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# Finite-Quark-Mass Effects on the Higgs Production Cross Section in the Gluon-Gluon Fusion Channel

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*Finite-Quark-Mass Effects on the Higgs Production Cross Section in the Gluon-Gluon Fusion Channel*



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# ABSTRACT

This is my Abstract

# ZUSAMMENFASSUNG

Dies ist meine Zusammenfassung

# ACKNOWLEDGMENTS

Here is where I thank god.

# PUBLICATIONS

During my PhD studies, I co-authored the following publications:

[1] Michał Czakon, Felix Eschment, and Tom Schellenberger. "Revisiting the double-soft asymptotics of one-loop amplitudes in massless QCD." In: *JHEP* 04 (2023), p. 065. DOI: 10.1007/JHEP04(2023)065. arXiv: 2211.06465 [hep-ph]

[2] Michał Czakon, Felix Eschment, and Tom Schellenberger. "Subleading effects in soft-gluon emission at one-loop in massless QCD." In: *JHEP* 12 (2023), p. 126. DOI: 10.1007/JHEP12(2023)126. arXiv: 2307.02286 [hep-ph]

[3] Michał Czakon et al. "Top-Bottom Interference Contribution to Fully Inclusive Higgs Production." In: *Phys. Rev. Lett.* 132.21 (2024), p. 211902. DOI: 10.1103/PhysRevLett.132.211902. arXiv: 2312.09896 [hep-ph]

[4] Michał Czakon et al. "Quark mass effects in Higgs production." In: *JHEP* 10 (2024), p. 210. DOI: 10.1007/JHEP10(2024)210. arXiv: 2407.12413 [hep-ph]

Among these, only the last two are directly relevant to this dissertation.

# CONTENTS

# ACRONYMS

| | |
|---|---|
| SM | Standard model |
| VEV | Vacuum expectation value |
| SSB | Spontaneous symmetry breaking |
| PDF | Parton distribution function |
| QCD | Quantum chromodynamics |
| RGE | Renormalization group equation |
| LO | Leading order |
| DR | Dimensional regularization |
| UV | Ultraviolet |
| IR | Infrared |
| LO | Leading order |
| OS | On-shell renormalization |
| RG | Renormalization group |
| RGE | Renormalization group equation |

# NOTATION AND CONVENTIONS

- In this thesis, I will be using the *Einstein summation convention*, by which any index—be it a Lorentz, color or flavor index—which appears twice is implicitly summed over.
- I will be using natural units
$$\hbar = c = 1. \tag{0.1}$$
- The electron charge is
$$-e, \quad e > 0. \tag{0.2}$$
- The metric is
$$g_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}. \tag{0.3}$$
- The normalization of the Levi-Civita anti-symmetric tensor $\epsilon_{\mu\nu\rho\sigma}$ is
$$\epsilon_{0123} = +1 \tag{0.4}$$
- The Pauli matrices are defined as
$$\sigma^1 \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \tau^i \equiv \sigma^i. \tag{0.5}$$

# 1 INTRODUCTION

General introductions

# 2 THE STANDARD MODEL OF PARTICLE PHYSICS

## 2.1 ELECTROWEAK SYMMETRY BREAKING

The standard model (SM) of particle physics is a theory describing all known matter and their fundamental interactions except for gravity. It unifies the electromagnetic, weak, and strong forces under a single theoretical framework. The matter content of the SM is classified into two primary groups: *fermions* and *bosons*. The fermions have spin 1/2, they are further subcategorized into *quarks* and *leptons*. Quarks participate in strong interactions, while leptons interact only via the electromagnetic and weak forces. In contrast, bosons have integer spin. There exists a single particle with spin 0, the *Higgs* boson, and four vector bosons, namely the gluon, the photon, and the $W$ and $Z$ boson. The vector bosons act as force carrier for the strong, the electromagnetic and the weak force respectively. Fermions are organized into three generations, with each generation containing two types of quarks (up-type and down-type) and two leptons (a charged lepton and its corresponding neutrino). These generations are shown in Fig. 2.1, along with their masses, charges, and spins.

The interactions between SM particles are described by a non-abelian gauge theory of the $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ group. Here $\mathrm{SU}(3)_C$ governs the strong interactions. It applies to all *colored* particles, i.e. quarks and gluons. The quarks transform under the fundamental representation of the $\mathrm{SU}(3)_C$ group. $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ governs electroweak interactions. The $\mathrm{SU}(2)_L$ transformation acts non-trivially only on *left-handed* fermions which form doublets

$$\begin{gathered} L_{iL} \equiv \begin{pmatrix} \nu_{iL} \\ l_{iL} \end{pmatrix}, \quad Q_{iL} \equiv \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix}, \\ \nu_i = (\nu_e, \nu_\mu, \nu_\tau)\,, \quad l_i = (e, \mu, \tau)\,, \quad u_i = (u, c, t)\,, \quad d_i = (d, s, b)\,. \end{gathered} \tag{2.1}$$

The phase transformation $\mathrm{U}(1)_Y$ acts on all particles except neutrinos according to their quantum number, the *hypercharge* $Y$. The symmetry is spontaneously broken to $\mathrm{SU}(3)_C \times \mathrm{U}(1)_Q$, where the $\mathrm{U}(1)_Q$ group corresponds to gauge transformation of the electromagnetic interaction, hence the subscript $Q$ for the *electric charge*. To ensure that the particles have the correct charges, the hypercharge must satisfy the *Gell-Mann–Nishijima relation*:

$$\frac{Y}{2} = Q - I^3. \tag{2.2}$$

With the particle charges displayed in Fig. 2.1 we then get

| | $L_{iL}$ | $Q_{iL}$ | $\nu_{iR}$ | $l_{iR}$ | $u_{iR}$ | $d_{iR}$ |
|---|---|---|---|---|---|---|
| $\frac{Y}{2}$ | $-\frac{1}{2}$ | $\frac{1}{6}$ | $0$ | $-1$ | $\frac{2}{3}$ | $-\frac{1}{3}$ |

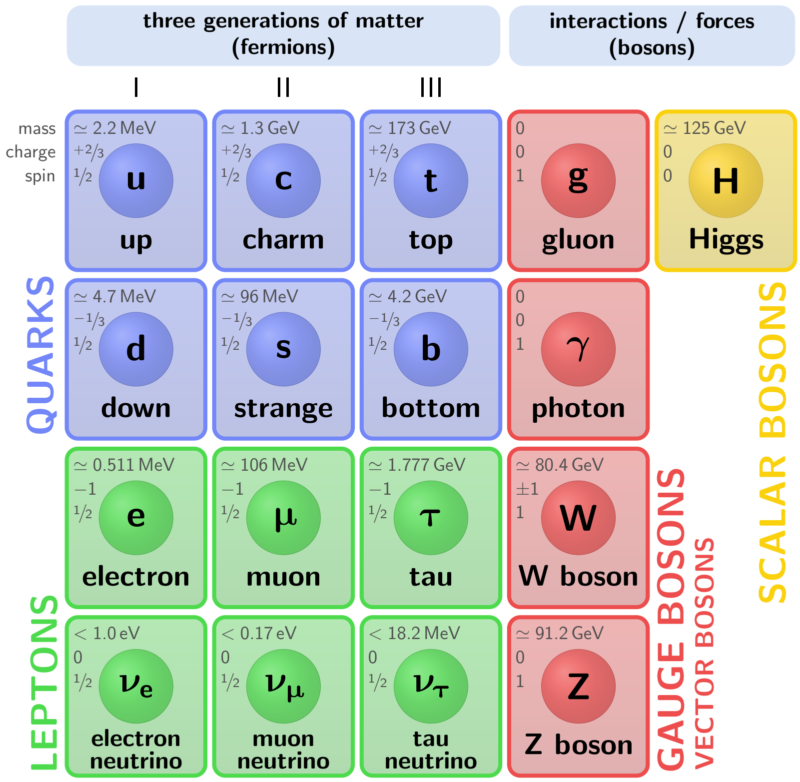



**Figure 2.1:** Elementary particles of the SM. The was image generated with the help of Ref. [5].

The transformation properties of the gauge bosons is dictated by the covariance of the covariant derivative

$$D_\mu \equiv \partial_\mu - igA^a_\mu T^a_R - ig_2 W^a_\mu I^a + ig_Y \frac{Y}{2} B_\mu$$

$$T^a_R = \begin{cases} T^a & \text{for quarks,} \\ 0 & \text{for leptons,} \end{cases} \qquad I^a = \begin{cases} \frac{\tau}{2} & \text{for left-handed fermions,} \\ 0 & \text{for right-handed fermions,} \end{cases} \tag{2.3}$$

where $T^a$ and $\tau^a$ are the *Gell-Mann* and *Pauli* matrices.

Before spontaneous symmetry breaking, the Lagrangian which governs the evolution of all matter fields must be invariant under the $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge group. Up to a $\mathbb{CP}$ violating term[1] the SM Lagrangian is the most general mass-dimension four Lagrangian for the described particle content

$$\mathcal{L}_{\mathrm{SM}} = \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}_Y + \mathcal{L}_H. \tag{2.4}$$

1 The absence of the $\mathbb{CP}$ violating term $\theta \frac{g^2}{64\pi^2} \epsilon^{\mu\nu\alpha\beta} F^a_{\mu\nu} F^a_{\alpha\beta}$ is an unsolved problem of particle physics, known as the strong CP problem.

The gauge-field Lagrangian $\mathcal{L}_G$ describes the free propagation and in the case of the non-abelian groups $\mathrm{SU}(3)_C$ and $\mathrm{SU}(2)_L$ also the self-interaction of the gauge bosons. It is given by

$$\begin{aligned} \mathcal{L}_G &= -\frac{1}{4}G^a_{\mu\nu}G^{a\,\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \\ G^a_{\mu\nu} &\equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc}A^b_\mu A^c_\nu, \\ W^a_{\mu\nu} &\equiv \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^{abc} W^b_\mu W^c_\nu, \\ B_{\mu\nu} &\equiv \partial_\mu B_\nu - \partial_\nu B_\mu. \end{aligned} \tag{2.5}$$

The propagation of the fermions and their interaction with the gauge bosons is described by

$$\mathcal{L}_F = \bar{L}_{iL} i \not{D} L_{iL} + \bar{\nu}_{iR} i \not{D} \nu_{iR} + \bar{l}_{iR} i \not{D} l_{iR} + \bar{Q}_{iL} i \not{D} Q_{iL} + \bar{u}_{iR} i \not{D} u_{iR} + \bar{d}_{iR} i \not{D} d_{iR}. \tag{2.6}$$

The Higgs field is a doublet of the $\mathrm{SU}(2)_L$ group. We want the field to have a non-vanishing vacuum expectation value (VEV) to dynamically generate the fermion and boson masses. Of course, the vacuum cannot carry an electric charge, which means that the Higgs field must be electrically neutral along the direction of *spontaneous symmetry breaking* (SSB). We choose this to be the second component of the doublet. With the Gell-Mann–Nishijima relation we can then deduce that hypercharge of the doublet must be $Y = +1$. The Higgs doublet field thus takes the form

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \tag{2.7}$$

where the superscript indicates the electic charge.

In order to generate a non-vanishing VEV, the Higgs field must be in a potential with a global minimum away from zero. Hence, the only gauge invariant mass-dimension four Lagrangian we can construct is

$$\begin{aligned} \mathcal{L}_H &= \left(D_\mu \Phi\right)^\dagger \left(D^\mu \Phi\right) - V(\Phi) \\ V(\Phi) &= \lambda(\Phi^\dagger\Phi)^2 - \mu^2 \Phi^\dagger \Phi, \quad \mu^2, \lambda > 0. \end{aligned} \tag{2.8}$$

The minimum of the Higgs potential $V$ is at

$$\Phi_0^\dagger \Phi_0 = \frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2} \neq 0. \tag{2.9}$$

After (SSB) we can expand the Higgs field around its minimum

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\xi). \end{pmatrix} \tag{2.10}$$

The real scalar field $H$ is the famous Higgs boson, whereas the fields $\phi^\pm$ and $\xi$ are unphysical since they can always be eliminated through a gauge transformation (*would-be Goldstone bosons*). After inserting the expansion in the Higgs Lagrangian, the mass of the Higgs can be read off from its square term

$$m_H = \sqrt{2}\mu. \tag{2.11}$$

SSB enables the generation of vector boson masses without breaking the gauge symmetry explicitly. If we insert the expanded Higgs field in the Higgs Lagrangian, we get quadratic terms of the gauge boson fields

$$\begin{aligned}\mathcal{L}_H \supsetneq& \frac{v^2}{2}\left\{ g_2^2 \begin{pmatrix} 0 & 1 \end{pmatrix} I^a I^b \begin{pmatrix} 0 \\ 1 \end{pmatrix} W_\mu^a W^{b\,\mu} - g_2 g_Y \begin{pmatrix} 0 & 1 \end{pmatrix} I^a \begin{pmatrix} 0 \\ 1 \end{pmatrix} W_\mu^a B^\mu + \frac{g_Y^2}{4} B_\mu B^\mu \right\} \\ =& \frac{v^2}{2}\left\{ \frac{g_2^2}{4}\left[(W^1)^2 + (W^2)^2\right] + \frac{1}{4}\begin{pmatrix} B^\mu & W^{3\,\mu} \end{pmatrix} \begin{pmatrix} g_Y^2 & g_Y g_2 \\ g_Y g_2 & g_2^2 \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} \right\}.\end{aligned} \tag{2.12}$$

By diagonalizing the mass matrix we obtain the physical states

$$\begin{pmatrix} A_\mu^\gamma \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}, \quad \cos\theta_W = \frac{g_2}{\sqrt{g_Y^2 + g_2^2}}, \sin\theta_W = \frac{g_Y}{\sqrt{g_Y^2 + g_2^2}}. \tag{2.13}$$

In this new basis, we have one massless boson $A_\mu^\gamma$, which we identify as the photon and a charge neutral boson of mass

$$m_Z = \frac{v}{2}\sqrt{g_Y^2 + g_2^2}. \tag{2.14}$$

The vector bosons $W^1$ and $W^2$ are not eigenstates of the charge operator. We therefore define the new states

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right), \qquad Q W_\mu^\pm = \pm W_\mu^\pm, \tag{2.15}$$

which are eigenstates of $Q$ and have mass

$$m_W = \frac{v}{2} g_2. \tag{2.16}$$

Last but not least, we discuss the Yukawa sector of the SM Lagrangian. Before SSB, fermions cannot generate masses because a mass term would mix the left- and right-handed components of the fields, thereby breaking the chiral gauge symmetry. Here, once again, the Higgs field comes to the rescue: by coupling the fermions with the Higgs field through a Yukawa interaction[2]

$$\mathcal{L}_Y = -\left( y_{ij}^\nu \bar{L}_{iL} \Phi^c \nu_{jR} + y_{ij}^l \bar{L}_{iL} \Phi l_{jR} + y_{ij}^d \bar{Q}_{iL} \Phi d_{jR} + y_{ij}^u \bar{Q}_{iL} \Phi^c u_{jR} \right) + \text{h.c.}, \tag{2.17}$$

where $\Phi^c$ is the charge-conjugated field to $\Phi$, we do not explicitly break the symmetry. However, after SSB this Lagrangian will generate exactly the required mixing between left- and right-handed fields to generate the fermion masses. The Yukawa-interaction matrices $y_{ij}^{\nu,l,d,u}$ can be shifted from the Yukawa sector to the fermion sector through a field redefinition. Indeed, if we apply the *singular value decomposition* of the Yukawa matrix

$$y = U_L^\dagger y_{\text{diag}} U_R, \quad \text{with} \quad (y_{\text{diag}})_{ij} = m_i \delta_{ij} \quad \text{and} \quad U_{L,R} \in \mathrm{U}(3), \tag{2.18}$$

and redefine our fermion fields to be

$$f_{iR} \longrightarrow U_{Rij} f_{jR}, \qquad f_{iL} \longrightarrow U_{Lij} f_{jL}, \qquad f = \nu, l, u, d \tag{2.19}$$

the Yukawa Lagrangian becomes

$$\mathcal{L}_Y = -\sum_i \left( m_{\nu_i} \bar{\nu}_i \nu_i + m_{l_i} \bar{l}_i l_i + m_{u_i} \bar{u}_i u_i + m_{d_i} \bar{d}_i d_i \right) \left(1 + \frac{H}{v}\right). \tag{2.20}$$

2 In the original formulation of the SM, there are no neutrino Yukawa interactions, since they were believed to be massless. Neutrino oscillation experiments have shown however, that neutrinos do in fact have finite masses.

As an immediate consequence, we observe that the Yukawa coupling of the Higgs to the fermions is proportional to the mass of that fermion. The field redefinition is a change from a flavor eigenbasis, which is diagonal in the couplings to the gauge bosons, to a mass eigenbasis. In the mass eigenbasis the part of fermion Lagrangian which contains the interaction to the electroweak gauge bosons after SSB is

$$\begin{aligned}\mathcal{L}_F \supsetneq & \sum_f (-Q_f) e \bar{f}_i \not{A}^\gamma f_i + \sum_f \frac{e}{\sin\theta_W \cos\theta_W} \bar{f}_i (I_f^3 \gamma^\mu P_L - \sin^2\theta_W Q_f \gamma^\mu) f_i Z_\mu \\ & + \frac{e}{\sqrt{2}\sin\theta_W}\left(\bar{u}_i \gamma^\mu P_L (V_{\text{CKM}})_{ij} d_j W_\mu^+ + \bar{d}_i \gamma^\mu P_L (V_{\text{CKM}}^\dagger)_{ij} u_j W_\mu^-\right) \\ & + \frac{e}{\sqrt{2}\sin\theta_W}\left(\bar{\nu}_i \gamma^\mu P_L (V_{\text{PMNS}}^\dagger)_{ij} l_j W_\mu^+ + \bar{l}_i \gamma^\mu P_L (V_{\text{PMNS}})_{ij} v_j W_\mu^-\right).\end{aligned} \tag{2.21}$$

Here we identified the electromagnetic coupling constant

$$e = \frac{g_2 g_Y}{\sqrt{g_2^2 + g_Y^2}}, \tag{2.22}$$

as the factor in front of the photon interaction term. The operators $P_{L,R}$ are just the projectors onto the left- and right-handed components

$$P_{L,R} = \frac{1 \mp \gamma^5}{2}. \tag{2.23}$$

The *CKM* and *PMNS matrices*[3] are the results of the field redefinitions

$$V_{\text{CKM}} \equiv U_L^{u\dagger} U_L^d, \qquad V_{\text{PMNS}} \equiv U_L^{l\,\dagger} U_L^\nu. \tag{2.24}$$

Typically, one prefers to work in the mass eigenbasis of the quarks, while the neutrinos are kept in the flavor eigenbasis, in which case one encounters flavor changes (*neutrino oscillations*) through propagation. This is why the PMNS matrix is defined in terms of the complex conjugate of the CKM matrix equivalent in the lepton sector.

## 2.2 CROSS SECTIONS

Many of the great successes of the SM are its *cross section* predictions. The cross section is simply defined as the probability to create some final state from some initial state per unit of time per target particle divided by the incoming particle flux. This means that the cross section can be easily measured with a simple counting experiment. Experiments like `CMS` of `Atlas` do exactly that: they smash particles together and count how many times a certain final state was produced in some time interval. On the theory side, the cross section can be calculated with

$$\mathrm{d}\hat{\sigma}_{ij\to n} = \frac{1}{F} \mathrm{d}\Phi_n |M_{ij\to n}|^2, \tag{2.25}$$

where $F$ is the *flux factor*[4]

$$F \equiv 4 p_1 \cdot p_2, \tag{2.26}$$

3 Named after Cabibbo, Kobayashi and Maskawa, and Pontecorvo, Maki, Nakagawa and Sakata.
4 In the following we assume that the initial state particles are massless.

$\mathrm{d}\Phi_n$ is the *Lorentz invariant phase space measure*

$$\mathrm{d}\Phi_n = \prod_{i=1}^{n} \frac{\mathrm{d}^3\mathbf{q}_i}{(2\pi)^3 2q_i^0}(2\pi)^4\delta^{(4)}\left(p_1 + p_2 - \sum_{i=1}^{n} q_i\right), \tag{2.27}$$

and $M_{fi}$ is the *scattering amplitude* describing the hard interactions.

The computation of cross sections involves three basic steps:

1. the calculation of the hard scattering amplitude,
2. the phase-space integration,
3. and the convolution with *parton distribution functions* (PDFs).

In the following we will discuss them one-by-one.

### 2.2.1 The Hard Scattering Amplitude

The Hard Scattering Amplitude describes the transition probability from a certain initial state to a specific finial state. Since the scattering is **hard**, the energy transfer between the particles during the scattering process is large compared to the QCD scale. This means we are in the perturbative regime of QCD, and we can perform an expansion in the coupling constant

$$M_{ij\to n} = \alpha_s^{n_{\mathrm{Born}}}\left(M^{(0)}_{ij\to n} + \frac{\alpha_s}{\pi}M^{(1)}_{ij\to n} + \left(\frac{\alpha_s}{\pi}\right)^2 M^{(2)}_{ij\to n} + \mathcal{O}\big(\alpha_s^3\big)\right). \tag{2.28}$$

$n_{\mathrm{Born}}$ is the power of the coupling constant at *leading order* (LO). The coefficients of the series are calculable graphically using *Feynman rules*. These are the set of all allowed propagators and vertices together with the corresponding mathematical prescription. The Feynman rules for the complete SM are listed in Appendix A. To calculate the coefficient $M^{(l)}_{ij\to n}$ for a specific process, one just has to draw all possible (connected and amputated) Feynman diagrams with the initial state $(i, j)$ and final state $n$, that contain $2(n_{\mathrm{Born}} + l)$ vertices[5]. Then one uses the Feynman rules to get the mathematical translation, keeping in mind that momentum must be conserved at every vertex and also taking into account possible symmetry factors.

Starting from $M^{(1)}_{ij\to n}$, but for some processes, called *loop induced processes*, even from $M^{(0)}_{ij\to n}$, we will encounter loops in the diagrams. Inside a loop, the momentum of the edges cannot be uniquely determined through momentum conservation. In consequence, we have to leave the momentum unspecified and integrate over all values. Typically, it is the computation of these *loop integrals* that makes the calculation of hard scattering amplitudes so challenging. A plethora of powerful techniques has been developed over the years to tackle this daunting task. Still, the computation of loop integrals remains a highly active field and two-loop integrals with 5 or more scales are only just becoming available. A detailed description of modern techniques is beyond the scope of this thesis. For a comprehensive overview see Ref. [6].

Loop integrals are notorious for exhibiting divergences. To tame these, we introduce *regulators*, i.e. we introduce a parameter, such that the integral becomes function of that parameter with a singularity at the physical value. The most commonly used regularization scheme is

5 Quartic vertices are counted twice.

*dimensional regularization* (DR), here we make the loop-integral a function of the dimension by replacing

$$\int \frac{\mathrm{d}^4 k}{(2\pi)^4}(\cdots) \longrightarrow \bar{\mu}^{2\epsilon}\int \frac{\mathrm{d}^d k}{(2\pi)^d}(\cdots), \quad d \equiv 4-2\epsilon \in \mathbb{C}, \quad \gamma_E = 0.5772\ldots. \tag{2.29}$$

The dimensionally regularized integral satisfies the usual integral properties like linearity, translation invariance and rescaling. The mass scale,

$$\bar{\mu}^2 = \frac{\mu^2}{4\pi}e^{\gamma_E} \tag{2.30}$$

was introduced to fix the mass dimension of the measure to 4, whereas the other factor was merely introduced to absorb some common factors of loop integrals. The physical limit then corresponds to $\epsilon \to 0$. A divergent integral in four dimensions will hence have $\epsilon$-poles in DR. The poles are categorized as *ultraviolet* poles if their origin comes from large loop momenta, i.e. the four dimensional integral diverges for $k \to \infty$. Further we categorize poles as *infrared* poles if the singularity arises from loop momenta which are either soft ($k \to 0$) or *collinear* ($k \cdot p_i \to 0$) to one of the external massless legs. UV and IR singularities are mutually exclusive since they originate from different regions of the phase space. This means that the poles do not multiply. Therefore, we may only get a single UV pole per loop integration. The soft and collinear singularities are **not** exclusive, meaning that IR singularities can develop one double pole per loop integration.

The IR singularities cancel for inclusive observables as we shall discuss in detail in section 2.2.2. UV poles on the other hand, are removed through a method called *renormalization*. The basic idea here is that the fields, constants and masses we observe in nature are not the same as the one in our Lagrangian. Instead, they are related through a *renormalization constant*

$$\begin{aligned}
W_\mu^{B,a} &= \left(Z_3^W\right)^{1/2} W_\mu^{R,a}\\
B_\mu^B &= \left(Z_3^Z\right)^{1/2} B_\mu^R\\
A_\mu^{B,a} &= \left(Z_3^A\right)^{1/2} A_\mu^{R,a}\\
\Phi^B &= \left(Z^\Phi\right)^{1/2} \Phi^R\\
Q_{iL}^B &= \left(Z_{2i}^L\right)^{1/2} Q_{iL}^R\\
u_{iR}^{B,a} &= \left(Z_{2i}^{u,R}\right)^{1/2} u_{iR}^{B,a}\\
d_{iR}^{B,a} &= \left(Z_{2i}^{d,R}\right)^{1/2} d_{iR}^{B,a}\\
g^B &= Z_g g^R\\
g_Y^B &= Z_Y g_Y^R\\
g_2^B &= Z_2 g_2^R\\
\left(\mu^2\right)^B &= Z_\mu \left(\mu^2\right)^R\\
\lambda^B &= Z_\lambda \lambda^R\\
y_{ij}^{d,B} &= Z_{y,ij}^d y_{ij}^{d,R}\\
y_{ij}^{u,B} &= Z_{y,ij}^u y_{ij}^{u,R}
\end{aligned} \tag{2.31}$$

Here the $B$ indicates the *bare* quantities appearing in the Lagrangian, and $R$ denotes the renormalized ones we observe in nature. In the SM, it can be shown [7, 8] that we can choose renormalization constants, such that *Green's functions*, i.e. vacuum expectation values of time ordered products of local renormalized fields, are free of UV divergences. Scattering amplitudes, generally do not depend on the unphysical fields, which is why the fields can be kept unrenormalized in this case. Since at LO Green's functions do not require renormalization, all renormalization constants are equal to the identity at this order.

The definition of the renormalization constants is not unique. Indeed, the renormalization constants were designed to absorb singularities, but the finite part is a priori unconstrained. We call a prescription which uniquely determines the renormalization constants a *renormalization scheme*. The most widely used renormalization scheme is the $\overline{\text{MS}}$ scheme. Here, beyond the leading 1 and a universal factor of $\bar{\mu}^{2\epsilon}$, the renormalization constants **only** consist of poles, i.e. the renormalization constants have the structure

$$Z_i(\alpha) = \bar{\mu}^{2\epsilon}\left(1 + \frac{z_1}{\epsilon}\frac{\alpha}{4\pi} + \left(\frac{z_{22}}{\epsilon^2} + \frac{z_{21}}{\epsilon}\right)\left(\frac{\alpha}{4\pi}\right)^2 + \dots\right). \tag{2.32}$$

$\overline{\text{MS}}$-renormalized masses are generally different from the pole mass. The *on-shell renormalization* (OS) scheme, is an alternative to the $\overline{\text{MS}}$ scheme specifically designed, such that the renormalized mass matches the pole mass. It is therefore the suitable choice for external particles that are asymptotically free. Bare quantities are independent of the chosen renormalization scheme. The invariance under the change of the renormalization scheme defines a group, the *renormalization group* (RG). In the $\overline{\text{MS}}$ scheme, the change from one scale $\bar{\mu}$ to another defines a continuous subgroup of the RG. This means we can formulate the invariance in terms of a differential equation

$$0 = \frac{\mathrm{d}}{\mathrm{d}\log\mu}a^B = a^R\frac{\mathrm{d}Z_a^{\overline{\text{MS}}}}{\mathrm{d}\log\mu} + Z_a^{\overline{\text{MS}}}\frac{\mathrm{d}a^R}{\mathrm{d}\log\mu}, \tag{2.33}$$

where $a$ could be a mass or a coupling. Eq. (2.33) is called the *renormalization group equation* (RGE) and it can be leveraged to determine the scale dependence, also called the *running*, of the observable.

The last step in calculating the hard scattering amplitude is applying the *Lehmann-Symanzik-Zimmermann* (LSZ) reduction formula. It relates the scattering amplitudes to Green's functions, and it is the reason why we only considered amputated Feynman diagrams. In practice, one just has to multiply each external field with the square root of the corresponding LSZ constant. These constants are defined as the proportionality factor between the propagator of the interacting and the free theory[6]. As such, they are numerically identical to the OS field renormalization constants.

### 2.2.2 The Phase-Space Integration

### 2.2.3 The Parton Distribution Functions

In hadron collisions, the initial state is not made up of elementary particles, but are bound states thereof. This means that during an inelastic scattering event, the partons which take part in the short-range interaction only carry a fraction of the original hadron momentum

$$p_1 = x_1 P_1, \qquad p_2 = x_2 P_2. \tag{2.34}$$

6 The interacting field theory might have an additional continuous spectrum.

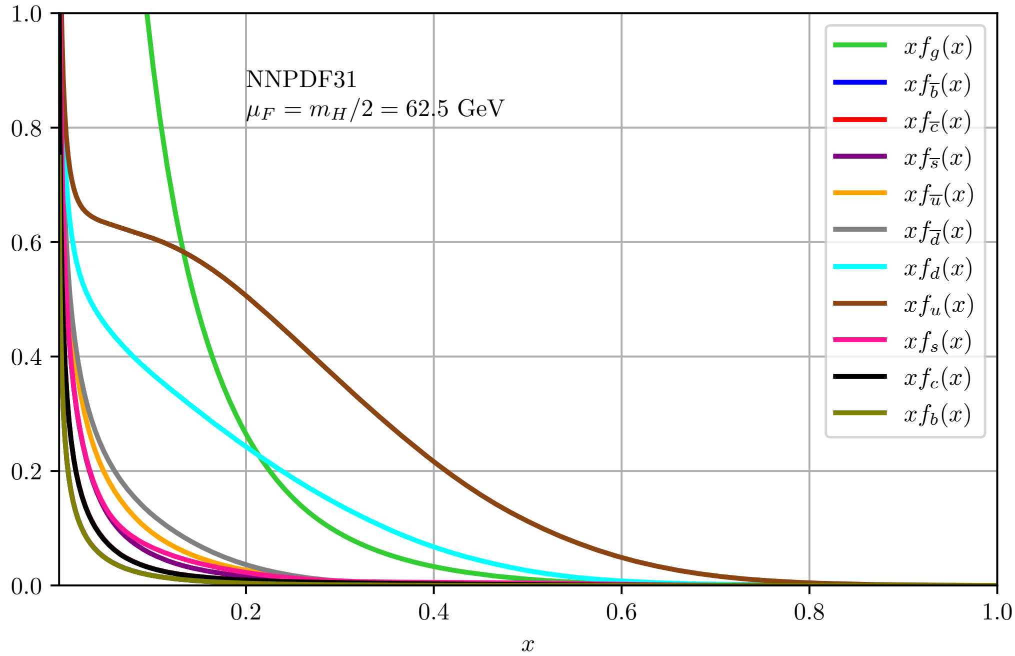



**Figure 2.2:** The various PDFs multiplied by $x$ as a function of $x$. The plot was created using the `LHAPDF6` [9] interface to the `NNPDF31_nnlo_as_0118` [10] PDF set at a scale of $\mu_F = m_H/2$.

Here $p_1$ and $p_2$ are the momenta of the partons and $P_1$ and $P_2$ are the momenta of the hadrons. Since the momentum of the parton can not be larger than that of the hadron, $x_{1,2}$ is restricted to be less than one. Furthermore, since the energy of the parton must be positive the momentum fraction must also be positive. Otherwise, the momentum fraction is a priori unconstrained, we therefore integrate over all allowed values of $x_1$ and $x_2$

$$\mathrm{d}\sigma_{H_1H_2\to n} = \sum_{i,j}\int_0^1 \mathrm{d}x_1\mathrm{d}x_2\, f_{H_1,i}(x_1,\mu_F)f_{H_2,j}(x_2,\mu_F)\mathrm{d}\hat{\sigma}_{ij\to n}(x_1P_1,x_2P_2,\mu_F,\mu_R). \quad (2.35)$$

$f_{H_k,i}(x_k,\mu_F)$ are the PDFs. They descibe the probability of finding a parton $i$ with momentum fraction $x_k$ inside the hadron $H_k$. Momentum conservation then requires the normalization

$$\sum_i \int_0^1 \mathrm{d}x\, x f_{H,i}(x,\mu_F) = 1. \quad (2.36)$$

The factorization theorem (2.35) is central in the SM as it tells us that the PDFs are universal quantities, i.e. they are not specific to any one process. It is a postulate of the parton model, in which hadrons are thought of as collection of the free elementary particles. In QCD however, the theorem requires proof [11]! The PDF for all light partons are displayed in Fig. 2.2. PDFs describe long range interactions, a regime in which QCD is non-perturbative. As such, PDFs are non-perturbative objects which have to be measured in experiments or be calculated non-pertubatively, e.g. on the lattice.

The scale $\mu_F$ marks the boundary, over which we treat interactions perturbatively. This scale is unphysical in the sense that it is not a parameter in our theory, nor can it be measured in an experiment. This independents can be precisely formulated in terms of the *renormalization group equation* (RGE)

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu_F} f^B_{H,i} = 0, \quad (2.37)$$

which states that the *bare, unrenormalized* PDF can not depend on the unphysical scale $\mu_F$. If we now express the unrenormalized PDF in terms of its renormalized counterpart

$$f^B_{H,i} \equiv Z_{ij} \otimes f^R_{H,j} \equiv \int_0^1 \mathrm{d}y\mathrm{d}z\, Z_{ij}(y) f^R_{H,j}(z)\delta(x-yz), \tag{2.38}$$

then we can use the RGE to get the factorization scale dependents of the renromalized PDF

$$\begin{aligned} 0 &= \frac{\mathrm{d}}{\mathrm{d}\ln\mu_F} f^B_{H,i} = \frac{\mathrm{d}\alpha_s}{\mathrm{d}\ln\mu_F}\frac{\mathrm{d}Z_{ij}}{\mathrm{d}\alpha_s} \otimes f^R_{H,j} + Z_{ij} \otimes \frac{\mathrm{d}f_{H,j}}{\mathrm{d}\ln\mu_F} \\ \Rightarrow \frac{\mathrm{d}f^R_{H,i}}{\mathrm{d}\ln\mu_F} &= -Z^{-1}_{ij} \otimes (4\pi\beta - 2\epsilon\alpha_s)\frac{\mathrm{d}Z_{jk}}{\mathrm{d}\alpha_s} \otimes f^R_{H,k} = 2\alpha_s Z^{-1}_{ij} \otimes \frac{\mathrm{d}Z^{(1)}_{ij}}{\mathrm{d}\alpha_s} \otimes f^R_{H,k}. \end{aligned} \tag{2.39}$$

Here we used the definition of the $d$-dimensional $\beta$*-function*

$$\overline{\beta} \equiv \frac{1}{4\pi}\frac{\mathrm{d}\alpha_s}{\mathrm{d}\ln\mu_F} = \beta - \epsilon\frac{\alpha_s}{2\pi}, \quad \beta = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}Z^{(1)}}{\mathrm{d}\ln\alpha_s}, \tag{2.40}$$

where $Z^{(1)}_{\alpha_s}$ and $Z^{(1)}_{ij}$ are the residues of the renormalization constants of the coupling constant and the PDFs respectively. At one loop, the PDF renormalization constant is designed to absorb the singularities from tree-level initial-state-collinear radiation, it therefore reads

$$Z_{ij}(z) = \delta(1-z)\delta_{ij} + \frac{\alpha_s}{2\pi}\frac{1}{\epsilon}P^{(0)}_{ij}(z), \tag{2.41}$$

with the *Altarelli-Parisi splitting kernels*

$$\begin{aligned} P^{(0)}_{qq}(z) &= P^{(0)}_{\bar{q}\bar{q}}(z) = C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right], \\ P^{(0)}_{qg}(z) &= T_F\left[z^2 + (1-z)^2\right], \\ P^{(0)}_{gq}(z) &= C_F\left[\frac{1+(1-z)^2}{z}\right], \\ P^{(0)}_{gg}(z) &= 2C_A\left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right] + \delta(1-z)\frac{\beta_0}{2}, \end{aligned} \tag{2.42}$$

and

$$\beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_l. \tag{2.43}$$

So even though the PDFs are non-perturbative, their dependence on the factorization scale is perturbative and calculable with the *Dokshitzter-Gribow-Lipatow-Altarelli-Parisi-evolution equation* [12, 13, 14]

$$\frac{\mathrm{d}f^R_{H,i}}{\mathrm{d}\ln\mu_F} = \frac{\alpha_s}{\pi}P^{(0)}_{ij} \otimes f^R_{H,j}. \tag{2.44}$$

# 3 THE HIGGS AS A WINDOW TO NEW PHYSICS

## 3.1 STABILITY OF THE HIGGS POTENTIAL

## 3.2 THE HIERARCHY PROBLEM

# 4 HADRONIC HIGGS PRODUCTION

## 4.1 THE LEADING-ORDER CROSS SECTION

Here I compute the leading-order cross section for Higgs Production in the gluon-gluon fusion channel.

## 4.2 THE HEAVY-TOP LIMIT

Here I explain the heavy-top limit.

## 4.3 HIGHER-ORDER CORRECTIONS

Here I outline how to perform higher order corrections.

## 4.4 THEORY STATUS

Here I describe what is already known about the gluon-gluon fusion channel. I explain the theory uncertainties.

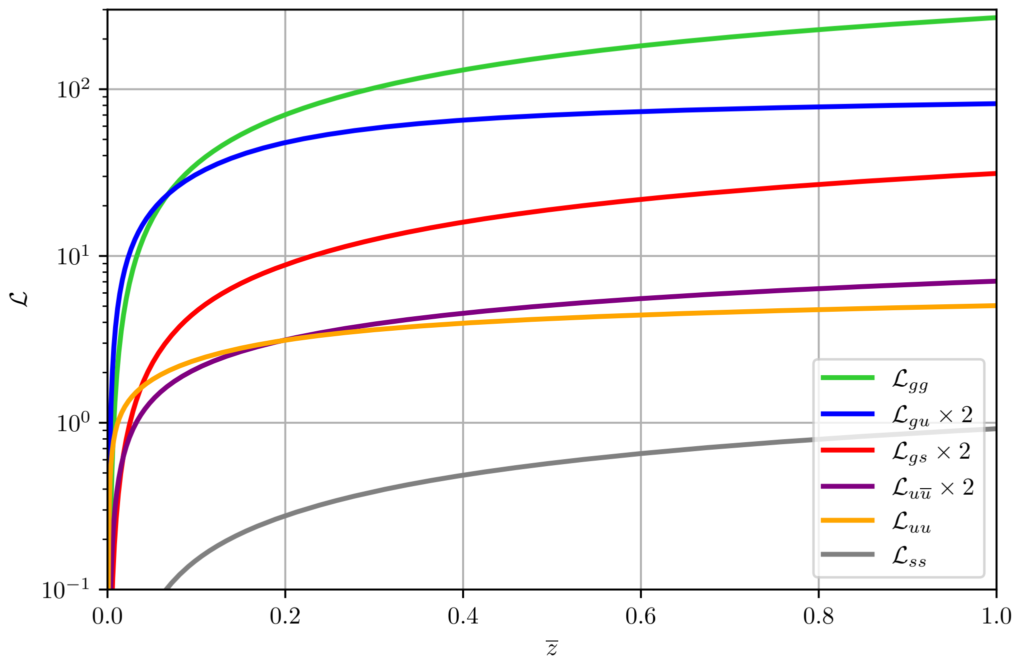



**Figure 4.1:** Illustration of the partonic luminosity of parton $i$ and $j$. We choose exemplary parton combinations to represent gluon–gluon, gluon–valence-quark, gluon–sea-quark, valence-quark–sea-quark, valence-quark–valence-quark and sea-quark–sea-quark luminosities. For the luminosity of two different partons we include a factor 2 to account for the different flavor permutations. The luminosity was calculated using the `LHAPDF6` [9] interface to the `NNPDF31_nnlo_as_0118` [10] PDF set at a scale of $\mu_F = m_H/2$ with a fixed collider energy of $\sqrt{S} = 13$ TeV.

# 5 COMPUTATIONAL DETAILS

Description of this chapter.

## 5.1 COMPUTING THE AMPLITUDES

Here I

### 5.1.1 The Real-Real Corrections

### 5.1.2 The Real-Virtual Corrections

### 5.1.3 The Virtual-Virtual Corrections

## 5.2 $\overline{\mathrm{MS}}$-SCHEME

## 5.3 THE 4-FLAVOUR SCHEME

## 5.4 PERFORMING THE PHASE-SPACE INTEGRATION

# 6 RESULTS AND DISCUSSION

Description of this chapter.

## 6.1 TOTAL CROSS SECTION

### 6.1.1 Effects of Finite Top-Quark Masses

### 6.1.2 Effects of Finite Bottom-Quark Masses

## 6.2 DIFFERENTIAL CROSS SECTION

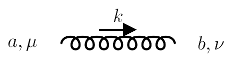
# 7 CONCLUSIONS

Here are my conclusions.

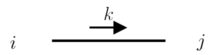
# A FEYNMAN RULES OF THE STANDARD MODEL

In this chapter we list all Feynman rules of the SM. We choose to work in a unitary gauge, meaning that the all Goldstone-bosons decouple and there are no unphysical particles in the electroweak sector. In the QCD sector, we work in the $R_\xi$ gauge, i.e. we have unphysical particles in the form of *Faddeev-Popov ghosts*. If not otherwise specified, the momenta on every line are defined as incoming.

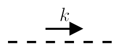
**Propagators:**



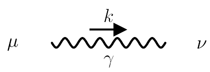

$$= i\frac{\delta^{ab}}{k^2+i0^+}\left[-g_{\mu\nu}+(1-\xi)\frac{k_\mu k_\nu}{k^2+i0^+}\right]$$



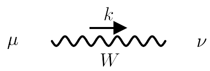

$$= i\delta_{ij}\frac{\not{k}+m}{k^2-m^2+i0^+}$$



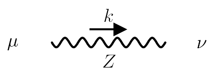

$$= i\frac{1}{k^2-m_H^2+i0^+}$$



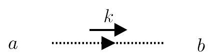

$$= i\frac{-g_{\mu\nu}}{k^2+i0^+}$$




$$= i\frac{1}{k^2-m_W^2+i0^+}\left(-g_{\mu\nu}+\frac{k_\mu k_\nu}{m_W^2+i0^+}\right)$$

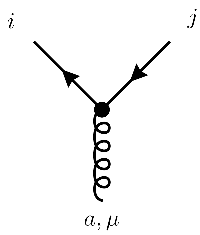



$$= i\frac{1}{k^2-m_Z^2+i0^+}\left(-g_{\mu\nu}+\frac{k_\mu k_\nu}{m_Z^2+i0^+}\right)$$

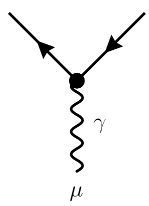



$$= i\frac{\delta^{ab}}{k^2+i0^+}$$

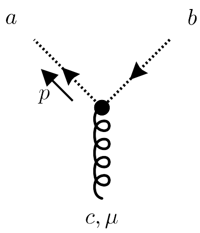
**Fermion–Gauge-Boson Vertices:**



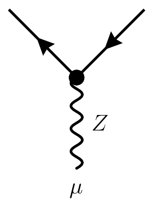

$$= ig\gamma^\mu T^a_{ij}$$



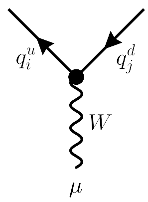

$$= -ie\gamma^\mu Q$$



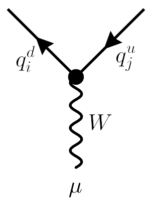

$$= gf^{abc}p^\mu$$




$$= i\frac{e}{\sin\theta_W\cos\theta_W}\gamma^\mu \times\left(I^3 P_L - Q\sin^2\theta_W\right)$$

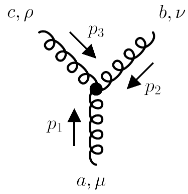



$$= i\frac{e(V_{\mathrm{CKM}})_{ij}}{\sqrt{2}\sin\theta_W}\gamma^\mu P_L$$

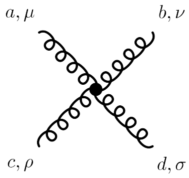



$$= i\frac{e(V_{\mathrm{CKM}})^*_{ji}}{\sqrt{2}\sin\theta_W}\gamma^\mu P_L$$

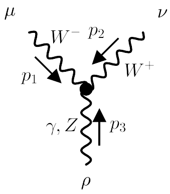
**Gauge-Boson Self Interactions:**



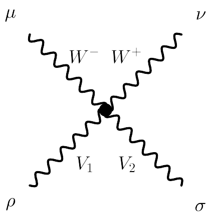

$$\begin{aligned} &= g f^{abc}((p_1^\rho - p_2^\rho)g^{\mu\nu} \\ &\qquad + (p_2^\mu - p_3^\mu)g^{\nu\rho} + (p_3^\nu - p_1^\nu)g^{\rho\mu}) \end{aligned}$$



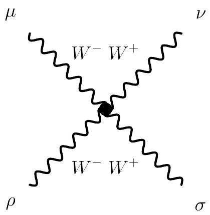

$$\begin{aligned} &= -ig^2\big(f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ &\qquad + f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ &\qquad + f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})\big) \end{aligned}$$




$$\begin{aligned} &= i\tfrac{e}{\sin\theta_W}((p_1^\rho - p_2^\rho)g^{\mu\nu} + (p_2^\mu - p_3^\mu)g^{\nu\rho} \\ &\qquad + (p_3^\nu - p_1^\nu)g^{\rho\mu}) \times \begin{cases} -\sin\theta_W & \text{for } \gamma \\ \cos\theta_W & \text{for } Z \end{cases} \end{aligned}$$

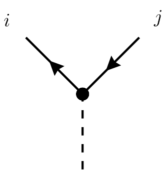



$$= i\frac{e^2}{\sin^2\theta_W}\left(g^{\mu\sigma}g^{\nu\rho} + g^{\mu\rho}g^{\nu\sigma} - 2g^{\mu\nu}g^{\rho\sigma}\right) \times \prod_{i=1}^{2}\begin{cases} -\sin\theta_W & \text{if } V_i = \gamma \\ \cos\theta_W & \text{if } V_i = Z \end{cases}$$

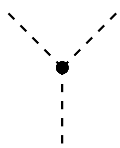



$$= i\frac{e^2}{\sin^2\theta_W}\left(2g^{\mu\rho}g^{\nu\sigma} - g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}\right)$$

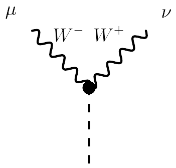
**Higgs Interactions:**



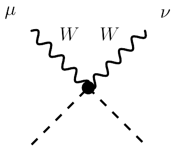

$$= -i\frac{m_q}{v}\delta_{ij}$$



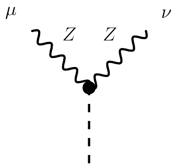
$$= -i\frac{3m_H^2}{v}$$



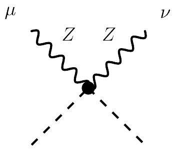

$$= i\frac{e}{\sin\theta_W} m_W g^{\mu\nu}$$



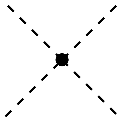

$$= i\frac{e^2}{2\sin^2\theta_W} g^{\mu\nu}$$




$$= i\frac{e}{\sin\theta_W \cos\theta_W} m_Z g^{\mu\nu}$$




$$= i\frac{e^2}{2\sin^2\theta_W \cos^2\theta_W} g^{\mu\nu}$$



$$= -i\frac{3m_H^2}{v^2}$$

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