstrated that even small 2-qubit Hamiltonians can exhibit structured degeneracies, where eigenstates are dominated by a small number of computational basis states. Experiment v0.4 extends this approach to 3-qubit systems.

The specific goals of v0.4 are:
• to scan an ensemble of random 3-qubit Hamiltonians,
• to detect exact and near-degenerate eigenvalue pairs,
• to analyse the dominant basis-state structure of the corresponding eigenvectors,
• and to evaluate whether these patterns support the hypothesis of physics-native subspaces.

2. Methods

Each Hamiltonian H is constructed as a sum of 3-qubit Pauli strings with real coefficients:

$H = \Sigma\_i c\_i P\_i$

where $P\_i \in \{I, X, Y, Z\}^3$.

For a given random seed, a fixed number of Pauli terms is generated, and coefficients c_i are drawn from a continuous distribution. The resulting SparsePauliOp is diagonalised classically to obtain eigenvalues $\{E_k\}$ and eigenvectors $\{|\psi_k\rangle\}$.

A near-degenerate eigenpair is defined by the threshold

$|E\_i - E\_j| < \varepsilon$

with $\varepsilon = 0.05$ in this experiment.

For each candidate pair (i, j), the eigenvectors are expanded in the computational basis and the largest amplitudes are reported. This reveals whether the degeneracy supports a simple low-dimensional subspace (e.g., dominated by two or a few bitstrings).

3. Python Implementation (v0.4)

The following code snippet shows the core v0.4 implementation used to generate Hamiltonians, diagonalise them, and extract near-degenerate eigenpairs. The full file is stored in the repository under experiments/v0_4.

```
import numpy as np
from qiskit.quantum_info import SparsePauliOp, Statevector
from datetime import datetime
import sys


# ==========================================================
# Helper: redirect console output to a text file
# ==========================================================
output_file = "v0_4_output.txt"
sys.stdout = open(output_file, "w")
```

```python
print("Physics-Native Subspace Discovery – Prototype v0.4")
print("Run timestamp:", datetime.now())
print("------------------------------------------------------\n")




# ============================================================
# 1. Build a random Hamiltonian (general version)
# ============================================================
def build_random_hamiltonian_sparse(n_qubits=3, num_terms=6, seed=None):
    """
    Builds a random Hamiltonian using Pauli strings.
    Uses only I, X, Z to keep the landscape interpretable.
    """
    rng = np.random.default_rng(seed)
    pauli_letters = ["I", "X", "Z"]
    terms = []

    for _ in range(num_terms):
        label = "".join(rng.choice(pauli_letters) for _ in range(n_qubits))

        # skip pure identity
        if set(label) == {"I"}:
            continue

        coeff = rng.uniform(-1.0, 1.0)
        terms.append((label, coeff))

    if not terms:
        # fallback Hamiltonian if everything filtered out
        terms = [("Z" + "I" * (n_qubits - 1), 1.0)]

    labels = [t[0] for t in terms]
    coeffs = [t[1] for t in terms]

    H = SparsePauliOp.from_list(list(zip(labels, coeffs)))
    return H




# ============================================================
# 2. Find eigenvalues and near-degenerate pairs
# ============================================================
def find_near_degenerate_pairs(hamiltonian_op, tolerance=0.05):
```

```python
    """
    Finds eigenvalue pairs |Ei - Ej| < tolerance.
    Returns eigenvalues, eigenvectors, and detected pairs.
    """
    H_mat = hamiltonian_op.to_matrix()
    evals, evecs = np.linalg.eigh(H_mat)

    pairs = []
    for i in range(len(evals)):
        for j in range(i + 1, len(evals)):
            if abs(evals[i] - evals[j]) < tolerance:
                pairs.append((i, j, evals[i], evals[j]))

    return evals, evecs, pairs



# ===============================================================
# 3. Display eigenvectors in computational basis
# ===============================================================
def print_statevector(evec, n_qubits=3, max_terms=6):
    """
    Prints the dominant computational basis states.
    """
    sv = Statevector(evec).data
    idx_sorted = np.argsort(-np.abs(sv))

    print("  Dominant basis states:")
    shown = 0
    for idx in idx_sorted:
        amp = sv[idx]
        if np.abs(amp) < 1e-3:
            continue

        bitstring = format(idx, f"0{n_qubits}b")
        print(f"   |{bitstring}> : amplitude {amp:.3f}")
        shown += 1
        if shown >= max_terms:
            break



# ===============================================================
# 4. MAIN EXECUTION LOOP
# ===============================================================
```

```python
if __name__ == "__main__":
    n_qubits = 3
    num_terms = 6
    tolerance = 0.05
    max_seed = 50

    print(f"Scanning {max_seed} random Hamiltonians for near-degenerate eigenpairs...")
    found_any = False

    for seed in range(max_seed):
        H = build_random_hamiltonian_sparse(
            n_qubits=n_qubits,
            num_terms=num_terms,
            seed=seed
        )

        evals, evecs, pairs = find_near_degenerate_pairs(H, tolerance=tolerance)

        if pairs:
            found_any = True
            print("\n======================================")
            print(f"Seed {seed} produced near-degenerate pairs")
            print("Hamiltonian:")
            print(H)
            print("Eigenvalues:")
            for i, ev in enumerate(evals):
                print(f"  {i}: {ev:.4f}")

            print("\nCandidate near-degenerate pairs:")
            for (i, j, ei, ej) in pairs:
                print(f"  Pair ({i}, {j}) with energies {ei:.4f}, {ej:.4f}")
                print("   State", i)
                print_statevector(evecs[:, i], n_qubits=n_qubits)
                print("   State", j)
                print_statevector(evecs[:, j], n_qubits=n_qubits)
                print()

    if not found_any:
        print("\nNo near-degenerate pairs found. Try increasing tolerance.")

    print("\n--- End of v0.4 experiment ---")
```

sys.stdout.close()

4. Experimental Results

The v0.4 run scanned 50 random 3-qubit Hamiltonians. Multiple seeds produced clear or near-clear degeneracies. For each such seed, the console output reports:
• the Hamiltonian in SparsePauliOp form,
• the full set of eigenvalues,
• and, for each near-degenerate pair, the dominant computational basis states of the corresponding eigenvectors.

4.1 Overview of Degeneracies

The following seeds produced at least one near-degenerate eigenpair in this run:
   2, 6, 12, 14, 16, 22, 31, 32, 34, 35, 38, 45, 46, 47, 48.

Typical patterns include:
• Pairs with almost identical energies at both low and high ends of the spectrum.
• Eigenstates dominated by two complementary bitstrings (e.g., |110> and |001>).
• Symmetric structures where partner states in a pair exchange roles of bitstrings.
• Multi-state manifolds with 4–6 significant amplitudes but clear internal symmetries.

4.2 Representative Examples

A few examples illustrate the typical structure observed:

• Seed 2: Four exactly degenerate pairs with energies grouped at −1.4563, −0.4828, 0.1994 and 1.7397. Each pair is built from two dominant bitstrings, such as |110> / |100> or |011> / |001>, with partner states swapping roles between basis states.

• Seed 6: Degeneracies at several energy levels, with eigenstates showing balanced support over up to six basis states. The patterns remain highly structured, not random.

• Seed 31: A full ladder of exact degeneracies (±1.5305, ±0.2679), with eigenstates organised in pairs that reflect clear bitstring symmetries across the computational basis.

• Seed 35: Near-degenerate pairs across the spectrum with rich symmetry between states, including paired patterns like |010> / |001> vs. |100> / |111> and their complements.

4.3 Full Console Log (v0.4 Run)

For transparency and reproducibility, the full console output of the v0.4 run is included below. It contains the exact numerical values of eigenvalues and the dominant basis-state amplitudes for each near-degenerate pair.

Physics-Native Subspace Discovery ⬚ Prototype v0.4
Run timestamp: 2025-12-04 14:37:43.061516

Scanning 50 random Hamiltonians for near-degenerate eigenpairs...

==
Seed 2 produced near-degenerate pairs
Hamiltonian:
SparsePauliOp(['ZII', 'IXI', 'ZZZ', 'IXI', 'IXI', 'IZZ'],
        coeffs=[0.62845148+0.j, 0.20020105+0.j, -0.88970675+0.j, 0.31486603+0.j,
 -0.13473842+0.j, -0.15443065+0.j])
Eigenvalues:
 0: -1.4563
 1: -1.4563
 2: -0.4828
 3: -0.4828
 4: 0.1994
 5: 0.1994
 6: 1.7397
 7: 1.7397

Candidate near-degenerate pairs:
 Pair (0, 1) with energies -1.4563, -1.4563
  State 0
 Dominant basis states:
  |110> : amplitude -0.972+0.000j
  |100> : amplitude 0.236+0.000j
  State 1
 Dominant basis states:
  |101> : amplitude 0.972+0.000j
  |111> : amplitude -0.236+0.000j

 Pair (2, 3) with energies -0.4828, -0.4828
  State 2
 Dominant basis states:
  |011> : amplitude -0.985+0.000j
  |001> : amplitude 0.174+0.000j
  State 3
 Dominant basis states:
  |000> : amplitude -0.985+0.000j
  |010> : amplitude 0.174+0.000j

 Pair (4, 5) with energies 0.1994, 0.1994
  State 4

Dominant basis states:
 |100> : amplitude -0.972+0.000j
 |110> : amplitude -0.236+0.000j
 State 5
Dominant basis states:
 |111> : amplitude -0.972+0.000j
 |101> : amplitude -0.236+0.000j

 Pair (6, 7) with energies 1.7397, 1.7397
 State 6
Dominant basis states:
 |001> : amplitude 0.985+0.000j
 |011> : amplitude 0.174+0.000j
 State 7
Dominant basis states:
 |010> : amplitude -0.985+0.000j
 |000> : amplitude -0.174+0.000j

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Seed 6 produced near-degenerate pairs
Hamiltonian:
SparsePauliOp(['XXX', 'XXX', 'IXX', 'ZZZ', 'XIZ', 'ZZZ'],
        coeffs=[-0.26186552+0.j, 0.97488998+0.j, -0.34007309+0.j, -0.75405525+0.j,
 -0.98220839+0.j, 0.65400605+0.j])
Eigenvalues:
 0: -1.4452
 1: -1.4452
 2: -1.0531
 3: -1.0531
 4: 1.0531
 5: 1.0531
 6: 1.4452
 7: 1.4452

Candidate near-degenerate pairs:
 Pair (0, 1) with energies -1.4452, -1.4452
 State 0
 Dominant basis states:
  |011> : amplitude 0.480+0.000j
  |101> : amplitude -0.462+0.000j
  |111> : amplitude -0.452+0.000j
  |001> : amplitude 0.435+0.000j

|000> : amplitude 0.236+0.000j
|110> : amplitude -0.227+0.000j
 State 1
Dominant basis states:
 |101> : amplitude 0.480+0.000j
 |011> : amplitude 0.462+0.000j
 |001> : amplitude -0.452+0.000j
 |111> : amplitude -0.435+0.000j
 |110> : amplitude 0.236+0.000j
 |000> : amplitude 0.227+0.000j

Pair (2, 3) with energies -1.0531, -1.0531
 State 2
Dominant basis states:
 |110> : amplitude 0.600+0.000j
 |010> : amplitude 0.585+0.000j
 |000> : amplitude -0.365+0.000j
 |100> : amplitude -0.356+0.000j
 |101> : amplitude -0.152+0.000j
 |011> : amplitude 0.092+0.000j
 State 3
Dominant basis states:
 |000> : amplitude -0.600+0.000j
 |100> : amplitude -0.585+0.000j
 |110> : amplitude -0.365+0.000j
 |010> : amplitude -0.356+0.000j
 |011> : amplitude 0.152+0.000j
 |101> : amplitude 0.092+0.000j

Pair (4, 5) with energies 1.0531, 1.0531
 State 4
Dominant basis states:
 |001> : amplitude 0.702+0.000j
 |101> : amplitude 0.684+0.000j
 |010> : amplitude 0.178+0.000j
 |110> : amplitude 0.081+0.000j
 State 5
Dominant basis states:
 |111> : amplitude 0.702+0.000j
 |011> : amplitude 0.684+0.000j
 |100> : amplitude 0.178+0.000j
 |000> : amplitude 0.081+0.000j

Pair (6, 7) with energies 1.4452, 1.4452
 State 6
Dominant basis states:
  |100> : amplitude -0.666+0.000j
  |000> : amplitude 0.627+0.000j
  |111> : amplitude 0.327+0.000j
  |011> : amplitude -0.237+0.000j
 State 7
Dominant basis states:
  |010> : amplitude 0.666+0.000j
  |110> : amplitude -0.627+0.000j
  |001> : amplitude -0.327+0.000j
  |101> : amplitude 0.237+0.000j

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Seed 12 produced near-degenerate pairs
Hamiltonian:
SparsePauliOp(['XIZ', 'ZII', 'XIZ', 'ZIZ', 'ZIX', 'XZI'],
        coeffs=[-0.62135923+0.j, -0.30022152+0.j, -0.76984124+0.j, 0.71626098+0.j,
-0.78629745+0.j, -0.16620792+0.j])
Eigenvalues:
 0: -2.3652
 1: -2.3287
 2: -0.9727
 3: -0.8803
 4: 0.8803
 5: 0.9727
 6: 2.3287
 7: 2.3652

Candidate near-degenerate pairs:
 Pair (0, 1) with energies -2.3652, -2.3287
 State 0
 Dominant basis states:
  |011> : amplitude 0.711+0.000j
  |111> : amplitude -0.423+0.000j
  |110> : amplitude 0.411+0.000j
  |010> : amplitude 0.382+0.000j
 State 1
 Dominant basis states:
  |001> : amplitude -0.610+0.000j
  |100> : amplitude -0.530+0.000j

|000> : amplitude -0.476+0.000j
|101> : amplitude 0.348+0.000j

Pair (6, 7) with energies 2.3287, 2.3652
 State 6
Dominant basis states:
 |101> : amplitude 0.610+0.000j
 |000> : amplitude -0.530+0.000j
 |100> : amplitude 0.476+0.000j
 |001> : amplitude 0.348+0.000j
 State 7
Dominant basis states:
 |111> : amplitude -0.711+0.000j
 |011> : amplitude -0.423+0.000j
 |010> : amplitude 0.411+0.000j
 |110> : amplitude -0.382+0.000j

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Seed 14 produced near-degenerate pairs
Hamiltonian:
SparsePauliOp(['IZX', 'XXZ', 'IXI', 'ZZX', 'ZZZ', 'IZZ'],
        coeffs=[0.40547861+0.j, 0.28263496+0.j, 0.43262877+0.j, 0.14491788+0.j,
 0.29375209+0.j, -0.20248231+0.j])
Eigenvalues:
 0: -0.9766
 1: -0.9766
 2: -0.4535
 3: -0.4535
 4: 0.4535
 5: 0.4535
 6: 0.9766
 7: 0.9766

Candidate near-degenerate pairs:
 Pair (0, 1) with energies -0.9766, -0.9766
 State 0
 Dominant basis states:
  |100> : amplitude 0.458+0.000j
  |111> : amplitude 0.448+0.000j
  |011> : amplitude -0.440+0.000j
  |010> : amplitude -0.420+0.000j
  |001> : amplitude 0.358+0.000j

|101> : amplitude -0.297+0.000j
 State 1
Dominant basis states:
 |111> : amplitude -0.458+0.000j
 |100> : amplitude 0.448+0.000j
 |000> : amplitude 0.440+0.000j
 |001> : amplitude -0.420+0.000j
 |010> : amplitude -0.358+0.000j
 |110> : amplitude -0.297+0.000j

Pair (2, 3) with energies -0.4535, -0.4535
 State 2
Dominant basis states:
 |111> : amplitude 0.546+0.000j
 |000> : amplitude 0.516+0.000j
 |001> : amplitude -0.431+0.000j
 |100> : amplitude -0.393+0.000j
 |010> : amplitude -0.215+0.000j
 |110> : amplitude 0.175+0.000j
 State 3
Dominant basis states:
 |100> : amplitude -0.546+0.000j
 |011> : amplitude -0.516+0.000j
 |010> : amplitude -0.431+0.000j
 |111> : amplitude -0.393+0.000j
 |001> : amplitude 0.215+0.000j
 |101> : amplitude 0.175+0.000j

Pair (4, 5) with energies 0.4535, 0.4535
 State 4
Dominant basis states:
 |110> : amplitude -0.650+0.000j
 |000> : amplitude 0.480+0.000j
 |001> : amplitude 0.459+0.000j
 |010> : amplitude 0.243+0.000j
 |100> : amplitude -0.177+0.000j
 |101> : amplitude 0.171+0.000j
 State 5
Dominant basis states:
 |101> : amplitude 0.650+0.000j
 |011> : amplitude 0.480+0.000j
 |010> : amplitude -0.459+0.000j
 |001> : amplitude 0.243+0.000j

|111> : amplitude 0.177+0.000j
|110> : amplitude 0.171+0.000j

Pair (6, 7) with energies 0.9766, 0.9766
 State 6
Dominant basis states:
  |110> : amplitude 0.639+0.000j
  |000> : amplitude 0.552+0.000j
  |001> : amplitude 0.335+0.000j
  |010> : amplitude 0.286+0.000j
  |100> : amplitude 0.235+0.000j
  |111> : amplitude -0.190+0.000j
 State 7
Dominant basis states:
  |101> : amplitude 0.639+0.000j
  |011> : amplitude -0.552+0.000j
  |010> : amplitude 0.335+0.000j
  |001> : amplitude -0.286+0.000j
  |111> : amplitude 0.235+0.000j
  |100> : amplitude 0.190+0.000j

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Seed 16 produced near-degenerate pairs
Hamiltonian:
SparsePauliOp(['XXZ', 'XXX', 'XII', 'ZZI', 'IIZ', 'ZIZ'],
        coeffs=[-0.81185235+0.j, 0.24301808+0.j, 0.70809906+0.j, 0.60484193+0.j,
-0.68999949+0.j, 0.91726766+0.j])
Eigenvalues:
 0: -2.7768
 1: -1.5425
 2: -1.2336
 3: -0.0007
 4: 0.0007
 5: 1.2336
 6: 1.5425
 7: 2.7768

Candidate near-degenerate pairs:
 Pair (3, 4) with energies -0.0007, 0.0007
  State 3
  Dominant basis states:
   |010> : amplitude -0.845+0.000j

|000> : amplitude -0.405+0.000j
|110> : amplitude -0.252+0.000j
|100> : amplitude 0.192+0.000j
|011> : amplitude 0.109+0.000j
|001> : amplitude 0.069+0.000j
 State 4
 Dominant basis states:
  |001> : amplitude -0.845+0.000j
  |011> : amplitude 0.405+0.000j
  |101> : amplitude 0.252+0.000j
  |111> : amplitude 0.192+0.000j
  |000> : amplitude -0.109+0.000j
  |010> : amplitude 0.069+0.000j

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Seed 22 produced near-degenerate pairs
Hamiltonian:
SparsePauliOp(['ZXX', 'IIX', 'IZI', 'ZII', 'IXX', 'IZI'],
        coeffs=[-0.82288325+0.j, -0.08132591+0.j, 0.67392265+0.j, 0.11069031+0.j,
 -0.04534409+0.j, -0.56702864+0.j])
Eigenvalues:
 0: -0.9769
 1: -0.8454
 2: -0.8142
 3: -0.6828
 4: 0.5928
 5: 0.7555
 6: 0.9041
 7: 1.0668

Candidate near-degenerate pairs:
 Pair (1, 2) with energies -0.8454, -0.8142
  State 1
  Dominant basis states:
   |011> : amplitude 0.530+0.000j
   |010> : amplitude 0.530+0.000j
   |000> : amplitude 0.468+0.000j
   |001> : amplitude 0.468+0.000j
  State 2
  Dominant basis states:
   |111> : amplitude 0.533+0.000j
   |110> : amplitude -0.533+0.000j

|100> : amplitude -0.465+0.000j
|101> : amplitude 0.465+0.000j

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Seed 31 produced near-degenerate pairs
Hamiltonian:
SparsePauliOp(['XZX', 'IIX', 'ZII', 'IZZ', 'IXZ', 'ZII'],
        coeffs=[0.34546163+0.j, 0.35485555+0.j, -0.64693972+0.j, -0.39568315+0.j,
 0.41017168+0.j, -0.15121728+0.j])
Eigenvalues:
 0: -1.5305
 1: -1.5305
 2: -0.2679
 3: -0.2679
 4: 0.2679
 5: 0.2679
 6: 1.5305
 7: 1.5305

Candidate near-degenerate pairs:
 Pair (0, 1) with energies -1.5305, -1.5305
  State 0
 Dominant basis states:
  |011> : amplitude -0.872+0.000j
  |001> : amplitude -0.345+0.000j
  |010> : amplitude 0.298+0.000j
  |110> : amplitude -0.134+0.000j
  |100> : amplitude 0.090+0.000j
  |111> : amplitude 0.078+0.000j
  State 1
 Dominant basis states:
  |000> : amplitude 0.872+0.000j
  |010> : amplitude -0.345+0.000j
  |001> : amplitude -0.298+0.000j
  |101> : amplitude -0.134+0.000j
  |111> : amplitude -0.090+0.000j
  |100> : amplitude 0.078+0.000j

 Pair (2, 3) with energies -0.2679, -0.2679
  State 2
 Dominant basis states:
  |001> : amplitude 0.620+0.000j

|011> : amplitude -0.467+0.000j
|010> : amplitude -0.451+0.000j
|100> : amplitude -0.353+0.000j
|111> : amplitude -0.257+0.000j
|110> : amplitude 0.051+0.000j
 State 3
Dominant basis states:
 |010> : amplitude 0.620+0.000j
 |000> : amplitude 0.467+0.000j
 |001> : amplitude 0.451+0.000j
 |111> : amplitude 0.353+0.000j
 |100> : amplitude -0.257+0.000j
 |101> : amplitude 0.051+0.000j

Pair (4, 5) with energies 0.2679, 0.2679
 State 4
Dominant basis states:
 |111> : amplitude -0.557+0.000j
 |100> : amplitude 0.527+0.000j
 |101> : amplitude -0.468+0.000j
 |010> : amplitude 0.317+0.000j
 |001> : amplitude 0.300+0.000j
 |000> : amplitude 0.051+0.000j
 State 5
Dominant basis states:
 |100> : amplitude 0.557+0.000j
 |111> : amplitude 0.527+0.000j
 |110> : amplitude -0.468+0.000j
 |001> : amplitude 0.317+0.000j
 |010> : amplitude -0.300+0.000j
 |011> : amplitude -0.051+0.000j

Pair (6, 7) with energies 1.5305, 1.5305
 State 6
Dominant basis states:
 |110> : amplitude 0.871+0.000j
 |100> : amplitude 0.335+0.000j
 |111> : amplitude 0.308+0.000j
 |011> : amplitude -0.134+0.000j
 |001> : amplitude 0.088+0.000j
 |010> : amplitude -0.081+0.000j
 State 7
Dominant basis states:

|101> : amplitude 0.871+0.000j
|111> : amplitude -0.335+0.000j
|100> : amplitude 0.308+0.000j
|000> : amplitude 0.134+0.000j
|010> : amplitude 0.088+0.000j
|001> : amplitude 0.081+0.000j

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Seed 32 produced near-degenerate pairs
Hamiltonian:
SparsePauliOp(['ZIZ', 'XII', 'XZZ', 'ZXZ', 'XIZ', 'XIZ'],
        coeffs=[-0.24572889+0.j, 0.37322778+0.j, 0.34259684+0.j, -0.08057632+0.j,
 0.20244675+0.j, -0.18308528+0.j])
Eigenvalues:
 0: -0.7847
 1: -0.7492
 2: -0.2468
 3: -0.2401
 4: 0.2401
 5: 0.2468
 6: 0.7492
 7: 0.7847

Candidate near-degenerate pairs:
 Pair (0, 1) with energies -0.7847, -0.7492
  State 0
 Dominant basis states:
  |000> : amplitude 0.806+0.000j
  |100> : amplitude -0.578+0.000j
  |010> : amplitude 0.117+0.000j
  |110> : amplitude 0.040+0.000j
  State 1
 Dominant basis states:
  |111> : amplitude -0.810+0.000j
  |011> : amplitude 0.571+0.000j
  |101> : amplitude -0.129+0.000j
  |001> : amplitude -0.045+0.000j

 Pair (2, 3) with energies -0.2468, -0.2401
  State 2
 Dominant basis states:
  |010> : amplitude -0.985+0.000j

|110> : amplitude 0.118+0.000j
  |100> : amplitude -0.108+0.000j
  |000> : amplitude 0.060+0.000j
 State 3
 Dominant basis states:
  |101> : amplitude -0.990+0.000j
  |011> : amplitude -0.114+0.000j
  |111> : amplitude 0.075+0.000j
  |001> : amplitude 0.042+0.000j

 Pair (4, 5) with energies 0.2401, 0.2468
 State 4
 Dominant basis states:
  |001> : amplitude 0.990+0.000j
  |111> : amplitude -0.114+0.000j
  |011> : amplitude -0.075+0.000j
  |101> : amplitude 0.042+0.000j
 State 5
 Dominant basis states:
  |110> : amplitude -0.985+0.000j
  |010> : amplitude -0.118+0.000j
  |000> : amplitude 0.108+0.000j
  |100> : amplitude 0.060+0.000j

 Pair (6, 7) with energies 0.7492, 0.7847
 State 6
 Dominant basis states:
  |011> : amplitude 0.810+0.000j
  |111> : amplitude 0.571+0.000j
  |001> : amplitude 0.129+0.000j
  |101> : amplitude -0.045+0.000j
 State 7
 Dominant basis states:
  |100> : amplitude -0.806+0.000j
  |000> : amplitude -0.578+0.000j
  |110> : amplitude -0.117+0.000j
  |010> : amplitude 0.040+0.000j

====================================
Seed 34 produced near-degenerate pairs
Hamiltonian:
SparsePauliOp(['ZII', 'IXI', 'ZXZ', 'XZI', 'ZXZ'],

coeffs=[0.3021226 +0.j, 0.76144513+0.j, 0.04198579+0.j, 0.35553159+0.j,
-0.6425825 +0.j])
Eigenvalues:
 0: -1.7220
 1: -1.1814
 2: -0.5208
 3: -0.0198
 4: 0.0198
 5: 0.5208
 6: 1.1814
 7: 1.7220

Candidate near-degenerate pairs:
 Pair (3, 4) with energies -0.0198, 0.0198
  State 3
 Dominant basis states:
  |111> : amplitude 0.669+0.000j
  |101> : amplitude 0.669+0.000j
  |011> : amplitude -0.229+0.000j
  |001> : amplitude 0.229+0.000j
  State 4
 Dominant basis states:
  |000> : amplitude 0.669+0.000j
  |010> : amplitude -0.669+0.000j
  |110> : amplitude -0.229+0.000j
  |100> : amplitude -0.229+0.000j

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Seed 35 produced near-degenerate pairs
Hamiltonian:
SparsePauliOp(['IXZ', 'XIZ', 'XIX', 'XIZ', 'ZZZ', 'IZX'],
        coeffs=[0.8700137 +0.j, -0.80642937+0.j, 0.81665193+0.j, 0.80873803+0.j,
 0.95796527+0.j, -0.99794816+0.j])
Eigenvalues:
 0: -2.4878
 1: -2.4869
 2: -0.7000
 3: -0.6969
 4: 0.6969
 5: 0.7000
 6: 2.4869
 7: 2.4878

Candidate near-degenerate pairs:
 Pair (0, 1) with energies -2.4878, -2.4869
  State 0
 Dominant basis states:
  |010> : amplitude 0.549+0.000j
  |001> : amplitude -0.549+0.000j
  |011> : amplitude -0.318+0.000j
  |000> : amplitude -0.318+0.000j
  |111> : amplitude -0.301+0.000j
  |100> : amplitude 0.301+0.000j
  State 1
 Dominant basis states:
  |100> : amplitude 0.549+0.000j
  |111> : amplitude 0.549+0.000j
  |101> : amplitude 0.318+0.000j
  |110> : amplitude -0.318+0.000j
  |001> : amplitude -0.300+0.000j
  |010> : amplitude -0.300+0.000j

 Pair (2, 3) with energies -0.7000, -0.6969
  State 2
 Dominant basis states:
  |001> : amplitude -0.630+0.000j
  |010> : amplitude -0.630+0.000j
  |110> : amplitude 0.249+0.000j
  |101> : amplitude -0.249+0.000j
  |100> : amplitude -0.188+0.000j
  |111> : amplitude -0.188+0.000j
  State 3
 Dominant basis states:
  |111> : amplitude -0.630+0.000j
  |100> : amplitude 0.630+0.000j
  |000> : amplitude 0.249+0.000j
  |011> : amplitude 0.249+0.000j
  |010> : amplitude -0.189+0.000j
  |001> : amplitude 0.189+0.000j

 Pair (4, 5) with energies 0.6969, 0.7000
  State 4
 Dominant basis states:
  |101> : amplitude -0.630+0.000j
  |110> : amplitude -0.630+0.000j

|010> : amplitude 0.249+0.000j
  |001> : amplitude -0.249+0.000j
  |000> : amplitude 0.189+0.000j
  |011> : amplitude 0.189+0.000j
 State 5
 Dominant basis states:
  |011> : amplitude -0.630+0.000j
  |000> : amplitude 0.630+0.000j
  |100> : amplitude 0.249+0.000j
  |111> : amplitude 0.249+0.000j
  |110> : amplitude 0.188+0.000j
  |101> : amplitude -0.188+0.000j

 Pair (6, 7) with energies 2.4869, 2.4878
 State 6
 Dominant basis states:
  |101> : amplitude -0.549+0.000j
  |110> : amplitude 0.549+0.000j
  |100> : amplitude 0.318+0.000j
  |111> : amplitude 0.318+0.000j
  |000> : amplitude -0.300+0.000j
  |011> : amplitude 0.300+0.000j
 State 7
 Dominant basis states:
  |011> : amplitude -0.549+0.000j
  |000> : amplitude -0.549+0.000j
  |010> : amplitude -0.318+0.000j
  |001> : amplitude 0.318+0.000j
  |110> : amplitude -0.301+0.000j
  |101> : amplitude -0.301+0.000j

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Seed 38 produced near-degenerate pairs
Hamiltonian:
SparsePauliOp(['IXI', 'IXX', 'IIX', 'ZZX', 'ZII', 'IXZ'],
        coeffs=[0.44076233+0.j, 0.86514348+0.j, -0.0068059 +0.j, -0.03082899+0.j,
 0.47112787+0.j, -0.20213048+0.j])
Eigenvalues:
 0: -1.8074
 1: -0.9131
 2: -0.8651
 3: -0.0159

4: 0.0291
5: 0.8518
6: 0.9264
7: 1.7941

Candidate near-degenerate pairs:
 Pair (1, 2) with energies -0.9131, -0.8651
 State 1
 Dominant basis states:
  |100> : amplitude -0.576+0.000j
  |110> : amplitude -0.532+0.000j
  |101> : amplitude 0.449+0.000j
  |111> : amplitude 0.428+0.000j
 State 2
 Dominant basis states:
  |001> : amplitude 0.556+0.000j
  |011> : amplitude -0.551+0.000j
  |000> : amplitude 0.449-0.000j
  |010> : amplitude -0.431+0.000j

 Pair (3, 4) with energies -0.0159, 0.0291
 State 3
 Dominant basis states:
  |110> : amplitude -0.575+0.000j
  |100> : amplitude 0.531+0.000j
  |111> : amplitude 0.450+0.000j
  |101> : amplitude -0.430+0.000j
 State 4
 Dominant basis states:
  |010> : amplitude -0.576+0.000j
  |000> : amplitude -0.532+0.000j
  |011> : amplitude 0.449+0.000j
  |001> : amplitude 0.428+0.000j

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Seed 45 produced near-degenerate pairs
Hamiltonian:
SparsePauliOp(['ZXZ', 'XZZ', 'XZX', 'ZIX', 'XXZ', 'ZZX'],
        coeffs=[0.52730048+0.j, 0.02045771+0.j, 0.18951001+0.j, 0.34255288+0.j,
 0.44832698+0.j, 0.92813184+0.j])
Eigenvalues:
 0: -1.7046

1: -1.6672
2: -0.3984
3: -0.3643
4: 0.3643
5: 0.3984
6: 1.6672
7: 1.7046

Candidate near-degenerate pairs:
 Pair (0, 1) with energies -1.7046, -1.6672
  State 0
 Dominant basis states:
  |101> : amplitude -0.473+0.000j
  |000> : amplitude 0.473-0.000j
  |100> : amplitude -0.380+0.000j
  |001> : amplitude -0.380+0.000j
  |110> : amplitude -0.331+0.000j
  |011> : amplitude -0.331+0.000j
  State 1
 Dominant basis states:
  |001> : amplitude 0.473+0.000j
  |100> : amplitude -0.473+0.000j
  |000> : amplitude -0.379+0.000j
  |101> : amplitude -0.379+0.000j
  |010> : amplitude 0.334+0.000j
  |111> : amplitude 0.334+0.000j

 Pair (2, 3) with energies -0.3984, -0.3643
  State 2
 Dominant basis states:
  |010> : amplitude -0.503+0.000j
  |111> : amplitude 0.503+0.000j
  |100> : amplitude 0.343+0.000j
  |001> : amplitude 0.343+0.000j
  |011> : amplitude -0.339+0.000j
  |110> : amplitude -0.339+0.000j
  State 3
 Dominant basis states:
  |011> : amplitude -0.511+0.000j
  |110> : amplitude 0.511+0.000j
  |000> : amplitude -0.348+0.000j
  |101> : amplitude -0.348+0.000j
  |010> : amplitude -0.326+0.000j

|111> : amplitude -0.326+0.000j

Pair (4, 5) with energies 0.3643, 0.3984
 State 4
Dominant basis states:
 |111> : amplitude -0.511+0.000j
 |010> : amplitude -0.511+0.000j
 |100> : amplitude -0.348+0.000j
 |001> : amplitude 0.348+0.000j
 |110> : amplitude -0.326+0.000j
 |011> : amplitude 0.326+0.000j
 State 5
Dominant basis states:
 |011> : amplitude 0.503+0.000j
 |110> : amplitude 0.503+0.000j
 |000> : amplitude 0.343+0.000j
 |101> : amplitude -0.343+0.000j
 |010> : amplitude -0.339+0.000j
 |111> : amplitude 0.339+0.000j

Pair (6, 7) with energies 1.6672, 1.7046
 State 6
Dominant basis states:
 |101> : amplitude 0.473+0.000j
 |000> : amplitude 0.473+0.000j
 |100> : amplitude -0.379+0.000j
 |001> : amplitude 0.379+0.000j
 |110> : amplitude 0.334+0.000j
 |011> : amplitude -0.334+0.000j
 State 7
Dominant basis states:
 |001> : amplitude -0.473+0.000j
 |100> : amplitude -0.473+0.000j
 |000> : amplitude -0.380+0.000j
 |101> : amplitude 0.380+0.000j
 |010> : amplitude -0.331+0.000j
 |111> : amplitude 0.331+0.000j

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Seed 46 produced near-degenerate pairs
Hamiltonian:
SparsePauliOp(['XZI', 'IIX', 'ZIZ', 'XIX', 'ZZX', 'XZI'],

```
        coeffs=[-0.45486049+0.j,  0.86805682+0.j,  0.55114349+0.j,  0.49701544+0.j,
  0.58981023+0.j,  0.46071836+0.j])
```
Eigenvalues:
 0: -1.7639
 1: -1.7569
 2: -0.4567
 3: -0.4497
 4: 0.4497
 5: 0.4567
 6: 1.7569
 7: 1.7639

Candidate near-degenerate pairs:
 Pair (0, 1) with energies -1.7639, -1.7569
  State 0
  Dominant basis states:
   |110> : amplitude -0.765+0.000j
   |111> : amplitude 0.516+0.000j
   |011> : amplitude 0.352+0.000j
   |010> : amplitude -0.155+0.000j
  State 1
  Dominant basis states:
   |001> : amplitude 0.767+0.000j
   |000> : amplitude -0.516+0.000j
   |100> : amplitude -0.349+0.000j
   |101> : amplitude 0.151+0.000j

 Pair (2, 3) with energies -0.4567, -0.4497
  State 2
  Dominant basis states:
   |100> : amplitude 0.918+0.000j
   |000> : amplitude -0.297+0.000j
   |001> : amplitude 0.239+0.000j
   |101> : amplitude -0.108+0.000j
  State 3
  Dominant basis states:
   |011> : amplitude -0.917+0.000j
   |111> : amplitude 0.296+0.000j
   |110> : amplitude -0.243+0.000j
   |010> : amplitude 0.107+0.000j

 Pair (4, 5) with energies 0.4497, 0.4567
  State 4

Dominant basis states:
　|101> : amplitude 0.917+0.000j
　|001> : amplitude -0.296+0.000j
　|000> : amplitude -0.243+0.000j
　|100> : amplitude 0.107+0.000j
　State 5
Dominant basis states:
　|010> : amplitude 0.918+0.000j
　|110> : amplitude -0.297+0.000j
　|111> : amplitude -0.239+0.000j
　|011> : amplitude 0.108+0.000j

　Pair (6, 7) with energies 1.7569, 1.7639
　State 6
Dominant basis states:
　|111> : amplitude -0.767+0.000j
　|110> : amplitude -0.516+0.000j
　|010> : amplitude -0.349+0.000j
　|011> : amplitude -0.151+0.000j
　State 7
Dominant basis states:
　|000> : amplitude -0.765+0.000j
　|001> : amplitude -0.516+0.000j
　|101> : amplitude -0.352+0.000j
　|100> : amplitude -0.155+0.000j

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Seed 47 produced near-degenerate pairs
Hamiltonian:
SparsePauliOp(['IZX', 'ZII', 'XIZ', 'IIX', 'IZZ', 'ZZZ'],
　　　coeffs=[-0.06963892+0.j, 0.9337037 +0.j, 0.71551838+0.j, -0.64151989+0.j,
　0.10765781+0.j, 0.22173769+0.j])
Eigenvalues:
　0: -1.8018
　1: -1.6765
　2: -0.8130
　3: -0.7646
　4: 0.7014
　5: 0.7368
　6: 1.7527
　7: 1.8650

Candidate near-degenerate pairs:
 Pair (2, 3) with energies -0.8130, -0.7646
  State 2
 Dominant basis states:
  |110> : amplitude 0.625+0.000j
  |111> : amplitude -0.561+0.000j
  |010> : amplitude -0.443+0.000j
  |011> : amplitude -0.315+0.000j
  State 3
 Dominant basis states:
  |101> : amplitude -0.580+0.000j
  |100> : amplitude 0.539+0.000j
  |001> : amplitude -0.491+0.000j
  |000> : amplitude -0.363+0.000j

 Pair (4, 5) with energies 0.7014, 0.7368
  State 4
 Dominant basis states:
  |001> : amplitude 0.683+0.000j
  |101> : amplitude -0.491+0.000j
  |000> : amplitude 0.401-0.000j
  |100> : amplitude 0.364+0.000j
  State 5
 Dominant basis states:
  |010> : amplitude 0.741+0.000j
  |110> : amplitude 0.452+0.000j
  |011> : amplitude 0.394+0.000j
  |111> : amplitude -0.303+0.000j

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Seed 48 produced near-degenerate pairs
Hamiltonian:
SparsePauliOp(['IXX', 'XIZ', 'ZZX', 'XZZ', 'ZIX', 'IIX'],
        coeffs=[0.02760109+0.j, 0.3049935 +0.j, 0.05594665+0.j, -0.57891708+0.j,
-0.04933467+0.j, -0.64070433+0.j])
Eigenvalues:
 0: -1.1984
 1: -0.9889
 2: -0.7010
 3: -0.6888
 4: 0.6888
 5: 0.7010

6: 0.9889
7: 1.1984

Candidate near-degenerate pairs:
 Pair (2, 3) with energies -0.7010, -0.6888
  State 2
 Dominant basis states:
  |101> : amplitude 0.690+0.000j
  |100> : amplitude 0.690+0.000j
  |001> : amplitude -0.141+0.000j
  |000> : amplitude 0.141+0.000j
  |110> : amplitude 0.057+0.000j
  |111> : amplitude 0.057+0.000j
  State 3
 Dominant basis states:
  |000> : amplitude 0.692+0.000j
  |001> : amplitude 0.692+0.000j
  |100> : amplitude 0.141+0.000j
  |101> : amplitude -0.141+0.000j
  |011> : amplitude 0.032+0.000j
  |010> : amplitude 0.032+0.000j

 Pair (4, 5) with energies 0.6888, 0.7010
  State 4
 Dominant basis states:
  |000> : amplitude 0.692+0.000j
  |001> : amplitude -0.692+0.000j
  |100> : amplitude -0.141+0.000j
  |101> : amplitude -0.141+0.000j
  |011> : amplitude -0.032+0.000j
  |010> : amplitude 0.032+0.000j
  State 5
 Dominant basis states:
  |101> : amplitude 0.690+0.000j
  |100> : amplitude -0.690+0.000j
  |001> : amplitude 0.141+0.000j
  |000> : amplitude 0.141+0.000j
  |111> : amplitude 0.057+0.000j
  |110> : amplitude -0.057+0.000j

--- End of v0.4 experiment ---

5. Interpretation

The v0.4 results are consistent with earlier 2-qubit experiments and strengthen the core hypothesis of this project:

• Even for small 3-qubit systems, random Hamiltonians frequently exhibit structured degeneracies.
• The associated eigenvectors are not arbitrary noise; they display clear patterns in the computational basis, often dominated by a small number of bitstrings.
• Partner states in a near-degenerate pair often show mirrored amplitude patterns, suggesting an underlying symmetry of the Hamiltonian.

This behaviour supports the idea that the Hilbert space contains naturally occurring, low-dimensional invariant subspaces that could be used as physics-native information units, rather than imposing hand-crafted qubit encodings on top of the dynamics.

6. Limitations

The current v0.4 experiment has several important limitations:

• System size: Only 3-qubit Hamiltonians were explored in this run.
• Classical simulation: All diagonalisation was performed classically; no quantum backend was used.
• No noise modelling: The analysis does not yet include decoherence, control errors, or hardware noise.
• No stability metrics: We have not quantified how robust each near-degenerate subspace is to perturbations of the Hamiltonian.

7. Roadmap toward v0.5

The next prototype (v0.5) will build directly on v0.4 and aim to:

1. Integrate execution on a quantum backend (e.g., IBM Quantum) for selected Hamiltonians.
2. Introduce perturbation-based stability analysis of near-degenerate subspaces.
3. Apply clustering or other AI methods to group eigenstates by structural similarity.
4. Expand the Hamiltonian library to include both random and physically motivated models.
5. Define preliminary metrics for "physics-native" information quality (e.g., robustness, locality, and implementability on real devices).

8. Conclusion

Experiment v0.4 demonstrates that structured near-degenerate subspaces are not rare exceptions but recurring features of random 3-qubit Hamiltonians. The eigenstate patterns observed here provide concrete, data-driven support for the broader programme of discovering physics-native quantum information units using systematic scanning and AI-

assisted analysis.

TO BE CONTINUED.