Assignment 1, written by Nanyang Tang

1. Solution to problem1:

Variables:

Random variables:

Constant: a

Probability: [], L[]

Distributions: Pareto(a,b)

(a).

(b).

Jeffreys’ prior has the key feature that its functional dependence on the likelihood L is invariant under reparametrization of the parameter vector, which makes this prior non-informative.

1. Solution to problem2:

Variables:

Random variables:

Training data:

Probability: [], L[]

Distributions: ,

(a).

(b).

where

so

Assume which is the variance of the observed data.

Since and are proportional to , they can be cancelled.

Therefore,

(c).

as have been known.

Assume , ,

()

Since , ().

Therefore, .

(d).

Assume where

, ,

Since ,

1. Solution to problem3:

Variables:

Random variables:

Training data:

Probability: [], L[]

Distributions: ,

(a).

Firstly,

Inv- and

n=5, ,

Inv-.

So, we could sample from Inv- using R.

Then given , we could sample from

Finally given and , we could obtain y from , and Figure 1 shows the distribution of , the distribution of and the distribution of.

Rplot.pdf

Figure 1

The sample variance of new sample is about 1.66.

(b).

The . I think that the reason why the sample variance of new sample is not completely similar to is that the number of is too few to simulate the original model but they are approximately same.

(c).

, , ,

Therefore, .

In fact,

This is up to scaling factor same as , and thus

Figure 2 shows the formula for pdf and kernel density estimate.

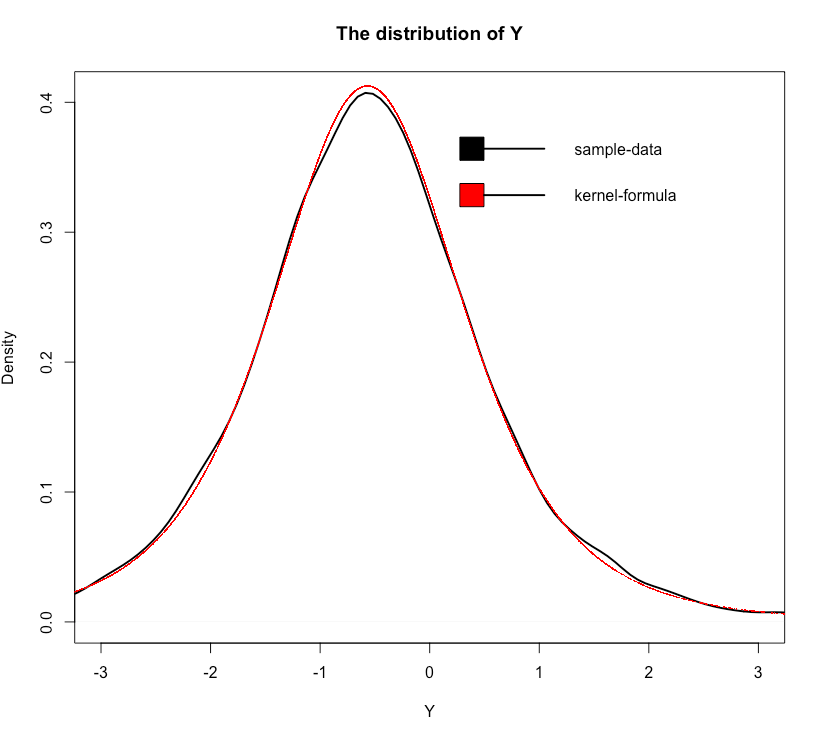


Figure 2

The R code for question Glory is posted below.

rm(list=ls())

library(metRology)

library(LaplacesDemon)

# sample data

y=c(-0.43,-1.67, 0.12, 0.28,-1.14)

# sample mean

y\_mean = mean(y)

# the number of sample

n=length(y)

# the variance of sample

s\_2 = 1/(n-1) \* sum((y-y\_mean)^2)

# use inverse chi-Squared distribution to obtain variance

variance = rinvchisq(10000, n-1, s\_2)

# use sample data and variance obtained to sample mean.

u = rnorm(10000,y\_mean,sqrt(variance/n))

# use mean and variance obtained to sample predictive data Y

Y = rnorm(10000,mean = u,sd=sqrt(variance))

# get the variance of Y

Y\_sigma = var(Y)

# get the mean variance from P(sigma|y)

E\_variance = mean(variance)

par(mfrow=c(2,2))

#Plot the density of variance

plot(density(variance),xlim=c(0,3),xlab="variance",main="The distribution of variance")

#Plot the density of mean

plot(density(u),xlim=c(-4,3),xlab="u",main="The distribution of mean")

#plot the density of Y

plot(density(Y),xlim=c(-3,3),xlab="Y",main="The distribution of Y")

par(mfrow=c(1,1))

# the scale of Y

t\_sigma = sqrt(1+1/n)\*sqrt(s\_2)

# sample Y from scaled t distribution with position=y\_mean, scale=sqrt(t\_sigma)

tr = rt.scaled(10000,df=(n-1),mean=y\_mean,sd=t\_sigma)

#plot two figure

plot(density(Y),xlim=c(-3,3),xlab="Y",main="The distribution of Y",lwd=2)

points(tr,dt.scaled(tr,df=(n-1),mean=y\_mean,sd=t\_sigma),pch='.',col=2)

legend(x=0,y=0.4,

legend=c("sample-data","kernel-formula"),

fill=c(1,2),bty="n",lty=1,lwd=2,cex=1,text.width = 2)