Lehigh University
ME 343:
Control Systems

Professor Hart

12/4/2024

Design Project - Satellite Boom Arm

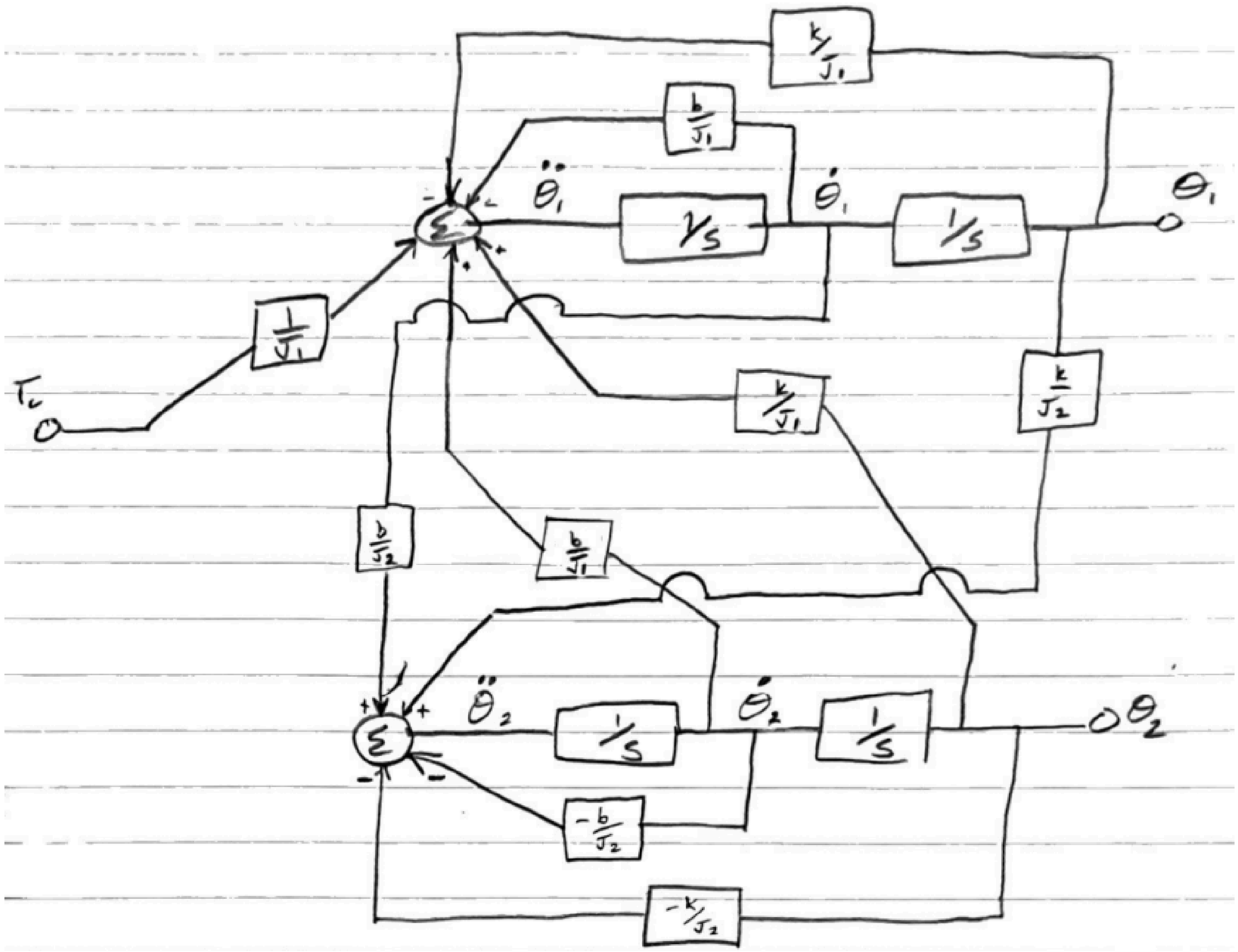
Completed by: Thomas Todaro

Part 1)

System of interest: Observing satellites with boom arms used for sensitive imaging equipment. Boom arm has flexibility to it that causes wobbling of equipment when boom is extended and satellite repositions to face a particular star. This system is 4th order with a pz excess of 3.

Part 2)

Block Diagram:



Limitations:

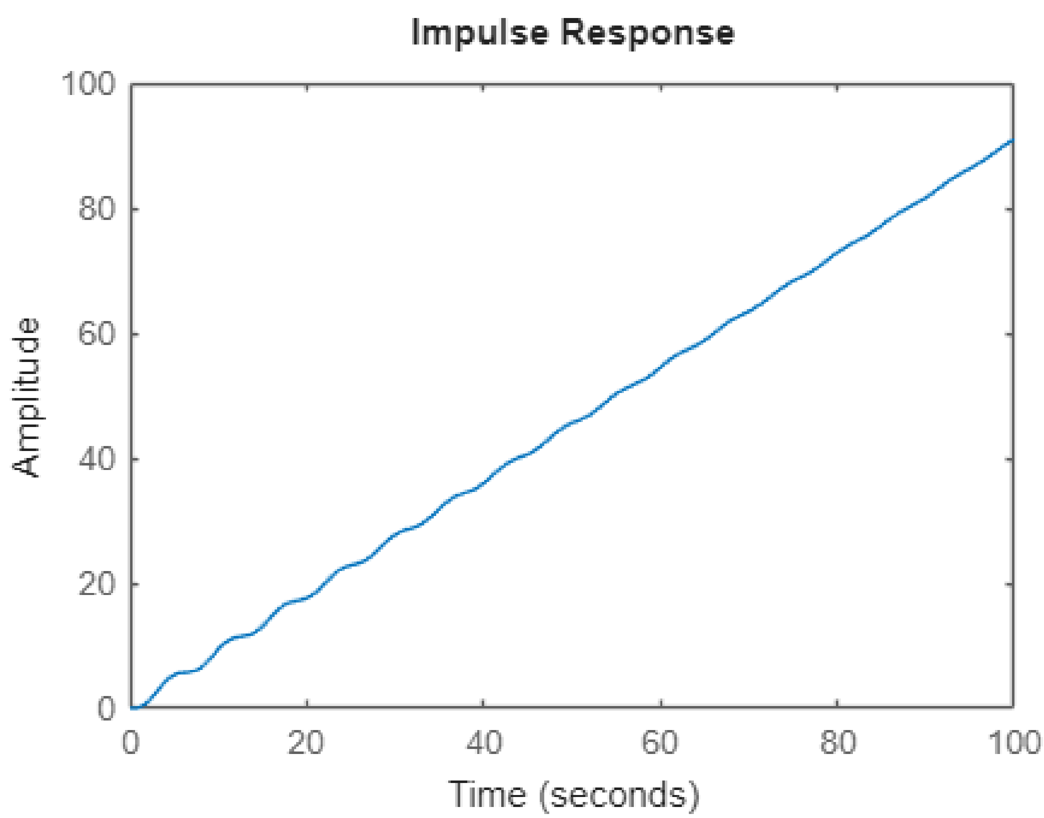
- Boom:
    - Spring Constant, k:
        - Values range from 0.09<=k<=0.4
        - I selected 0.09 as the worst case stiffness
    - Damping Constant, b :
        - Values range from 0.038*sqrt(k/10) <= b <= 0.2*sqrt(k/10)
        - I selected 0.038*sqrt(k/10) as the worst case damping
    - Max deflection:
        - Modern booms are made of aluminum or carbon fiber composite tubing
    - To prevent boom from being damaged while repositioning, book suggests keeping overshoot <= 15%
- Satellite:
    - Toque Mechanism: Book recommended Cold Gas Jets
        - Max Torque: 20mN per unit, assuming 4 units, thus 80mN
            - Assuming 1 meter from center of mass
            - Max Torque: 80mN*m
        - Solenoid Control
            - Some cold gas jets are throttled by solenoids that has a saturation limit
            - Saturation Time: <=~5ms, based

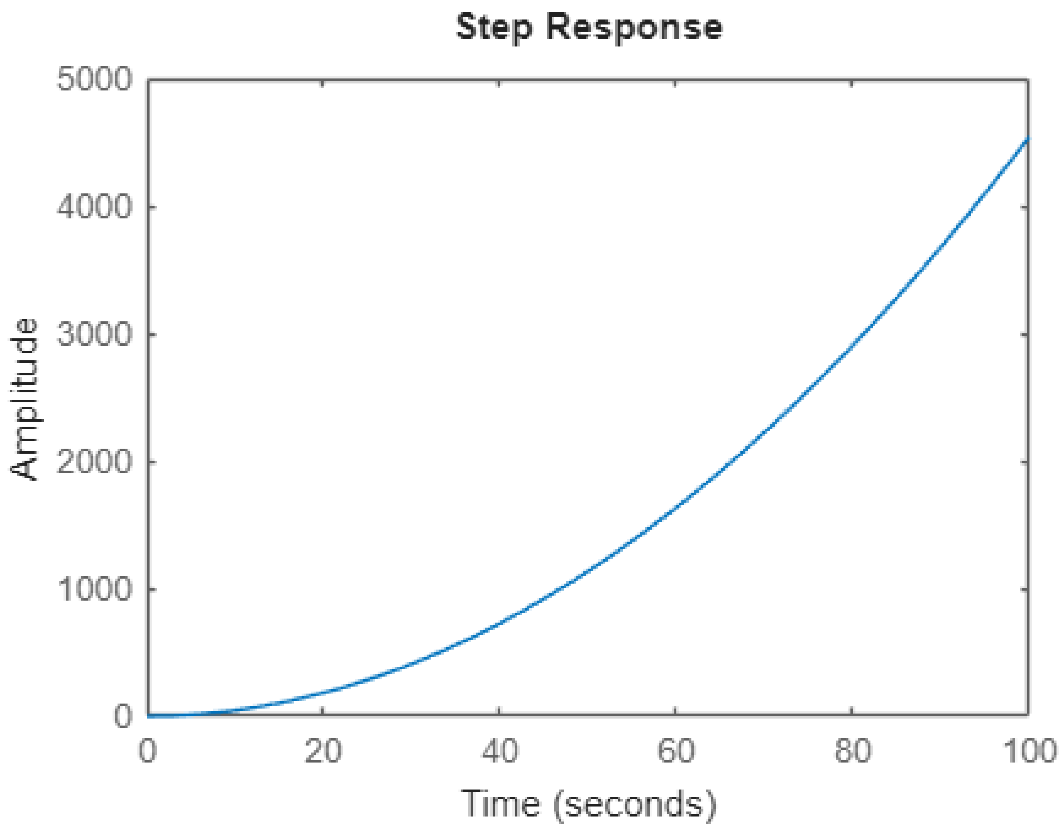
Part 3) Open Loop Performance

Transfer Function calculated through manipulating the block diagram(verified by book):

```
systf =

          0.003605 s + 0.09
  ----------------------------------
  0.1 s^4 + 0.003965 s^3 + 0.099 s^2
```

System Responses:

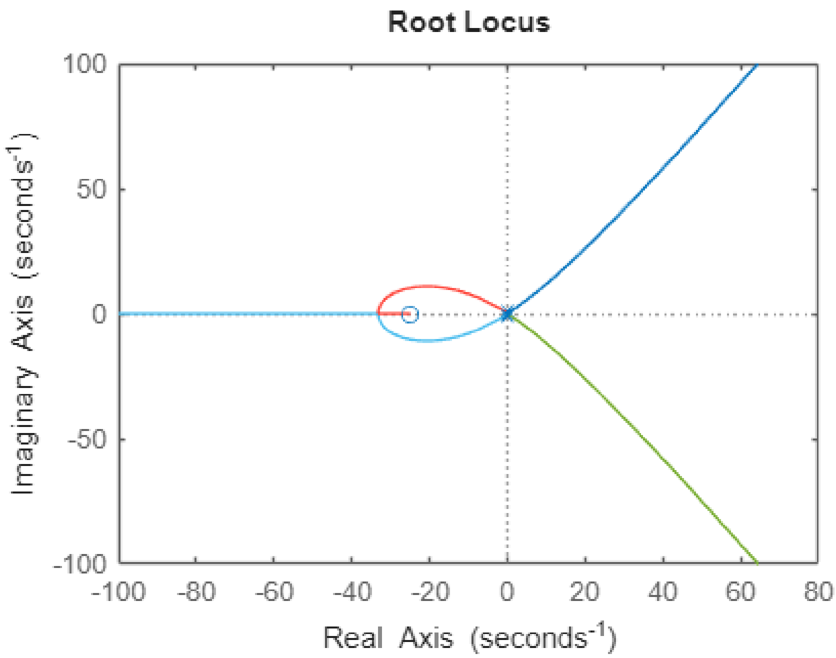




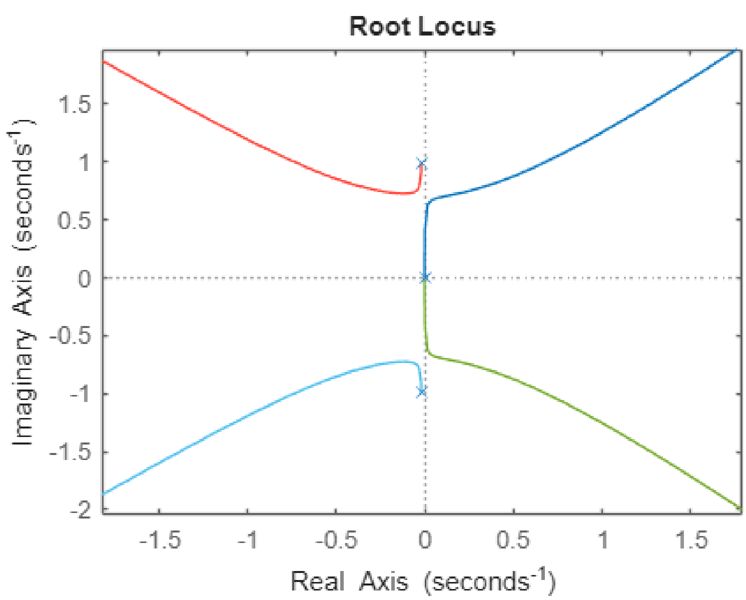
Stability: This system is completely unstable without compensators, with impulse and step disturbances the response shown is to continue to spin out of control, never reaching its star point direction.

Part 4) Root Locus Analysis

Total System



Zoomed in View of Origin



Analysis: Root locus display branches that can only extend into the right hand plane, meaning for any gain, this uncompensated system is highly uncontrolled. Based on this graph, the system will likely require a lead compensator to pull the RHP branches into the LHP and a notch compensator for the poles that exist above and below the real axis.

Open Loop Poles:

0.0000 + 0.0000i

0.0000 + 0.0000i

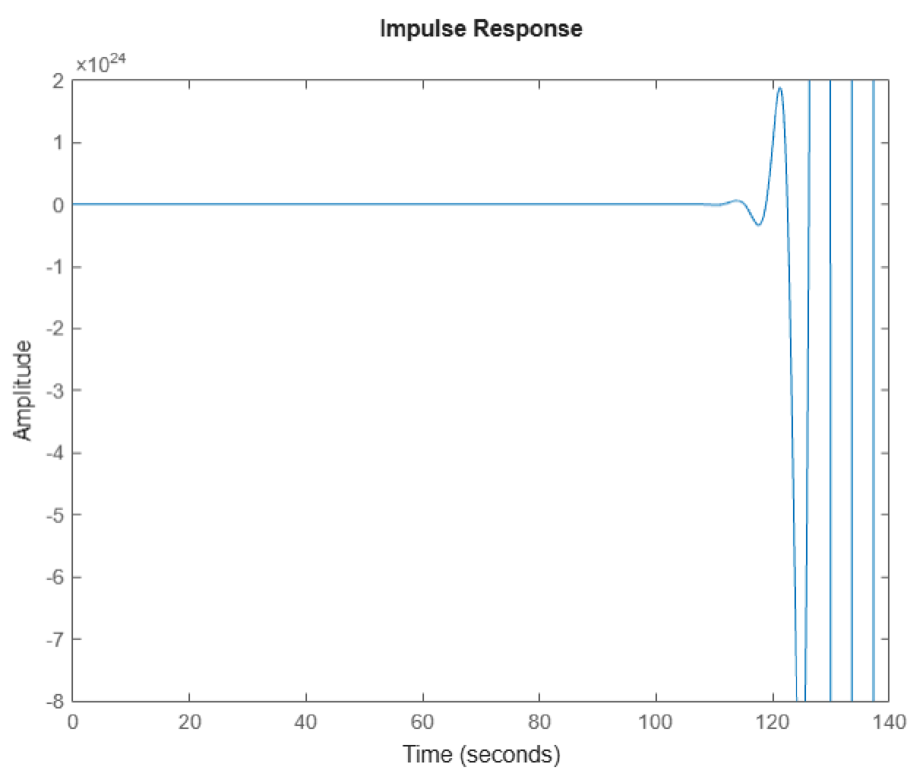
-0.0198 + 0.9948i

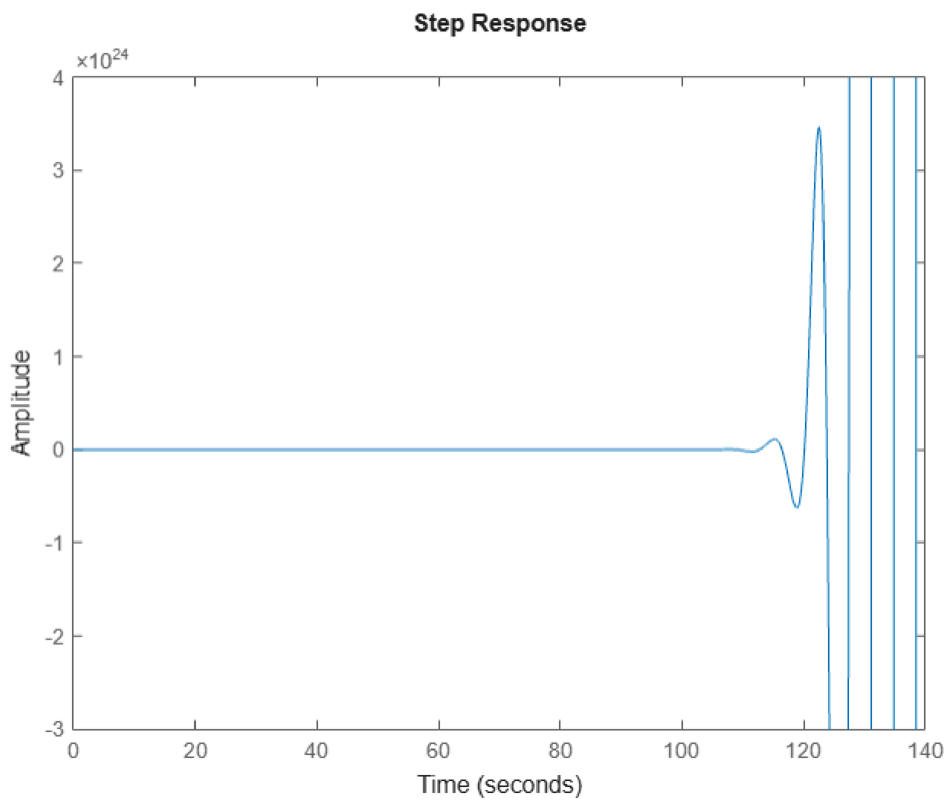
-0.0198 - 0.9948i

Open Loop Zeros:

-24.9653

Closed Loop Unity Feedback Performance:

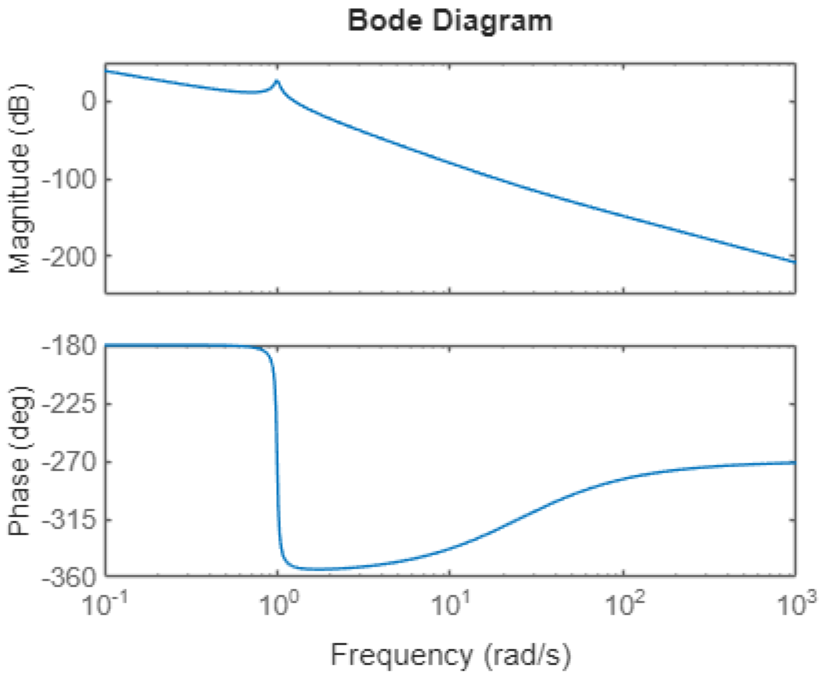




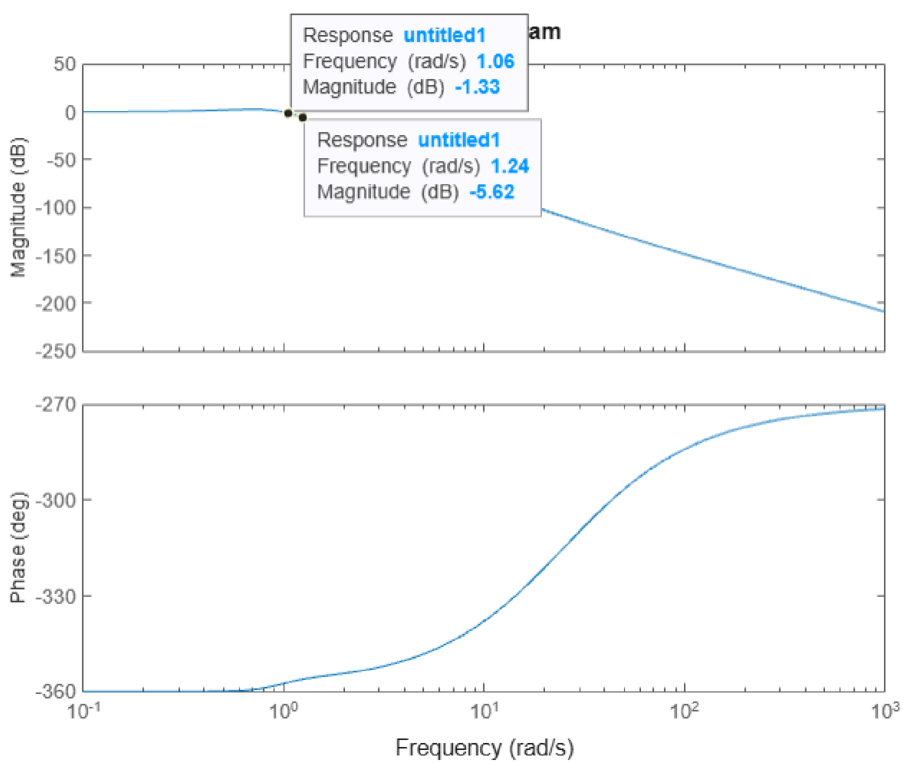
Closed Loop Response Analysis: System is obviously unstable, resulting in growing deviation due to undamped attempts at control.

Part 5) Bode Analysis

(Open Loop)



(Closed Loop)



Gain Margin:0

Phase Margin: -172.1944

Bandwidth of Closed Loop: ~1.1

Time Delay: -2.4031

Analysis: Because the system is unstable by default, values like a maximum time delay of -2.4 seconds reflects that the system is unusable without compensators. Bode diagram indicates that the compensated system will likely require a lower gain and a notch compensator.

Part 6) Performance Improvements:

Goal:

Create a reasonable, controlled response

<15% overshoot

Shortest settling time

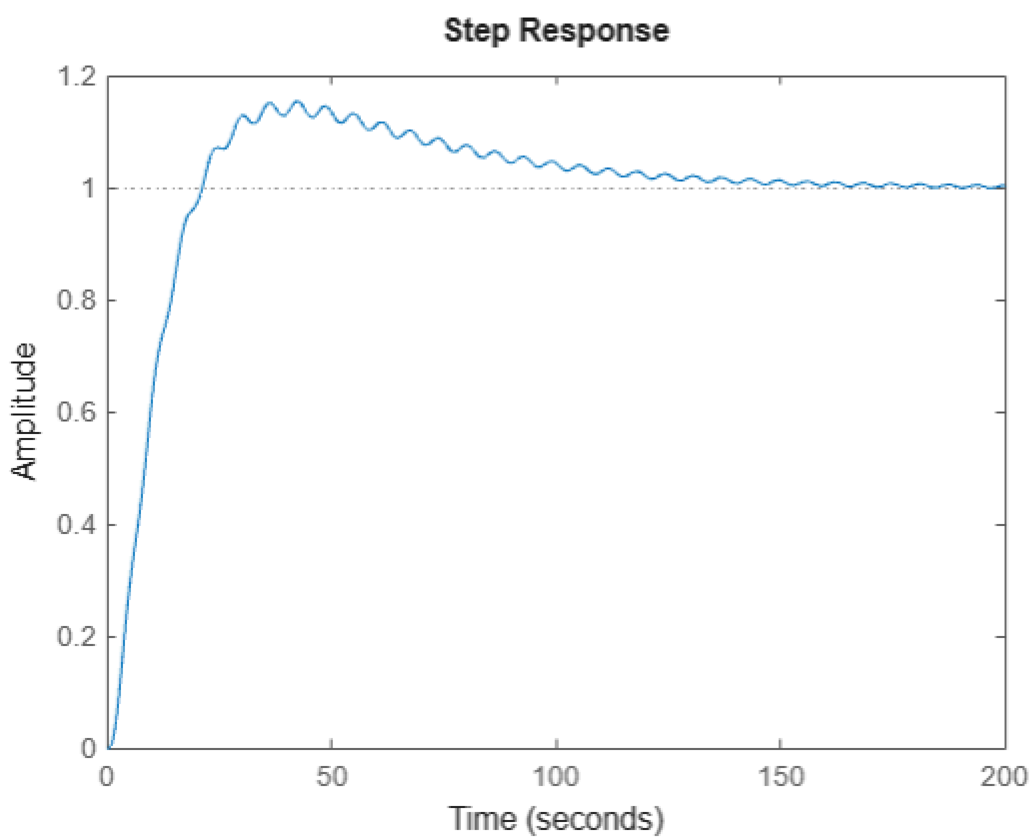
Method:

Lead+Notch Compensator: Using SISO Tool the following compensator was designed

```
   60 s^3 + 26.2 s^2 + 50.5 s + 1
  ----------------------------------
  0.0288 s^3 + 1.488 s^2 + 2.42 s + 1
```

This compensator was made to avoid changing the pz excess

Gain, K: 0.002

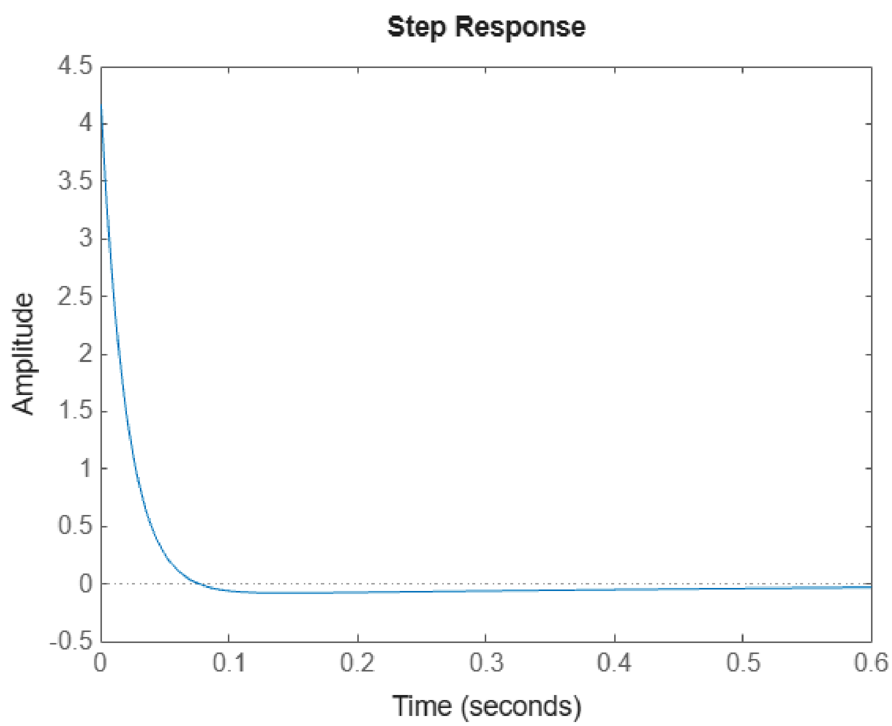


This is the best response I could manage with this system. Having selected the worst possible case of physical properties for the boom as well as the lack of natural damping in space. This solution does manage to keep overshoot under 15% and settling time (<3% from target) to around 118 seconds. In order to keep overshoot at a minimum to preserve the structure of the boom itself, resonance was allowed to continue for quite a long duration. Despite this, a satellite in this scenario would simply have to wait 230 seconds for vibration to end before imaging, which in space, should not be very significant.

Part 7) Hardware Limitations

Boom: Because overshoot was kept to a minimum, the worry of collapsing the boom is minimized.

Input Response(Torque): The following graph was obtained by rearranging the block diagram and transfer function to reflect the response of the input, putting the gain and compensator in the forward path and the plant transfer function in the feedback path.



This response shows that with an initial input of a torque from the cold gas jets, the system will then correct for the theta of the boom end by reversing torque direction to account for the boom's overshoot. Since in this scenario the input has an initial step value it begins at a toque of 40 mN*m but then slows down to adjust over a short time frame. While this response is fast, it would likely not saturate the solenoid actuators for the jets since this response does not indicate a <5ms sudan change of value except at t=0 which was set at an initial condition.

8) State Space Analysis

A

$$\begin{bmatrix} -0.0397 & -0.9900 & 0 & 0 \\ 1.0000 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix}$$

B

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

C

$$\begin{bmatrix} 0 & 0 & 0.0360 & 0.9000 \end{bmatrix}$$

D

$$0$$

Such that:

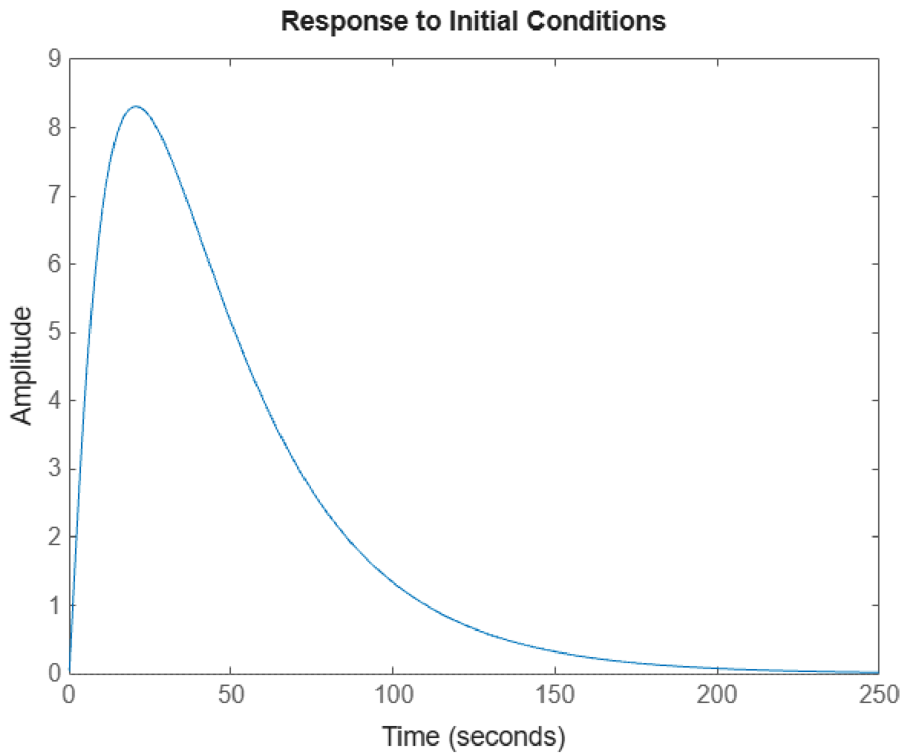
x=[Theta 2

Theta dot 2

Theta 1

Theta dot 1]

Theta 2 defines the angle between the end of the boom and the star while theta 1 is the angle between the satellite itself and the star.



Doing an initial condition of 1 degree difference of the satellite corrects the boom in a very extreme manner. The extreme overshoot implies that the flexibility of the boom plus the overshoot of the satellite body correcting sends the end of the boom far off course before slowly correcting. This graph is surprising,however, since there is no observable resonance but it is possible the gain matrix adjusted the response to take much longer to settle with no resonance. Because the response is so fast and with so much overshoot, it will likely cause stress in the boom arm, likely requiring the gain matrix to be scaled down more similarly to how I tuned it with root locus and bode.

K Matrix: Using Acker

```
K =

    1.6459   -0.2664    0.0635    0.0013
```

Part 9) Discrete System Analysis

Full System (Original Transfer Function+Combined Compensator):

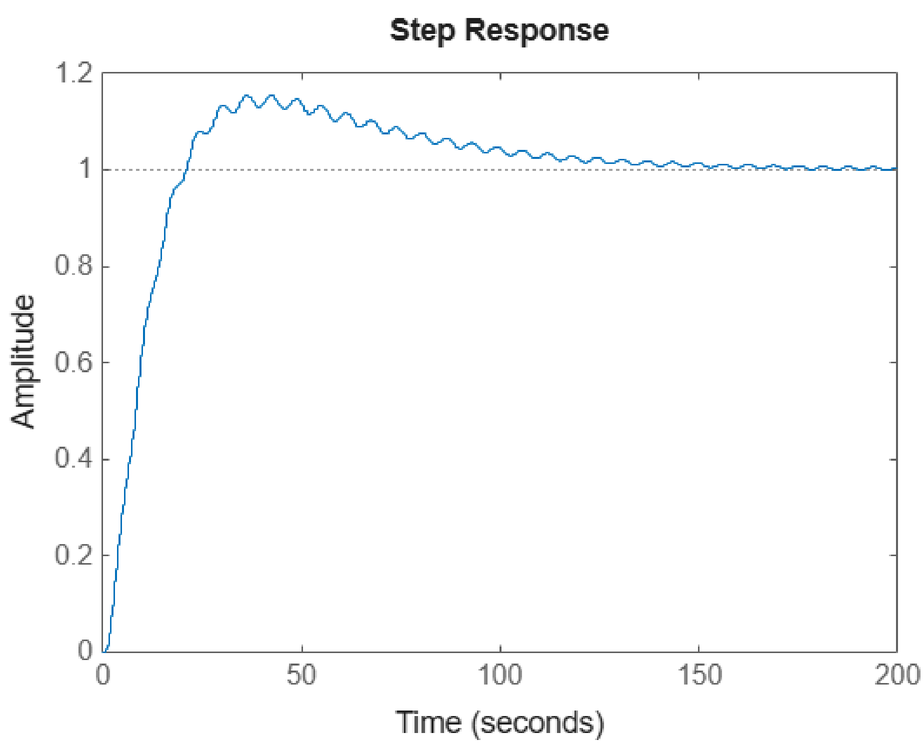
```
G_cl_digi =

  0.0007501 z^6 + 0.004707 z^5 - 0.01014 z^4 + 0.001392 z^3 + 0.00723 z^2 - 0.003603 z - 0.0003104
  -------------------------------------------------------------------------------------------------
          z^7 - 4.198 z^6 + 6.494 z^5 - 3.28 z^4 - 2.441 z^3 + 4.097 z^2 - 2.03 z + 0.3582
```

Plant Transfer function was calculated with ZOH and the compensator with Tustin

Calculated Sampling Rate:

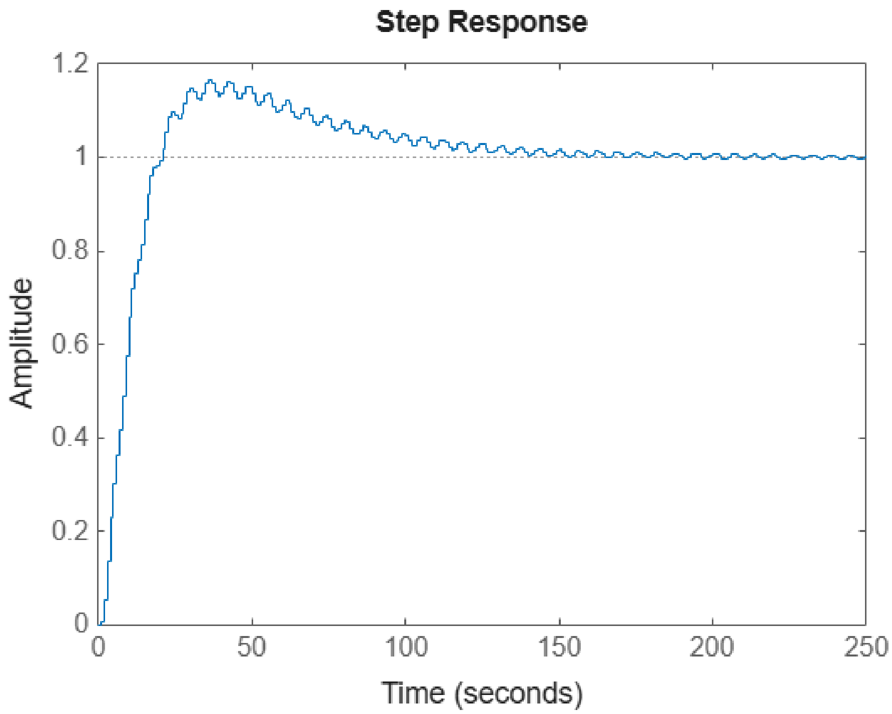
Using the method of finding the frequency at the closed loop system dropping 3db, I determined the reasonable sampling rate to be 0.2856 seconds

Digital Response at T=0.2856:



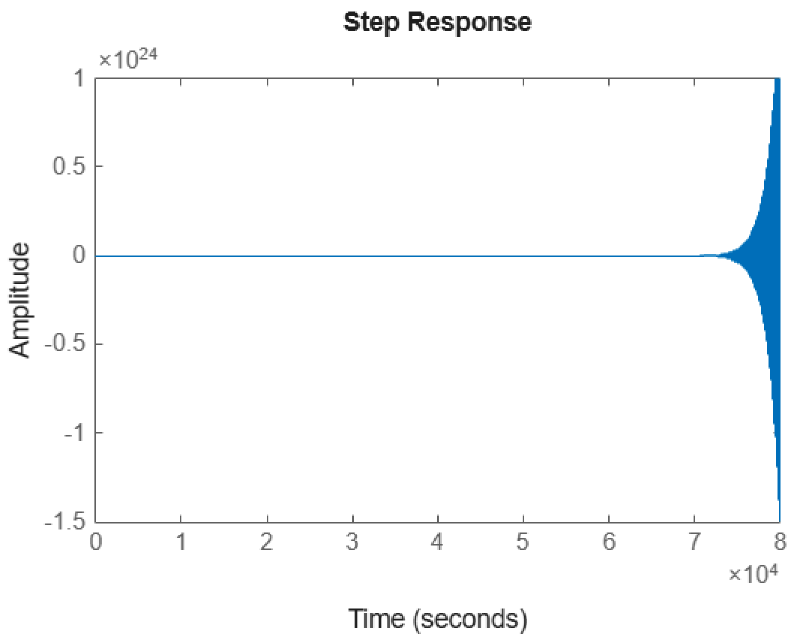
Clearly the previous response is very clear since the system operates over such a large time scale, however the system can likely afford to be slowed down marginally.

Beginning of Degradation Sample Rate: ~1 second



The system can still be controlled but data is beginning to become hard to read towards the end of the response and the overshoot goes to 16%. A reasonable time response would likely be between the two, somewhere around 0.5 seconds.

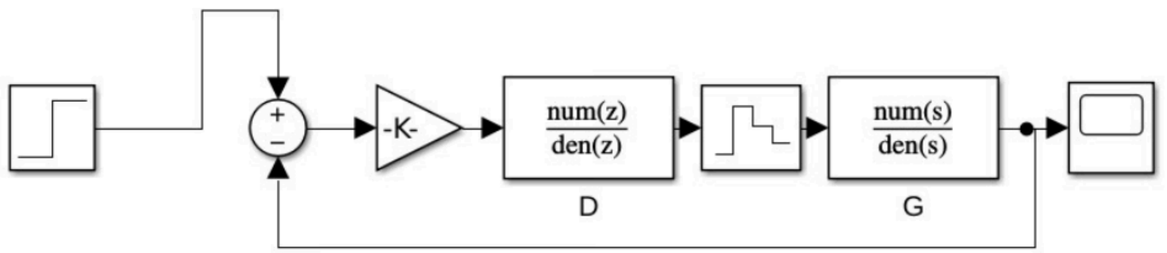
Uncontrollable Sample Rate: ~1.8 Seconds



For curiosity, I wanted to find the sampling time where the system became completely uncontrollable and for what I designed, anything larger than ~1.7 seconds cannot be traced sufficiently to be controlled.

Part 10) Simulink Analysis

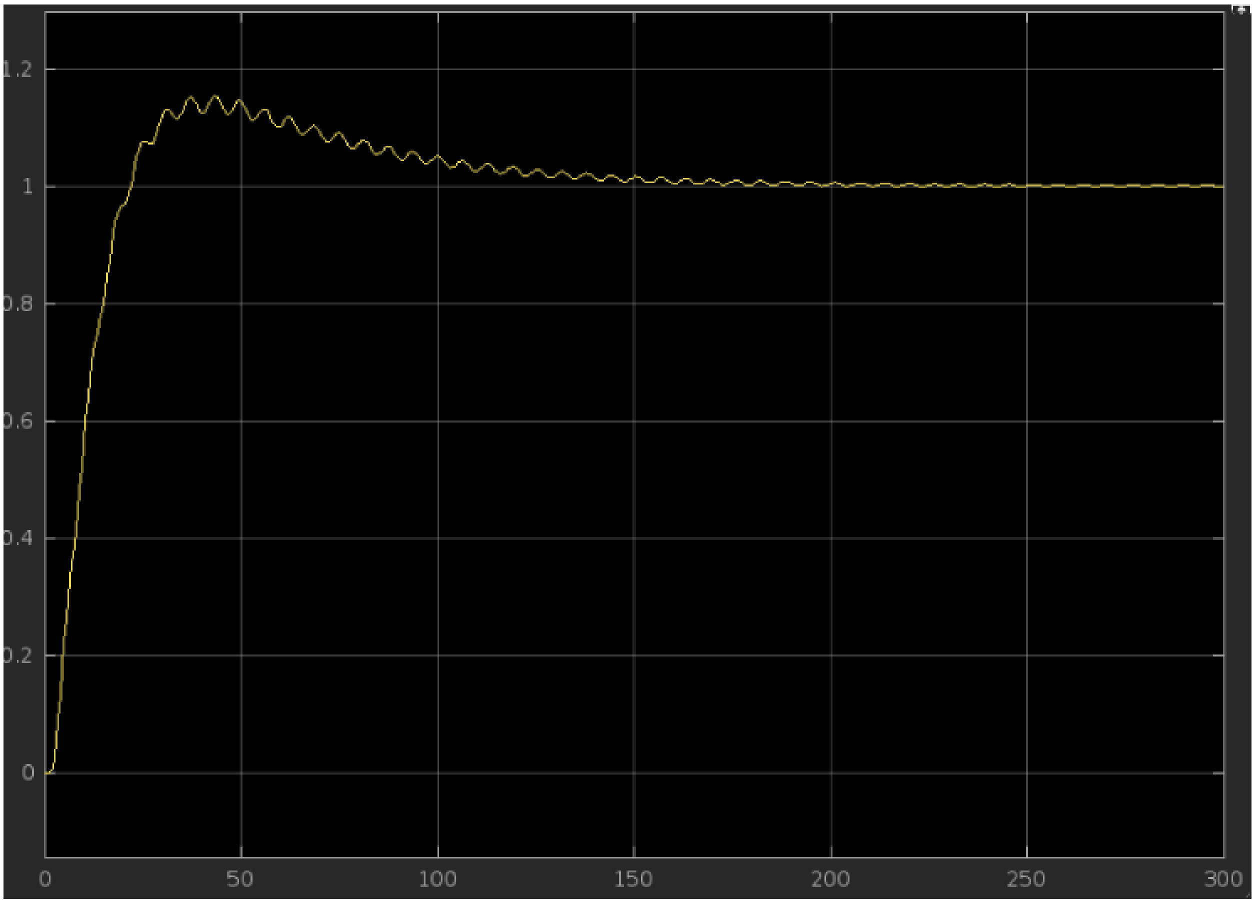
Block Diagram Set Up:



Such that D and G are simply the transfer functions for each as stated above, with D being discrete such that D is:

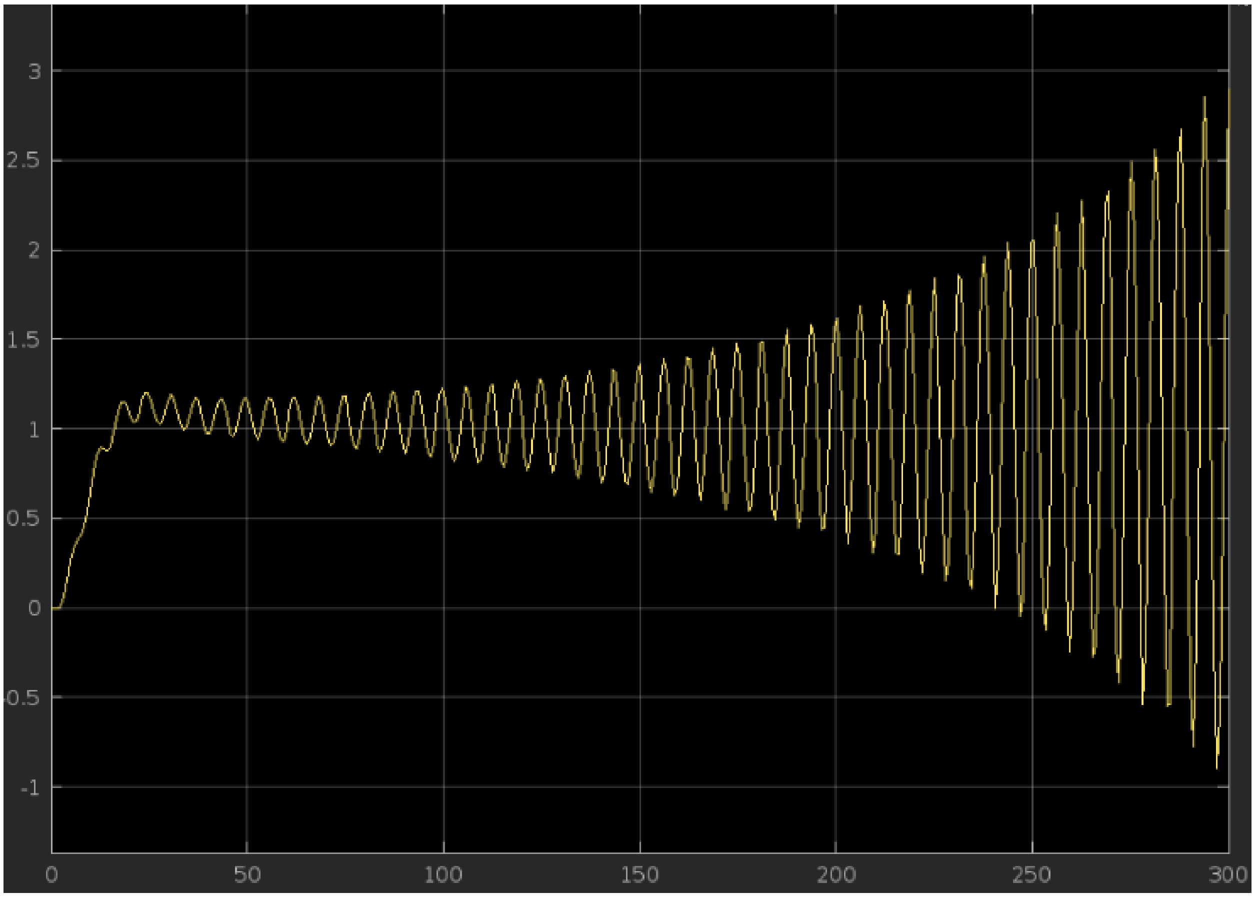
	Source	Value
Numerator:	Dialog	[122.8 -323 300.1 -99.69]
Denominator:	Dialog	[1 -0.4585 -0.687 0.3657]
Initial states:	Dialog	0

Simulink response (T=0.5 seconds)



At T=0.5 seconds the response is still extremely clean and readable. For comparison the following response is at 0.75 seconds, where the simulink lost track of the system after the initial step because it could not track the small peaks of the remaining resonance.

Simulink Response (T=0.75 seconds)



Project Code:

```
%Thomas Todaro
%ME 343 - Controls Systems
%Final Design Project
clear all
close all
clc

%% 1) System Selection

    %Satellite Attitude Control - Found in book section 10.2

%% 2) Block Diagram of Laplace

    %Block Diagram Variables
    J1=1; %Inertia 1
    J2=0.1; %Inertia 2



    %Variable Limitation
        %Boom arm - https://digitalcommons.usu.edu/cgi/viewcontent.cgi?referer=&l
            %Spring constant, k: 0.09<=k<=0.4
                k=0.09; %Worst case torque stiffness
            %Damping Constant, b: 0.038*sqrt(k/10) <= b <= 0.2*sqrt(k/10)
                b=0.038*sqrt(k/10); %Worst case damping
            %Max deflection slope: defined as Theta 2 - Theta 1
                %Assuming the boom is 10 meters long and made of carbon
                %fiber composite tubing...Assume max deflection slope<
                %10 degrees

        %Torque Mechanism - Cold Gas Jets (as reccomended in the book)
            %Thrust - 20 mN per unit maximum (Can be throttled)
            %Torque = 20 mN*1m*4 units = 80 mN*m

%% 3) Open Loop Performance
    %Transfer function observes Theta 2 as our target
    num=[b k];
    denom=conv([J1 b k],[J2 b k])-[0 0 b^2 k*b 0]-[0 0 0 k*b k^2];
    systf=tf(num,denom);

    t=0:0.1:100;
    %Response to Impulse:
    figure(1)
    impulse(systf,t)
    %Response to Step:
```

```
%% 4) Root Locus Analysis
    figure(3)
    rlocus(systf)
    %Poles and Zeros
    disp("Poles:")
    disp(pole(systf));
    disp("Zeros:")
    disp(zero(systf));
    %Response to varied gain of closed loop unity feedback
    sys_cl=feedback(systf,1);
    K=1;
    %Response to Impulse:
    figure(4)
    impulse(sys_cl*K)
    %Response to Step:
    figure(5)
    step(sys_cl*K)
%% 5) Bode Analysis

    figure(6)
    bode(systf)
    [GM, PM, w_gm, w_pm]=margin(systf);

    %Gain Margin
    disp("Gain Margin:")
    disp(GM)
    %Phase Margin
    disp("Phase Margin:")
    disp(PM)
    %Max delay time
    Td=PM*pi/180/w_pm;
    disp("Max Time Delay:")
    disp(Td)
    %figure(7)
    %bode(feedback(systf,1))
%% 6) Performance Improvements
    %Aim to make system stable, remain under 10 percent overshoot and 20
    %settling time
    %Compensator must be notching
    %numd=[conv([50 1],[1.1 0.18 1])];
    %denomd=[conv(conv([1.1 1],[1.1 1]),[0.02 1])];
    numd=[conv([50 1],[1.2 0.5 1])];
    denomd=[conv(conv([1.2 1],[1.2 1]),[0.02 1])];
```

```
    D1=tf(numd,denomd);
    L=tf([1 0.02],[1 50]);
    figure(7)
    %K=0.003933;
    K1=0.002;
    sysclosed=feedback(systf*D1*K1,1);
    step(sysclosed)
    %figure(10)
    %bode(sysclosed)
    %sisotool(systf)
```

```
%% 7) Physical Variable Analysis

    figure(8)
    step(feedback(D1*K1,systf))
    %Check how close you are
    % e to physical limitations of the system
    %How fast can you make the system
```

```
%% 8) State Space
    [A,B,C,D]=tf2ss(num,denom);
    disp('A')
    disp(A)
    disp("B")
    disp(B)
    disp("C")
    disp(C)
    disp("D")
    disp(D)
    %Find K matric that will put poles where previous compensator put them
    pls=pole(sysclosed);
    K=acker(A,B,pls([4,5,6,7]));
    Acl=A-B*K;
    sys_cl=ss(Acl,B,C,D);
    xo=[0 0 1 0];
    figure(9)
```

```
%% 8) State Space
    [A,B,C,D]=tf2ss(num,denom);
    disp('A')
    disp(A)
    disp("B")
    disp(B)
    disp("C")
    disp(C)
    disp("D")
    disp(D)
    %Find K matric that will put poles where previous compensator put them
    pls=pole(sysclosed);
    K=acker(A,B,pls([4,5,6,7]));
    Acl=A-B*K;
    sys_cl=ss(Acl,B,C,D);
    xo=[0 0 1 0];
    figure(9)
    initial(sys_cl,xo)

%% 9) Digititization

    %Z transform

    %Based on bode at 3db
    %Minimum sampling time
    w_bw=1.1;
    w_sample=20*w_bw;
    w_hz=w_sample/(2*pi);
    T1=1/w_hz;
    %Reasonable sampling time - should be faster since oscilatibng
    T=1;%sec
    Dz=c2d(D1,T,'tustin');
    Gz=c2d(systf,T,'zoh');
    G_cl_digi=feedback(Gz*Dz*K1,1);
    figure(10)
    step(G_cl_digi)
%% 10) SIMULINK

    %step input of digital system
```