

Dispersive shock wave solutions of the KdV equation

w/ Deniz Bilman

The KdV equation

Consider solving the KdV equation

$$q_t + 6qq_x + q_{xxx} = 0, \quad (x, t) \in \mathbb{R} \times (0, \infty),$$

with initial data

$$q(x, 0) = q_0(x) = u_0(x) - c^2 H(x), \quad H(x) = \begin{cases} 1 & x \geq 0, \\ 0 & x < 0. \end{cases}$$

Assume that $u_0(x)$ has its only discontinuity at $x = 0$ and decays rapidly as $|x| \rightarrow \infty$.

For $u_0 = 0$, we have the so-called dispersive Riemann problem for the KdV equation.

General data can be treated using the Galilean boost.

