# Understanding Equilibrium

### OVERVIEW & OBJECTIVES

The concept of equilibrium is foundational for understanding game theory, so we're going to dedicate an entire chapter to it before we start considering poker explicitly. We'll consider instead a few "games" so simple that, although you have played them may times, you probably never thought of as exercises in game theory. They are, though!

By investigating how equilibrium applies in these simple cases, we'll build your understanding of the concept in general, leaving you better equipped to make use of it in the more complicated scenarios that poker presents. We'll also talk a bit about other important game theoretic concepts such as strategy, and about how computers can assist us in solving the equilibria of more complex games.

By the end of this chapter, you should be able to:

* Define "equilibrium" and "strategy" as these terms are used in game theory.
* Understand what it means for two strategies to be in equilibrium.
* Recognize when a pair of strategies is not an equilibrium.
* Explain the difference between pure strategies and mixed strategies and know when each arises.
* Randomize your play in an unpredictable and unexploitable fashion.

### WHAT IS A STRATEGY?

Strategy is a term with a very specific meaning in game theory. Wikipedia defines it as "a complete algorithm for playing the game, telling a player what to do for every possible situation throughout the game." For our purposes, a poker strategy will describe not just a vague description of style ("tight and aggressive", for instance) but rather exactly what you will do in every possible situation, with every hand you could possibly have in that situation.

For example, my pre-flop strategy for playing in first position at a nine-handed table might be to raise three times the big blind with a pair of 7s or better, AQo or better, and any suited broadway hand, to raise 20% of the time that I have suited connectors T9s through 76s, and to fold everything else. To really be complete, I'd also need to describe how I'd respond to three-bets of various sizes from each opponent. This is more information than most human brains can store and process at once, which is why computers are such valuable tools for studying game theory rigorously. When actually playing poker, we won't be able to formulate strategies with nearly the same precision, but we still must consider how we will handle future situations we could encounter, both before and after the flop, in order to make good deci- sions in the current situation.

We'll also need to have some idea about our opponent's likely strategy, or at least about what strategies will be available to him. Knowing that we'll be vulnerable to bluffs on certain rivers, for instance, might inform which hands we call with on the turn.

### 

### NASH EQUILIBRIUM

An equilibrium, sometimes called a Nash Equilibrium after mathematician John Nash, is a set of strategies for all players in a game such that no player has incentive to deviate unilaterally from his strategy. For example, if you and I were playing equilibrium strategies in a heads up poker game, I could tell you exactly how I would play every hand in every possible situation, and you could give me the same information, and neither of us would want to change anything about our strategies even after we had perfect information about how the other would play. That formal definition makes it sound more complicated than it is, so let's look at a simple example.

Two cars approach each other on a road, traveling in opposite directions. Each driver can choose to stay to the right side of the road (from his own perspective), to stay to the left side of the road, or to proceed straight down the middle. Neither driver has a preference for which part of the street he drives on, but both would strongly prefer to avoid a collision.

This is not a game in the ordinary sense, but for our purposes it is. There are multiple players, each can choose from multiple moves, and the payoff for each player depends on both his own choice and that of the other player. In other words, without knowing what the other player will do, a player cannot predict what the result of his own choice will be. If he moves to the right, he might continue safely on his way, or he might get a collision - the result depends on what the other driver chooses.

The grid below represents the entire game. The options along the left side are the choices that the first driver can make, and the options along the top are the choices that the second driver can make. The box at the intersection of two options represents the payoffs for both players if they choose those options. The first number in each pair represents the payoff for the first driver, and the second number the payoff for the second driver. Successfully passing each other is scored as a 1, while a collision, which would be very bad news for both drivers, is scored as a -10.

#### ### Table: Chicken Game Payoff Matrix

meta:

id: chicken\_payoff\_matrix

rows: "Driver 1 actions"

cols: "Driver 2 actions"

cell: "(Row payoff, Column payoff)"

| | Driver 2: Right | Driver 2: Middle | Driver 2: Left |

|-----------------|------------------|------------------|----------------|

| Driver 1: Right | (1,1) | (-10,-10) | (-10,-10) |

| Driver 1: Middle| (-10,-10) | (-10,-10) | (-10,-10) |

| Driver 1: Left | (-10,-10) | (-10,-10) | (1,1) |

The highlighted boxes represent two different strategy pairs that yield equally good results for both players: each can drive on the right or the left side of the road. If both players consistently make the same choice, then that's an equilibrium. They both get what they want, which is to keep on driving without a collision, and neither has any incentive to do anything different.

There's no intrinsic reason why consistently staying to the right is any better or worse than consistently staying to the left, and indeed in the real world there are places where both equilibria prevail. In the United States, the convention is to drive on the right. In the UK and some of its former colonies, the convention is to drive on the left. It doesn't really matter which you choose, as long as everyone is on the same page about what to do when the situation arises.

A rational player might choose to drive either to the right or to the left, but she would never drive straight down the middle. That can only lead to collision, which is not in either player's interest. Although it's technically a move in the game, there's no reason to ever use it. In the vocabulary of game theory, this is called a dominated strategy.

### PROVING AN EQUILIBRIUM

These two equilibria - both players stay to the right or both players stay to the left - are so clear and intuitive that you probably don't need me to prove them to you. It's good to know how to prove or disprove that a certain strategy pair is an equilibrium, though, because in the future it won't always be so obvious.

Recall that the definition of equilibrium requires that neither player can unilaterally deviate from his strategy and achieve a better result. To prove that two strategies are at equilibrium, then, we must experiment with changing one player's strategy at a time to see whether that player can get a higher payoff. If both players change strategies at the same time, that could lead to a higher payoff, but it wouldn't prove any- thing about whether the original strategies were at equilibrium.

Looking at the grid, we can see that it is not possible for any player to achieve a payoff higher than 1. Thus, any box where both players have a payoff of 1 is an equilibrium.

Now let's consider the case where Driver 1 stays to his right while Driver 2 stays to his left. This produces a payoff of -10 for both players. We know that a higher payoff is possible, but that by itself isn't enough to disprove equilibrium. We must demonstrate that one player can achieve a higher payoff solely by changing his own strategy. That is, we will keep the other player's strategy fixed and try to find a way for just one player to change his own strategy and achiever a higher payoff as a result.

In this case, either player can do so. Driver 1 could change his strategy and drive on the left. That would improve his payoff (and his opponent's, though that isn't important for this purpose) to 1. That right there is enough to prove that this is not an equilibrium, but of course Driver 2 could also unilaterally improve his payout by driving on the right. If both players changed their strategies, there would be a problem, but that's not the test of equilibrium.

### SCENARIO: GOING TO THE MOVIES

Now we're going to look at some more simple scenarios drawn from everyday interactions (though they may require a little suspension of disbelief). I'll explain each scenario to you, then ask you to answer some questions about it, based on what you've learned about the concept of equilibrium so far.

Two friends, Andrew and Blanca, want to go to a movie together. Andrew would prefer to see a drama, and Blanca would prefer to see a comedy, but the most important thing to both of them is that whatever they see, they see it together. That is, Andrew would rather see a com- edy with Blanca than a drama by himself, and vice versa.

Like the previous example, this may not fit the colloquial idea of a “game”, but it has multiple players, each of whom can choose from two different moves (go to a comedy or go to a drama), and the payoff for each player depends what that the other chooses.

One small suspension of disbelief is required to make this example work: you must imagine that each player chooses which movie to see independently, without consulting the other. Andrew may know Blanca's strategy for choosing a movie, but he will not know her actual choice. That is, if Blanca's strategy were to flip a coin, you can assume Andrew knows she will flip a coin but not the result of the flip. (If it helps, you could imagine the two agreed to go to the movies without deciding exactly what to see, and that Andrew subsequently lost his phone, thereby preventing them from coordinating their choices.)

We'll want to do some simple math with this scenario, so we need to put numeric values on our players' preferences. For Andrew, going to a drama with Blanca is the best-case scenario, so we'll make that worth five points. Going to a comedy with Blanca is nearly as good, call it four points. Going to a drama by himself is one point, and going to a comedy by himself is the worst outcome for him, so we'll call that zero points. For Blanca, it's the opposite: going to a comedy with Andrew is five points, and going to a drama with Andrew is four points. Going to a com- edy alone is one point, and going to a drama alone is zero points.

Now we can represent the entire game, including both players' strategic options and their payoffs (see table “### Table: Andrew vs Blanca — Movie Choice (Payoff Matrix)”):

#### ### Table: Andrew vs Blanca — Movie Choice (Payoff Matrix)

meta:

id: movie\_choice\_andrew\_blanca

rows: "Andrew actions"

cols: "Blanca actions"

cell: "(Andrew payoff, Blanca payoff)"

source\_format: "column-by-column blocks"

| | Blanca: Go to Comedy | Blanca: Go to Drama |

|---------------------|----------------------|---------------------|

| Andrew: Go to Comedy| (4,5) | (0,0) |

| Andrew: Go to Drama | (1,1) | (5,4) |

### Questions & Answers: Going to the Movies

Q1: Suppose both players play a strategy of always going to the comedy. Is this an equilibrium? How can you tell?

A1: Yes, this is an equilibrium. To test it, we check whether either player can unilaterally change his or her strategy to achieve a higher payoff. Blanca already has her highest possible payout with this set of strategies, so she's got no incentive to change.

What about Andrew? He was more interested in the drama than the comedy, so let's see what happens if he changes his strategy and goes to see the drama. Then the players end up in in the lower left quadrant, where Andrew has a payoff of 1, which is lower than what it would be if he stuck with the comedy. Even though Andrew wanted to see the drama, what he wanted most of all was to go to the movies with Blanca, and that won't happen if he unilaterally changes his strategy.

The "unilaterally" qualifier is important here. Even though this set of strategies does not give Andrew his highest possible payoff, the only way he can achieve a better outcome is if both players change their strategies, so it is an equilibrium, and he has no incentive to deviate from it.

Q2: What if Andrew's strategy is always to go to the drama and Blanca's is to always go to the comedy? Is this an equilibrium? How can you tell?

A2: This is not an equilibrium, because either player could achieve a better payoff by changing his or her strategy. If Andrew changes strategies and goes to the comedy, his payoff increases from 1 to 4. If Blanca changes strategies and goes to the drama, her payoff increases from 1 to 4. The only way they wouldn't achieve a better outcome would be if they both changed strategies, but that wouldn't be a unilateral change.

### EQUITY AND EXPECTED VALUE

Poker is full of uncertainty; you don't need a book to tell you that. We're going to talk about how to deal with that uncertainty, how to make good decisions despite it, and even how to quantify the value of the choices available to you when you cannot be sure about the results of those choices.

Many poker players have unrealistic expectations of certainty. They do not want to bluff unless they are sure that their opponent will fold. They do not want to value bet the river unless they are sure that their hand is best. Some do not even want to get all-in with Kings before they see the flop to be sure it does not contain an Ace!

Probably you recognize the fallacy of at least some of this logic already. Poker is not about certainty, it is about learning to make good decisions under conditions of uncertainty. The most successful players accept - embrace, even - the fact that they will frequently be unsure of whether a given decision will result in a win, or even that it will have a positive expectation.

Uncertainty in poker comes in many forms. Most fundamentally, before the river, you do not know which cards will come. Nor do you know the cards that your opponents hold. Even if you could determine an opponent's range - the set of all hands that he might hold given his actions so far - you wouldn't know which hand he actually holds. And even if you knew an opponent's exact hand, you still might not be able to pre- dict his future actions, such as whether he will call a large bet, or whether he will bet if you check.

Equity and Expected Value are two different ways of quantifying uncertainty in poker, putting a price on a situation where we don't know which cards are going to come or what actions other players will take.

When choosing between different options, it helps to put a dollar value on each to enable a direct comparison. However, when your payoff depends on the choices that another player makes, you can't know what your payoff will be. You may know the possibilities - going to a drama could yield a payoff of either 1 or 5 for Andrew, depending on what Blanca chooses - but that by itself is not enough to determine what it is worth to Andrew to choose the drama.

Equity and Expected Value can help us to work around this uncertainty and determine what a choice is worth to a player even when we don't have perfect information about future cards or actions.

Equity Quantifies Uncertainty About Future Board Cards

Equity, at least as the term is widely used in the poker world, is a measure of how much of the pot a given hand or range would win if there were no further betting.

The advantage of equity as a means of estimating value is that it can be calculated easily and precisely with the help of equity calculator software. An equity calculator takes two or more hands and/or ranges as inputs, generates 100,000 or more possible boards, and tracks how often each wins, loses, and ties. In less than a second, it returns the average win or loss per trial for each hand or range.

For example, an equity calculator can tell us that before the flop, AKo has about 30% equity versus KK. Versus a range of {AK,KK,AA}, it has about 37% equity. If you were to get all-in before the flop with AKo versus KK, you would win about 30% of the final pot, on average. If you were all-in against a range of {AK,KK,AA}, you'd average about 37% of the pot.

By measuring our hand's equity, we remove uncertainty about what the final board will be. Of course, we don't know whether our AKo will win or lose in any particular instance, but we can still put a price on what it is worth to get all-in against a particular hand or range.

Even if you are not all-in, it can still be useful to know how much of the pot you could expect to win in the absence of additional betting. Because there will be additional betting, however, equity is only a very rough measure of what your hand will actually win. You may end up folding to a bluff, or successfully bluffing yourself, or paying off a value bet, or making a value bet. All of these possibilities and more will affect what it's actually worth to you to hold AKo versus KK in a given pre-flop situation.

Expected Value Quantifies Uncertainty About Future Actions

Expected Value, or EV, is a measure that attempts to take future actions by both you and your opponents into account. For this reason, though, it requires more estimation and is harder to calculate than equity.

For example, if I bluff $50 into a $100 pot on the river, I don't know whether you are going to call or fold. However, I can express the value of this bluff as a function of your raising, calling, and folding frequencies. When you fold, I win $100. When you call or raise, I lose $50. So, my EV is equal to ($100 \* %Fold) - ($50\* %Call) - ($50\* %Raise).

With the help of this equation, if I can predict your folding frequency, then I can predict the EV of this bluff. If you were to fold 40% of the time, my EV would be ($100 \* .4) - ($50 \* .6) = $10. I collapsed the %Call and %Raise variables into a single 60%, because when you choose either of these, the result is the same: I lose $50. Even if I don't know how often you'll raise rather than call, it won't make a difference in this case.

With the help of a little algebra, an equation like this is also useful to get a sense of how often I need you to fold if I am to have an EV greater than $0 - to show a profit, in other words. We'll see how exactly that works in the next chapter.

### Questions & Answers: Equity and Expected Value

Q1: How much equity does T♥ 9♥ have against A♣ K♣ pre-flop?

A1: It has about 41% equity.

Q2: How much equity does T♥ 9♥ have against a range of {55+,AJ+,KQ}?

A2: It has about 37% equity.

Q3: How much equity does T♥ 9♥ have against A♣ A♣ on an 8 ♥ 7 ♥ 3 ♣ flop?

A3: It has about 56% equity.

Q4: Suppose that you make a $100 bluff into a $100 pot on the river. If we assume that your opponent will fold 40% of the time and that you will lose any time he does not fold, what is the EV of this bet?

A4: It's -$20, meaning you would lose an average of $20 each time you attempted this bluff.

To arrive at this answer, we can use the same equation as the example above. We just need to change the amount that you will lose if your opponent calls or raises from $50 to $100. That gives us EV = $100\* %Fold - $100\* %Call - $100\* %Raise.

Once again, we can combine %Call and %Raise into 60%, since we don't care what your opponent does if he doesn't fold. So, EV = $100\*.4 - $100 \*.6 = -$20.

If your opponent will only fold 40% of the time, then you would lose an average of $20 every time you made this bet, even though you would sometimes get lucky and win the pot.

### SCENARIO: DODGING A DEBT

Christina owes David $100. It's Saturday night, and they are both going to one of two poker rooms, Room A or Room B. If they show up at the same room, Christina will have to pay David the $100 she owes him. David hopes this will happen, while Christina hopes it will not. Here's the grid showing their strategic options and their payoffs:

#### ### Table: Christina vs David — Room Choice (Payoff Matrix)

meta:

id: room\_choice\_christina\_david

rows: "Christina choices"

cols: "David choices"

cell: "(Christina payoff, David payoff) in USD"

source\_format: "column-by-column blocks"

| | David: Room A | David: Room B |

|---------------------|-------------------|-------------------|

| Christina: Room A | (-$100, $100) | ($0, $0) |

| Christina: Room B | ($0, $0) | (-$100, $100) |

### Questions & Answers Dodging a Debt

Q1: Explain or demonstrate why none of the boxes in the grid above represents an equilibrium.

A1: If Christina goes to Room A and David goes to Room B, then David's payoff would be $0. This would not be an equilibrium, because David could unilaterally increase his payoff by going to Room A to collect his $100. But both players going to Room A is also not an equilibrium, as Christina could get a better payoff by changing her strategy and going to Room B so as not to pay the $100.

Any pure strategy pair would result in the players always going to the same room or always going to a different room, and in each case there would be a player who could change his or her strategy to get a better payoff.

Unlike in the other scenarios we've looked at, pure strategies will never produce an equilibrium for this game. The players need to mislead each other about their intentions, and that requires mixed strategies, some method of choosing a room that the opponent cannot predict.

Q2: What is the equilibrium for this game?

A2: The equilibrium is for each player to choose randomly, whether by flipping a coin or some other method, with a 1/2 chance of going to Room A and a 1/2 chance of going to Room B.

The equilibrium requires mixed strategies because the available pure strategies are predictable. If Christina just picked a room and went there, she would be guessing about where David was likely to show up. If she guesses wrong - if David correctly predicts where she will be - she gets a bad outcome. Likewise if David tries to just pick a room.

In the scenarios we looked at previously, predictability was a good thing. When you're sharing a road with other drivers or going out with friends, you typically want them to know what your strategy is going to be, because it is possible to arrive at win-win solutions. Some out- comes are better for everyone involved and some are worse for everyone involved, and all players are incentivized to arrive at the former. The Dodging a Debt scenario, like most two-player poker situations, is a zero-sum game. A gain for one player is a loss for another, so an outcome that is good for one player is not good for the other. Christina is actively trying to avoid David, so she needs to be unpredictable. That's where a mixed strategy comes in handy.

As for why each player's strategy is to choose each room with 1/2 probability, this has to do with the player's payoffs. Christina's only interest is in dodging David; she doesn't otherwise care which room she goes to. Nor does David have any preference other than trying to catch up with Christina. By being unpredictable in exactly this way, each player creates a situation where there is no good strategy for his or her opponent. Whatever the opponent chooses, he or she will have only a 1⁄2 chance of choosing the right room. If David went to either room with greater than 1⁄2 frequency, this would be exploitable. That is, Christina could have a better than 1⁄2 chance of dodging him if she could predict what his preference would be.

In the next chapter, we'll focus more on how to find the exact frequencies for mixed strategies. Right now, you just need to understand what a mixed strategy is and why they are often necessary to arrive at an equilibrium.

Q3: What is each player's EV at equilibrium?

A3: To find a player's EV, you must look at all possible outcomes of his action, how likely each is, and what each is worth to him.

This game has four possible outcomes: 1) Both players go to Room A, David gets $100; 2) Christina goes to A and David goes to B, David gets $0; 3) Both players go to B, David gets $100; 4) Christina goes to B and David goes to A, David gets $0.

The probability of each of these outcomes is the probability of Christina going to the given room multiplied by the probability of David going to the given room. In this example, those probabilities are all 50%, so the equation for David's EV is (.5 \* .5 \* $100) + (.5 \* .5 \* $0) + (.5 \* .5 \* $100) + (.5 \* .5 \* $0) = $50

Essentially, David has a 50% chance of choosing the same room as Christina, whichever room that happens to be, and collecting his $100. That amounts to an EV of $50.

Because this is a zero-sum game, Christina's EV must be -$50. For David to make money, she must lose an equivalent amount. There is a 1/2 chance that she will have to pay her debt to David and a 1/2 chance that she dodges him.

Q4: Prove that neither player has incentive to deviate from his or her equilibrium strategy.

A4: To prove an equilibrium, we can experiment with changing a single player's strategy and see whether any change that player makes could improve his payoff relative to the supposed equilibrium that we are testing. Let's try keeping Christina's strategy the same - she'll still go to each room with 1/2 probability - and changing David's.

If Christina flips a coin to decide which room to play at, while David plays a pure strategy of going to Room A, then the probabilities of each outcome change, and David's new equation looks like this: (.5 \* 1 \* $100) + (.5 \* 1 \* $0) + (.5 \* 0 \* $100) + (.5 \* 0 \* $0) = $50

In this case, David always goes to Room A, and there is a 50% chance that Christina also chooses Room A, in which case David gets his $100. His EV is still $50, so this change does not improve his payoff.

What if David played a more complicated strategy, with an 80% chance of going to Room A and a 20% chance of going to Room B? Then the equation would look like this:

(.5 \*.8 \* $100) + (.5 \* .8 \* $0) + (.5 \* .2 \* $100) + (.5 \* .2 \* $0) = $50

David could change his strategy in lots of ways: he could play a pure strategy of going to Room B, he could flip a coin, or he could randomize in some other proportion. None of these improves his payoff, though. No matter what he does, he has a 1/2 chance of choosing the room where Christina will be.

We'd get the same result by keeping David's coin flipping strategy static and experimenting with different strategies for Christina. No matter what change she made, if David played his half of the equilibrium, she would have an EV of -$50.

It's important to note that even though the EVs remain the same when we change one player's strategy, there is no longer an equilibrium. If Christina flips a coin while David consistently goes to Room A, their EVs remain the same, but now David's strategy is exploitable. The risk he takes is that if Christina also changes her strategy, she might end up with a better than 1/2 chance of evading him. There are infinitely many strategy pairs that would yield EVs of (-$50, $50) in this game, but only one of them is an equilibrium.

The crucial thing is that neither player can do better than the equilibrium strategy by unilaterally deviating from it. It's quite common that a change will produce an equally good payoff for one player, as it does here.

A player can even decrease his payoff by unilaterally deviating from the equilibrium; think about Andrew going to a drama by himself instead of seeing a comedy with Blanca. In real poker situations, it's quite common that a balanced strategy will do a good deal better against a weak opponent than it would against an opponent playing a balanced strategy, even if it doesn't do quite as well as a strategy specifically crafted to exploit that player's weaknesses. In fact, understanding the equilibrium can help us to craft more effective exploitative strategies, a goal that we'll return to each time we explore new concepts.

### INDIFFERENCE MEANS GIVING YOUR OPPONENTS NO GOOD OPTIONS

When a player has no preference between two or more of his strategic options, then he is indifferent between them.

Christina's equilibrium strategy is to randomize her choice of card room, with a 50% chance of ending up at either Room A or Room B, because this is the only way to avoid being predictable, or exploitable. If she shifted that frequency even a little bit, going to Room A at a 51% frequency, for instance, then there would be a clear best strategy for David: he should always go to Room A, as he'd have the best chance of catching her there.

By getting that frequency exactly right, Christina creates a situation where David's choice is not meaningful. Whether he always goes to A, always goes to B, or employs any mixed strategy, his payoff remains the same. He has a 50% of getting his $100, for an expected value, or average return, of $50.

The technical way of saying this is that at equilibrium, David is indifferent between the two rooms. Neither is better or worse for him. Not only that - he's indifferent between all possible strategies of choosing a room. Whether he always goes to A, goes to A 50% of the time, or goes to A 39.37568% of the time, his EV remains the same. By playing her equilibrium strategy, Christina makes David indifferent between all his potential strategies. The same is true if David plays his half of the equilibrium: no matter which strategy Christina chooses, she will have an EV of -$50.

Real poker players have lots of weird preferences. They'll tell you that their main goal is to make money, but when you watch them play and hear them talk about their decision-making, they often reveal other preferences of which they may or may not be aware. Many are risk- averse, meaning they try to avoid big losses even if it means they will make a little less money overall. Some are risk-loving; they're there to gamble, and they're willing to play losing strategies because, like people chasing jackpots at slot machines, they enjoy trying to hit hands and win big pots. Some want to be regarded as good players by their peers and so will not pursue unconventional strategies even if they are likely to be profitable.

Despite all of this, when we talk about game theory and equilibrium, we assume that all players' only preference is to maximize their expected value. This is not to say that other preferences are invalid, only that they are beyond the scope of this book. My goal here is to help you identify the most profitable choice at any decision point. How you weigh that against other considerations is entirely up to you. Regardless of your own preferences, you absolutely should consider your opponents' preferences. Identifying whether an opponent is risk- averse, interested in gambling, or obsessed with protecting her hand will help you determine how best to exploit that player.

In a game theoretic sense, though, a player would have no preference between two options only if they had the same EV for him. We will assume that when presented with a choice between two or more strategic options in a poker game, a player will choose the one with the highest EV. While it isn't always easy to determine which option has the highest EV, this is what most players are trying to do most of the time. For our purposes, indifference arises only when two or more options have the same EV.

Equilibrium strategies typically involve quite a bit of indifference. In fact, you can think of indifference as the goal of poker. You can't force your opponents to make mistakes - to call when they ought to fold, for instance, or to fold when they ought to raise. What you can do is put them in situations where there is no right play, or where the right play is difficult to identify because it is barely better than the wrong play. This not only denies them the opportunity to make good decisions but also increases the likelihood that they will make mistakes.

This may require some adjustment to how you currently think about poker. Most poker players are accustomed to thinking first about mistakes that they want or expect their opponents to make and then focusing on how to induce those mistakes. The game theoretic approach is to focus instead on putting opponents in difficult situations where they will have trouble identifying the correct play. The mistakes will follow.

### YOUR BRAIN IS NOT CAPABLE OF RANDOMIZATION

Suppose that you have a bluff-catcher on the river, and you believe that you should call 1/2 the time with it. You need to find some way of generating a random number. You could look at the final digit of a digital watch or clock and call if it is less than 5. If you're playing online, you could flip a coin or roll a die or use a website such as Random.org to generate a random number.

What you should not do is say, "Well, I folded the last time this player bet the river, so I'll call this time. That way I'll be calling 1/2 the time." For one thing, the last time you folded was a different situation. The board was different, the action was different, and your hand was different. From an exploitative perspective, you might be able to guess whether the fact that you folded that previous hand would make your opponent more or less likely to bluff this time, but if you're aiming for randomness, then what happened in that other hand should have no bearing on how you play this one. After all, your opponent knows that you folded to his last bet, so he may also try to predict how that will in- fluence your decision this time around.

Even if by some miracle of probability you were to find yourself in precisely the same situation a second time, though, you'd still need a random number generator. There must be a 50% chance that you will call each time you are in this situation. Whatever you did last time should have no bearing on the current situation. If it did, then an opponent who knows that you folded the last time could predict that you would call this time. That's not random.

Consider flipping a fair coin. There is a 50% chance that any given flip will be heads, but just because you got heads on the first flip doesn't mean that the second will be tails. It's an independent event, and you can't predict anything about the next flip based on the previous flips.

### SOLVING COMPLEX POKER SCENARIOS REQUIRES COMPUTER ASSISTANCE

Human brains aren't capable of true randomness, but they face an even bigger challenge applying game theory to poker. Finding equilibrium strategies for real poker games is, to put it mildly, a computationally intensive task. In the Dodging a Debt scenario, we worked with a neat and tidy 2x2 grid, but in no limit hold 'em, the game space - the number of potential situations in which you could find yourself - is millions of times larger.

The only equilibria that humans can solve without the aid of computers are for extremely simplified toy games like the ones we've looked at so far. Hopefully, solving those has helped you better understand what equilibria are, why they are relevant, and how to find them. When it comes to solving actual poker situations, we need the assistance of computers and specialized software. These programs, called game theory solvers or equilibrium solvers or just solvers, enable us to approximate equilibrium solutions to these much more complex games. I say "approximate" because even when working with solvers, we're still forced to restrict the game space by limiting the number of avail- able bets. Even a computer can't handle the hundreds of different betting options available to you in a real game. After all, in a $1/$2 game with $200 effective stacks, you could bet $2 or $3 or $4 or any amount you wish up to $200. That's nearly two hundred strategic options! Then your opponent could respond to that bet with a raise of many different sizes, you'd have the choice to re-raise, etc. The number of pos- sible game situations explodes quickly.

When working with a solver, you must decide how to limit the bet sizes. You could, for instance, allow bets of 50% of the pot on the flop, 50% or 100% on the turn, and 50%, 100%, or all-in on the river. Then, a solver could give you an equilibrium solution within these constraints. Restricting players to three bet sizes probably won't result in an output that's too far off from the true equilibrium, but strictly speaking, it isn't a solution to a no-limit game, it's a solution to a closely related game with limited betting options.

Solvers have other limitations besides bet size restrictions. If you're solving for a situation on the flop, for instance, you'll still have to do some guesswork to input the ranges with which each player will see the flop. There actually are now consumer-facing solvers capable of solving pre-flop situations, but these are extremely computationally intensive, often requiring hundreds if not thousands of dollars in pro- cessing power to do substantive work.

Choosing the right inputs when working with a solver is a skill unto itself. Think of a scientist designing a laboratory experiment: if she designs it poorly, she will end up testing something other than the hypothesis she intended, and any conclusions she draws from the results will be flawed.

You won't need access to a solver to use this book. We will look at some solver solutions, but I've already done the set-up for you; we'll just analyze the outputs. After studying the material in this book, however, you should be capable of using a solver independently should you wish to do so.