The KdV equation

Consider solving the KdV equation

$$q_t + 6qq_x + q_{xxx} = 0, \quad (x,t) \in \mathbb{R} \times (0,\infty),$$

with initial data

$$q(x,0) = q_0(x) = u_0(x) - c^2 H(x), \quad H(x) = \begin{cases} 1 & x \geq 0, \\ 0 & x < 0. \end{cases}$$

Assume that $u_0(x)$ has its only discontinuity at $x = 0$ and decays rapidly as $|x| \to \infty$.

For $u_0 = 0$, we have the so-called dispersive Riemann problem for the KdV equation.

General data can be treated using the Galilean boost.

Other work

- Schrödinger scattering theory for such potentials:

A Cohen and T Kappeler. Scattering and inverse scattering for steplike potentials in the Schrodinger equation. *Indiana Univ. Math. J.*, 34:127–180, 1985

- Existence of solutions using GLM:

A Cohen. Solutions of the Korteweg–de Vries equation with steplike initial profile. *Commun. Partial Differ. Equations*, 9(8):751–806, jan 1984

T Kappeler. Solutions of the Korteweg-deVries equation with steplike initial data. *J. Differ. Equ.*, 63(3):306–331, jul 1986

- Riemann–Hilbert approach to rarefaction:

K Andreiev, I Egorova, T L Lange, and G Teschl. Rarefaction waves of the Korteweg–de Vries equation via nonlinear steepest descent. *J. Differ. Equ.*, 261(10):5371–5410, 2016

K. Andreiev and I. Egorova. On the Long-Time Asymptotics for the Korteweg-de Vries Equation with Steplike Initial Data Associated with Rarefaction Waves. *J. Math. Physics, Anal. Geom.*, 13(4):325–343, dec 2017

- DSWs and long-time asymptotics:

M J Ablowitz and D E Baldwin. Dispersive shock wave interactions and asymptotics. *Phys. Rev. E*, 87(2):022906, feb 2013

- Riemann–Hilbert approach to dispersive shock waves:

I Egorova, Z Gladka, V Kotlyarov, and G Teschl. Long-time asymptotics for the Korteweg–de Vries equation with step-like initial data. *Nonlinearity*, 26(7):1839–1864, jul 2013