CHARLES UNIVERSITY
FACULTY OF SOCIAL SCIENCES
Institute of Economic Studies

Measuring Air Transport Industry Linkages: Input-Output Analysis

Bachelor thesis

Author: Tomáš Tax
Study program: Ekonomie a finance
Supervisor: Ing. Vilém Semerák, M.A., Ph.D.
Year of defense: 2021

Declaration of Authorship

The author hereby declares that he compiled this thesis independently, using only the listed resources and literature, and the thesis has not been used to obtain any other academic title.

The author grants to Charles University permission to reproduce and to distribute copies of this thesis in whole or in part and agrees with the thesis being used for study and scientific purposes.

Prague, May 2, 2021

Tomas Tax

Abstract

This thesis examines linkages of the Czech air-transport sector to the rest of the Czech economy using the hypothetical extraction method (HEM) within the input-output analysis. In order to estimate air transport total linkages on the Czech economy together with linkages air transport has with its upstream and downstream industries, three different forms of hypothetical extractions using two input-output models (the Leontief and the Ghosh models) were applied on the World Input-Output Tables (WIOTs). The Leontief model enabled quantification of the total and backward (upstream) linkages, whereas the Ghosh model was used to measure forward (downstream) linkages of air transport. Moreover, upstream and downstream domestic industries, that would be hit the most by the extraction of air transport from the Czech economy, were identified using the corresponding models. The results of the research allow to estimate how the rest of the economy would be affected by the Czech air transport's shutdown (for example due to travel restrictions connected to the Covid-19 pandemic) and help to identify industries hit the most by such industry closure.

Keywords

Input-output analysis, hypothetical extraction method, Leontief model, Ghosh model, World Input-Output Tables, air transport, inter-industry linkages

Abstrakt

Tato práce zkoumá vazby českého leteckého odvětví na zbytek české ekonomiky za použití hypotetické extrakce v rámci input-output analýzy. K odhadu celkových vazeb na českou ekonomiku a vazeb, které má letecké odvětví na dodavatelský a navazující průmysl, byly uvažovány tři různé formy hypotetické extrakce za použití dvou input-output modelů (Leontiefova a Ghoshova modelu) aplikovaných na Světové input-output tabulky (World Input-Output Tables, zkr. WIOT). Leontiefův model umožnil kvantifikovat celkové a zpětné vazby, zatímco Ghoshův model byl použit k výpočtu vazeb na navazující odvětví. Dále byla za pomocí odpovídajících modelů identifikována dodavatelská i navazující odvětví, která by byla zasažena nejvíce v případě, že by došlo k uzavření českého leteckého sektoru. Výsledky této analýzy odpovídají na otázky, jak by byla ovlivněna zbývající část české ekonomiky zastavením letecké dopravy (například kvůli cestovním restrikcím spojeným s pandemií Covid-19) a pomáhají určit, která odvětví ekonomiky by byla takovou uzávěrou zasažena nejvíce.

Klíčová slova

Input-output analýza, metoda hypotetické extrakce, Leontiefův model, Ghoshův model, Světové input-output tabulky (World Input-Output Tables), letecká doprava, mezisektorové vazby

Acknowledgments

I would like to express my gratitude to Ing. Vilém Semerák, M.A., Ph.D. for providing me with his time, knowledge and patient support throughout this research.

Bibliographic Record
Tax, Tomas: *Measuring Air Transport Industry Linkages: Input-Output Analysis.* Bachelor thesis. Charles University, Faculty of Social Sciences, Institute of Economic Studies, Prague. 2021, pages 90. Advisor: Ing. Vilém Semerák, M.A., Ph.D.

Bachelor Thesis Proposal

Author:	Tomáš Tax
Supervisor:	Ing. Vilém Semerák, M.A., Ph.D.
Proposed topic:	Measuring Air Transport Industry Linkages: Input-Output Analysis

Research question and motivation

At the beginning of 2020, news about potential outbreak of a new coronavirus (later called Covid-19 or simply Covid) spread across the globe. In order to "flatten the curve", world's governments came up with unprecedented social distancing measures. Together with people's fear and panic, these restrictions have had devastating effect on the world's economy and especially on industry sectors that depend on people's movement. Not only in Europe was air-transport one of the most affected industries for which the European Commission[1] reported over 60% decrease in the number of passengers (for the most affected airports) for March 2020 (YoY comparison).

This extreme downfall in air-transport volume brings up a question: What would happen to the rest of the economy if there was a complete shutdown of the air transportation industry. Moreover, how would the shock spread over the economy?

In my thesis, I will try to answer the precedent questions using the Hypothetical Extraction Method (HEM) that was used in the past to measure inter-industry linkages. As an example of such analysis, let me mention the paper written by Guerra and Sancho (2010) in which they show that the Electricity sector is stronger interconnected with the whole Energy industry than other energy-related sub-sectors, such as Extraction of gas or Extraction of coal. Consequently, the authors suggest the Electricity sector to be the "key sector" for energy efficiency policies.

I further describe the HEM in the upcoming Methodology section.

[1]ec.europa.eu

Contribution

As the HEM extracts an industry from the economy, it can be useful, for example, for policymakers to make well-informed relevant decisions in case of an industry shutdown. A potential policymaker could prepare a support for industries affected by other industry's close down.

To support my statement, I can relate, among others, again to Guerra and Sancho (2010) who used HEM to suggest which sector is the most effective to regulate in order to increase energy efficiency in the Spanish economy.

Methodology

Firstly, it is necessary to state the dataset used in this thesis. HEM works based on Input-Output (IO) tables. Based on an overview conducted by Tukker and Dietzenbacher (2013), tables available on (WOID's website) are adequate for this task. I will use the 2014 release of World Input-Output Tables (WOIT) as they are the most recent ones accessible. Moreover, in favour of the WOIT speaks the fact that they diversify between singular transportation types (Land, Water, and Air Transport) which is crucial for the topic, as well as the fact, that they are described transparently and in detail e.g. by Dietzenbacher et al. (2013).

To evaluate the given dataset, Input-Output analysis will be conducted based (not exclusively) on the book Input-Output Analysis: Foundations and Extensions by Miller and Blair (2009) and based on the "Illustrated User Guide to the World Input–Output Database" presented by Timmer et al. (2015).

As mentioned above the Hypothetical Extraction Method (HEM) will be used. HEM measures inter-industry linkages simply put in following steps: HEM takes into consideration a situation in which one industry does not produce anymore. It estimates new level of outputs through the economy. By taking the difference between the initial outputs and the HEM outputs it enables to measure the linkages of the extracted industry (Dietzenbacher et al., 2019).

Further, I will assess potential use of HEM's adaption - the Global Extraction Method (GEM) described by Dietzenbacher et al. (2019) - and compare the results, if possible.

Finally, I will apply the above-mentioned methods to extract the Czech Air Transport industry and report the results.

Bibliography

Dietzenbacher, E., Los, B., Stehrer, B., Timmer, M. & De Vries, G., 2013. The Construction Of World Input-Output Tables In The Wiod Project. Economic Systems Research, 25(1), p.71.

Dietzenbacher, E. & Lahr, M.L., 2013. Expanding Extractions. Economic Systems Research, 25(3), pp.341-360. Available at: tandonline.com [Accessed August 17, 2020].

Dietzenbacher, E., van Burken, B. & Kondo, Y., 2019. Hypothetical extractions from a global perspective. Economic Systems Research, 31(4), pp.505-519.

Guerra, A.-I. & Sancho, F., 2010. Measuring energy linkages with the hypothetical extraction method: An application to Spain. Energy Economics, 32(4), pp.831-837.

Miller, R.E. & Blair, P.D., 2009. Input-Output Analysis: Foundations and Extensions 2nd ed., Cambridge: Cambridge University Press.

Miller, R.E. & Lahr, M.L., 2001. A taxonomy of extractions. Regional Science Perspectives in Economic Analysis, 249, pp.407-441.

Timmer, M.P. Dietzenbacher, E., Los, B., Stehrer, B. & De Vries, G., 2015. An Illustrated User Guide to the World Input-Output Database: the Case of Global Automotive Production. Review of International Economics, 23(3), pp.575-605. Available at: http://doi.wiley.com/10.1111/roie.12178 [Accessed October 23, 2020].

Tukker, A. & Dietzenbacher, E., 2013. Global Multiregional Input-Output Frameworks: An Introduction And Outlook. Economic Systems Research, 25(1)

Contents

List of Abbreviations

DIIM	Dynamic inoperability input-output model
EU	European Union
GDP	Gross domestic product
GEM	Global extraction method
GMRIO	Global multiregional input-output tables
GNI	Gross national income
GNP	Gross national product
HEM	Hypothetical extraction method
ICIO	Inter-Country Input-Output Tables released by OECD
IIM	Inoperability input-output model
IMF	International Monetary Fund
IOT	Input-output table
IRIO	Interregional input-output (model)
ISIC	International Standard Industrial Classification of All Economic Activities
MERS	Middle East respiratory syndrome-related coronavirus
MRIO	Multiregional input-output (model)
OECD	Organisation for Economic Co-operation and Development
RoW	Rest of the World
SARS	Severe acute respiratory syndrome-related coronavirus
SUT	Supply and Use Tables
USD	United States dollar
VA	Value added
WIOD	World Input-Output Database
WIOT	World Input-Output Tables released by WIOD
WHO	World Health Organization

1. Introduction

At the end of 2019, a new coronavirus (later called *Covid-19* or *SARS-CoV-2*) was detected in Wuhan, China, for the first time. It became known worldwide around 20 January 2020 when WHO issued its first Novel Coronavirus Situation Report identifying 282 confirmed cases. Ten days later WHO reported almost 8 thousand confirmed cases across 19 countries. By the end of February, WHO reported approximately 85 thousand cases across 53 countries, and by the end of March, it identified 750 thousand cases across the globe.[1] In order to slow down this exponentially-growing epidemic outbreak, governments around the world introduced restrictive measures, including stay-at-home policies, lockdowns, or travel bans.

According to Gössling et al. (2020), restrictive measures together with uncertainty related to the Covid-19 pandemic itself have caused the most severe disruption of the global economy since World War II. In fact, in its World Economic Outlook Update in January 2021, IMF (2021) has estimated world real GDP to decrease by 3.5% in 2020.[2] Gössling et al. (2020) claim that international travel bans affected over 90% of the world population. The restrictions and policies hit particularly hard industries related to travelling and tourism, for example, hotels and restaurants or air travel.

Before the Covid-19 crisis, in 2019, the number of air-traffic passengers reached 4.3 billion with more than 100 thousand commercial flights operated every day (Ryšavá, 2020). However, as soon as governments had introduced travel restrictions in the spring of 2020, the volume of air traffic dropped immediately. Figure 1.1 compares the number of monthly flights in Europe in 2020 to the the monthly number of flights in 2019 as reported by EUROCONTROL.[3] From the figure, it is clear that the number of flights decreased significantly hitting the bottom in April 2020 when the number of flights decreased by 88% compared to April 2019. Throughout the rest of 2020, the number of flights in Europe remained less than half compared to the preceding year. According to Gössling et al. (2020), such dramatic shock is unparalleled to any of the past crises that

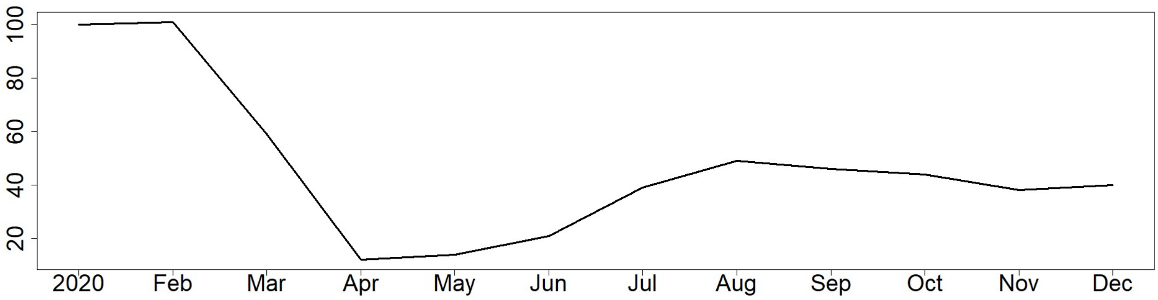
[1]By the end of 2020, WHO reported almost 80 million confirmed cases of Covid-19 together with 1.8 million deaths.

[2]IMF (2021) reports an even greater decrease of 4.9% in the real GDP of advanced economies.

[3]EUROCONTROL is an organization dedicated to supporting European aviation which covers 41 European countries and two non-European countries.

have hit air travel.[4]

Figure 1.1: Number of Flights in Europe in 2020 as a Percentage of 2019



Source: EUROCONTROL (n.d.)

However, as the industries in the world economy are interconnected, any shock to air travel affects also other industries (connected to air travel) in the economy. As of its nature, the air-travel industry is deeply interconnected with other sectors as one of its purposes is to move intermediate and finished goods around the globe. According to Eurostat (2020), air transport accounted for 28.6% of the EU-27's exports value and 19.4% of its imports value in 2019.[5] In case of the Czech Republic, it accounted for 18.7% of Czech exports value and 6% of imports value making air transport relatively less significant to the Czech economy than to the EU economy overall. However, the transportation sector, generally, is an integral part of modern global supply chains implying that it is interconnected with many other industries.

This interconnectedness of the air-transport industry together with the above-described drop in air traffic brings up the question of *How much would the economy be hit by air transport collapse*? This general question can be further developed into three more detailed (sub)questions. The ultimate goal of this thesis is to answer these three questions for the Czech Republic:

1. What would be the overall effect on the Czech economy's output in case of air transport breakdown? In other words, what are air transport's *total linkages* to the rest of the economy?

[4] Gössling et al. (2020) mention, for example, the September 11 terrorist attacks or SARS and MERS epidemics.

[5] Note, that this holds for the value of imports, not quantity. In terms of quantity, air travel accounts for less than 2% of both, EU-27's exports and imports, which suggests that air transport is usually used for transportation of goods with high per-unit value.

2. What would be the overall backward (demand-side) effect on the economy and what *upstream industries* would be hit the most in case the air transport shut down? In other words, what industries does air transport have the strongest *backward linkages* with?

3. What would be the forward (supply-side) effect on the economy and what *downstream industries* would be hit the most in case of air transport shutdown? That is, what industries does air transport have the biggest *forward linkages* with?

Upstream industries supply air transport with their products. Said differently, air transport uses inputs from its upstream industries in order to produce its output. On the other hand, downstream industries of air transport are industries that use air transport output in their production. Additionally, assuming the air-transport industry to be fully extracted from the economy, upstream industries would be hit by lack of demand from air transport, whereas downstream industries would be hit by lack of supplies from the extracted sector. An example of the upstream industry to the air transport might be an industry that extracts and manufactures petroleum which is used to run airplanes. On the other hand, tourism activities might be downstream as they takes air transport as their input.

To answer the research questions, an input-output framework will be used. When introducing the input-output framework, this thesis sticks mainly to a textbook written by Miller and Blair (2009) that gives an excellent overview of the input-output analysis together with its extensions. The input-output framework describes the use one industry's outputs by other industries in the economy and by final consumers. As this is done for every industry in the economy, a complete table of transactions within the economy is constructed. As described in chapters 2 and 3, assuming that the inter-industry relations in the economy are fixed, it is possible to model a reaction of the economy to a change in demand for final goods or a shutdown of an industry. One such method is the Hypothetical extraction method (HEM) which assumes that an industry is removed from the economy.[6] We can therefore quantify the effects of such extraction on the whole economy, in other words, to measure the extracted industry's linkages. After developing a framework capable of identifying total, backward and forward industry linkages, the framework is to be applied on real-world empirical data which are available in the World Input-Output Database (WIOD, 2016).

[6]Note that in practice it is possible to extract more than one industry.

The result of this research is a framework capable of quantifying the effect of air transport collapse on the Czech economy (thus enabling to estimate the sector's importance) and of identifying the domestic industries, which have the strongest backward and forward interconnections with the Czech air transport. The framework provided by this thesis can be easily transformed to identify linkages of any industry in the Czech economy for which WIOD provides data. Furthermore, these measures can be aggregated to examine industry groups instead of single industries. Finally, the same framework is suitable to analyze inter-industry linkages within any of the countries contained in WIOD tables.

How can the above-described knowledge be used in the real world? In the specific case of Covid-19 and air transport in the Czech Republic, the results suggest other sectors that are damaged the most apart from air transport. This type of information can be used, for example, by the government in order to identify sectors that need state subsidies the most and what sectors do not. In general, the framework developed in this thesis is a fast easy-to-apply tool for analysis of short-term effects of an industry shutdown. Note that a full industry shutdown is hypothetical as it is unlikely to happen in the real world. Nonetheless, it might prove useful to know the worst-scenario effects instead of an approximate effect of a certain scenario. Moreover, the primary goal of this research is to identify sectors that would be the most affected by a shock in air transport. As input-output framework assumes fixed relations in the economy, the model would identify the same industries even if assuming partial extraction. It is possible to develop models which enable quantifying the effects of a certain scenario more precisely. These models are discussed briefly in chapter 2 as they are of no use to the goal of this research.

This text proceeds with chapter 2 which provides a literature review of the topic and introduces the input-output models and extraction methods to the reader. Next, chapter 3 discusses data and methods that were used in order to get empirical results, which are described later in chapter 4. Finally, chapter 5 summarizes the research and results and provides suggestions for further research.

2. Literature Review

The input-output table represents transactions within an economic area whereas the input-output model enables analysis such data. This chapter firstly briefly introduces the history of the input-output analysis. Next, section 2.2 describes the construction of and math behind the input-output framework for a single economy. Moreover, it introduces the Leontief model, which is the main tool used in the input-output analysis. Next, section 2.3 extends the input-output framework to multiple regions. After that, section 2.4 introduces an alternative approach to the Leontief model - the Ghosh model. Section 2.5 then explains how the Leontief and Ghosh models can be used in order to estimate industry linkages using hypothetical extraction. Finally, section 2.6 discusses available input-output tables in terms of their content and their suitability for several types of analysis.

2.1 History of the Input-Output Framework

Input-output analysis in the form in which it is known today was firstly introduced by Wassily Wassilievich Leontief in the late 1930s, however, there were several predecessors to Leontief's ideas who are to be mentioned briefly before this text proceeds to the Leontief model.

The idea of circular flows in the economy was firstly developed by the French physician François Quesnay (1759). Quesnay's Tableau Economique depicts income flows between economic sectors. It describes a situation of a landlord investing 600 livres[1] equally between agriculture and artisans. The Tableau assumes agriculture to yield 100% surplus meaning that for every 300 invested the landlord gets 300 back every given period. From the remaining 300, half is directly consumed, and half is used to buy goods from artisans. On the other hand, artisans also produce 300, however with no yield to the landlord. Similarly to agriculture, artisans consume goods worth 150 directly and spend 150 to buy agricultural goods. In other words, both farmers and artisans supply each other. At the end of the period, there are artisans and farmers, both with 150 livres in their hands and the whole circle starts again with 150 livres instead of 300. After the next period, both groups have 75. Next, they have 37.5 and so on. It is necessary to point out that Quesnay believed that the value of the output of all sectors apart from agriculture is the

[1] French livre was a currency of the Kingdom of France at Quesnay's time.

same as the value of their input. It means that they yield no value added, which is not what the modern economy suggests. Another flaw to the Tableau is that Quesnay was not able to express his theory mathematically.

Secondly, according to Miller and Blair (2009), it was Achille-Nicholas Isnard (1781) who further developed the theory of circular flows. Isnard named surplus "disposable wealth", and more importantly, was probably the first to use a system of algebraic equations to describe income and expenditure flows in the economy.

Miller and Blair (2009) continue with Karl Marx who, they claim, got inspired from Quesnay's Tableau and who developed a process of determining profits and subsequently prices. Though, Marx was soon proved wrong (not exclusively) by Ladislaus von Bortkiewicz. Von Bortkiewicz was an advocate of general equilibrium theory and more importantly to the topic of this text, he was expressing his ideas also mathematically. Von Bortkiewicz lived in Berlin where he was teaching at the Berlin University and where he helped young Wassily Wassilievich Leontief to get his doctorate.

Finally, in 1936 Leontief revealed his own input-output framework in form of a system of linear equations which he further developed later in his career. The main advantage of Leontief's analysis was the stress on its empirical application as Leontief himself felt that economists of that time focused more on theoretical than on empirical proofs (DeBresson, 2004). Leontief (1936) presented "*an attempt to construct, based on available statistical materials, a Tableau Economique of the United States for the year 1919*" in which he developed his idea that each revenue (output) item in the economy must have an expense (input) item elsewhere in the economy. Based on this assumption, Leontief found it possible to express the whole economy as a system of interconnected accounts in just one table. The author suggests that it would be possible but chaotic to create a table containing all units in the economy, thus he provides simplification in form of grouping into industries, and describes the methodology of doing so.

To sum up, the idea of circular economy was firstly presented by the French physicist Quesnay in 1759 in his Tableau Economique. Input-output framework as it is known to us today was developed by Wassily W. Leontief and firstly published in 1936. For further reading on Leontief's life and discoveries see, for example, Leontief's necrology written by Polenske (1999). Leontief model, input-output framework, and their applications are described in the following chapters.

2.2 Basics of the Input-Output Framework

This section provides an introduction to the input-output framework together with its mathematical foundations and notation used later in this thesis. Table 2.1 presents an illustrative input-output table. Briefly said, the upper-left part represents inter-industry transactions, the lower-left part value added of production and the right side presents final demand fore each sector's products. The goal of the upcoming chapter is to define the mathematical notation used throughout this text and to explain the logic behind input-output tables and models. This section summarizes selected ideas described by Miller and Blair (2009) who give an extensive overview of the input-output analysis.

Table 2.1: Illustrative Input-Output Table

<table>
<tr><th colspan="2" rowspan="2"></th><th colspan="8">Producers as Consumers</th><th colspan="4">Final Demand</th></tr>
<tr><th>Agric.</th><th>Mining</th><th>Const.</th><th>Manuf.</th><th>Trade</th><th>Trans.</th><th>Serv.</th><th>Other</th><th>Pers. Cons.</th><th>Gross Private Domest. Invest.</th><th>Govt. Purch.</th><th>Net Exp.</th></tr>
<tr><td rowspan="8">Pro-du-cers</td><td>Agriculture</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Mining</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Construct.</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Manufactur.</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Trade</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Transport.</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Services</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Other</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td rowspan="3">Value Added</td><td>Employees</td><td colspan="8">Employee Compensation</td><td colspan="4" rowspan="3">Gross Domestic Product</td></tr>
<tr><td>Business Owners and Capital</td><td colspan="8">Profit-type Income and Capital Consumption</td></tr>
<tr><td>Government</td><td colspan="8">Indirect Business Taxes</td></tr>
</table>

Source: Miller and Blair (2009)

2.2.1 Input-Output Tables

An input-output table (IOT) represents empirical data for a given economic area (a region, a state, an economic union, etc.) over a specified period of time (usually a year). It groups economic activities across the economic area (for simplicity a country) into a number of producing sectors/industries. For these groups, it is necessary to have information about flows of products between them (inter-industry flows or transactions) in the given period. The flows can be measured by monetary units, which is usually preferred for its easier interpretation and comparability, or by physical units (Miller and Blair, 2009).

Let z_{ij} be a transaction from sector i to sector j, then z_{ij} is related to sector's j's demand for input used to produce sector j's product. For example, a furniture-making sector's demand for wood will be related to the amount of furniture it outputs. Moreover, industries do not produce solely to supply other industries' products but they also produce for external consumption. This endogenous consumption constitutes of household and government consumption and exports, it is called *final demand*. Let f_i denote total final demand for sector i's products and let x_i be the total output of sector i in a country with n industries, then

$$x_i = z_{i1} + \cdots + z_{ij} + \cdots + z_{in} + f_i = \sum_{j=1}^{n} z_{ij} + f_i \tag{2.1}$$

where f_i is the total final demand for sector i's products and z_{ij}'s are inter-industry sales to all sectors j including industry i.
This equation can be written for all industries i, where $i \in (1,...,n)$, into a system of n linear equations:

$$\begin{aligned} x_1 &= z_{11} + \cdots + z_{1j} + \cdots + z_{1n} + f_1 \\ &\vdots \\ x_i &= z_{i1} + \cdots + z_{ij} + \cdots + z_{in} + f_i \\ &\vdots \\ x_n &= z_{n1} + \cdots + z_{nj} + \cdots + z_{nn} + f_n \end{aligned} \tag{2.2}$$

These equations can be further rewritten into matrix notation using the following vectors and matrix.

$$\mathbf{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, \mathbf{Z} = \begin{bmatrix} z_{11} & \dots & z_{1n} \\ \vdots & \ddots & \vdots \\ z_{n1} & \dots & z_{nn} \end{bmatrix} \text{ and } \mathbf{f} = \begin{bmatrix} f_1 \\ \vdots \\ f_n \end{bmatrix} \tag{2.3}$$

where $\mathbf{Z}$ is the *transaction matrix* and $\mathbf{x}$ and $\mathbf{f}$ are vectors of *total outputs* and total *final demands* of all industries, respectively. Additionally, defining $\mathbf{i}$ to be a unit vector of dimension n enables for a simple summary of the transaction matrix $\mathbf{Z}$ as $\mathbf{Zi}$ gives a column vector with sums of the rows in the $\mathbf{Z}$ matrix. Similarly, $\mathbf{i}'\mathbf{Z}$ gives a row vector with sums of each column of the transaction matrix. Because of this summation ability, vector $\mathbf{i}$ is often called *summation vector* (Miller and Blair, 2009). Altogether, it is possible to obtain aggregated information about the whole system as in (2.2) with the following matrix equation:

$$\mathbf{x} = \mathbf{Zi} + \mathbf{f}. \tag{2.4}$$

Table 2.2 provides general structure of the inter-industry transaction table. For example, jth column of the table represents all of the sector j's purchases from other sectors, that is it provides information about the sector's inputs. On the other hand, ith row gives out information about ith sector's outputs.

Table 2.2: Generalized Inter-industry Transaction Table

		Buying Sector				
		1	$\dots$	j	$\dots$	n
Selling	1	z_{11}	$\dots$	z_{11}	$\dots$	z_{11}
Sector	$\vdots$	$\vdots$		$\vdots$		$\vdots$
	i	z_{i1}	$\dots$	z_{ij}	$\dots$	z_{in}
	$\vdots$	$\vdots$		$\vdots$		$\vdots$
	n	z_{n1}	$\dots$	z_{nj}	$\dots$	z_{nn}

Source: Miller and Blair (2009)

2.2.2 National Accounts

The table of transactions (Table 2.2) can be extended to include all information about the whole economy. Miller and Blair (2009) provide an illustrative example of such table of a two-sector economy as presented in Table 2.3.

Firstly, final demand can be split into foreign (f_F) and domestic(f_D). Domestic final demand consists of household consumption (C), private investment (I), government expenditure (G), and foreign demand in other words means exports (E). Thus $f_1 = f_{1D} + f_{1F} = c_1 + i_1 + g_1 + e_1$ for sector 1 and $f_2 = f_{2D} + f_{2F} = c_2 + i_2 + g_2 + e_2$ analogously. Secondly, value added can also be decomposed. Here, for simplicity, it consists of labour payments (l) and other payments (n) such as rent, taxes, interest, etc. Value added of sector 1 is thus $v_1 = l_1 + n_1$, similarly for v_2. Next, it is assumed that some of the sectors use imported goods (m) to produce their inputs, thus total expenditure in the payments sector for a given sector are equal to $l_i+n_i+m_i = v_i+m_i$. However, Miller and Blair (2009) note that imports can as well be reported together with exports in which case the final demand consists of consumption, investment, government expenditure, and *net exports* which, therefore, can be negative. Payments in range l_C to m_E are interpreted similarly to the ones belonging to the producing sectors. For example, m_C might represent imports to satisfy households' consumption or g_L represents government labor costs.

Table 2.3: Input-Output Table for Simplified Economy

	S1	S2	Final Demand				Total Output ($\mathbf{x}$)
Sector 1	z_{11}	z_{12}	c_1	i_1	g_1	e_1	x_1
Sector 2	z_{21}	z_{22}	c_2	i_2	g_2	e_2	x_2
Payment Sector 1	l_1	l_2	l_C	l_I	l_G	l_E	L
Payment Sector 2	n_1	n_2	n_C	n_I	n_G	n_E	N
Imports	m_1	m_2	m_C	m_I	m_G	m_E	M
Total Outlays ($\mathbf{x}'$)	x_1	x_2	C	I	G	E	X

Source: Miller and Blair (2009)

In addition, the table gives more interesting information. Summing a whole row gives a total outputs of both sectors, the total amount of labor, and other value-added payments and total imports. Moreover, when summing all of these (i.e. the last column of Table 2.3), we get the total gross output of the economy

$$X = x_1 + x_2 + L + N + M$$

From the other perspective, summing the table by columns gives us total outlays of both sectors and of all components of the final demand. Again, by summing these we obtain total outlay of the economy

$$X = x_1 + x_2 + C + I + G + E$$

Additionally, when setting the two equations equal, we get

$$x_1 + x_2 + L + N + M = x_1 + x_2 + C + I + G + E$$

$$L + N + M = C + I + G + E$$

$$L + N = C + I + G + (E - M)$$

The right-hand side represents the gross national product (GNP) and the left-hand side denotes gross national income (GNI) (Miller and Blair, 2009).

2.2.3 Input-Output Model

In this subsection, the technical coefficient together with the Leontief matrix will be introduced in order to develop a basic input-output model.

The input-output model works under the assumption of fixed ratios of inputs and outputs in an industry. Let's assume industry i supplies the industry j which then produces the total output x_j. Let's denote this ratio of input from i to j (z_{ij}) to total output x_j as a_{ij}, then

$$a_{ij} = \frac{z_{ij}}{x_j} \tag{2.5}$$

where a_{ij} is called a *technical coefficient* by Miller and Blair (2009). Subsequently, it holds that $a_{ij}x_j = z_{ij}$, which will be later substituted into the transaction matrix in order to get the Leontief matrix. It is assumed that the input-output ratios do not change with the amount produced (economies of scale do not take place within the input-output framework) which implies that the technical coefficients are constant. Moreover, a fixed proportion (p_{ik}) of inputs from industries i and k to the industry j is assumed in way that $p_{ik} = z_{ij}/z_{kj} = a_{ij}x_j/a_{kj}x_j = a_{ij}/a_{kj}$.
Having the technical coefficient defined, it can be substituted into an equation (2.2) as

$$\begin{aligned}
&x_1 = a_{11}x_1 + \cdots + a_{1i}x_i + \cdots + a_{1n}x_n + f_1 \\
&\vdots \\
&x_i = a_{i1}x_1 + \cdots + a_{ii}x_i + \cdots + a_{in}x_n + f_i \\
&\vdots \\
&x_n = a_{n1}x_1 + \cdots + a_{ni}x_i + \cdots + a_{nn}x_n + f_n
\end{aligned} \tag{2.6}$$

where final demands and technical coefficients are the observed data and x's are unknown. Further, by moving all multiplicands $a_{ii}x_i$ to one side of each equation, we have

$$\begin{aligned}
&x_1 - a_{11}x_1 - \cdots - a_{1i}x_i - \cdots - a_{1n}x_n = f_1 \\
&\vdots \\
&x_i - a_{i1}x_1 - \cdots - a_{ii}x_i - \cdots - a_{in}x_n = f_i \\
&\vdots \\
&x_n - a_{n1}x_1 - \cdots - a_{ni}x_i - \cdots - a_{nn}x_n = f_n
\end{aligned} \tag{2.7}$$

Now $x_i - a_{ii}x_i$ can be factored out into $(1 - a_{ii})x_i$ for all ith equations to obtain the following form:

$$\begin{aligned}
&(1 - a_{11})x_1 - \cdots - a_{1i}x_i - \cdots - a_{1n}x_n = f_1 \\
&\vdots \\
&- a_{11}x_1 - \cdots + (1 - a_{ii})x_i - \cdots - a_{in}x_n = f_i \\
&\vdots \\
&- a_{n1}x_1 - \cdots - a_{ni}x_i - \cdots + (1 - a_{nn})x_n = f_n
\end{aligned} \tag{2.8}$$

Let $\hat{\mathbf{x}}$ be a matrix with vector $\mathbf{x} = (x_1, \ldots, x_n)$ on its diagonal[2] such that $\hat{\mathbf{x}} = \begin{bmatrix} x_1 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & x_n \end{bmatrix}$, then it holds that $(\hat{\mathbf{x}})(\hat{\mathbf{x}})^{-1} = \mathbf{I}$ where $\mathbf{I}$ is an identity matrix. Furthermore, post-multiplication of the matrix $\mathbf{Z}$ from (2.3) with $\hat{\mathbf{x}}^{-1}$ gives matrix of technical coefficients $\mathbf{A} = [a_{ij}]$.

$$\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1} \tag{2.9}$$

Here, every a_{ij} can be read as a ratio of output and input in the respective industry. For example, $a_{ij} = 0.8$ would mean that for every dollar of output of industry j, industry i supplied an intermediate product worth 0.8 dollar.

[2]Throughout this text, objects denoted with hat (in this case $\hat{\mathbf{x}}$) usually represent a diagonal matrix with the corresponding vector ($\mathbf{x}$ in this case) on the diagonal.

Next, using the vector $\mathbf{f}$ from (2.3) and equation (2.9), it is possible to express the system of equations in (2.6) as

$$\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{f} \tag{2.10}$$

which can be further transformed into a form corresponding to (2.8) as

$$(\mathbf{I} - \mathbf{A})\mathbf{x} = \mathbf{f} \tag{2.11}$$

Finally, when $(\mathbf{I} - \mathbf{A})$ is invertible (which it is as long as $|\mathbf{I} - \mathbf{A}| \neq 0$), it holds that

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f} = \mathbf{L}\mathbf{f} \tag{2.12}$$

where $\mathbf{L} = [l_{ij}] = (\mathbf{I} - \mathbf{A})^{-1}$ is called a *Leontief inverse*. It is possible to rewrite the equation back from the matrix notation such that

$$\begin{aligned}
x_1 &= l_{11}f_1 + \cdots + l_{1j}f_i + \cdots + l_{1n}f_n \\
&\vdots \\
x_i &= l_{i1}f_1 + \cdots + l_{ij}f_i + \cdots + l_{in}f_n \\
&\vdots \\
x_n &= l_{n1}f_1 + \cdots + l_{nj}f_i + \cdots + l_{nn}f_n
\end{aligned} \tag{2.13}$$

where $l_{ij} = \frac{\partial x_i}{\partial f_j}$. The partial derivative notation can be read as "how much the output of the industry i (x_i) changes with respect to a change in the final demand for sector j's product (f_j)."

2.2.4 Basic Calculations Using the Input-Output Framework

In the previous section, it was shown how to solve the initial set of equations (2.2), specific to the data in the input-output table, using technical coefficients and subsequently Leontief matrix. In this section, a simple example of how to use such knowledge is introduced.

Let's assume a complete input-output table is already constructed. This means we know total outputs and outlays (x_is), final demands (f_is), value added, and intersectoral flows (z_{ij}s). It is possible to proceed as described in the preceding section. Firstly, we obtain a matrix of technical coefficients $\mathbf{A}$ using definitions (2.5) and (2.9). Next, $(\mathbf{I} - \mathbf{A})$ and its inverse - the Leontief matrix $\mathbf{L}$ - can be computed. At this point, we have estimated the structure of the whole economy and we can start asking questions. For example, how would total outputs and intermediate production change if there was an increase or decrease in final demand for products of one or more sector(s)?

The following example is inspired by a similar illustration presented by Miller and Blair (2009) who describe the issue in greater detail. Let's assume two sectors, agriculture and manufacturing - both supplying each other to some extent. We are interested in what happens when demand for both increases. Common sense suggests that both will increase their production at least to the level of new final demand. However, both sectors increase production by more than by the final demand increase as they both also supply the other sector. But this extra increase of their output means that both sectors will need even more inputs to produce their extra outputs. And yet again, this leads to another demand for the other sector's outputs. These are called second and third-round effects and they are followed by fourth-round and fifth-round effects and so on to eternity. However, as this multiplication effect gets smaller every round, it soon converges to zero and can be neglected.[3] The whole series of effects can be summed to get the new outputs of both sectors.

To sum up, from the observed data, an economy's structure has been estimated and this knowledge was used to estimate how the volume of individual transactions in the economy will change in reaction to a change in final demand.

It is possible to repeat this exercise using the notation from this chapter. At the beginning (period 0), we know all z_{ij} (intersectoral transactions) or in other words we know the matrix $\mathbf{Z}$. Moreover, we know final outputs $\mathbf{x}$ from which we get $\hat{\mathbf{x}}$ and $\hat{\mathbf{x}}^{-1}$ and subsequently technical matrix $\mathbf{A}$ using (2.9). Subsequently, using $\mathbf{A}$ to get $(\mathbf{I} - \mathbf{A})$ and $\mathbf{x}$ together with the observed final demand $\mathbf{f}$ as in (2.12), Leontief inverse $\mathbf{L}$ is found, for which holds $\mathbf{x^0} = \mathbf{Lf^0}$.
Now let us assume a change in final demand such that $\Delta f = f^1 - f^0$ where f^0 is the initial observed final demand and f^1 is the new final demand. Then post-multiplication of Leontief inverse by new final demand gives a vector of new outputs $\mathbf{x^1} = \mathbf{Lf^1}$. This

[3]Miller and Blair (2009) state that the effect becomes insignificant on a seventh or eighth level.

calculation already includes secondary and other-round effects. Moreover, it holds that

$$\mathbf{x^1} = \mathbf{Lf^1} = \mathbf{Lf^0} + \mathbf{L\Delta f} = \mathbf{x^0} + \Delta\mathbf{x}$$

or perhaps more intuitively

$$\Delta\mathbf{x} = \mathbf{x^1} - \mathbf{x^0} = \mathbf{Lf^1} - \mathbf{Lf^0} = \mathbf{L\Delta f} \tag{2.14}$$

To sum up, this section has simplistically shown the kind of calculations that the input-output framework often used for. This means that the the structure of the economy is calculated from an observed input-output table using the Leontief inverse and, later, the consequences of a change in final demand on the economic system can be estimated.

2.2.5 Open and Closed Models

The input-output framework as it has been described above considers the household consumption to be a part of the (exogenous) final demand. However, it is possible to include it explicitly into the transaction matrix as an additional endogenous sector. This is called closing the model with respect to household consumption. Moreover, the model can be closed with respect to other components of the final demand as well (Miller and Blair, 2009).

Making the household consumption endogenous means that the transaction, technical, and Leonftief matrices will all have one more row and one more column (it makes them $(n+1)\times(n+1)$ matrices) which catch households' labor and consumption, respectively. The analysis is done correspondingly to section 2.2.3 only with one more column and row. Such adjustment makes the model account for changes in labor and consumption relative to the change in demands of the initial n industries.[4] According to Miller and Blair (2009), the model can be closed also for other components of final demand, however,

[4]Miller and Blair (2009) introduce an example of two plus one sector economy (agriculture and manufacturing plus households' labor and consumption). An increase in final demand leads to an increase in outputs of agriculture and manufacturing. However, the increased production of these producing sectors translate into a need for more human labor in the process. Including households in the transaction matrix enables for such analysis. For the particular example given in the previous section, the labor payments and households' consumption will increase. Again, as discussed in section 2.2.4, this increase will have a further effect on producing sectors outputs and so on. Hence, closing the model will lead to a bigger increase in all sectors output in reaction to increase in final demand than in the initial model.

in case of, for example, government, it does not make much sense as the government decides about its inputs and outputs based also on other than economic factors.

On the other side, Miller and Blair (2009) claim that there are drawbacks to the model closure. By including household consumption in the model, it is implicitly assumed that the relationship between households' labor and expenditure is fixed, which might be unrealistic as different groups of people demand different things.[5] Drawbacks to the closed models can be overcome by disaggregation of the household sector into more detailed homogeneous (in income or specialization) groups of workers but will not be discussed here further as it is of little concern to the goal of this thesis. See chapter 2.5 in Miller and Blair (2009) for more detail.

2.2.6 Monetary versus Physical Units

Most of the input-output tables contain data measured in monetary units, however, it is also possible to develop a table in physical units. According to Miller and Blair (2009), tables in physical units are particularly useful when the research is interested in the environmental or energy effects.[6] Examples of such type of research are Sajid et al. (2020), Sajid et al. (2019), Zhao et al. (2016), or Guerra and Sancho (2010).

Input-output tables can be originally developed in both, physical and monetary units. Yet, when prices per physical unit are known for each sector's output, it is easily possible to translate a table from physical units into monetary units and vice versa. A problem with such conversion comes up when an industry produces more than one good, though. However, as there is no such conversion and only monetary units are used in this thesis, this topic is not discussed further in this text. For more detail on monetary and physical unit framework see chapter 2.6 in Miller and Blair (2009).

2.2.7 Section Summary

In this section, firstly, the content and framework of input-output tables were presented. Secondly, mathematical concepts behind the input-output framework were explained using matrix algebra. Next, an illustration of modelling the change in

[5] Additionally, increased demand for labor might attract new people into the economy which will again have different spending preferences to the initial residents.

[6] For example, when a scientist is interested in measuring of an impact of a certain policy or restriction on carbon emissions produced which are measured in tons (thus in physical units).

outputs in relation to the change in final demand was given. Lastly, the difference between open and closed models and tables that use physical or monetary units were discussed. The following section provides a discussion on regional and multiregional models.

2.3 Regional and Multiregional Models

In the preceding section, the input-output model was assumed on a national level. However, it is possible to take into consideration flows on a more disaggregated (regional) level in order to get into more detail. This section introduces such framework as presented by Miller and Blair (2009).

2.3.1 Single-regional Framework

In order to model a sub-national economy, the development of a regional model is needed. This model will most probably depend more on exogenous imports and exports as the region might be too little to cover its demands by its supplies fully.

Nonetheless, from a mathematical point of view not that much changes. It is necessary to introduce new notation, though. Let z_{ij}^{rr} be the flow from sector i in region r to sector j in region r and let x_j^r be the total gross output of sector j in region r then, analogously to (2.5) regional technical coefficients are derived as

$$a_{ij}^{rr} = \frac{z_{ij}^{rr}}{x_j^r} \tag{2.15}$$

Next, let $\mathbf{Z}^{rr} = [z_{ij}^{rr}]$ be a $n \times n$ matrix, $\mathbf{x}^r = [x_j^r]$ be a vector of length n and $\hat{\mathbf{x}}^r$ be a $n \times n$ diagonal matrix with vector $\mathbf{x}^r$ on its diagonal, then, as in (2.9),

$$\mathbf{A}^{rr} = \mathbf{Z}^{rr}(\hat{\mathbf{x}}^r)^{-1} \tag{2.16}$$

and accordingly to (2.12), regional outputs based on final demand in the region are obtained as

$$\mathbf{x}^r = (\mathbf{I} - \mathbf{A}^{rr})^{-1}\mathbf{f}^r \tag{2.17}$$

The single-regional model assumes a region to be cut off its connections with the rest of the country, which is usually unrealistic. As regional models fail to describe interconnection between regions, the many-regional models are to be introduced in the following section.

2.3.2 Many-regional Framework

There are two approaches to expressing interconnections between several regions[7] - interregional and multiregional - which are both described in the following section. For more detail on many-regional models, see chapter 3 in Miller and Blair (2009).

Interregional Approach

The first approach to the many-regional framework is the interregional input-output model (IRIO). IRIO is the simpler one from a theoretical point of view, however, it requires (ideally) perfect information about interregional and intraregional flows which is getting hard to get with a growing number of regions. This is why IRIO is very difficult to implement under real-world conditions.

For simplicity, assume a two-region (regions r and s) framework. IRIO assumes that we have complete information about all the flows between regions. In other words, $\mathbf{Z}^{rr} = [z_{ij}^{rr}]$,$\mathbf{Z}^{rs} = [z_{ij}^{rs}]$, $\mathbf{Z}^{sr} = [z_{ij}^{sr}]$, and $\mathbf{Z}^{ss} = [z_{ij}^{ss}]$ are known, where z_{ij}^{rs} reads as flow from industry i in region r to industry j in region s for all r, s, i, and j. Additionally, all x_i^r's and x_i^s's are also known. Then all the transactions can be summarized within one matrix

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z}^{rr} & \mathbf{Z}^{rs} \\ \mathbf{Z}^{sr} & \mathbf{Z}^{ss} \end{bmatrix}$$

Knowing these, it is possible to obtain regional output coefficients for all types of flows similar to (2.15) but in three more alternations.

$$a_{ij}^{rs} = \frac{z_{ij}^{rs}}{x_j^s},\ a_{ij}^{sr} = \frac{z_{ij}^{sr}}{x_j^r},\ and\ a_{ij}^{ss} = \frac{z_{ij}^{ss}}{x_j^s} \tag{2.18}$$

[7]Note that region does not necessarily mean a subnational level. A region in the whole system can for example mean a single country that is part of an economic area such as the EU or the whole World. World input output tables are discussed later in this chapter.

Input coefficients from equation (2.18) give matrices $\mathbf{A}^{rr}$, $\mathbf{A}^{rs}$, $\mathbf{A}^{sr}$, and $\mathbf{A}^{ss}$, which all together give a complete coefficient matrix for this two-region illustrative economy

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}^{rr} & \mathbf{A}^{rs} \\ \mathbf{A}^{sr} & \mathbf{A}^{ss} \end{bmatrix} \tag{2.19}$$

Then equation (2.17) takes the following form

$$(\mathbf{I} - \mathbf{A})\mathbf{x} = \mathbf{f} \tag{2.20}$$

where

$$\mathbf{I} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \ \mathbf{x} = \begin{bmatrix} \mathbf{x}^{r} \\ \mathbf{x}^{s} \end{bmatrix}, \text{ and } \mathbf{f} = \begin{bmatrix} \mathbf{f}^{r} \\ \mathbf{f}^{s} \end{bmatrix}$$

The solution to this system is in the same form as described in (2.12), that is $\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f}$.

Multiregional Approach

According to Miller and Blair (2009), the Multiregional input-output approach (MRIO) was developed with an intention to get a model for which data are more easily obtainable than for IRIO. Its basics were introduced by Chenery (1953) and Moses (1955) and further developed by Polenske (1970).

The first difference between IRIO and MRIO is that MRIO does not calculate the input coefficients directly for each sector and industry but estimates the technical coefficients from the national-level model. Thus, it assumes that ratios of inputs to total outputs of individual industries do not change across regions. The technical coefficients are defined as $a_{ij}^{r} = \frac{z_{ij}^{\cdot r}}{x_{j}^{r}}$ where $z_{ij}^{\cdot r}$ denotes the industry j in the region r inputs from all industries i across all regions. Subsequently $\mathbf{A}^{r} = [a_{ij}^{r}]$. Next, z_{i}^{rs} is interpreted as a flow of the good i from the region r to the region s. By summing inputs of i to the region s from all regions r (where $r \in (1, \ldots, p)$ where p is the total number of regions), we get the total shipments of the good i into region s which is denoted T_{i}^{s} and

$$T_{i}^{s} = z_{i}^{1s} + \cdots + z_{i}^{rs} + \cdots + z_{i}^{ps} \tag{2.21}$$

Now it is possible to obtain a proportion of the good i used in s that comes from each region r and denote them as c_i^{rs}:

$$c_i^{rs} = \frac{z_i^{rs}}{T_i^s} \tag{2.22}$$

Moreover, by getting this information for every $r \in (1,...,p)$, it is possible to create a vector $\mathbf{c}^{rs}$ of all goods:

$$\mathbf{c}^{rs} = \begin{bmatrix} c_1^{rs} \\ \vdots \\ c_n^{rs} \end{bmatrix} \text{ and matrix } \hat{\mathbf{c}}^{rs} = \begin{bmatrix} c_1^{rs} & 0 & \dots & 0 \\ 0 & c_2^{rs} & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & c_n^{rs} \end{bmatrix} \tag{2.23}$$

Then it is possible to represent the whole system with

$$(\mathbf{I} - \mathbf{CA})\mathbf{x} = \mathbf{Cf} \tag{2.24}$$

and for which solution is

$$\mathbf{x} = (\mathbf{I} - \mathbf{CA})^{-1}\mathbf{Cf} \tag{2.25}$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}^1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^2 & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{A}^p \end{bmatrix}, \ \mathbf{C} = \begin{bmatrix} \hat{\mathbf{c}}^{11} & \dots & \hat{\mathbf{c}}^{1p} \\ \hat{\mathbf{c}}^{21} & \dots & \hat{\mathbf{c}}^{1p} \\ \vdots & \ddots & \vdots \\ \hat{\mathbf{c}}^{p1} & \dots & \hat{\mathbf{c}}^{pp} \end{bmatrix}, \ \mathbf{x} = \begin{bmatrix} \mathbf{x}^1 \\ \mathbf{x}^2 \\ \vdots \\ \mathbf{x}^p \end{bmatrix}, \text{ and } \mathbf{f} = \begin{bmatrix} \mathbf{f}^1 \\ \mathbf{f}^2 \\ \vdots \\ \mathbf{f}^p \end{bmatrix} \tag{2.26}$$

To sum up, this section provided a brief introduction to the construction of multiregional tables using IRIO and MRIO. IRIO requires perfect data which is usually unrealistic, so, in order to allow for less strict data MRIO was developed. For a more detailed discussion on multiregional models see Miller and Blair (2009).

2.4 Ghosh Model

Before this literature overview proceeds to industry linkages measures, there is a need for an introduction of another input-output model. The Leontief model calculates output levels based on changes in final demand, it is demand-driven. However, in certain cases (as discussed later in section 2.5.5) a supply-driven model might suit better for certain purposes. Such supply-side model was developed by Ghosh (1958) (Miller and Blair, 2009) and it is described based on Miller and Blair (2009) and Miller and Lahr (2001) in the following section.

2.4.1 Construction of the Model

The Ghosh model is built on the same data as the Leontief model. The Leontief model uses Leontief inverse which links gross outputs of sectors to final demand (to production leaving the system). The Ghosh model transposes these ideas. It relates gross outputs to the primary inputs - that is to a unit value entering the transaction system, not leaving it.

The Ghosh model can be perceived as a rotated or transposed Leontief model. Similarly to (2.5) there is a need to set up technical coefficients which are, however, called *direct-output coefficients* in case of the Ghosh model (Miller and Blair, 2009). In the Leontief model, equation (2.5), technical coefficients are obtained by dividing each column of $\mathbf{Z}$ by its corresponding total output. The Ghosh coefficients are obtained by dividing rows by their respective output such that

$$\begin{aligned}\mathbf{B} &= \begin{bmatrix} b_{11} & \dots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{n1} & \dots & b_{nn} \end{bmatrix} = \begin{bmatrix} z_{11}/x_1 & \dots & z_{1n}/x_1 \\ \vdots & \ddots & \vdots \\ z_{n1}/x_n & \dots & z_{nn}/x_n \end{bmatrix} \\ &= \begin{bmatrix} 1/x_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & 1/x_n \end{bmatrix} \begin{bmatrix} z_{11} & \dots & z_{1n} \\ \vdots & \ddots & \vdots \\ z_{n1} & \dots & z_{nn} \end{bmatrix} = \hat{\mathbf{x}}^{-1}\mathbf{Z} \end{aligned} \tag{2.27}$$

where $\hat{\mathbf{x}}$ is matrix with vector $\mathbf{x}$ on its diagonal. The direct-output coefficients b_{ij} represent distribution of the sector i's outputs across all industries j that use it as their input. Now, using $\mathbf{x}' = \mathbf{i}'\mathbf{Z} + \mathbf{v}'$, which is transposed form of $\mathbf{x} = \mathbf{Zi} + \mathbf{f}$ from (2.10), $\mathbf{Z} = \hat{\mathbf{x}}\mathbf{B}$ from (2.27), and $\mathbf{i}'\hat{\mathbf{x}} = \mathbf{x}'$ we have

$$\mathbf{x}' = \mathbf{i}'\hat{\mathbf{x}}\mathbf{B} + \mathbf{v}' = \mathbf{x}'\mathbf{B} + \mathbf{v}' \tag{2.28}$$

From this

$$\mathbf{x}' = \mathbf{v}'(\mathbf{I} - \mathbf{B})^{-1} \tag{2.29}$$

Define matrix $\mathbf{G}$ as

$$\mathbf{G} = (\mathbf{I} - \mathbf{B})^{-1};\ \mathbf{G} = [g_{ij}] \tag{2.30}$$

then

$$\mathbf{x}' = \mathbf{v}'\mathbf{G} \tag{2.31}$$

Matrix $\mathbf{G}$ is the *output inverse* oppositely to Leontief's input inverse. Change in $\mathbf{x}'$ is computed parallelly to (2.14) as

$$\boldsymbol{\Delta}\mathbf{x}' = \boldsymbol{\Delta}\mathbf{v}'\mathbf{G} \tag{2.32}$$

As the Leontief model assumes fixed proportions of outputs, the Ghosh model assumes fixed proportions of inputs. That means that if output of industry i triples, input of i coming from j also triples. This notation can be transposed using basic matrix algebra so to total output $\mathbf{x}$ corresponded to the Leontief model. In that case (2.28) gives

$$\mathbf{x} = \mathbf{B}'\mathbf{x} + \mathbf{v} \tag{2.33}$$

and (2.29) is, therefore, equivalent to

$$\mathbf{x} = (\mathbf{I} - \mathbf{B}')^{-1}\mathbf{v} \tag{2.34}$$

Then as $\mathbf{G}' = (\mathbf{I} - \mathbf{B}')^{-1}$, equation (2.31) takes form of

$$\mathbf{x} = \mathbf{G}'\mathbf{v} \tag{2.35}$$

and equation (2.32) follows with

$$\boldsymbol{\Delta}\mathbf{x} = \boldsymbol{\Delta}\mathbf{G}'\mathbf{v} \tag{2.36}$$

2.4.2 Interpretation

In the Leontief model, column sums of **L** give total increase in output in the economy with one dollar increase in final demand for the sector j products. Opposite to that, row sums of **L** suggest how much more intermediate goods sold by i to all sectors were needed in case of one dollar increase in each of the n sectors final demand (Miller and Blair, 2009). Matrix **G** is interpreted similarly but with different outcome. Note the similarities between solutions to both models. The solution in the Leontief case took form of (2.13), that is

$$x_i = l_{i1} f_1 + \cdots + l_{ij} f_i + \cdots + l_{in} f_n$$

whereas the Ghosh solution for supply-driven model takes form of

$$x_j = v_1 g_{1j} + \cdots + v_i g_{ij} + \cdots + v_n g_{nj} \tag{2.37}$$

where $g_{ij} = \frac{\partial x_j}{\partial v_i}$. From this, the Ghosh model can be interpreted correspondingly to the Leontief model. Let $g_{ij} = 0.6$, for example. Assume one dollar decrease of labor available to sector i, then the output of j will decrease by 0.6 dollar. This is because decrease of labor available to i causes i's output to decrease. However, i's output is needed as an input for sector j, thus, decrease in i's output will lead to a decrease in j's output which is expressed by g_{ij}.

Also column and row sums are interpreted correspondingly to the Leonief model. Row sums of **G** represent the effect on total output among all sectors which takes place after an one dollar change in primary inputs for the sector i. This is the supply-driven model analog to a column sum of **L**. On the other hand, column sums of **G** represent total effect on j's output in case of one dollar change in the supply of the primary factors to all of the n sectors (Miller and Blair, 2009).

This interpretation, however, is not flawless as noted by Miller and Blair (2009), because in the Ghosh model, the increase in j's output translates into an increase in all other sectors' (connected to j) output. But this increase in their output should also mean that these sectors consume more inputs, which is not translated into the model. These issues connected to the Ghosh model lead to a proposal of a different interpretation, which is introduced in the following section.

2.4.3 Ghosh Price Model

Dietzenbacher (1997) introduced an alternative interpretation to the one above. He suggested the Ghosh model to be looked upon as a price model, not a quantity model. This means, that keeping all quantities fixed, the Ghosh model analyzes a situation in response to the change of *costs* of primary inputs. Change $\mathbf{\Delta v}$ is thus a change in *value* not in quantity of inputs and $\mathbf{\Delta x}$ is a change in *value* of outputs. This simulates a situation, where the cost of primary input(s) such as labor increases, and producers subsequently increase prices of their products. Then the increased priced are pushed through the economy while increasing the value of the total output in the economy (Miller and Blair, 2009).

2.4.4 Parallels to the Leontief Model

As mentioned in 2.4.1, the Leontief and Ghosh model work based on the same data but in a rotated manner. From (2.9) $\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1}$ and from (2.27) $\mathbf{B} = \hat{\mathbf{x}}^{-1}\mathbf{Z}$ which is equivalent to $\mathbf{Z} = \hat{\mathbf{x}}\mathbf{B}$. Putting these together gives

$$\mathbf{A} = \hat{\mathbf{x}}\mathbf{B}\hat{\mathbf{x}}^{-1} \iff \mathbf{B} = \hat{\mathbf{x}}^{-1}\mathbf{A}\hat{\mathbf{x}} \tag{2.38}$$

Then, as $\hat{\mathbf{x}}\mathbf{I}\hat{\mathbf{x}}^{-1} = \mathbf{I}$,

$$(\mathbf{I} - \mathbf{A}) = \hat{\mathbf{x}}(\mathbf{I} - \mathbf{B})\hat{\mathbf{x}}^{-1} \tag{2.39}$$

and

$$(\mathbf{I} - \mathbf{A})^{-1} = \hat{\mathbf{x}}(\mathbf{I} - \mathbf{B})^{-1}\hat{\mathbf{x}}^{-1} \tag{2.40}$$

which gives

$$\mathbf{L} = \hat{\mathbf{x}}\mathbf{G}\hat{\mathbf{x}}^{-1} \iff \mathbf{G} = \hat{\mathbf{x}}^{-1}\mathbf{L}\hat{\mathbf{x}} \tag{2.41}$$

From these equations, it can be seen that any measures defined for $\mathbf{A}$ are also defined for $\mathbf{B}$ provided that $\mathbf{x}$ is known. This interconnection between the two models makes it easier to work with them interchangeably.

On the other hand, there are also drawbacks to this connection. The Leontief model assumes fixed technical coefficients, $\mathbf{A}$, in order to get $\mathbf{\Delta x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{\Delta f}$. But from the interconnection between $\mathbf{A}$ and $\mathbf{B}$ from (2.38) (and also $\mathbf{F}$ and $\mathbf{G}$), $\mathbf{B}$ (and $\mathbf{G}$, respectively) cannot remain unchanged as $\mathbf{x}$ jumps to a new level, thus breaking the assumption of fixed proportions of inputs.[8] This is called *joint stability* problem and Miller and Blair (2009) claim that it led to a lot of empirical research[9] concluding, however, that in real world it is not a major issue.

2.5 Industry Linkages and Extraction Methods

The following section provides selected topics considering interindustry linkages. First, the intuition and theory behind hypothetical extractions are explained. Secondly, a matrix partitioning structure is introduced to enable easier mathematical interpretation of logic behind hypothetical extractions. Next, measures of the total, backward, and forward interindustry linkages are presented. It is argued that to measure forward linkages, the Ghosh model is a better choice than the classical Leontief model. Later in this section, an alternative to the hypothetical extraction method at a global level - the Global extraction method - is introduced. Lastly, multiple methods suitable for disaster analysis are mentioned together with the relevant literature.

2.5.1 Hypothetical Extraction Method

According to Miller and Lahr (2001), the Hypothetical extraction method (HEM) was firstly used by Paelinck et al. (1965) and Strassert (1968) with an intention to quantify how much would the total output of an economy decrease if one sector shut down. This extraction of one sector from the economy is quite straightforward as it follows the math developed in section 2.2. For an n-sector economy, nullifying one sector (let it be sector j) means having a new economy with only $n-1$. Then it is possible to calculate a $(n-1)\times(n-1)$ matrix of technical coefficients $\tilde{\mathbf{A}}$ for this downsized economy and to obtain vector $\tilde{\mathbf{f}}$ of the new final demands by crossing out jth position. Then the total output of the economy is found using equation (2.12) but with the extraction-adjusted matrices and vectors

$$\tilde{\mathbf{x}} = (\tilde{\mathbf{I}} - \tilde{\mathbf{A}})^{-1}\tilde{\mathbf{f}}$$

As the total output of the initial setup was found similarly using $\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f}$, the change in economy's output after the loss of sector j can be measured as $\mathbf{i}'\mathbf{x} - \mathbf{i}'\tilde{\mathbf{x}}$.

[8]This problem also comes up when conducting the impact analysis using the supply-driven model. In that scenario $\mathbf{B}$ is fixed, new output goes as $\mathbf{\Delta x}' = \mathbf{\Delta v}'\mathbf{G}$, and consequently $\mathbf{A}$ changes.

[9]Miller and Blair (2009) mention Rose and Allison (1989) and Chen et al. (1991).

Generally, HEM is often used in order to measure industry linkages in an economy,[10] although is also possible to express industry linkages without hypothetical extractions. Such measures, however, do not enable for quantification of the linkage effect and thus are not used in the empirical part of this thesis (see appendix A.6 for a brief introduction of such methods). According to Miller and Lahr (2001), there are several ways of measuring total, backward, and forward linkages of an industry, which authors also describe in detail. Selected parts of their discussion about different linkages are summarized in three upcoming sections 2.5.3, 2.5.4, and 2.5.5.

2.5.2 Partitioning Structure

Firstly, in order to enable a mathematical expression of the demanded methodology, it is necessary to define several mathematical objects. A similar notation as in Miller and Lahr (2001) is used as other authors publishing about HEM use it as well, thus developing different notation would only cause confusion.

Following matrix algebra, Miller and Lahr (2001) show that it is possible to rewrite $\mathbf{A}$ as

$$\mathbf{A} = \left[\begin{array}{c|c} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \hline \mathbf{A}_{21} & \mathbf{A}_{22} \end{array}\right] \tag{2.42}$$

where $\mathbf{A}_{11}$ is a submatrix of $\mathbf{A}$ including information about the first k ($k < n$) sectors.[11] Then it is possible to express the partitioned Leontief matrix as

$$\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1} = \left[\begin{array}{c|c} \mathbf{H} & \mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \\ \hline \boldsymbol{\alpha}_{22}\mathbf{A}_{21}\mathbf{H} & \boldsymbol{\alpha}_{22}(\mathbf{I} + \mathbf{A}_{21}\mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22}) \end{array}\right] \tag{2.43}$$

where $\mathbf{H} = (\mathbf{I} - \mathbf{A}_{11} - \mathbf{A}_{12}\boldsymbol{\alpha}_{22}\mathbf{A}_{21})^{-1}$ and where $\boldsymbol{\alpha}_{22} = (\mathbf{I} - \mathbf{A}_{22})^{-1}$. The gross output and final demand vectors can be partitioned in a similar way as

$$\mathbf{x} = \left[\begin{array}{c} \mathbf{x}_1 \\ \hline \mathbf{x}_2 \end{array}\right], \text{ and } \mathbf{f} = \left[\begin{array}{c} \mathbf{f}_1 \\ \hline \mathbf{f}_2 \end{array}\right]$$

[10] Subsequently, it is also used to identify key sector(s) in the economy, that is the sectors with the biggest linkages. See Temurshoev (2010) for a detailed discussion about key sector analysis.

[11] This is called *matrix partitioning* and in the case of HEM, it was firstly used by Cella (1984).

Then from equation (2.12) holds that

$$\mathbf{x} = \left[\begin{array}{c} \mathbf{x}_1 \\ \hline \mathbf{x}_2 \end{array}\right] = \left[\begin{array}{c|c} \mathbf{H} & \mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \\ \hline \boldsymbol{\alpha}_{22}\mathbf{A}_{21}\mathbf{H} & \boldsymbol{\alpha}_{22}(\mathbf{I} + \mathbf{A}_{21}\mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22}) \end{array}\right] \left[\begin{array}{c} \mathbf{f}_1 \\ \hline \mathbf{f}_2 \end{array}\right] \tag{2.44}$$

It is possible to use this structure in order to extract the jth sector as described in the previous section. Assume that the extracted sector lies in the first row and column and that $\mathbf{A}_{11}$ is 1×1 matrix. Then extracting the jth sector (perhaps more intuitively the first sector) means replacing matrices $\mathbf{A}_{11}, \mathbf{A}_{12}, \mathbf{A}_{21}$ in matrix $\mathbf{A}$ from (2.42) by $\mathbf{0}$ (null vectors). Consequently, it is possible to calculate new outputs using (2.44) and total linkages of the jth industry using $\mathbf{i}'\mathbf{x} - \mathbf{i}'\tilde{\mathbf{x}}$ as shown later in section 2.5.3.

However, extracting all three $\mathbf{A}_{11}$, $\mathbf{A}_{12}$, and $\mathbf{A}_{21}$ from $\mathbf{A}$ is only one of seven overall options. This is because it is possible to extract $\mathbf{A}_{11}$ and/or $\mathbf{A}_{12}$ and/or $\mathbf{A}_{21}$ making it seven combinations possible. As Miller and Lahr (2001) show, there is a different motivation behind every possibility and some combinations are better suited for the estimation of specific types of linkages than others.

The three upcoming sections cover only the three scenarios which are to be used later in the empirical part of this thesis. These three scenarios were chosen based on Miller and Lahr (2001), who claim that

- scenario where $\mathbf{A}^t = \left[\begin{array}{c|c} \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{A}_{22} \end{array}\right]$ suits well the total linkages estimation using the Leontief model;
- scenario where $\mathbf{A}^b = \left[\begin{array}{c|c} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \hline \mathbf{0} & \mathbf{A}_{22} \end{array}\right]$ suits the backward linkages estimation using the Leontief model;
- and scenario with $\mathbf{B}^f = \left[\begin{array}{c|c} \mathbf{B}_{11} & \mathbf{0} \\ \hline \mathbf{B}_{21} & \mathbf{B}_{22} \end{array}\right]$ suits best the forward linkages estimation using a Ghosh model introduced in section 2.4.[12]

For an extensive discussion about all the possible extraction combinations, see Miller and Lahr (2001).

[12]Note that superscript t stands for total linkages, b for the backward linkages, and f for the forward linkages.

2.5.3 Total Linkages

Total linkages of a sector are the most "straightforward" to obtain as they are equal to the total decrease in output after an industry full extraction. As stated before, Miller and Lahr (2001) recommend extracting all of the extracted sector inputs and outputs as introduced by Paelinck et al. (1965), that is

$$\mathbf{A}^t = \left[\begin{array}{c|c} \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{A}_{22} \end{array}\right] \tag{2.45}$$

Then plugging into (2.43) gives

$$\mathbf{L}^t = \left[\begin{array}{c|c} \mathbf{I} & \mathbf{0} \\ \hline \mathbf{0} & \boldsymbol{\alpha}_{22} \end{array}\right] \tag{2.46}$$

and using equation (2.44) gives

$$\mathbf{x}^t = \left[\begin{array}{c} \mathbf{x}_1^t \\ \hline \mathbf{x}_2^t \end{array}\right] = \left[\begin{array}{c|c} \mathbf{I} & \mathbf{0} \\ \hline \mathbf{0} & \boldsymbol{\alpha}_{22} \end{array}\right] \left[\begin{array}{c} \mathbf{f}_1 \\ \hline \mathbf{f}_2 \end{array}\right] \tag{2.47}$$

where $\boldsymbol{\alpha}_{22} = (\mathbf{I} - \mathbf{A}_{22})^{-1}$. Then also holds that $\boldsymbol{\Delta}\mathbf{x}^t$ is a difference between outputs from equations (2.44) and (2.47), that is

$$\boldsymbol{\Delta}\mathbf{x}^t = \left[\begin{array}{c} \mathbf{x}_1 - \mathbf{x}_1^t \\ \hline \mathbf{x}_2 - \mathbf{x}_2^t \end{array}\right] = \left[\begin{array}{c} \boldsymbol{\Delta}\mathbf{x}_1^t \\ \hline \boldsymbol{\Delta}\mathbf{x}_2^t \end{array}\right] = \left[\begin{array}{c|c} \mathbf{H} - \mathbf{I} & \mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \\ \hline \boldsymbol{\alpha}_{22}\mathbf{A}_{21}\mathbf{H} & \boldsymbol{\alpha}_{22}\mathbf{A}_{21}\mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \end{array}\right] \left[\begin{array}{c} \mathbf{f}_1 \\ \hline \mathbf{f}_2 \end{array}\right] \tag{2.48}$$

Here, as sector 1 has been extracted completely, $\mathbf{x}_1^t = \mathbf{0}$ and thus $\boldsymbol{\Delta}\mathbf{x}_1^t = \mathbf{x}_1$. As there still is the same final demand for sector 1 products, this demand needs to be satisfied completely by imports. The total economic importance of the sector for the whole economy is measured as

$$\mathbf{i}'\boldsymbol{\Delta}\mathbf{x}^t + \mathbf{f}_1 = \mathbf{i}'\mathbf{x} - \mathbf{i}'\mathbf{x}_2^t = \mathbf{x}_1 - \mathbf{i}'\boldsymbol{\Delta}\mathbf{x}_2^t$$

To measure the importance of the extracted sector solely to the remaining sectors means computing $\boldsymbol{\Delta}\mathbf{x}_2^t = \mathbf{x}_2 - \mathbf{x}_2^t$.

The total linkage provides a measure of the overall economic importance of a sector for the economic system. However, scientists have researched the question of whether it is possible to decompose such total metric into forward and backward linkages.

2.5.4 Backward Linkages

First, what is a backward linkage? A backward linkage of an industry j is a linkage that the industry j has with its *upstream* industries. It means a linkage with industries that provide intermediate goods (inputs) to the jth industry. From another perspective, upstream industries will be the ones to be hit by the lack of demand for their products in the case of the jth industry extraction. Due to this hypothetical decrease in demand from industry j, the upstream industries will decrease their output respectively. Thus, the backward linkage is to be measured on the demand side of the input-output table.

In order to measure backward linkages with HEM, Miller and Lahr (2001) recommend using a scenario where $\mathbf{A}_{21} = \mathbf{0}$, that is

$$\mathbf{A}^b = \left[\begin{array}{c|c} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \hline \mathbf{0} & \mathbf{A}_{22} \end{array}\right] \tag{2.49}$$

This scenario represents a situation where the extracted industry is not able to obtain any inputs from the system and relies solely on imported goods. The respective Leontief inverse for such scenario is

$$\mathbf{L}^b = \left[\begin{array}{c|c} \boldsymbol{\alpha}_{11} & \boldsymbol{\alpha}_{11}\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \\ \hline \mathbf{0} & \boldsymbol{\alpha}_{22} \end{array}\right] \tag{2.50}$$

where, again, $\boldsymbol{\alpha}_{22} = (\mathbf{I} - \mathbf{A}_{22})^{-1}$ and $\boldsymbol{\alpha}_{11} = (\mathbf{I} - \mathbf{A}_{11})^{-1}$. Then using equation (2.44) similarly as in (2.48) gives the change in total output as

$$\boldsymbol{\Delta}\mathbf{x}^b = \left[\begin{array}{c} \mathbf{x}_1 - \mathbf{x}_1^b \\ \hline \mathbf{x}_2 - \mathbf{x}_2^b \end{array}\right] = \left[\begin{array}{c} \boldsymbol{\Delta}\mathbf{x}_1^b \\ \hline \boldsymbol{\Delta}\mathbf{x}_2^b \end{array}\right] = \left[\begin{array}{c|c} \mathbf{H} - \boldsymbol{\alpha}_{11} & (\mathbf{H} - \boldsymbol{\alpha}_{11})\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \\ \hline \boldsymbol{\alpha}_{22}\mathbf{A}_{21}\mathbf{H} & \boldsymbol{\alpha}_{22}\mathbf{A}_{21}\mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \end{array}\right] \left[\begin{array}{c} \mathbf{f}_1 \\ \hline \mathbf{f}_2 \end{array}\right] \tag{2.51}$$

Again, similarly to the total linkages, the backward linkages are measured as a difference between the initial and the new output. This time, however, there is no need to add final demand for air transport's products to the output effect as, under this scenario, the industry keeps producing normally and is thus able to satisfy the final demand. Moreover, according to Miller and Blair (2009), when more detail is demanded, it is possible to look at each sector one by one to see how much they were affected by such extraction. Using the same notation with the sector 1 being extracted, then for each industry i from vector $\mathbf{x}_2^b$, the difference $x_{2i} - x_{2i}^b$ is a backward dependence of the extracted sector on the sector i.

2.5.5 Forward Linkages

What are forward linkages? Forward linkages can be described oppositely to backward linkages. They affect the supply side of the input-output table. In the case of forward linkages, we are interested in the effect of the jth industry extraction on its *downstream* industries, that is the industries that suffer from the lack of supplies from the industry j. This definition indicates that forward linkages are driven by supply and not by demand. As the Leontief model is demand-driven, there is a need for the introduction of another supply-driven model - the Ghosh model (this was suggested by Dietzenbacher et al. (1993) or Miller and Lahr (2001)). The Ghosh model basics were summarized in section 2.4.

In order to use the same notation as with the Leontief model, the partitioning structure of the Ghosh model is developed. Partitioned $\mathbf{B}$ can be related to the Leontief model using (2.38) as

$$\mathbf{B} = \left[\begin{array}{c|c} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \hline \mathbf{B}_{21} & \mathbf{B}_{22} \end{array}\right] = (\hat{\mathbf{x}})^{-1}\mathbf{A}(\hat{\mathbf{x}}) = \left[\begin{array}{c|c} (\hat{\mathbf{x}}_1)^{-1}\mathbf{A}_{11}(\hat{\mathbf{x}}_1) & (\hat{\mathbf{x}}_1)^{-1}\mathbf{A}_{12}(\hat{\mathbf{x}}_2) \\ \hline (\hat{\mathbf{x}}_2)^{-1}\mathbf{A}_{21}(\hat{\mathbf{x}}_1) & (\hat{\mathbf{x}}_2)^{-1}\mathbf{A}_{22}(\hat{\mathbf{x}}_2) \end{array}\right] \tag{2.52}$$

Where

$$\hat{\mathbf{x}} = \left[\begin{array}{c|c} \hat{\mathbf{x}}_1 & \mathbf{0} \\ \hline \mathbf{0} & \hat{\mathbf{x}}_2 \end{array}\right] \text{ and } (\hat{\mathbf{x}})^{-1} = \left[\begin{array}{c|c} (\hat{\mathbf{x}}_1)^{-1} & \mathbf{0} \\ \hline \mathbf{0} & (\hat{\mathbf{x}}_2)^{-1} \end{array}\right]$$

Then

$$\mathbf{G} = (\mathbf{I} - \mathbf{B})^{-1} = (\hat{\mathbf{x}})^{-1}(\mathbf{I} - \mathbf{A})^{-1}(\hat{\mathbf{x}}) = \left[\begin{array}{c|c} \mathbf{K} & \mathbf{K}\mathbf{B}_{12}\boldsymbol{\beta}_{22} \\ \hline \boldsymbol{\beta}_{22}\mathbf{B}_{21}\mathbf{K} & \boldsymbol{\beta}_{22}(\mathbf{I} + \mathbf{B}_{21}\mathbf{K}\mathbf{B}_{12}\boldsymbol{\beta}_{22}) \end{array}\right] \tag{2.53}$$

where

$$\begin{aligned} \mathbf{K} &= (\mathbf{I} - \mathbf{B}_{11} - \mathbf{B}_{12}\boldsymbol{\beta}_{22}\mathbf{B}_{21})^{-1} = (\hat{\mathbf{x}}_1)^{-1}\mathbf{H}(\hat{\mathbf{x}}_1), \text{ and} \\ \boldsymbol{\beta}_{22} &= (\mathbf{I} - \mathbf{B}_{22})^{-1} = (\hat{\mathbf{x}}_2)^{-1}\boldsymbol{\alpha}_{22}(\hat{\mathbf{x}}_2) \end{aligned} \tag{2.54}$$

Moreover, the value-added vector can be partitioned similarly to final demand vector in the Leontief model in section 2.5.2, thus $\mathbf{v}' = \left[\begin{array}{c|c} \mathbf{v}'_1 & \mathbf{v}'_2 \end{array}\right]$ and therefore it gives

$$\mathbf{x}' = \left[\begin{array}{c|c} \mathbf{x}'_1 & \mathbf{x}'_2 \end{array}\right] = \left[\begin{array}{c|c} \mathbf{v}'_1 & \mathbf{v}'_2 \end{array}\right] \left[\begin{array}{c|c} \mathbf{K} & \mathbf{K}\mathbf{B}_{12}\boldsymbol{\beta}_{22} \\ \hline \boldsymbol{\beta}_{22}\mathbf{B}_{21}\mathbf{K} & \boldsymbol{\beta}_{22}(\mathbf{I} + \mathbf{B}_{21}\mathbf{K}\mathbf{B}_{12}\boldsymbol{\beta}_{22}) \end{array}\right] \tag{2.55}$$

Additionally, the relations between Leontief and the Ghosh model in the partitioning notation can be generalized as follows:

$$\boldsymbol{\alpha}_{ii} = (\mathbf{I} - \mathbf{A}_{ii})^{-1} \tag{2.56}$$

$$\boldsymbol{\beta}_{jj} = (\mathbf{I} - \mathbf{B}_{jj})^{-1} = (\hat{\mathbf{x}}_i)^{-1}\boldsymbol{\alpha}_{ii}(\hat{\mathbf{x}}_i) \tag{2.57}$$

$$\mathbf{H} = (\mathbf{I} - \mathbf{A}_{11} - \mathbf{A}_{12}\boldsymbol{\alpha}_{22}\mathbf{A}_{21})^{-1} \tag{2.58}$$

$$\mathbf{K} = (\mathbf{I} - \mathbf{B}_{11} - \mathbf{B}_{12}\boldsymbol{\beta}_{22}\mathbf{A}_{21})^{-1} = (\hat{\mathbf{x}}_1)^{-1}\mathbf{H}(\hat{\mathbf{x}}_1) \tag{2.59}$$

Considering the case where $\mathbf{B} = \left[\begin{array}{c|c} \mathbf{B}_{11} & \mathbf{0} \\ \hline \mathbf{B}_{21} & \mathbf{B}_{22} \end{array}\right]$ gives a forward linkage using the Ghosh model in the form of

$$\boldsymbol{\Delta}\mathbf{x}^f = \left[\begin{array}{c|c} \mathbf{x}_1^f & \mathbf{x}_2^f \end{array}\right] = \left[\begin{array}{c|c} \mathbf{v'}_1 & \mathbf{v'}_2 \end{array}\right] \left[\begin{array}{c|c} \mathbf{K} - \boldsymbol{\beta}_{11} & (\mathbf{K}\mathbf{B}_{12})\boldsymbol{\beta}_{22} \\ \hline \boldsymbol{\beta}_{22}\mathbf{B}_{21}(\mathbf{K} - \boldsymbol{\beta}_{11}) & \boldsymbol{\beta}_{22}\mathbf{B}_{21}\mathbf{K}\mathbf{B}_{12}\boldsymbol{\beta}_{22} \end{array}\right] \tag{2.60}$$

This scenario represents a situation in which the extracted industry no longer supplies other industries in the system and exports its intermediate outputs instead. Similarly to the backward linkages, forward linkages are calculated as a difference in output before and after the extraction. Moreover, for the forward linkages using the Ghosh model also holds, that they can be looked at in greater detail by looking at the difference in the output of each industry individually. Such knowledge enables to evaluate the importance of each sector i as an intermediate buyer (of the extracted industry intermediate outputs) to the extracted sector (Miller and Blair, 2009).

2.5.6 Global Extraction Method

What was not stressed out previously in this chapter is that by extracting an industry from an economy using HEM, one implicitly assumes the extracted industry to be replaced by some "outside-of-system" imports so the other sectors receive inputs they need anyway (Dietzenbacher and Lahr, 2013). This assumption is valid as long as HEM is applied on national-level tables for which it was developed. Additionally, it can be used to analyze inter-regional relations on a supranational level as done by Dietzenbacher et al. (1993) who measured linkages within a part of the EU. However, Dietzenbacher et al. (2019) argue, that in the case of applying HEM on *global* MRIO tables, this assumption becomes flawed as there is no "outside-of-system" anymore. Tables such as WIOT (which are further described in section 2.6) include over 40 countries explicitly with supplemental "Rest of the World" region, which all together catch complete global GDP in one table. In such case, there is nothing left outside of this system. Therefore, assuming that the extracted industry is replaced by industry from a country that is not included in the input-output tables does not make sense anymore.

HEM's behavior at the national level

Assume HEM at the national level. Then, following Dietzenbacher et al. (2019), the implicit assumption described in the preceding paragraph can be quantified explicitly. Apart from the standard technical coefficients $\mathbf{A}$, value-added coefficients $\boldsymbol{\pi'} = \mathbf{v}'\hat{\mathbf{x}}^{\mathbf{-1}}$ and total value added $VA = \boldsymbol{\pi'x} = \boldsymbol{\pi'Lf}$ can be obtained alongside with import coefficients $\boldsymbol{\mu'} = \mathbf{m}'\hat{\mathbf{x}}^{\mathbf{-1}}$ and total imports of intermediate (internal) inputs $IMPINT = \boldsymbol{\mu'x} = \boldsymbol{\mu'Lf}$. Next, assume that HEM is applied, and the industry k is extracted. This means that there is a vector of new final demand $\tilde{\mathbf{f}}$ where $\tilde{f}_k = 0$, and new matrix $\tilde{\boldsymbol{A}}$ in which all $\tilde{a}_{ik} = \tilde{a}_{kj} = 0$ but other a_{ij} don't change ($a_{ij} = \tilde{a_{ij}}$ for $j \neq k$). Now, because technical coefficients are fixed, the missing intermediate goods are imported, thus making $\tilde{\mu}_j = \mu_j + a_{kj}$ for $j \neq k$ ($\mu_k = 0$ as k does not operate). Then satisfying the same final demands (except for f_k) gives

$$\tilde{\mathbf{x}} = (\tilde{\mathbf{I}} - \tilde{\mathbf{A}})^{-1} = \tilde{\mathbf{L}}\tilde{\boldsymbol{f}} \tag{2.61}$$

$$\widetilde{VA} = \boldsymbol{\pi'}\tilde{\mathbf{x}} = \boldsymbol{\pi'}\tilde{\mathbf{L}}\tilde{\mathbf{f}} \tag{2.62}$$

$$\widetilde{IMPINT} = \tilde{\boldsymbol{\mu}}'\tilde{\mathbf{x}} = \tilde{\boldsymbol{\mu}}'\tilde{\mathbf{L}}\tilde{\mathbf{f}} \tag{2.63}$$

As already mentioned, after an industry extraction with HEM, the output in the system decreases in all[13] other industries as they do not need to supply the sector k anymore. What has not been mentioned is that also VA and IMP change - VA negatively and IMP positively. Here $\Delta VA = \widetilde{VA} - VA = \boldsymbol{\pi'}\tilde{\boldsymbol{x}} - \boldsymbol{\pi'x}$ and $\Delta IMP = \Delta IMPINT + \Delta IMPFIN = \tilde{\boldsymbol{\mu}}'\tilde{\boldsymbol{x}} - \boldsymbol{\mu'x} + f_k$ (as f_k is the initial final demand that now has to be substituted by imports). Therefore, there is a decrease in domestic GDP, which is given by ΔVA. However, Dietzenbacher et al. (2019) show that ΔIMP

[13]More precisely put, the output of other industries in the economy decreases for industries connected to the extracted industry and stays the same for industries that have no interactions with the extracted sector.

correspondingly increases the foreign value added by the same amount and leaves the world GDP unchanged. At the national level, the process described above makes sense. Nonetheless, it is not the case when applying HEM at the global level as there is no space left to compensate for the VA domestic decrease, that is the global GDP would decrease.

Introduction to Global Extraction Method

This section introduces the Global extraction method (GEM) as described in Dietzenbacher et al. (2019). For an empirical example, see Dietzenbacher et al. (2019). Assume n industries within N countries. Define

$$\boldsymbol{Z}=\begin{bmatrix}\boldsymbol{Z}^{11} & \dots & \boldsymbol{Z}^{1N}\\ \vdots & \ddots & \vdots\\ \boldsymbol{Z}^{N1} & \dots & \boldsymbol{Z}^{NN}\end{bmatrix},\ \boldsymbol{F}=\begin{bmatrix}\boldsymbol{f}^{11} & \dots & \boldsymbol{f}^{1N}\\ \vdots & \ddots & \vdots\\ \boldsymbol{f}^{N1} & \dots & \mathbf{f}^{NN}\end{bmatrix},\ \mathbf{x}=\begin{bmatrix}\mathbf{x}^{1}\\ \vdots\\ \mathbf{x}^{N}\end{bmatrix},\ \mathbf{v}=\begin{bmatrix}\mathbf{v}^{1}\\ \vdots\\ \mathbf{v}^{N}\end{bmatrix} \tag{2.64}$$

where $\mathbf{Z}$ is $Nn \times Nn$ matrix of transactions, $\mathbf{F}$ is $Nn \times N$ matrix of final demands, and $\mathbf{x}$ and $\mathbf{v}$ are Nn elements vectors of output and value added, respectively. Let R,$S \in (1,...N)$ and i,$j \in (1,...n)$. Then z_{ij}^{RS} in $\mathbf{Z}^{RS}$ is a transaction from the industry i in the country R to the industry j in the country S, f_i^{RS} is a n-element vector of deliveries from the industry i in the country R for final demands in S, x_i^R represents final output of the industry i in the country R, and v_j^R describes value added of industry j in country R. Then $\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1}$ and $\mathbf{A}^{RS} = \mathbf{Z}^{RS}(\hat{\mathbf{x}}^S)^{-1}$ give technical coefficients. Moreover, $\mathbf{f}^R = \sum_S \mathbf{f}^{RS}$ and $\mathbf{f} = \mathbf{Fi}$.

Now, extract industry k in country H, that is

$$\tilde{a}_{kj}^{HS} = \tilde{a}_{ik}^{TH} = 0 \ \ \forall i,j,\ \forall S,T; \tag{2.65}$$

$$\tilde{f}_k^{HS} = 0 \ \ \forall S \tag{2.66}$$

This means that any industry that was using inputs from Hk needs to find a new source. In HEM, this is where outside-the-system imports step in, however, GEM proceeds differently.

Assume a hypothetical example, where the automobile industry in the Czech Republic uses iron to make cars. From all the iron supplied to the industry, 40% come from the Czech Republic, 40% from Poland, and the rest from Germany. Now, let's extract the sector producing iron in the Czech Republic. Then GEM assumes the German and Polish iron-making sectors to increase their output in order to satisfy the Czech automobile sector's demand. Both, Polish and German, sectors will maintain the same proportion of production so now 66.7% of iron supplied to the Czech car-making industry will come from Poland and 33.3% will come from Germany. These percentages must always add up to 100% no matter how many supplying sectors there are.

Another point of view comes up when extracting a "foreign" sector. That is, assume the same situation but from a perspective of a German car-producing sector. Let's say, for example, that this sector was getting 60% of the iron from Germany, 20% from the Czech Republic, 10% from Poland and 10% from France. Dietzenbacher et al. (2019) argue, that, again, the non-extracted industries will replace the extracted (Czech) industry, with one difference - the German-supplied iron ratio stays the same (i.e. 60%)[14]. So in the end, Poland and France both end up supplying 20% of the demanded iron.

The same assumptions hold for final demands. In the example, Czech Republic was country H and iron producing industry is industry k. Putting down all of the mentioned assumptions mathematically gives

$$\tilde{a}_{kj}^{TS} = a_{kj}^{TS} + a_{kj}^{HS} \frac{a_{kj}^{TS}}{\sum_{R \neq H,S} a_{kj}^{RS}} \quad \forall j, \forall S, \forall T \neq H, S \tag{2.67}$$

$$\tilde{f}_{k}^{TS} = f_{k}^{TS} + f_{k}^{HS} \frac{f_{k}^{TS}}{\sum_{R \neq H,S} f_{k}^{RS}} \quad \forall S, \forall T \neq H, S \tag{2.68}$$

Further calculations are similar to the ones with HEM. For a country T, the change in output is computed as $\tilde{\mathbf{x}}^T - \mathbf{x}^T$ and change in value added as $\Delta VA^T = \widetilde{VA}^T - VA^T = (\boldsymbol{\pi}^T)'(\tilde{\mathbf{x}}^T - \mathbf{x}^T)$. In both, HEM (at the national level) and GEM (at the global level) holds that global value added (GDP) does not change with the extraction taking place.

[14]Dietzenbacher et al. (2019) argue that there must have been a reason for the German car producer to use imported goods before the extraction. This might be that German iron is too expensive or there is a limited supply of it. These constraints will not disappear after the extraction, so the imports from the Czech Republic are replaced completely by more foreign imports. See Dietzenbacher et al. (2019) for further justification of such assumption.

For a mathematical proof of such a statement, see Dietzenbacher et al. (2019) page 7.

Conclusion

To sum up, Dietzenbacher et al. (2019) came up with an alternative to the HEM and its theoretical shortcoming when applied on a global scale. GEM's objective is to avoid HEM's implicit assumption according to which the missing outputs from the extracted sector are replaced by endogenous imports as this assumption does not make sense on a global level. Instead of exogenous imports, GEM magnifies existing imports from other countries in the system.

An advantage of GEM is that it calculates effects in foreign countries relatively realistically, whereas applying HEM on global data results in a decrease (or no effect) of all the outputs in the system. However, GEM is also more data demanding and requires more assumptions (specifically about how the extracted outputs are replaced). For both methods (GEM on global and HEM on national-level data) holds that global GDP remains the same and redistribution of value-added happens.

"Which model is superior?" one may ask. Dietzenbacher et al. (2019) argue, that when one is interested in international effects, GEM should perform better. In the case of domestic effects, there is no straightforward answer to such a question. The authors argue that when available, the global model should be used as it catches possible feedback effects. On the other hand, HEM is more straightforward to apply as GEM requires more assumptions and even its authors claim that GEM's empirical applications yet have to be proven worth the extra assumptions and data requirements. In the end, Dietzenbacher et al. (2019) conclude that "*GEM is an interesting alternative to HEM (...). The choice between GEM and HEM in empirical applications, however, should depend on the research question (...) in combination with data quality and availability.*"

2.5.7 Disaster Impact Analysis

As this thesis touches the ground of a disaster impact analysis, it seems relevant to briefly mention how the input-output analysis is used in order to investigate a disaster impact. Okuyama and Santos (2014) claim that the input-output framework has been the most the frequently used tool in a short term disaster analysis. Furthermore, the authors state that the input-output model's advantage is that it catches economic interdependencies together with its relative simplicity.

The first of the existing models is an inoperability input-output model (IIM). IIM analyzes a situation of a decrease in economic activity with an inoperability vector (which gives percentage changes in output due to the catastrophe). Very briefly said, the main difference between the classic input-output model and IIM is that IIM gives inoperability index in a range between 1 (ideal state) and 0 (failure) as its output. Dietzenbacher and Miller (2015) claim IIM to be only a "relabeled" version of the classical input-output model that was introduced in section 2.2. As an empirical example of such a model, research by Santos and Haimes (2004) can be mentioned as the authors model an economic impact of a decrease in demand for the air-transport industry due to terrorism on the U.S. economy.

Next, Dietzenbacher and Miller (2015) consider HEM to be an alternative to the IIM. They suggest HEM as it works under a scenario in which one or more industries shut down which might be a realistic assumption in case of a natural catastrophe. Moreover, the authors suggest using a generalized hypothetical extraction method as developed by Dietzenbacher and Lahr (2013). This method considers only a partial closedown of an industry which in the real world might represent the extraction of a sub-industry from the system or partial slowdown due to capacity constraints.

Lastly, Dietzenbacher and Miller (2015) mention dynamic IIM (DIIM) and mixed input-output models as methods suitable to analyze the recovery path from a disaster. The mixed model is a model in which some sector's outputs or gross outputs are specified exogenously (oppositely to the demand-driven input-output model). Introduction to such a model can be found in Miller and Blair (2009), chapter 13. Dynamic input-output models describe how is the gap between produced output and demanded output closed in time. Dynamic models basics are described, again, in Miller and Blair (2009), chapter 13. Dynamic IIM was described in detail by Lian and Haimes (2006). For more detailed overview of a disaster impact analysis methods, see Dietzenbacher and Miller (2015) and Okuyama and Santos (2014).

2.5.8 Section Summary

This chapter introduced the hypothetical extraction method, under which one or more sectors are hypothetically extracted in order to analyze other sectors dependence on the extracted sector. The total linkage measure describes sector's importance to the whole economy as a decrease of total output after the sector's extraction. Backward linkage provides such measure but solely for upstream industries of the extracted sector. Such industries decrease their output because of a lack of demand from the extracted sector. On the contrary, forward linkages measure the importance of the extracted sector to its downstream industries, it means industries that decrease their production due to a loss of supplies. Lastly, an alternative to the hypothetical extraction method on a global level - the global extraction method - was introduced. It was argued that HEM at the national level assumes the extracted sector to be replaced by foreign (out-of-system) imports, thus not changing world GDP. However, when applying HEM on global input-output tables, the assumption of out-of-system imports is no longer possible. Instead, GEM assumes the extracted sector to be replaced by imports from other countries in the system.

2.6 World Input-Output Tables

This section describes input-output tables (IOTs) used later in the empirical part of this thesis. Nowadays, there are ready-to-use input-output tables freely available on the internet so one does not have to invest time and resources in order to construct their own tables.[15] The tables to be used depend on purpose of the research. For example, some tables account for environmental point of interest and thus would be preferable to scientists interested in measuring, for example, carbon footprint or energy efficiency. Examples of such research are, among others, Guerra and Sancho (2010), Zhao et al. (2016), Sajid et al. (2019), or Sajid et al. (2020).

One of the free sources of input-output data is OECD (2018) with its Inter-Country Input-Output Tables (ICIO)[16]. Its latest 2018 release contains information about flows between 36 sectors within 64 countries. However, this section pays attention mainly to World Input-Output Tables, used in the empirical part of this thesis.

[15]For a detailed overview of global multiregional input–output projects, see Tukker and Dietzenbacher (2013).

[16]Available at oecd.org

2.6.1 World Input-Output Database

Wiedmann et al. (2011) argue that a modern tool enabling for analysis of social, environmental and economic problems must be global. It must cover socioeconomic indicators and it must describe over-time changes. World Input-Output Database (WIOD) project tries to respond all of the above-mentioned requirements (Dietzenbacher et al., 2013).

WIOD used publicly available data from official national statistical offices and national input-output tables in order to ensure high data quality and full transparency in creation of World Input-Output Tables (WIOTs). Dietzenbacher et al. (2013) point out that a process of harmonization and standardization of Supply and Use Tables (SUT)[17] needed to be developed as WIOTs consists of 17 consecutive years but SUTs are usually not provided on a yearly basis. Moreover, the participants of the WIOD project needed to develop international SUTs. Overall, the creation of WIOTs was a highly technical process which, however, is not so important for this thesis. For more detail on how were the WIOTs created see Dietzenbacher et al. (2013) and Timmer et al. (2015). Also, social and environmental accounts are not described here any further as they are not used in this research; see Genty et al. (2012) and Erumban et al. (2012) for more detail on environmental and socieconomic accounts, respectively.

Table 2.4: World Input-Output Table's Scheme

<table>
<tr><td colspan="3" rowspan="3"></td><td colspan="7">Use by Country-Industries</td><td colspan="3">Final Use by Countries</td><td rowspan="3">Total Use</td></tr>
<tr><td colspan="3">Country 1</td><td>...</td><td colspan="3">Country M</td><td rowspan="2">Country 1</td><td rowspan="2">...</td><td rowspan="2">Country N</td></tr>
<tr><td>Indus. 1</td><td>...</td><td>Indus. N</td><td>...</td><td>Indus. 1</td><td>...</td><td>Indus. N</td></tr>
<tr><td rowspan="7">Supply from Country-Industries</td><td rowspan="3">Country 1</td><td>Indus. 1</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>...</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Indus. N</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>...</td><td>...</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td rowspan="3">Country M</td><td>Indus. 1</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>...</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Indus. N</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="3">Value Added</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="3">Gross Output</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

Source: Timmer et al. (2015)

World Input-Output Tables Content

What is important, though, is the content of the WIOTs. Table 2.4 depicts WIOTs outline. The transaction matrix captures supply of goods from producing industries

[17]SUTs are a core source of information for WIOTs.

by country to consuming industries by country. Moreover, the final demand for each industry's products is available for each country separately. Additionally to the explicit countries, the Rest of the World (RoW) region is also covered. According to Timmer et al. (2015) and Dietzenbacher et al. (2013), the explicit part covers aproximately 85 % of the world GDP (as in 2008) and the rest lies within the RoW region. All of the content of the WIOTs is denoted in U.S. dollars million.

WIOT Compared to Other Tables

The biggest advantage of WIOD tables over other available tables is that they are constituted as a time series of such tables, thus they enable for comparability in time. Timmer et al. (2015) claim that there was not any other project enabling for this feature available in the world by 2015. Another advantage is that WIOTs are built on publicly available data so it is possible to replicate them or to use the same underlying data to construct similar tables using different assumptions. Moreover, WIOTs are based on SUTs which are the core input for constructing national IOTs using additional assumptions, and WIOTs are, therefore, free of these needless extra assumptions and use mainly the core data. Last but not least, WIOD provides also socioeconomic and environmental[18] accounts to their tables.

Compared to OECD ICIO, WIOTs provide information about more industries, but on the other hand, ICIO explicitly covers approximately 20 more countries than WIOTs.

2013 and 2016 Releases

World Input-Output Tables (WIOTs) were first created in 2013 within the WIOD project. The WIOD (2013) release consists of input-output tables for 35 industry groups within 40 countries for years 1995-2011. Compared to the later 2016 release, its advantage is that it also consist of environmental accounts and thus might be preferred by researchers interested in such topics as, for example, Sajid et al. (2019) and Sajid et al. (2020). The WIOD (2016) release of WIOTs captures 56 industry groups[19] using International International Standard Industrial Classification of All Economic Activities (ISIC)[20] within 43 countries (see Appendix A.1) making it more detailed

[18]Environmental accounts are available only for the 2013 release as discussed later in this section.
[19]For further detail on industries included in WIOT 2016 see Appendix A.2.
[20]For more information about ISIC, see United Nations (2008).

than the 2014 release. It covers years 2000-2014. Socioeconomic accounts are also part of both releases. For further commentary on both releases of WIOTs, see Timmer et al. (2014).

2.7 Chapter Summary

This chapter introduced the input-output framework together with models used to analyze such framework as described in the existing literature. Besides other sources, two main sources were used. First, textbook written by Miller and Blair (2009) gives an excellent introduction to the input-output analysis. Second, Miller and Lahr (2001) give a broad overview of the hypothetical extraction method that was described in section 2.5.

This chapter began with a brief introduction into the historical development of ideas considering the input-output framework in section 2.1. Secondly, section 2.2 introduced transaction and the final-use framework for a n-industry economy as described in a textbook written by Miller and Blair (2009). Subsequently, the Leontief model was developed. Such model enables analysis of the input-output framework. Later in this section, selected details about the input-output framework and analysis were discussed in sections 2.2.5 and 2.2.6. In section 2.3, the national-level framework introduced in section 2.2 was expanded to enable multiregional analysis.

Next, section 2.4, introduced an alternative to the Leontief model described in section 2.2. Such a model was developed by Ghosh and it can be viewed as a transposed version of the Leontief model. The main difference between the models is that the Leontief model is demand-driven, whereas the Ghosh model is supply-driven. Such supply-side model is particularly useful for forward linkages estimation as argued in section 2.5.

Section 2.5 introduced three main inter-industry linkage measures. First, the total linkage is a measure of total economic importance of a sector to other sectors. Secondly, backward linkages describe only the interconnections between an industry and its upstream industries. On contrary to that, forward linkages describe interconnections with downstream industries only. In this section, two methods useful for linkage measurement were introduced - the hypothetical extraction method and the alternative global extraction method. Simply put, the extraction methods extract an industry from the economy and observe what happens to the output in the economy after such extraction.

Finally, section 2.6 introduced and described creation and contents of World Input-Output Tables which are used in the empirical part of this thesis.

3. Methodology and Data

This chapter describes the methodology and data that were used to answer the three research questions stated in chapter 1. Once again, the research questions are:

1. What would be the overall effect on the Czech economy's output in case of air transport's breakdown? In other words, what are air transport's *total linkages* to the rest of the economy?

2. What would be the overall backward (demand-side) effect on the economy and what *upstream industries* would be hit the most in case of the shutdown of air transport? In other words, what industries does air transport have the strongest *backward linkages* with?

3. What would be the forward (supply-side) effect on the economy and what *downstream industries* would be hit the most in case of the shutdown of air transport? That is, with what industries does air transport have the biggest *forward linkages*?

The chapter is divided into 5 sections. The first section (3.1) describes in detail WIOD's tables and explains why they were chosen for this research. Secondly, section 3.2 argues what extraction method (HEM or GEM) was used in order to estimate the air-transport industry linkages and why. Next, sections 3.3 and 3.4 describe how the Leontief model was used to estimate the air-transport total and backward (upstream) linkages, respectively. Lastly, section 3.5 depicts how the air-transport industry forward (downstream) linkages were determined using the Ghosh model.

3.1 Data Used

3.1.1 Input-Output Tables Used

In the empirical part of this thesis, the 2016 release of World Input-Output Tables is used. With respect to the OECD ICIO tables, WIOTs were chosen particularly because of their detail as they cover more industries (specifically air-transport which is of main concern to this thesis). WIOT for the year 2014 is used as it is the latest table available. It might seem that the 2014 table could be outdated due to the economic growth of the world, however, as this analysis is interested mainly in the structure of the world economy and as a more recent alternative is unavailable, 2014 tables should be appropriate.

The 2014 WIOT consists of 2690 columns and 2472 rows. The first 2464 rows and columns make a symmetrical *transaction matrix* that contains information about transactions between 56 industries within 44 regions (43 countries and Rest of the World - RoW). Next, row 2465 contains *total intermediate consumption* of each sector, that is it sums all elements of the transaction matrix column-vise. Rows 2466 to 2471 cover individual components of *value added*.[1] The last row contains columns sums of all the above mentioned, that is *total outlays* of all industries in the table.

From the column perspective, the first 5 columns contain a description of countries and industries. Then 2464 columns with transactions follow. Next, columns 2469 to 2689 consist of *final demand* by countries. Again, when summing row elements of the matrix (that is transactions and final use), one gets *total outputs* of all sectors. For more illustration on WIOTs, see Table 2.4 in chapter 2.6.

3.1.2 National versus World Tables

When conducting such a type of research as in this thesis, it is necessary to think about whether to use national or international (world) input-output tables. Miller and Blair (2009) argue, that when the interest lies in the effects within a particular economy, national tables for the demanded economy should be used. On the other hand, Dietzenbacher et al. (2019) suggest that the global model is superior to the national one as it also accounts for the international feedback effects.

For the empirical part of this thesis, a global model was estimated. As the national economies are deeply interconnected, the global model should better reflect such situation, and leaving out the rest of the world would cause a loss of detail of the model. Moreover, one of the main disadvantages of the global model is data availability. However, as WIOD provides sufficiently detailed data, this possible weakness disappears.

[1] These consist namely of *taxes less subsidies on products*, *export adjustments*, *direct purchases abroad by residents*, *purchases on the domestic territory by non-residents*, *value added at basic prices*, and *international transport margins*.

3.2 Hypothetical versus the Global Extraction Method

As global tables and global models are used in the empirical part of the research, the potential use of GEM instead of classical HEM should be discussed. As mentioned in section 2.5.6, HEM implicitly assumes the extracted goods to be replaced by "outside-the-system" imports. However, as discussed in the previous section 3.1.2, global tables and global model are used in this thesis. For such model, this implicit assumption is flawed, as there is no "outside of the system". For such case, Dietzenbacher et al. (2019) suggest HEM's adaptation GEM to be considered. On the other hand, Dietzenbacher et al. (2019) also state, that HEM is simpler and enables for better comparability as GEM needs more assumptions to work.

Moreover, as shown in section 2.5.6, the main differences between results of HEM and GEM are in the economies foreign to the extracted industry. That means, for example, that as air transport in the Czech Republic is extracted, it does not supply to any domestic and foreign sectors. But the foreign sectors need the same inputs as before the extraction and thus they will demand more from the other air-transport industries in the world.[2] Consequently, air transport in some countries could profit from the extraction in the Czech Republic which would have some high-order feedback effects on some Czech industries in return. However, these foreign effects would affect the Czech economy only limitedly because of the Czech air transport's relatively small size. Moreover, the higher the order of the effect, the smaller the effect gets and as such the potential error would be insignificant. Therefore the potential of missing the positive feedback effect is of little concern to the topic of this thesis that targets the estimation of domestic linkages in the Czech economy solely.

In addition, the main concern of HEM is that it might overestimate the slowdown of the economy due to the extraction. However, in certain cases, it might be favourable to know the worst possible scenario. In the hypothetical example in the introduction, a country government was mentioned. In a Covid-like crisis, it might be advantageous for the government to know what maximum damage could air transport shutdown cause.

[2]For an example of such situation, see Dietzenbacher et al. (2019) who provide results for extraction of the Chinese car industry. According to their results, such extraction would significantly increase the output of car-making industries in Germany, Japan, the U.S., and many more countries.

Finally, Dietzenbacher et al. (2019) based GEM on the Leontief demand-driven model. But as argued by Miller and Lahr (2001) or Miller and Blair (2009), the Ghosh supply-driven model is better suited for the forward-linkages estimation than the Leontief model. Thus using GEM would mean rejecting the Ghosh model for estimation of forward linkages, which is not preferable.

To conclude, there is a trade-off between GEM's theoretical appeal and HEM's simplicity, its empirical background, and compatibility with the Ghosh model. As this thesis is domestically oriented and does not pay attention to foreign effects, HEM is considered better-suited than GEM for this research. However, in the case of interest in cross-border linkages or in case of extraction of a sector in more than one country, it might be preferable to use GEM instead of HEM.

3.3 Total Linkages

The first research question asks for a measurement of the overall effect of the air transport extraction from the Czech economy. To do so, total linkages of the air-transport industry are to be estimated. From the theoretical perspective, this is done as described in section 2.5.3. As suggested by Miller and Lahr (2001), in order to measure total linkages with HEM, both rows and columns that contain air transport's transactions are set to zero. Then the difference between pre-HEM and HEM outputs represent the total linkages of the air-transport industry.

In practice, it can be done as follows. Firstly, transaction matrix $\mathbf{Z}$, final use matrix, and final output vector $\mathbf{x}$ are obtained from WIOT. Before it is possible to continue, it is necessary to aggregate the final use matrix into a single-column vector $\mathbf{f}$. Now, from the transaction matrix, it is possible to compute the matrix of technical coefficients $\mathbf{A}$ by dividing each number in each row by the corresponding total output as described in section 2.2.3 (equations (2.5) to (2.9)). The next step is to subtract such matrix from the identity matrix $\mathbf{I}$ and invert the result in order to get the Leontief matrix $\mathbf{L}$ as in (2.10) to (2.13). At this point, $\mathbf{x}$, $\mathbf{L}$, and $\mathbf{f}$ are known and it holds that $\mathbf{x} = \mathbf{Lf}$. Moreover, $\mathbf{x}$ represents the initial output which is later to be compared with the post-HEM output to obtain total linkages.

After having the initial model set up, it is possible to proceed to the hypothetical extraction. Firstly, in order to catch all the transactions from and to the demanded industry, the row and the column that represent the industry in the $\mathbf{A}$ matrix are set to zero as in (2.45). To follow the same notation as in section 2.5.1, let $\tilde{\mathbf{A}}$ be the $\mathbf{A}$ matrix after the extraction.

The question is whether to also null the corresponding element of the final demand. By keeping the final demand on its initial level, it is assumed that the final demand is satisfied by foreign exports implying that foreign output should increase correspondingly in order to satisfy it. However, as the input-output model and HEM on a global level assume fixed relations in the system, this implication does not translate into the model. All in all, as long as the interest lies only in output effects, it should not matter, whether the final demand is kept the same and later added to the total linkage measure or set to zero straight away as the extracted sector does not produce anymore and is thus not able to react to any change in the final demand. However, when one is also interested in value added effects, the extraction of final demand matters. In this particular case, the final demand was set to zero in order to extract the sector "completely", and to enable for more realistic measurement of value added, which is discussed later in this section.

After adjusting $\mathbf{A}$, $\mathbf{x}$, and $\mathbf{f}$ for the extraction, the process goes as with the initial model. This means estimation of the new Leontief matrix using $(\tilde{\mathbf{I}} - \tilde{\mathbf{A}})^{-1}$ together with subsequent calculation of the new total output as $\tilde{\mathbf{x}} = \tilde{\mathbf{L}}\tilde{\mathbf{f}}$. The new Leontief matrix takes the form of $\tilde{\mathbf{L}} = (\tilde{\mathbf{I}} - \tilde{\mathbf{A}})^{-1}$ as in (2.46). Finally, total linkages are defined as a difference in output before and after the extraction also accounting for the final demand change of the extracted vector. Thus, $\tilde{\mathbf{x}}$ is subtracted from $\mathbf{x}$ as in (2.48). Summing the resulting vector $\Delta\mathbf{x} = \mathbf{x} - \tilde{\mathbf{x}}$ by elements gives the total linkage of the extracted industry to the whole economy.

In the case of total linkages, it is also possible to look at each sector individually in order to see which sectors were hit the most by air transport's extraction. However, it would be misleading to consider such disaggregation to measure the *total* effect on the non-extracted industries. This is because of the nature of the Leontief model, which is demand driven, and as such it is able to catch only the demand-side (i.e. the backward) effects of the extraction. Therefore, such effort to estimate the total effect on the other industries would lead to identical results to the backward linkages estimation (except for the air transport itself).

In addition to the effect of the air transport's extraction on the output of the economy, it is also possible to estimate an effect of such extraction on the *value added* of the economy. Such effect can be estimated using an extra assumption about a fixed ratio of the value added and total output for each industry. Using this assumption, a vector of value added coefficients can be obtained from the initial economy as in section 2.5.6, that is as $\boldsymbol{\pi'} = \mathbf{v}'\hat{\mathbf{x}}^{\mathbf{-1}}$, where total value added is $VA = \boldsymbol{\pi'}\mathbf{x}$. Later, after the extraction, the new output of all industries in the economy is known. Assuming the proportion of value added and total output to be fixed, it is possible to obtain post-extraction value added of the economy by multiplying the total output vector by the value added coefficients vector and summing the elements as $\widetilde{VA} = \boldsymbol{\pi'}\tilde{\mathbf{x}}$. Having such measure of change in value added is useful as such measure corresponds to the change in an economy's GDP.

3.4 Backward Linkages

The second research question considers backward linkages of air transport in the Czech Republic. The estimation of the initial model follows the previous section. This means that from $\mathbf{Z}$ and $\mathbf{x}$, $\mathbf{A}$ is obtained. From $\mathbf{A}$ using $\mathbf{I}$, Leontief matrix $\mathbf{L}$ is estimated for which holds that $\mathbf{x} = \mathbf{Lf}$. Next, the extraction takes place. This time, however, only the column of $\mathbf{A}$ containing air transport is nullified as suggested by Miller and Lahr (2001). It is to decide whether to null the intersection of air transport's column and row. Nullifying this element as well would mean measuring air transport's linkages to upstream industries and itself. As it is desired to measure linkages to upstream industries only, intra-sectoral flows are kept untouched. This scenario represents a situation in which air transport is no longer using inputs from its suppliers.[3] Thus the HEM matrix of technical coefficients $\tilde{\mathbf{A}}$ takes form as in (2.49). Similarly to total linkages, HEM Leontief matrix is obtained using $\tilde{\mathbf{L}} = (\tilde{\mathbf{I}} - \tilde{\mathbf{A}})^{-1}$ which corresponds to (2.50). Again, using (2.12), $\tilde{\mathbf{x}} = \tilde{\mathbf{L}}\mathbf{f}$, where $\tilde{\mathbf{x}}$ is the new total output under HEM. Finally, by comparing initial output $\mathbf{x}$ and the new output $\tilde{\mathbf{x}}$, one gets backward linkages of the air transport as in (2.51).

As suggested by Miller and Blair (2009), it is possible to examine every sector individually. Backward linkages, in this case, were obtained as $\mathbf{x} - \tilde{\mathbf{x}}$. Let $\boldsymbol{\Delta}\mathbf{x} = \mathbf{x} - \tilde{\mathbf{x}}$, then each element of $\boldsymbol{\Delta}\mathbf{x}$ gives the decrease of the corresponding industry's output caused by the lack of demand from the extracted air-transport industry. By examining

[3]This scenario counts with air transport still producing after the extraction and thus the final demand for its outputs is kept the same.

every sector, it is possible to identify industries with the strongest upstream (backward) interconnections with the air transport.

3.5 Forward Linkages

As mentioned already in section 2.5.5 and in the introduction to this chapter, the Ghosh model is to be used in order to answer the third research question about forward linkages of the air-transport industry. There are flaws to the Ghosh model (as mentioned in chapter 2.4), though, because as industries reach a new level of intermediate production (as, after the extraction, they are supplied with less inputs from the extracted industry), they should also demand a new level of inputs from other industries. Such adjustment, however, does not take place in the Ghosh model. Moreover, as discussed in chapter 2.4, the interpretation of such model was also subject to broad academic discussions in the past as it is not very clear. As discussed in section 2.4, these flaws lead to a reinterpretation of the Ghosh model as the price model. Even though it might be imperfect, the Ghosh model probably is the best option for forward linkages estimation anyways.

As mentioned already, the logic behind this choice is connected to the definition of forward linkages. Forward linkages are the linkages of an industry with its downstream industries. Downstream industries, however, won't be affected by lack of demand from the extracted industry as they are supplied by the extracted industry. Thus, downstream industries linkage is more likely to be supply-driven than demand-driven. The Leontief model is demand-driven thought, and therefore it is not well suited for this purpose. In chapter 2.4, the Ghosh supply-driven model was developed. As argued in chapter 2.4, this model is a kind of a transposed Leontief model. Thus, the following process of forward linkages estimation is very similar to the one presented in sections 3.3 and 3.4, although in a rotated manner. It follows methodology and notation from sections 2.4 and 2.5.5.

Firstly, it is necessary to set up the initial Ghosh model for the full data set. The procedure starts again with transaction matrix $\mathbf{Z}$ and total outlays $\mathbf{x}$. This time however, it is not columns that are divided by $\mathbf{x}$ as in the Leontief model but rows. The direct-output coefficients matrix $\mathbf{B}$ is computed according to (2.27). Next, the Ghosh output inverse is computed similarly to the Leontief inverse as $\mathbf{G} = (\mathbf{I} - \mathbf{B})^{-1}$ from (2.30). Before it is possible to proceed to the full Ghosh model setup, it is necessary to aggregate value added matrix into a row vector $\mathbf{v}'$. As mentioned in section 3.1.1, WIOTs contain 6 disaggregated value added components. These need to be aggregated into one vector simply by summing the 6 rows of the matrix. At this point, $\mathbf{x}$, $\mathbf{v}'$, and $\mathbf{G}$ are known for which (2.31) and (2.35) hold, i.e. $\mathbf{x}' = \mathbf{v}'\mathbf{G}$ and its transposed form $\mathbf{x} = \mathbf{G}'\mathbf{v}$ which corresponds to the form of results of the Leontief model.[4]

Hypothetical extraction in the Ghosh model follows the same logic as with the Leontief model. Again, the air-transport industry is extracted and the resulting outputs are compared to the initial outputs. To measure forward linkages, Miller and Lahr (2001) recommend nullifying the row corresponding to air transport (and similarly to backward linkages, keeping the intra-industry flows). This scenario could represent a situation in which air transport stops supplying other sectors within the system and produces only for exports.[5] From the new $\tilde{\mathbf{B}}$, using (2.30), new HEM direct-output matrix is obtained as $\tilde{\mathbf{G}} = (\tilde{\mathbf{I}} - \tilde{\mathbf{B}})^{-1}$. Finally, the new outputs are obtained using (2.35), i.e. $\tilde{\mathbf{x}} = \tilde{\mathbf{G}'}\mathbf{v}$. Forward linkages of the air-transport industry are then obtained by taking a difference between the initial and HEM outputs, analogously to total and backward linkages. Additionally, as with the backward linkages, it is possible to examine the output effect industry by industry identifying industries with the strongest downstream (forward) interconnections with the air transport.

[4]There is a possible "shortcut" to the whole process as, from the similarity of $\mathbf{A}$ and $\mathbf{B}$ in (2.39), it is possible to express $\mathbf{G}$ using (2.41) as $\mathbf{G} = \hat{\mathbf{x}}^{-1}\mathbf{L}\hat{\mathbf{x}}$. However, when conducting hypothetical extraction, it seems more transparent and simpler to go through the whole estimation step by step.

[5]Again, similarly to backward linkages estimation, it, therefore, makes sense to keep value added on its initial level.

4. Results

This chapter presents and discusses the results of the analysis of the total, backward, and forward linkages, respectively, using the methodology described in the previous chapter 3. The three different linkage measures should be looked at separately though, as they use distinct models and answer different questions. Moreover, note that backward and forward linkages *do not* add up to total linkages, which might be counter-intuitive but well justified. Firstly, as mentioned in chapters 2 and 3, total and backward linkages are estimated using the Leontief demand-driven model, whereas forward linkages are obtained via the Ghosh supply-side model. Secondly, neither in backward nor forward linkages estimation the intraindustry transactions were extracted contrarily to total linkages where all transactions (including intraindustry transactions) were extracted.

4.1 Total Linkages

This section presents the results of the total linkages analysis as described in section 3.3. The analysis is done mainly by analyzing the output of the economy with and without the air-transport sector in it. The change in output represents the total linkages of air transport with the rest of the economy. Moreover, the effect on value added is going to be presented.

After extracting air transport from the Czech economy, the intermediate output of the Czech economy decreases by USD 2353 million. Adding the air transport's final demand, which was also extracted, translates into a total linkage of USD 2678 million meaning a 0.54% of the economy's overall pre-extraction output. Out of the 2678 million, 1297 million comes from the air transport itself, whereas the rest 1381 million from all the other industries in the economy meaning that more than 50% of the decrease in total output originates in industries connected to air transport not in the air transport itself. Such effect in the non-extracted industries is quite large.[1]

[1] For illustration, in case of extraction of Land transport, approx. 60% of the decrease would come from Land transport itself. In case of Construction, almost 70% of the decrease comes from the sector itself. Thus in comparison with these two sectors, air transport is more interconnected with the rest of the economy. Please note, that these two sectors were selected randomly for illustrative purposes. Such a comparison can be done for any two or more sectors in the economy.

As discussed in chapter 3.3, estimation of change in value added can be done subsequently to the total-output estimation. Total value added of the Czech economy decreased from USD 185.4 billion by USD 688 million meaning a 0.37% decrease after air transport's extraction. Out of the 688 million decrease, approximately 20% (USD 152.5 million) originates in the value added of air transport and the remaining 80% of value added effect happens in the other industries.

Finally, as discussed in chapter 3.3, it is possible to look at each sector separately. However, as the Leontief model captures only the demand-side effects of the extraction on the non-extracted industries, it also yields the same results for these industries as the backward linkages model in the following chapter. Such disaggregation, therefore, is not presented in this section and is included in the following section only.

To sum up, the answer to the research question number one is that, according to the model, the Czech air transport's breakdown would result in USD 2678 million decrease in output of the Czech economy. In other words, the model suggests the total linkages of the air transport industry to the Czech economy to equal USD 2678 million. Moreover, assuming fixed value-added/total-output ratio, the model also estimated the Czech value added to decrease by 0.37% (USD 688 million).

4.2 Backward Linkages

The analysis of backward linkages answers the question about air transport's linkages with its upstream industries. As described in chapter 3.4, the main difference in computation of total and backward linkages is that in case of total linkages, all transactions were set to zero, whereas in case of backward linkages only the column containing inputs to air transport from other industries were nullified (except for intraindustry flows).

The overall backward linkage of air transport was estimated to USD 1387 million meaning that the output of the Czech economy would decrease by USD 1387 million in a scenario in which air transport imports all of its inputs decreasing demand for existing inputs (a scenario described in chapter 2.5.4). The decrease of USD 1387 million equals to 0.28% of the initial output. Contrarily to the total linkages, almost all of the effect comes from the non-extracted industries. Interestingly, the output of air transport itself decreases by USD 6 million even though air transport's outputs were not extracted. This is caused by the decrease in other industries' demand for air transport's inputs as they do not produce as much as before the extraction. This effect is therefore purely of a higher order. Moreover, at this point, we have enough information to see that the only difference between the total and backward linkage is in the initial output of air transport less the decrease in air transport's output in this scenario.[2]

Finally, backward linkages enable to answer the research question asking what upstream industries would be hit the most by air transport's collapse. Table 4.1 provides answers to such question. It is divided into two parts. First, the upper table presents ten industries with the strongest backward linkages to air transport in absolute terms. The lower table sorts the industries in relation to their initial output (i.e. their size). The first table corresponds to the industry linkages definition, which says that linkages are measured as the change in output after the extraction. On the other hand, the second table compares the change in an industry's output with its overall output before the extraction, meaning that the second table takes into consideration the industry's size. This part might detect industries which were hit significantly but would not appear in the primary search. It can be seen that the first table contains generally bigger industries than the table in relative terms. Thus, the choice of table depends on the required information. For the complete set of results of the backward-linkages analysis see appendix A.4.

From both absolute and relative perspectives, the four sectors hit the most by air transport's extraction are Warehousing and support activities for transportation (with linkage of USD 445.9 million), Manufacture of coke and refined petroleum products (with output change of USD 125.6 million), Financial service activities except insurance and pension funding (decrease by USD 110.9 million), and Land transport and transport via pipelines (USD 79.5 million). The four together account for approximately half of the output decrease in the economy. Together, the ten industries hit the most make up for 70% of air-transport's linkages which suggests air transport to be significantly connected to few industries in the economy (especially the first four), whereas not so much to the remaining industries.

[2]Total linkage was measured as decrease in total output including the final demand for air transport being $2353 + 325 = 2678$. The intermediate output of air transport was 972 which, after adding final demand makes total output of 1297. Finally, by adjusting backward linkage of 1387 by air transport's total output of 1297 minus the 6 that air transport decreased in backward linkages, one gets $1387 + 1297 - 6 = 2678$ which is the total linkage. All values are denoted in USD million.

Table 4.1: Backward linkages: Ten most hit industries in **(1)** absolute and **(2)** relative terms (in USD million)

(1)	Industry	Total output	HEM TO	Output change	Change (%)
1	Warehousing and support activities for transportation	10933.5	10487.6	-445.9	-4.08
2	Manufacture of coke and refined petroleum products	6563.5	6437.9	-125.6	-1.91
3	Financial service activities, except insurance and pension funding	9952.9	9842.0	-110.9	-1.11
4	Land transport and transport via pipelines	14517.2	14437.7	-79.5	-0.55
5	Wholesale trade, except of motor vehicles and motorcycles	23476.9	23408.5	-68.5	-0.29
6	Construction	35030.5	34968.1	-62.4	-0.18
7	Real estate activities	29880.9	29831.3	-49.7	-0.17
8	Administrative and support service activities	9186.7	9142.9	-43.8	-0.48
9	Other service activities	8254.1	8210.7	-43.3	-0.52
10	Electricity, gas, steam and air conditioning supply	19544.3	19501.5	-42.7	-0.22

(1)	Industry	Total output	HEM TO	Output change	Change (%)
1	Warehousing and support activities for transportation	10933.5	10487.6	-445.9	-4.08
2	Manufacture of coke and refined petroleum products	6563.5	6437.9	-125.6	-1.91
3	Financial service activities, except insurance and pension funding	9952.9	9842.0	-110.9	-1.11
4	Land transport and transport via pipelines	14517.2	14437.7	-79.5	-0.55
5	Other service activities	8254.1	8210.7	-43.3	-0.52
6	Repair and installation of machinery and equipment	4952.1	4928.2	-23.9	-0.48
7	Administrative and support service activities	9186.7	9142.9	-43.8	-0.48
8	Mining and quarrying	3696.1	3679.0	-17.1	-0.46
9	Postal and courier activities	1493.0	1486.2	-6.8	-0.46
10	Water transport	40.5	40.3	-0.2	-0.42

Note: TO stands for total output, HEM can be read as "after extraction".
For the complete set of results of the backward-linkages analysis, see appendix A.4.

Moreover, two of the four most severely hit industries belong to the transportation sector (ISIC Section H), which suggests transportation to be highly interconnected. This claim can be supported further by the fact that Water transport and Postal and courier activities (the remaining transport industries) made the top ten in relative terms as they are relatively small industries. Therefore, the results might suggest that goods transported via air are usually also transported by other modes of transportation (Land transport, Water transport, and Postal activities linkages) and stored in the meantime (Warehousing and support activities for transportation linkage).

Overall, some results might be intuitive, however, some might not. For example, the backward linkage of air transport to Manufacture of coke and petroleum could be expected as airplanes use petroleum in order to function. The same goes for Warehousing as the transported cargo needs to be stored somewhere before and after it is transported by air. On the other hand, for example, the Construction sector being the sixth most hit industry (with decrease of output worth USD 62.4 million) and real estate ranking seventh (decrease of USD 49.7 million) in absolute terms might be surprising. From Table 4.1, it can be seen that these are the two biggest industries in both parts of the table.[3] This is the main reason why HEM was chosen over the classical per-unit linkage measures which can be obtained directly from the Leontief matrix. Using such "simple" methods, would enable to obtain similar results to the lower part of the table, whereas HEM was chosen because of its ability to quantify the linkage effect in terms of monetary units and not only in per-unit (of output) terms. For an introduction to estimation of industry linkages without use of extractions see appendix A.6.

Finally, how can be the knowledge obtained from the backward-linkages analysis useful in the real world? In the introduction to this paper, Covid-19-related travel restrictions were mentioned. Envisage a policymaker wishing to answer such situation by providing state subsidies to the sectors most hit by air transport drop. Then the analysis conducted in here might help the policymaker to identify the most severely hit sectors and to decide what sectors to help. However, industry linkages can be also viewed from another perspective. Imagine another scenario, where the industry of interest is Warehousing. Knowing that air transport has strong linkages with Warehousing, a policymaker could decide to boost air transport in order to support Warehousing indirectly (by the demand-

[3]Overall, Construction is the second biggest sector in the Czech economy (with total output worth USD 35.0 billion) after Manufacture of motor vehicles (total output of USD 47.4 billion), according to WIOT classification. Real estate is the third biggest (with total output of roughly USD 29.9 billion).

driven spill-off effect). For such type of analysis, the researcher might, however, proceed differently. The scientist might go through all the sectors in the economy and estimate their linkages with Warehousing in order to find the key sector for Warehousing.[4]

To sum up, the answer to the second question is that sectors hit the most from the Czech air transport's failure would be Warehousing, Manufacture of coke and petroleum, Financial services, Land transport, and Wholesale trade. Table 4.1 presents some of the other most hit industries such as Construction, Real estate, Postal services, or Water transport.[5] See appendix A.4 for the complete set of backward linkages analysis results.

4.3 Forward Linkages

Oppositely to the backward linkages, forward linkages provide information about the effect of the air transport's extraction on its downstream industries. As argued in section 3.5, forward linkages are driven primarily by supply and therefore the Ghosh supply-side model was used to obtain the estimates presented in the following chapter. However, as mentioned in chapter 2.4, the interpretation of such model is not as straightforward as with the Leontief model. It was argued, that assuming the Ghosh model to be a quantity model leads to some trouble. Therefore, this text follows the suggestion by Dietzenbacher (1997) to consider the Ghosh model as a price model. This means assuming that the model describes how prices change across sectors after the extraction of air transport. This means assuming quantities to be fixed and enabling prices to adjust (oppositely to the Leontief model with fixed prices). Such assumption leads also to another interpretation of the extraction as in this case, extracting air transport means setting the prices of air transport's products to zero. Such extraction decreases the costs of production of the downstream industries and consequently translates to the rest of the economy. The outcome of such analysis is therefore to be described in change in *values* of output and not in *quantities*.

[4]Going through multiple industries in order to find the one with the strongest linkages is the basic principle of the *key sector analysis*.

[5]Additionally, in order to show how else can such a model be used, foreign sectors hit the most can also be identified. The most hit foreign sectors in absolute terms in case of air transport's collapse would be German Manufacture of coke and refined petroleum products (decrease of USD 51 million), Russian Mining and quarrying (USD 36 million decrease) or Mining and quarrying in the Rest of the World region (USD 52 million effect). These effects, however, are very small when compared to the size of the respective sectors, and in reality they might be even smaller as HEM depicts the worst outcome of an industry's extraction.

Table 4.2: Forward linkages: Ten most hit industries in **(1)** absolute and **(2)** relative terms (in USD million)

(1)	Industry	Total output	HEM TO	Output change	Change (%)
1	Administrative and support service activities	9185.7	9006.4	-179.3	-1.95
2	Warehousing and support activities for transportation	10932.7	10776.8	-155.9	-1.43
3	Wholesale trade, except of motor vehicles and motorcycles	23473.0	23421.4	-51.7	-0.22
4	Land transport and transport via pipelines	14516.2	14471.3	-44.9	-0.31
5	Manufacture of motor vehicles, trailers and semi-trailers	47414.9	47384.3	-30.7	-0.06
6	Construction	35027.5	34997.1	-30.4	-0.09
7	Real estate activities	29879.0	29850.8	-28.1	-0.09
8	Manufacture of food products, beverages and tobacco products	16874.5	16850.8	-23.7	-0.14
9	Public administration and defence; compulsory social security	17213.1	17193.2	-19.9	-0.12
10	Retail trade, except of motor vehicles and motorcycles	13896.5	13879.1	-17.4	-0.13

(2)	Industry	Total output	HEM TO	Output change	Change (%)
1	Administrative and support service activities	9185.7	9006.4	-179.3	-1.95
2	Warehousing and support activities for transportation	10932.7	10776.8	-155.9	-1.43
3	Water transport	40.5	40.2	-0.3	-0.77
4	Postal and courier activities	1492.9	1484.5	-8.4	-0.56
5	Land transport and transport via pipelines	14516.2	14471.3	-44.9	-0.31
6	Wholesale trade, except of motor vehicles and motorcycles	23473.0	23421.4	-51.7	-0.22
7	Mining and quarrying	3695.9	3687.9	-8.0	-0.22
8	Other professional, scientific, technical, and veterinary activities	3832.2	3825.8	-6.3	-0.17
9	Insurance, reinsurance and pension funding, except social security	3734.6	3728.7	-6.0	-0.16
10	Manufacture of other non-metallic mineral products	6305.0	6295.3	-9.7	-0.15

Note: TO stands for total output, HEM can be read as "after extraction".
For the complete set of results of the forward-linkages analysis, see appendix A.5.

Using the Ghosh model, the overall forward linkage of air transport was estimated to USD 838 million. Such decrease in value of outputs after the extraction equals to 0.17% of the initial output of the Czech economy. The model therefore suggest the total forward linkage of the air transport to be weaker than the backward linkage, although, as mentioned earlier, two different models were used and therefore it might be misleading to compare the results without caution.

Similarly to the Leontief model in case of backward linkages, the Ghosh model also enables to allocate each industry with the corresponding decrease in the output value. However, as the Ghosh model is supply-driven, the extraction effects are pushed forward and the identified industries are therefore downstream, oppositely to the backward linkages. Table 4.2 presents the ten industries which air transport has the strongest forward linkages with. Again, similarly to Table 4.1, the upper table presents the downstream industries that were hit the most in absolute terms, whereas the lower table contains ten industries hit the most relatively to their size.

The two industries hit the most from both absolute and relative perspectives are Administrative and support service activities (with decrease in value of output by USD 179.3 million) and Warehousing and support activities for transportation (USD 155.9 million). According to ISIC (United Nations, 2008), Administrative and support service activities contain, among other things,[6] travel agencies, tour operators, and reservation services. This seems to be relevant as all of these sub-industries use air transport as an input for their services. Travel agencies and tour operators book flights for their customers, whereas reservation services provide reservation of flights, hotels, tours, and other tourism-related activities. It could be that the linkage is driven also by the other sub-industries of Administrative and support service activities, but WIOTs do not allow for more detail. For the moment, let's assume that the air transport's linkage with Administrative and support service activities is driven by the tourism-related sub-industries. Then this industry might be an example of a mainly downstream industry of air transport. Tourism might be hit more likely by lack of supplies from air transport and not by lack of demand, as people who wish to travel demand air tickets, which are supplied by the air transport. Nonetheless, as WIOTs do

[6]Administrative and support service activities (ISIC Section N) consist of six sub-industries from which one consists of Travel Agencies, Tour Operators, Reservation Services And Related Activities (ISIC code N 79). See United Nations (2008) for more information.

not include further detail on the industry, the exact relationship remains uncertain.[7] The industry, that can be identified as almost purely downstream is Manufacture of motor vehicles, trailers and semi-trailers as it was not affected by the extraction when using the Leontief model in the previous section. On the opposite, an example of an industry linkage driven almost purely by demand is air transport's linkage with Manufacture of coke and refined petroleum products from the backward linkages estimation.[8] This suggests that air transport demands petroleum, however, does supply the sector with almost no goods.

Similarly to backward-linkages results, industries belonging to the rest of the transportation sector (Warehousing and support activities for transportation, Land transport and transport via pipelines, and Water transport together with Postal and courier activities) can be found in the Table 4.2. This means that, according to both Ghosh and Leontief model, air transport is strongly linked to the rest of its own industry group. This suggests that transportation modes are highly interconnected meaning that for goods transported via air, more than one mode of transport is (sometimes) used. This makes sense from both upstream and downstream perspectives as, for example, a simplified supply chain of goods transported by air transport might look as follows: *production of primary inputs → transportation to the manufacturer → manufacture of the final good → land transportation to a warehouse near the airport → air transportation to a warehouse near the place of consumption → transportation to the wholesaler who sells the good to the retailer → retailer supplies the final consumer.* The example illustrates why the input-output framework is often used to study (global) supply chains. Note that wholesale and retail traders were included in the simplified supply chain as Table 4.2 suggest them to be significantly linked to the air transport (Wholesale trade by USD 51.7 million and retail trade by USD 17.4 million). However, Wholesale trade was also one of the most hit industries from the backward perspective and as such it should be considered as one of the most affected sectors overall.

[7] In Table 4.1 and in the appendix A.4, a significant backward linkage of air transport to Administrative and support service activities can be found. However, such linkage might be driven by a different sub-industry than the forward linkage. Unfortunately further decomposition is not possible as WIOTs do not work in more detail.

[8] In the appendix A.5 it can be seen, that air transport has very small forward linkages to Manufacture of coke and refined petroleum products.

In the first part of Table 4.2, five out of the ten industries, which air transport has the strongest forward linkages with, can be found in Table 4.1 as well. Similarly to the backward linkages, Real estate (linkage of USD 28.1 million) and Construction (USD 30.4 million) were estimated to have strong forward linkages to the air transport. Again, these two sectors have large absolute linkage but insignificant per-unit linkage. Such claim is also valid for Manufacture of motor vehicles, trailers and semi-trailers (with linkage of USD 30.7 million) which is the biggest industry in the Czech economy (according to the industry classification used in WIOTs). As discussed, HEM's ability to account for industry size is its main advantage over the per-unit linkage measures.

To sum up, the answer to the last research question is that the downstream sectors hit the most from the Czech air transport's failure would be Administrative and support service activities (probably mainly tourism-related activities) and Warehousing and support activities for transportation. The other industries, that have strong linkages with air transport can be found in Table 4.2.[9] Similarly to backward linkages in the previous section, the model suggests the other industries from the transportation sector to be hit the most by the potential shutdown of the Czech air transport. Therefore, the policymaker desiring to help the industries damaged the most from air travel restrictions might conduct deeper analysis on what sub-industries of the Administrative and support service activities sector are hit the most and subsidise them together with the rest of the transportation sector.

4.4 Chapter Summary

In order to answer the three research questions, total, backward, and forward linkages of the Czech air transport were estimated. The Leontief model suggested the total linkage of the Czech air transport to equal USD 2678 million and the backward linkage of the same sector equal to USD 1387 million. Additionally, the Ghosh model estimated the forward linkages of the Czech air transport to be USD 838 million. In case of the

[9]Again, similarly to backward linkages, it is possible to present some indicative results for foreign industries as well. In absolute terms, the foreign industry which the Czech air transport has the strongest forward linkage with is the Belgian Administrative and support service activities with output effect of USD 27 million. From the relative perspective, Austrian air transport decreased the value of its output by 0.2% (USD 10.4 million). However, as discussed in chapter 2.5.6, the effect of the extraction on the Austrian air transport could be overestimated as the Austrian air transport might also benefit from some substitution effect of the extraction. Again, please note, that estimation of foreign effects was not the goal of this thesis and it is mentioned mainly to show further usage of the input-output analysis.

total-linkages estimation, the Leontief model suggested that the decrease of USD 2678 million would be accompanied with a USD 688 million decrease in the Czech value added.

Next, Leontief and Ghosh models were used to estimate the effect of the Czech air transport's shutdown on its upstream and downstream industries, respectively. The results for the upstream industries can be found in Table 4.1 and appendix A.4, whereas for the downstream industries analysis see Table 4.2 and appendix A.5. According to this research, some of the most hit industries act as both upstream and downstream, meaning that air transport has significant backward and forward linkages with them. Examples of such industries are Warehousing and support activities for transport and Land transport and transport via pipelines, which are together with the air transport part of the transportation sector. Additionally, two other industries, Construction and Real estate activities, also occur in both models as they are so large that they are affected more than other industries in absolute terms even though they do not suffer as much in relative terms. On the other hand, the research has found also industries that act mainly as upstream or downstream industries (they are affected by one of the models only). Examples of mainly downstream industries are Manufacture of coke and refined petroleum products and Financial service activities except insurance and pension funding, which ranked as the second and a third industries hit the most from air transport's extraction. On the contrary, Manufacture of motor vehicles, trailers and semi-trailers is an example of an industry, which is suggested to be to the air transport by the model.

Overall, the models suggest other transportation industries to be affected the most by the air transport's extraction together with Administrative and support service activities and large Construction and Real estate industries.

5. Conclusion

The aim of this research was to measure interindustry linkages of the Czech air transport to the remaining sectors in the Czech economy. It was argued, that Covid-19 related travel restrictions and other factors caused a 88% decline in number of flights in Europe in April 2020 compared to April 2019. Such a dramatic slowdown motivated to research a question about the effects of the air transport downturn to the rest of the economy. Moreover, the general question about the total effect on the Czech economy was further developed into two subquestions about what upstream and downstream industries would be hit the most by air transport collapse.

In order to answer the research questions, the hypothetical extraction method within the input-output framework was applied. In technical terms, the three research questions ask about the total, backward, and forward linkages of the air transport which can be estimated using the hypothetical extraction method (HEM). HEM assumes an industry to be extracted from the economy and measures the effects of such an extraction on the remaining industries in the economy. Such an assumption about a complete removal of the industry from the economy is hypothetical, however, as the number of flights decreased by almost 90% in April 2020, the assumption seems to be quite realistic. Nonetheless, in order to determine all the three types of linkages, three distinct versions of extraction were used based on recommendation by Miller and Lahr (2001) as discussed in chapter 2.5. As mentioned, HEM is a method withing the input-output framework which was described in detail in chapters 2.2 to 2.4. For the total and backward linkages estimation, the widely-used Leontief demand-driven model was utilized. On the other hand, forward linkages were measured using the Ghosh supply-side model which, although it has many imperfections, seems to be the best option for estimating the effect of air transport extraction on its downstream industries. The Ghosh model enabled to identify the most affected downstream industries oppositely to the Leontief model that allowed to recognize the most hit upstream industries.

The research suggests the total linkage of the air transport to the rest of the Czech economy to equal USD 2678 million, the overall backward linkage to be USD 1387 million, and the total forward linkage to equal USD 838 million. It was discussed that backward and forward linkages do not add up to the total linkage as distinct extractions and different models were applied in the estimation process. In chapters 4.2 and 4.3, ten upstream and downstream industries with the strongest linkages to the air transport were identified. From both downstream and upstream perspectives, Land transport and Warehousing (industries from the transportation sector) were among the most affected industries. The same can be said about Wholesale trade and Administrative and support service activities that also ranked high in both parts of the research. This suggests the transportation sector to be closely interlinked with the air transport. Next, Construction and Real estate activities can also be found among the most hit sectors, although they have significantly lower per-unit linkages than the industries mentioned above. It was argued that HEM's ability to account for industry size is one of its advantages over the simpler per-unit linkage measures. On the contrary, the models suggested some industries to behave more like downstream than upstream industries and vice versa. The results suggest Manufacture of coke and petroleum products to be mainly an upstream industry to air transport, which seems reasonable as air transport uses petroleum to fuel airplanes. On the opposite, Manufacture of motor vehicles was identified as a predominantly downstream industry to the air transport.

The information obtained in this research might prove useful to anyone with the desire to know the impact of air transport restrictions (motivated, for example, by the Covid-19 pandemic) on the Czech economy's total output and value added. As argued already, HEM assumes the complete extraction of a sector, which might be unrealistic. However, thanks to this assumption, HEM yields the worst possible effects from an industry shutdown, which might sometimes prove more useful than trying to quantify the approximate effect of a certain scenario. Moreover, the methods used in this paper enable for identification of the most severely hit sectors from the air transport's collapse. Such knowledge could be used when allocating subsidies to selected sectors that suffered the most from a certain policy or situation. The methods used in this thesis therefore allow for relatively simple and fast estimation of extraction effects. If the point of interest was to quantify the effects of a certain situation/policy as precisely as possible, the use of partial extractions or the Global extraction method (GEM) might be considered instead. These methods, however, require more specific assumptions to work. As its name suggests, partial extractions (introduced by Dietzenbacher and Lahr (2013)) assume only a partial closedown of a sector due to possible capacity constraints. Using partial extractions, it is possible to assume a percentage decrease in an industry's (intermediate) production. On the other hand, GEM (developed by Dietzenbacher et al. (2019)) reallocates the extracted production to foreign economies which moderates the high-order effects in the world economy. GEM might prove especially useful when the point of interest lies in the foreign effects of a domestic extraction. However, there might be a problem with exact quantification of the effect of a specific situation as in the real world the economic shocks are complex and thus hard to work with. In 2020, the shock from the Covid-19 pandemic did not hit the air transport sector exclusively. Therefore estimation of the effect of a certain percentage decrease of air transport's output could be biased as some of the high-order effect might be double-counted or misallocated. HEM's ability to provide the highest bound of the decrease that could have been caused might be advantageous. All in all, the most suitable method should be always chosen with regards to the specific scenario for which it is intended to be applied on to bring the most relevant results.

Bibliography

Cella, G. (1984), 'The input-output measurement of interindustry linkages', *Oxford Bulletin of Economics and Statistics* **46**(1), 73–84.

Chen, C.-Y., Rose, A. et al. (1991), 'The absolute and relative joint stability of input-output production and allocation coefficients', *Advances in Input-Output Analysis. Oxford University Press, New-York* pp. 25–36.

Chenery, H. B. (1953), 'Regional analysis', *The structure and growth of the Italian economy* pp. 98–139.

DeBresson, C. (2004), *Some highlights in the life of Wassily Leontief - an interview with Estelle and Wassily Leontief*, Cambridge University Press, Cambridge.

Dietzenbacher, E. (1997), 'In vindication of the ghosh model: a reinterpretation as a price model', *Journal of regional science* **37**(4), 629–651.

Dietzenbacher, E. and Lahr, M. L. (2013), 'Expanding extractions', **25**(3), 341–360.
URL: *https://www.tandfonline.com/doi/full/10.1080/09535314.2013.774266*

Dietzenbacher, E., Linden, J. A. v. d. and Steenge, A. E. (1993), 'The regional extraction method: Ec input–output comparisons', *Economic Systems Research* **5**(2), 185–206.

Dietzenbacher, E., Los, B., Stehrer, R., Timmer, M. and de Vries, G. (2013), 'The construction of the world input-output tables in the wiod project', **25**(1), 71–98.
URL: *http://www.tandfonline.com/doi/abs/10.1080/09535314.2012.761180*

Dietzenbacher, E. and Miller, R. E. (2015), 'Reflections on the inoperability input-output model', **27**(4), 478–486.
URL: *http://www.tandfonline.com/doi/full/10.1080/09535314.2015.1052375*

Dietzenbacher, E., van Burken, B. and Kondo, Y. (2019), 'Hypothetical extractions from a global perspective', **31**(4), 505–519.
URL: *https://www.tandfonline.com/doi/full/10.1080/09535314.2018.1564135*

Erumban, A. A., Gouma, R., de Vries, G., de Vries, K. and Timmer, M. (2012), 'WIOD socio-economic accounts (SEA): Sources and methods', p. 35.

EUROCONTROL (n.d.), 'Covid-19 impact on the european air traffic network'. Accessed: 2021-03-10.
URL: *https://www.eurocontrol.int/covid19*

Eurostat (2020), 'International trade in goods by mode of transport'. Accessed: 2021-03-12.
URL: *https://ec.europa.eu/eurostat/statistics-explained/index.php?title=International_trade_in_goods_by_mode_of_transport&oldid=494356#Trade_by_mode_of_transport_in_value_and_quantity*

Genty, A., Arto, I. and Neuwahl, F. (2012), 'Final database of environmental satellite accounts: Technical report on their compilation', p. 69.

Ghosh, A. (1958), 'Input-output approach in an allocation system', *Economica* **25**(97), 58–64.

Gössling, S., Scott, D. and Hall, C. M. (2020), 'Pandemics, tourism and global change: a rapid assessment of covid-19', *Journal of Sustainable Tourism* **29**(1), 1–20.

Guerra, A.-I. and Sancho, F. (2010), 'Measuring energy linkages with the hypothetical extraction method: An application to spain', **32**(4), 831–837.
URL: *https://linkinghub.elsevier.com/retrieve/pii/S014098830900200X*

IMF (2021), 'World economic outlook update: January 2021', *International Monetary Fund, World Economic Outlook Reports* . Accessed: 2021-03-10.
URL: *https://www.imf.org/en/Publications/WEO/Issues/2021/01/26/2021-world-economic-outlook-update*

Isnard, A. N. (1781), *Traité des richesses, contenant l'analyse de l'usage des richesses en général et de leurs valeurs, les principes et les loix naturelles de la circulation des richesses, de leur distribution, du commerce, de la circulation des monnoies et de l'impôt, et des recherches historiques sur les révolutions que les droits de propriété publics et particuliers ont éprouvées en France depuis l'origine de a monarchie...*, Chez François Grasset.

Leontief, W. W. (1936), 'Quantitative input and output relations in the economic systems of the united states', **18**(3), 105.
URL: *https://www.jstor.org/stable/1927837?origin=crossref*

Lian, C. and Haimes, Y. Y. (2006), 'Managing the risk of terrorism to interdependent infrastructure systems through the dynamic inoperability input–output model', *Systems Engineering* **9**(3), 241–258.

Miller, R. E. and Blair, P. D. (2009), 'Input–output analysis: Foundations and extensions, second edition', p. 784.

Miller, R. E. and Lahr, M. L. (2001), 'A taxonomy of extractions', p. 37.

Moses, L. N. (1955), 'The stability of interregional trading patterns and input-output analysis', **45**(5), 803–826. Publisher: American Economic Association.
URL: *https://www.jstor.org/stable/1821380*

OECD (2018), 'Inter-country input-output tables'. Accessed: 2021-02-07.
URL: *https://www.oecd.org/sti/ind/inter-country-input-output-tables.htm*

Okuyama, Y. and Santos, J. R. (2014), 'Disaster impact and input-output analysis', **26**(1), 1–12.
URL: *http://www.tandfonline.com/doi/abs/10.1080/09535314.2013.871505*

Paelinck, J., de Caevel, J. and Degueldre, J. (1965), 'Analyse quantitative de certaines phénoménes du développment régional polarisé: Essai de simulation statique d'itérarires de propogation', *Problémes de Conversion Éconmique: Analyses Théoretiques et Études Appliqueés* pp. 341–387.

Polenske, K. R. (1970), 'An empirical test of interregional input-output models: Estimation of 1963 japanese production', **60**(2), 76–82. Publisher: American Economic Association.
URL: *https://www.jstor.org/stable/1815789*

Polenske, K. R. (1999), 'Wassily w. leontief, 1905–99', *Economic Systems Research* **11**(4), 341–348.

Quesnay, F. (1759), 'Tableau economique'.

Rose, A. and Allison, T. (1989), 'On the plausibility of the supply-driven input-output model: Empirical evidence on joint stability', *Journal of regional science* **29**(3), 451–458.

Ryšavá, M. (2020), 'The fight between the covid-19 pandemic and air transport has a clear winner so far: Are we facing a historical change?', *Czech National Bank: Global*

Economic Outlook - November 2020 .
URL: *https://www.cnb.cz/en/monetary-policy/geo/*

Sajid, M. J., Cao, Q., Cao, M. and Li, S. (2020), 'Sectoral carbon linkages of indian economy based on hypothetical extraction model', **12**(3), 323–347.
URL: *https://www.emerald.com/insight/content/doi/10.1108/IJCCSM-11-2018-0075/full/html*

Sajid, M. J., Li, X. and Cao, Q. (2019), 'Demand and supply-side carbon linkages of turkish economy using hypothetical extraction method', **228**, 264–275.
URL: *https://linkinghub.elsevier.com/retrieve/pii/S0959652619313253*

Santos, J. R. and Haimes, Y. Y. (2004), 'Modeling the demand reduction input-output (i-o) inoperability due to terrorism of interconnected infrastructures', *Risk Analysis: An International Journal* **24**(6), 1437–1451.

Strassert, G. (1968), 'Zur bestimmung strategischer sektoren mit hilfe von input-output-modellen', *Jahrbucher fur Nationalokonomie und Statistik* pp. 211–215.

Temurshoev, U. (2010), 'Identifying optimal sector groupings with the hypothetical extraction method', **50**(4), 872–890.
URL: *http://doi.wiley.com/10.1111/j.1467-9787.2010.00678.x*

Timmer, M. P., Dietzenbacher, E., Los, B., Stehrer, R. and de Vries, G. J. (2015), 'An illustrated user guide to the world input-output database: the case of global automotive production: User guide to world input-output database', **23**(3), 575–605.
URL: *http://doi.wiley.com/10.1111/roie.12178*

Timmer, M. P., Erumban, A. A., Los, B., Stehrer, R. and de Vries, G. J. (2014), 'Slicing up global value chains', **28**(2), 99–118.
URL: *https://pubs.aeaweb.org/doi/10.1257/jep.28.2.99*

Tukker, A. and Dietzenbacher, E. (2013), 'Global multiregional input-output frameworks: An introduction and outlook', **25**(1), 1–19.
URL: *http://www.tandfonline.com/doi/abs/10.1080/09535314.2012.761179*

United Nations (2008), 'International standard industrial classification of all economic activities (ISIC)'. Accessed: 2021-04-24.
URL: *https://unstats.un.org/unsd/publication/seriesm/seriesm_4rev4e.pdf*

Wiedmann, T., Wilting, H. C., Lenzen, M., Lutter, S. and Palm, V. (2011), 'Quo vadis MRIO? methodological, data and institutional requirements for multi-region input–output analysis', **70**(11), 1937–1945.
URL: *https://linkinghub.elsevier.com/retrieve/pii/S0921800911002606*

WIOD (2013), 'World input-output tables'. Accessed: 2021-02-07.
URL: *http://www.wiod.org/database/wiots13*

WIOD (2016), 'World input-output tables'. Accessed: 2021-02-07.
URL: *http://www.wiod.org/database/wiots16*

Zhao, Y., Liu, Y., Wang, S., Zhang, Z. and Li, J. (2016), 'Inter-regional linkage analysis of industrial CO 2 emissions in china: An application of a hypothetical extraction method', **61**, 428–437.
URL: *https://linkinghub.elsevier.com/retrieve/pii/S1470160X15005300*

A. Appendices

A.1 Countries Included in the World Input-Output Tables

Overall, WOIT consist of 27 EU countries and 16 other major economies. Namely, these are the following:

Europe: Austria, Italy, Bulgaria, Belgium, Luxembourg, Czech Republic, Cyprus, Malta, Denmark, Estonia, Netherlands, Hungary, Finland, Portugal, Latvia, France, Slovakia, Lithuania, Germany, Slovenia, Poland, Greece, Spain, Romania, Ireland, Sweden, United Kingdom, Switzerland, Croatia

USMCA:[1] Canada, United States of America, Mexico

BRIIAT: Brazil, Russia, India, Indonesia, Australia, Turkey

Asia: China, Japan, Korea, Taiwan

[1]These are the states of the former NAFTA treaty. In mid-2020, NAFTA agreement was renegotiated and new agreement took effect on July 1, 2020. The new agreement between USA, Canada, and Mexico is called CUSMA (Canada-United States-Mexico Agreement) by Canada or USMCA (United States-Mexico-Canada Agreement) by the U.S.

A.2 Industries Included in the World Input-Output Tables

WOIT 2016 release covers 56 industries which are listed in this appendix. The industry description corresponds to the International Standard Industrial Classification of all Economic Activities (ISIC). For more information about the ISIC see United Nations (2008).

ISIC	Industry Description
A01	Crop and animal production, hunting and related service activities
A02	Forestry and logging
A03	Fishing and aquaculture
B	Mining and quarrying
C10-C12	Manufacture of food products, beverages and tobacco products
C13-C15	Manufacture of textiles, wearing apparel and leather products
C16	Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials
C17	Manufacture of paper and paper products
C18	Printing and reproduction of recorded media
C19	Manufacture of coke and refined petroleum products
C20	Manufacture of chemicals and chemical products
C21	Manufacture of basic pharmaceutical products and pharmaceutical preparations
C22	Manufacture of rubber and plastic products
C23	Manufacture of other non-metallic mineral products
C24	Manufacture of basic metals
C25	Manufacture of fabricated metal products, except machinery and equipment
C26	Manufacture of computer, electronic and optical products
C27	Manufacture of electrical equipment
C28	Manufacture of machinery and equipment n.e.c.
C29	Manufacture of motor vehicles, trailers and semi-trailers
C30	Manufacture of other transport equipment
C31-C32	Manufacture of furniture; other manufacturing
C33	Repair and installation of machinery and equipment
D35	Electricity, gas, steam and air conditioning supply
E36	Water collection, treatment and supply
E37-E39	Sewerage; waste collection, treatment and disposal activities; materials recovery; remediation activities and other waste management services
F	Construction
G45	Wholesale and retail trade and repair of motor vehicles and motorcycles
G46	Wholesale trade, except of motor vehicles and motorcycles
G47	Retail trade, except of motor vehicles and motorcycles

ISIC	Industry Description
H49	Land transport and transport via pipelines
H50	Water transport
H51	Air transport
H52	Warehousing and support activities for transportation
H53	Postal and courier activities
I	Accommodation and food service activities
J58	Publishing activities
J59-J60	Motion picture, video and television programme production, sound recording and music publishing activities; programming and broadcasting activities
J61	Telecommunications
J62-J63	Computer programming, consultancy and related activities; information service activities
K64	Financial service activities, except insurance and pension funding
K65	Insurance, reinsurance and pension funding, except compulsory social security
K66	Activities auxiliary to financial services and insurance activities
L68	Real estate activities
M69-M70	Legal and accounting activities; activities of head offices; management consultancy activities
M71	Architectural and engineering activities; technical testing and analysis
M72	Scientific research and development
M73	Advertising and market research
M74-M75	Other professional, scientific and technical activities; veterinary activities
N	Administrative and support service activities
O84	Public administration and defence; compulsory social security
P85	Education
Q	Human health and social work activities
R-S	Other service activities
T	Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use
U	Activities of extraterritorial organizations and bodies

A.3 Results of Total Linkages Analysis

	Industry	TO	TO HEM	ΔTO	ΔTO (%)
1	Crop and animal production, hunting and related service activities	9606.1	9601.0	-5.1	-0.05
2	Forestry and logging	2868.9	2867.1	-1.8	-0.06
3	Fishing and aquaculture	83.3	83.3	-0.0	-0.02
4	Mining and quarrying	3696.1	3679.0	-17.1	-0.46
5	Manufacture of food products, beverages and tobacco products	16876.9	16869.2	-7.8	-0.05
6	Manufacture of textiles, wearing apparel and leather products	3483.0	3482.6	-0.4	-0.01
7	Manufacture of wood and of products of wood and cork, except furniture	4162.4	4159.5	-2.9	-0.07
8	Manufacture of paper and paper products	3322.9	3321.2	-1.7	-0.05
9	Printing and reproduction of recorded media	1824.5	1821.1	-3.4	-0.19
10	Manufacture of coke and refined petroleum products	6563.5	6437.9	-125.6	-1.91
11	Manufacture of chemicals and chemical products	8302.8	8297.5	-5.3	-0.06
12	Manufacture of basic pharmaceutical products	1949.7	1949.4	-0.4	-0.02
13	Manufacture of rubber and plastic products	12596.1	12592.8	-3.4	-0.03
14	Manufacture of other non-metallic mineral products	6305.5	6301.1	-4.5	-0.07
15	Manufacture of basic metals	9497.8	9494.8	-3.0	-0.03
16	Manufacture of fabricated metal products, except machinery and equip.	15883.5	15876.3	-7.1	-0.04
17	Manufacture of computer, electronic and optical products	14587.7	14586.5	-1.2	-0.01
18	Manufacture of electrical equipment	12656.6	12651.6	-5.0	-0.04
19	Manufacture of machinery and equipment n.e.c.	15496.4	15491.1	-5.3	-0.03
20	Manufacture of motor vehicles, trailers and semi-trailers	47420.6	47413.4	-7.2	-0.02
21	Manufacture of other transport equipment	2942.1	2937.1	-5.1	-0.17
22	Manufacture of furniture; other manufacturing	4775.6	4774.4	-1.2	-0.03
23	Repair and installation of machinery and equipment	4952.1	4928.2	-23.9	-0.48
24	Electricity, gas, steam and air conditioning supply	19544.3	19501.5	-42.7	-0.22
25	Water collection, treatment and supply	1936.2	1934.5	-1.7	-0.09
26	Sewerage; waste collection, treatment and disposal activities	3863.6	3859.7	-3.9	-0.10

	Industry	TO	TO HEM	ΔTO	ΔTO (%)
27	Construction	35030.5	34968.1	-62.4	-0.18
28	Wholesale and retail trade and repair of motor vehicles	5214.5	5203.4	-11.1	-0.21
29	Wholesale trade, except of motor vehicles and motorcycles	23476.9	23408.5	-68.5	-0.29
30	Retail trade, except of motor vehicles and motorcycles	13900.3	13862.9	-37.4	-0.27
31	Land transport and transport via pipelines	14517.2	14437.7	-79.5	-0.55
32	Water transport	40.5	40.3	-0.2	-0.42
33	Air transport	1296.9	0.0	-1296.9	-100.00
34	Warehousing and support activities for transportation	10933.5	10487.6	-445.9	-4.08
35	Postal and courier activities	1493.0	1486.2	-6.8	-0.46
36	Accommodation and food service activities	7661.7	7647.3	-14.3	-0.19
37	Publishing activities	1926.8	1926.2	-0.6	-0.03
38	Motion picture, video and television programme production	2272.9	2271.9	-1.0	-0.04
39	Telecommunications	5149.0	5140.8	-8.2	-0.16
40	Computer programming; information service activities	7887.2	7871.1	-16.0	-0.20
41	Financial service activities, except insurance and pension funding	9952.9	9842.0	-110.9	-1.11
42	Insurance, reinsurance and pension funding	3735.1	3727.7	-7.4	-0.20
43	Activities auxiliary to financial services and insurance activities	1931.1	1926.0	-5.1	-0.26
44	Real estate activities	29880.9	29831.3	-49.7	-0.17
45	Legal and accounting activities	6539.6	6526.7	-12.8	-0.20
46	Architectural and engineering activities; technical testing and analysis	6992.2	6981.6	-10.6	-0.15
47	Scientific research and development	1849.6	1848.7	-0.9	-0.05
48	Advertising and market research	3772.6	3760.7	-11.9	-0.31
49	Other professional, scientific and technical activities; veterinary activities	3832.8	3824.5	-8.3	-0.22
50	Administrative and support service activities	9186.7	9142.9	-43.8	-0.48
51	Public administration and defence; compulsory social security	17213.8	17181.5	-32.3	-0.19
52	Education	10290.6	10286.3	-4.3	-0.04
53	Human health and social work activities	13195.5	13194.2	-1.3	-0.01
54	Other service activities	8254.1	8210.7	-43.3	-0.52
55	Activities of households as employers	144.9	144.9	0.0	0.00
56	Activities of extraterritorial organizations and bodies	0.0	0.0	0.0	n.a.

Note: Industry names were shortened. For the complete list of industries and their respective ISIC codes, see A.2.
TO stands for total output, HEM reads as "after extraction", Δ reads as "change".

A.4 Results of Backward Linkages Analysis

	Industry	TO	TO HEM	ΔTO	ΔTO (%)
1	Crop and animal production, hunting and related service activities	9606.1	9601.0	-5.1	-0.05
2	Forestry and logging	2868.9	2867.1	-1.8	-0.06
3	Fishing and aquaculture	83.3	83.3	-0.0	-0.02
4	Mining and quarrying	3696.1	3679.0	-17.1	-0.46
5	Manufacture of food products, beverages and tobacco products	16876.9	16869.2	-7.8	-0.05
6	Manufacture of textiles, wearing apparel and leather products	3483.0	3482.6	-0.4	-0.01
7	Manufacture of wood and of products of wood and cork, except furniture	4162.4	4159.5	-2.9	-0.07
8	Manufacture of paper and paper products	3322.9	3321.2	-1.7	-0.05
9	Printing and reproduction of recorded media	1824.5	1821.1	-3.4	-0.19
10	Manufacture of coke and refined petroleum products	6563.5	6437.9	-125.6	-1.91
11	Manufacture of chemicals and chemical products	8302.8	8297.5	-5.3	-0.06
12	Manufacture of basic pharmaceutical products	1949.7	1949.4	-0.4	-0.02
13	Manufacture of rubber and plastic products	12596.1	12592.8	-3.4	-0.03
14	Manufacture of other non-metallic mineral products	6305.5	6301.1	-4.5	-0.07
15	Manufacture of basic metals	9497.8	9494.8	-3.0	-0.03
16	Manufacture of fabricated metal products, except machinery and equip.	15883.5	15876.3	-7.1	-0.04
17	Manufacture of computer, electronic and optical products	14587.7	14586.5	-1.2	-0.01
18	Manufacture of electrical equipment	12656.6	12651.6	-5.0	-0.04
19	Manufacture of machinery and equipment n.e.c.	15496.4	15491.1	-5.3	-0.03
20	Manufacture of motor vehicles, trailers and semi-trailers	47420.6	47413.4	-7.2	-0.02
21	Manufacture of other transport equipment	2942.1	2937.1	-5.1	-0.17
22	Manufacture of furniture; other manufacturing	4775.6	4774.4	-1.2	-0.03
23	Repair and installation of machinery and equipment	4952.1	4928.2	-23.9	-0.48
24	Electricity, gas, steam and air conditioning supply	19544.3	19501.5	-42.7	-0.22
25	Water collection, treatment and supply	1936.2	1934.5	-1.7	-0.09
26	Sewerage; waste collection, treatment and disposal activities	3863.6	3859.7	-3.9	-0.10

	Industry	TO	TO HEM	ΔTO	ΔTO (%)
27	Construction	35030.5	34968.1	-62.4	-0.18
28	Wholesale and retail trade and repair of motor vehicles	5214.5	5203.4	-11.1	-0.21
29	Wholesale trade, except of motor vehicles and motorcycles	23476.9	23408.5	-68.5	-0.29
30	Retail trade, except of motor vehicles and motorcycles	13900.3	13862.9	-37.4	-0.27
31	Land transport and transport via pipelines	14517.2	14437.7	-79.5	-0.55
32	Water transport	40.5	40.3	-0.2	-0.42
33	Air transport	1296.9	1290.6	-6.3	-0.48
34	Warehousing and support activities for transportation	10933.5	10487.6	-445.9	-4.08
35	Postal and courier activities	1493.0	1486.2	-6.8	-0.46
36	Accommodation and food service activities	7661.7	7647.3	-14.3	-0.19
37	Publishing activities	1926.8	1926.2	-0.6	-0.03
38	Motion picture, video and television programme production	2272.9	2271.9	-1.0	-0.04
39	Telecommunications	5149.0	5140.8	-8.2	-0.16
40	Computer programming; information service activities	7887.2	7871.1	-16.0	-0.20
41	Financial service activities, except insurance and pension funding	9952.9	9842.0	-110.9	-1.11
42	Insurance, reinsurance and pension funding	3735.1	3727.7	-7.4	-0.20
43	Activities auxiliary to financial services and insurance activities	1931.1	1926.0	-5.1	-0.26
44	Real estate activities	29880.9	29831.3	-49.7	-0.17
45	Legal and accounting activities	6539.6	6526.7	-12.8	-0.20
46	Architectural and engineering activities; technical testing and analysis	6992.2	6981.6	-10.6	-0.15
47	Scientific research and development	1849.6	1848.7	-0.9	-0.05
48	Advertising and market research	3772.6	3760.7	-11.9	-0.31
49	Other professional, scientific and technical activities; veterinary activities	3832.8	3824.5	-8.3	-0.22
50	Administrative and support service activities	9186.7	9142.9	-43.8	-0.48
51	Public administration and defence; compulsory social security	17213.8	17181.5	-32.3	-0.19
52	Education	10290.6	10286.3	-4.3	-0.04
53	Human health and social work activities	13195.5	13194.2	-1.3	-0.01
54	Other service activities	8254.1	8210.7	-43.3	-0.52
55	Activities of households as employers	144.9	144.9	0.0	0.00
56	Activities of extraterritorial organizations and bodies	0.0	0.0	0.0	n.a.

Note: Industry names were shortened. For the complete list of industries and their respective ISIC codes, see A.2.
TO stands for total output, HEM reads as "after extraction", Δ reads as "change".

A.5 Results of Forward Linkages Analysis

	Industry	TO	TO HEM	ΔTO	ΔTO (%)
1	Crop and animal production, hunting and related service activities	9605.0	9595.4	-9.6	-0.10
2	Forestry and logging	2868.8	2866.1	-2.6	-0.09
3	Fishing and aquaculture	83.3	83.3	-0.0	-0.04
4	Mining and quarrying	3695.9	3687.9	-8.0	-0.22
5	Manufacture of food products, beverages and tobacco products	16874.5	16850.8	-23.7	-0.14
6	Manufacture of textiles, wearing apparel and leather products	3482.7	3480.4	-2.2	-0.06
7	Manufacture of wood and of products of wood and cork, except furniture	4162.2	4158.1	-4.0	-0.10
8	Manufacture of paper and paper products	3322.6	3319.2	-3.4	-0.10
9	Printing and reproduction of recorded media	1824.0	1821.7	-2.3	-0.13
10	Manufacture of coke and refined petroleum products	6563.3	6558.5	-4.8	-0.07
11	Manufacture of chemicals and chemical products	8301.8	8294.8	-7.0	-0.08
12	Manufacture of basic pharmaceutical products	1949.1	1947.0	-2.1	-0.11
13	Manufacture of rubber and plastic products	12594.7	12587.5	-7.2	-0.06
14	Manufacture of other non-metallic mineral products	6305.0	6295.3	-9.7	-0.15
15	Manufacture of basic metals	9497.1	9487.4	-9.7	-0.10
16	Manufacture of fabricated metal products, except machinery and equip.	15882.3	15870.1	-12.2	-0.08
17	Manufacture of computer, electronic and optical products	14586.3	14577.3	-9.0	-0.06
18	Manufacture of electrical equipment	12655.5	12648.1	-7.4	-0.06
19	Manufacture of machinery and equipment n.e.c.	15495.2	15479.7	-15.5	-0.10
20	Manufacture of motor vehicles, trailers and semi-trailers	47414.9	47384.3	-30.7	-0.06
21	Manufacture of other transport equipment	2941.9	2940.2	-1.7	-0.06
22	Manufacture of furniture; other manufacturing	4775.1	4768.8	-6.4	-0.13
23	Repair and installation of machinery and equipment	4951.8	4947.4	-4.3	-0.09
24	Electricity, gas, steam and air conditioning supply	19543.4	19529.3	-14.0	-0.07
25	Water collection, treatment and supply	1936.1	1934.1	-1.9	-0.10
26	Sewerage; waste collection, treatment and disposal activities	3863.3	3859.1	-4.2	-0.11

	Industry	TO	TO HEM	ΔTO	ΔTO (%)
27	Construction	35027.5	34997.1	-30.4	-0.09
28	Wholesale and retail trade and repair of motor vehicles and motorcycles	5213.8	5209.9	-3.9	-0.07
29	Wholesale trade, except of motor vehicles and motorcycles	23473.0	23421.4	-51.7	-0.22
30	Retail trade, except of motor vehicles and motorcycles	13896.5	13879.1	-17.4	-0.13
31	Land transport and transport via pipelines	14516.2	14471.3	-44.9	-0.31
32	Water transport	40.5	40.2	-0.3	-0.77
33	Air transport	1296.8	1290.5	-6.3	-0.48
34	Warehousing and support activities for transportation	10932.7	10776.8	-155.9	-1.43
35	Postal and courier activities	1492.9	1484.5	-8.4	-0.56
36	Accommodation and food service activities	7660.9	7653.3	-7.6	-0.10
37	Publishing activities	1925.3	1923.6	-1.7	-0.09
38	Motion picture, video and television programme production	2272.6	2271.0	-1.7	-0.07
39	Telecommunications	5148.4	5145.8	-2.6	-0.05
40	Computer programming; information service activities	7886.3	7876.3	-9.9	-0.13
41	Financial service activities, except insurance and pension funding	9951.7	9940.0	-11.6	-0.12
42	Insurance, reinsurance and pension funding	3734.6	3728.7	-6.0	-0.16
43	Activities auxiliary to financial services and insurance activities	1930.9	1929.3	-1.7	-0.09
44	Real estate activities	29879.0	29850.8	-28.1	-0.09
45	Legal and accounting activities	6538.6	6531.1	-7.5	-0.11
46	Architectural and engineering activities; technical testing and analysis	6991.1	6984.7	-6.3	-0.09
47	Scientific research and development	1849.4	1848.1	-1.2	-0.07
48	Advertising and market research	3765.4	3761.4	-4.1	-0.11
49	Other professional, scientific and technical activities; veterinary activities	3832.2	3825.8	-6.3	-0.17
50	Administrative and support service activities	9185.7	9006.4	-179.3	-1.95
51	Public administration and defence; compulsory social security	17213.1	17193.2	-19.9	-0.12
52	Education	10289.6	10286.1	-3.4	-0.03
53	Human health and social work activities	13194.6	13187.6	-7.1	-0.05
54	Other service activities	8252.9	8243.8	-9.1	-0.11
55	Activities of households as employers	144.9	144.9	0.0	0.00
56	Activities of extraterritorial organizations and bodies	0.0	0.0	0.0	n.a.

Note: Industry names were shortened. For the complete list of industries and their respective ISIC codes, see A.2.
TO stands for total output, HEM reads as "after extraction", Δ reads as "change".

A.6 Industry Linkages without Extractions

Linkages can be also measured in a more straightforward fashion without using HEM. Using the same notation as in chapter 2, simplified measures for both, forward and backward linkages are summarized in this appendix. The relative advantage of such measures is that they are easy to calculate as shown below. This is because they are obtained directly from the basic input-output matrices ($\mathbf{A}$, $\mathbf{B}$, $\mathbf{L}$, $\mathbf{G}$). On the other hand, they are expressed in "per-unit" terms and interpreted similarly to the matrix from which they were computed not taking into account the size of each sector oppositely to HEM, which enables for quantification of an industry linkage.

Backward Linkages

According to Miller and Blair (2009), to measure direct backward linkage of an industry j in the most straightforward way means to sum the jth column in the matrix $\mathbf{A}$. However, this measures only first-order (direct) effects, the direct backward linkages, as jth column in $\mathbf{A}$ represents the value of total intermediate inputs of a sector j as a proportion of total output of j (x_j). When going one step further in terms of the Leontief model, total backward linkages of sector j capturing all-order (direct and indirect) effects are measured by summing the jth column of the Leontief matrix

$$BL_j = \sum_{i=1}^{n} l_{ij}$$

In matrix notation, backward linkages of each sector are denoted by vectors

$$\mathbf{b} = \mathbf{i}'\mathbf{L}$$

Miller and Blair (2009) suggest normalizing such measure for easier comparison of importance between sectors in an economy. An example of such measure is

$$\overline{BL_j} = \frac{BL_j}{(1/n)\sum\limits_{j=1}^{n} BL_j} = \frac{\sum\limits_{i=1}^{n} l_{ij}}{(1/n)\sum\limits_{i=1}^{n} l_{ij} \sum\limits_{j=1}^{n} l_{ij}}$$

Or, if put in to the matrix notation, the following

$$\overline{\mathbf{b}} = \frac{\mathbf{i}'\mathbf{L}}{\mathbf{i}'\mathbf{L}\mathbf{i}/n} = \frac{n\mathbf{i}'\mathbf{L}}{\mathbf{i}'\mathbf{L}\mathbf{i}}$$

where, the linkage of each sector ($\mathbf{i}'\mathbf{L}$) is compared to the average of all the linkages ($\mathbf{i}'\mathbf{L}\mathbf{i}/n$) meaning that the average value of vector $\overline{\mathbf{b}}$ is unity. Thus industries with above-average strong linkages score above one and below-average score between zero and one. This measure also enables for easier ranking of the (extracted) industries.[2]

Forward Linkages

Forward linkages can be easily found parallelly to the backward linkages but using the Ghosh model. That is that direct forward linkages can be measured by row-summing the $\boldsymbol{B}$ matrix of input coefficients. When measuring the total forward linkages of the jth industry, again, one would compute row-sums of the Ghosh inverse, that is

$$FL_i = \sum_{j=1}^{n} g_{ij} \quad \text{or} \quad \mathbf{FL} = \mathbf{G}\mathbf{i}$$

Again, normalization of such measure is possible in order to enable easier comparison between the extracted sectors

$$\overline{\mathbf{FL}} = \frac{\mathbf{G}\mathbf{i}}{\mathbf{i}'\mathbf{G}\mathbf{i}/n} = \frac{n\mathbf{G}\mathbf{i}}{\mathbf{i}'\mathbf{G}\mathbf{i}}$$

[2] Such normalized measure is particularly useful in key sector analysis. As its name suggests, key sector analysis' goal is to find a sector that affects the economy the most by extracting industries one by one. Thus having such normalized measure makes such comparison easier.