Benefits and complications

$$\Phi^+(s) = \Phi^-(s)\begin{bmatrix} 1-|R(s)|^2 & -\overline{R}(s)e^{-2i\theta(s)}, \\ R(s)e^{2i\theta(s)} & 1 \end{bmatrix}, \quad \Phi(\infty) = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

The (x,t) dependence is explicit via $\theta(s) = xs + 4ts$ — plug in (x,t) and solve numerically!

This is not free: $e^{2i\theta(s)}$ is highly oscillatory.

The current method to treat this involves contour deformations.

Steepest descent

To illustrate the complications that arise in solving this RH problem for all x and t, consider solving the linear equation

$$-\mathrm{i}q_t + q_{xx} = 0, \quad q(x,0) = q_0(x), \quad q(x,t) = \frac{1}{2\pi}\int_{\mathbb{R}} \mathrm{e}^{\mathrm{i}sx + \mathrm{i}s^2 t}\hat{q}_0(s)\mathrm{d}s.$$

$\mathrm{e}^{\mathrm{i}sx+\mathrm{i}s^2t}$ for $s \in \mathbb{R}$

$$z_0 = -\frac{x}{2t}$$

$\mathrm{e}^{\mathrm{i}sx+\mathrm{i}s^2t}$ for $s \in \Gamma$