Can multi-scale image pyramid processing concepts improve migration results?

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Agenda

Motivation

Project objectives

Theory

Results

Motivation

Image pyramids decompose by scale

Backscattering artifacts are obnoxious long wavelength features

Can image pyramids be used to target and remove backscattering artifacts?

Project objectives

1. Analyze migrated results using image pyramids
2. Migrated image pyramids levels independently
3. Propose and test new multi-scale imaging condition

Results

1. CIC then decompose
2. Decompose then CIC
3. Multiscale imaging condition application

Modeling assumptions

- Acoustic variable density wave equation
- Smoothly varying media
- Finite differences used for implementation

Building an image pyramid theory

1. Begin with an image
2. Smooth image
3. Discard even rows and columns
4. Repeat 1-3 until resulting image is size 1-by-1

Image pyramids size is manageable

$$N^2 \left[\frac{1}{4} + \frac{1}{4^2} + \ldots + \frac{1}{4^P} + \right] =$$

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$$N^2 \left[\frac{1}{4} + \frac{1}{4^2} + \ldots + \frac{1}{4^P} + \right] = N^2 \sum_{k=0}^{P} \left(\frac{1}{4}\right)^k$$

Image pyramids size is manageable

$$N^2\left[\frac{1}{4}+\frac{1}{4^2}+\ldots+\frac{1}{4^P}+\right]=N^2\sum_{k=0}^{P}\left(\frac{1}{4}\right)^k\leq\frac{4}{3}N^2 \tag{1}$$

Conventional imaging condition (CIC)

$$\mathbf{R}(\mathbf{x}) = \sum_t \mathbf{W}_s(\mathbf{x}, t)\mathbf{W}_r(\mathbf{x}, t) \quad (2)$$

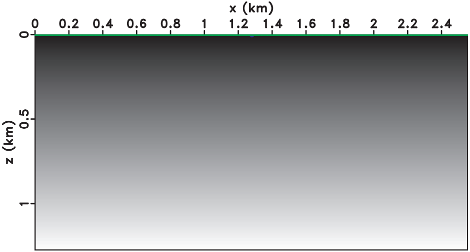
- $\mathbf{x}$ space coordinates
- t time coordinate
- $\mathbf{W}_s$ source wavefield
- $\mathbf{W}_r$ receiver wavefield
- $\mathbf{R}(\mathbf{x})$ migrated image

Conventional imaging condition (CIC) on pyramid levels

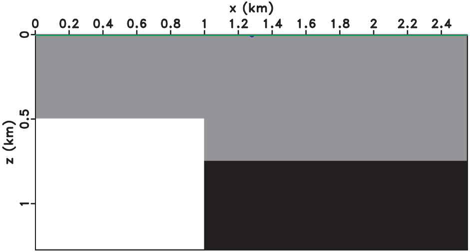
$$\hat{\mathbf{R}}_1(\mathbf{x}, \mathbf{l}) = \sum_t \mathbf{W}_s(\mathbf{x}, t, \mathbf{l}) \mathbf{W}_r(\mathbf{x}, t, \mathbf{l}) \tag{3}$$

- $\mathbf{x}$ space coordinates
- t time coordinate
- $\mathbf{W}_s$ source wavefield
- $\mathbf{W}_r$ receiver wavefield
- $\hat{\mathbf{R}}_1(\mathbf{x}, \mathbf{l})$ migrated image for all levels

Model structure

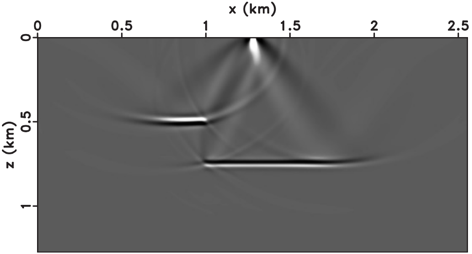


velocity model

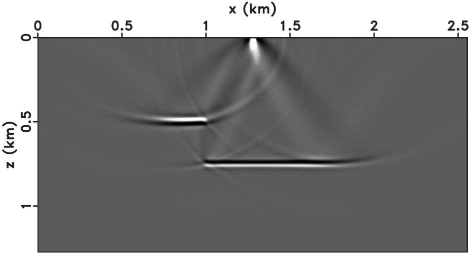


density model

Level 1 comparison

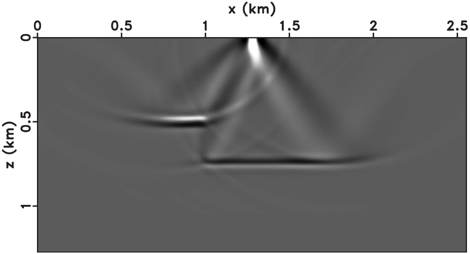


CIC → pyramid

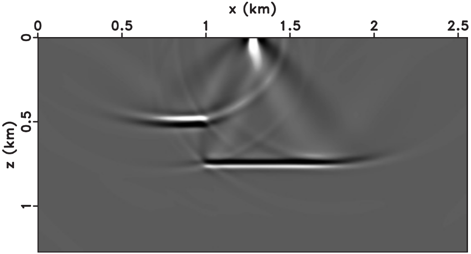


pyramid → CIC

Level 2 comparison

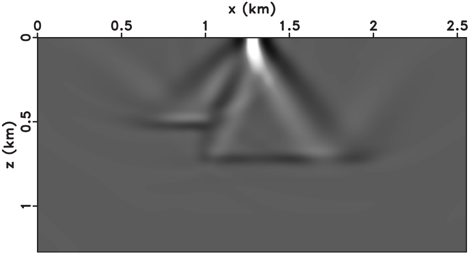


CIC → pyramid

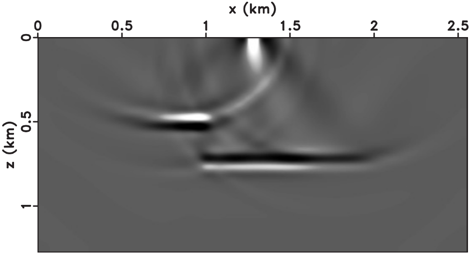


pyramid → CIC

Level 3 comparison

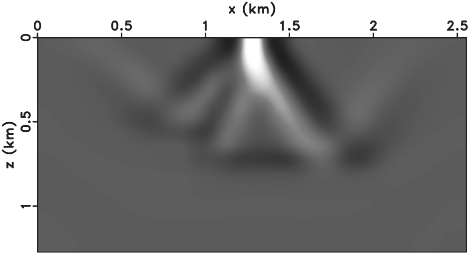


CIC $\rightarrow$ pyramid

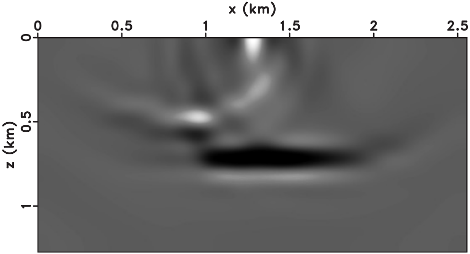


pyramid $\rightarrow$ CIC

Level 4 comparison

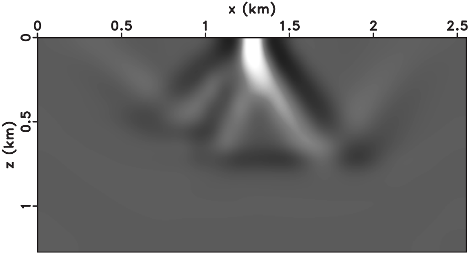


CIC → pyramid

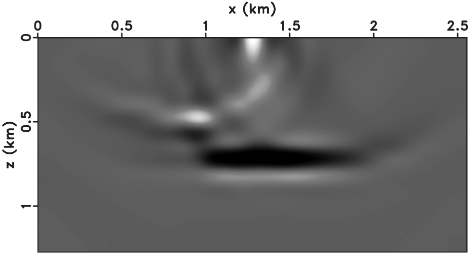


pyramid → CIC

Level 4 comparison



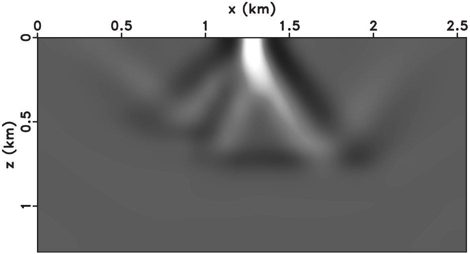
CIC $\rightarrow$ pyramid



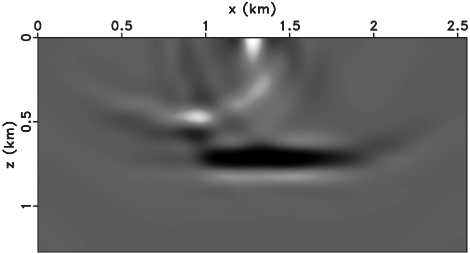
pyramid $\rightarrow$ CIC

Building and collapsing pyramid is a nonlinear operation

Level 4 comparison



CIC → pyramid



pyramid → CIC

Building and collapsing pyramid is a nonlinear operation

What can we gain by comparing and combining wavefields by scale before applying CIC?

Multi-scale imaging condition

$$\hat{\mathbf{R}}_1(\mathbf{x},\mathbf{I}) = \sum_t \mathbf{W}_s(\mathbf{x},t,\mathbf{I})\mathbf{W}_r(\mathbf{x},t,\mathbf{I}) \tag{4}$$

Multi-scale imaging condition

$$\hat{\mathbf{R}}_1(\mathbf{x}, \mathbf{l}) = \sum_t \mathbf{W}_s(\mathbf{x}, t, \mathbf{l}) \mathbf{W}_r(\mathbf{x}, t, \mathbf{l}) \tag{4}$$

$$\hat{\mathbf{W}}_s(\mathbf{x}, t, l_i) = \beta_i \mathbf{W}_s(\mathbf{x}, t, l_i) \tag{5}$$

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$$\hat{\mathbf{W}}_r(\mathbf{x}, t, l_i) = \beta_i \mathbf{W}_r(\mathbf{x}, t, l_i) \tag{6}$$

$$\hat{\mathbf{R}}_2(\mathbf{x}, l_i) = \sum_t \hat{\mathbf{W}}_s(\mathbf{x}, t, l_i) \hat{\mathbf{W}}_r(\mathbf{x}, t, l_i)$$

Multi-scale imaging condition

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$$\begin{aligned} \hat{\mathbf{R}}_2(\mathbf{x}, l_i) &= \sum_t \hat{\mathbf{W}}_s(\mathbf{x}, t, l_i) \hat{\mathbf{W}}_r(\mathbf{x}, t, l_i) \\ &= \beta_i^2 \sum_t \mathbf{W}_s(\mathbf{x}, t, l_i) \mathbf{W}_r(\mathbf{x}, t, l_i) \end{aligned}$$

Multi-scale imaging condition

$$\hat{\mathbf{R}}_1(\mathbf{x}, \mathbf{l}) = \sum_t \mathbf{W}_s(\mathbf{x}, t, \mathbf{l})\mathbf{W}_r(\mathbf{x}, t, \mathbf{l}) \tag{4}$$

$$\hat{\mathbf{W}}_s(\mathbf{x}, t, l_i) = \beta_i \mathbf{W}_s(\mathbf{x}, t, l_i) \tag{5}$$

$$\hat{\mathbf{W}}_r(\mathbf{x}, t, l_i) = \beta_i \mathbf{W}_r(\mathbf{x}, t, l_i) \tag{6}$$

$$\begin{aligned}\hat{\mathbf{R}}_2(\mathbf{x}, l_i) &= \sum_t \hat{\mathbf{W}}_s(\mathbf{x}, t, l_i)\hat{\mathbf{W}}_r(\mathbf{x}, t, l_i) \\ &= \beta_i^2 \sum_t \mathbf{W}_s(\mathbf{x}, t, l_i)\mathbf{W}_r(\mathbf{x}, t, l_i) \\ &= \beta_i^2 \hat{\mathbf{R}}_1(\mathbf{x}, l_i)\end{aligned} \tag{7}$$

Multi-scale imaging condition

$$\hat{\mathbf{R}}_1(\mathbf{x}, \mathbf{l}) = \sum_t \mathbf{W}_s(\mathbf{x}, t, \mathbf{l})\mathbf{W}_r(\mathbf{x}, t, \mathbf{l}) \tag{4}$$

$$\hat{\mathbf{W}}_s(\mathbf{x}, t, l_i) = \beta_i \mathbf{W}_s(\mathbf{x}, t, l_i) \tag{5}$$

$$\hat{\mathbf{W}}_r(\mathbf{x}, t, l_i) = \beta_i \mathbf{W}_r(\mathbf{x}, t, l_i) \tag{6}$$

$$\begin{aligned}\hat{\mathbf{R}}_2(\mathbf{x}, l_i) =& \sum_t \hat{\mathbf{W}}_s(\mathbf{x}, t, l_i)\hat{\mathbf{W}}_r(\mathbf{x}, t, l_i) \\ =& \beta_i^2 \sum_t \mathbf{W}_s(\mathbf{x}, t, l_i)\mathbf{W}_r(\mathbf{x}, t, l_i) \\ =& \beta_i^2 \hat{\mathbf{R}}_1(\mathbf{x}, l_i)\end{aligned} \tag{7}$$

- β_i scalar $\in$ (0,1)
- $\sum_i \beta_i = 1$

Multi-scale imaging condition

$$\hat{\mathbf{R}}_2(\mathbf{x}, l_i) = \beta_i^2 \hat{\mathbf{R}}_1(\mathbf{x}, l_i) \tag{8}$$

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Multi-scale imaging condition

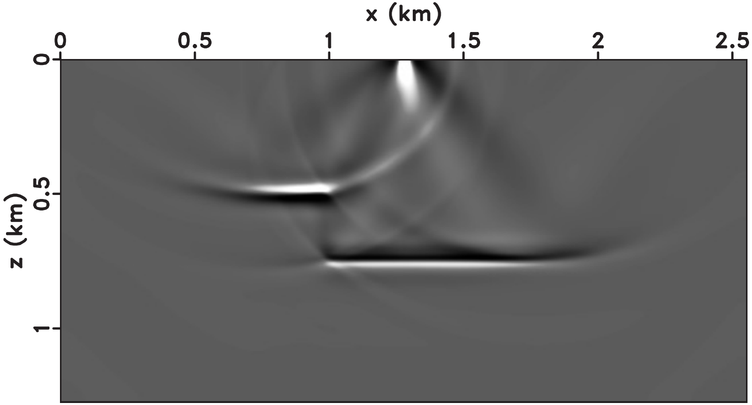
$$\hat{\mathbf{R}}_2(\mathbf{x}, l_i) = \beta_i^2 \hat{\mathbf{R}}_1(\mathbf{x}, l_i) \tag{8}$$

$$\beta_i^2 \rightarrow \alpha_i \in (-1, 1) \tag{9}$$

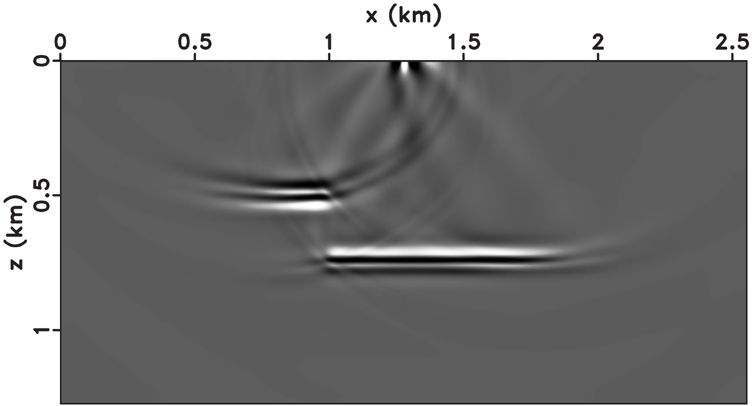
$$\hat{\mathbf{R}}_3(\mathbf{x}) = \sum_{\alpha_i} \alpha_i \hat{\mathbf{R}}_1(\mathbf{x}, l_i) \tag{10}$$

- Allows for truncation or subtraction of pyramid levels.

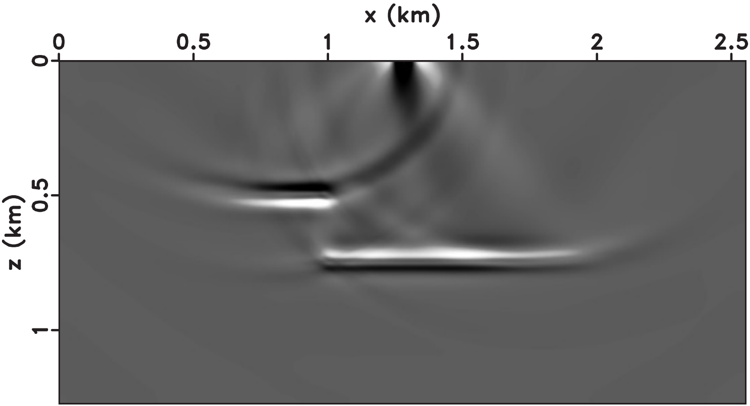
Migrated image: truncate at level 4 (include all levels)



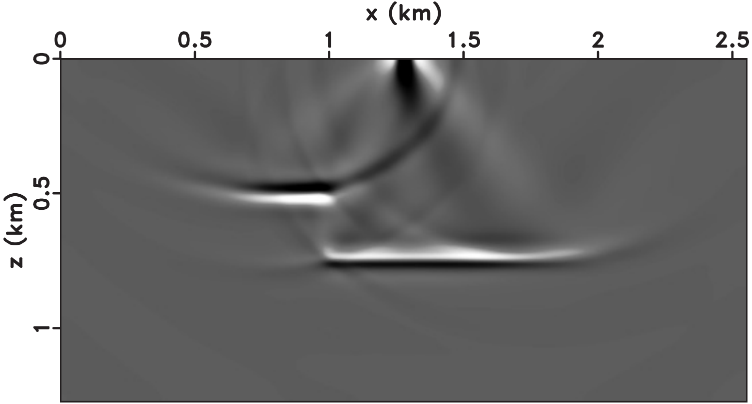
Migrated image: truncate at level 3



Migrated image: truncate at level 2



Migrated image: truncate at level 1



Conclusions

- Backscattering not attenuated using proposed multi-scale imaging condition
- Sharpening of horizontal reflectors can occur at certain truncation levels
- Truncating levels can reverse polarity of image events
- Future work: more complex image pyramids, other decompositions

Backup slides follow

Image pyramid theory

$$\mathbf{G}_j = \mathbf{B}_{j-1}\mathbf{A}_{j-1}\mathbf{G}_{j-1}. \tag{11}$$

$$\mathbf{G}_{j+1} = \mathbf{A}_j^T\mathbf{B}_j^T\mathbf{G}_j. \tag{12}$$

$$\mathbf{L}_k = \mathbf{G}_k - \mathbf{G}_{k+1,1}. \tag{13}$$