The KdV equation

The evolution of the KdV flow represents an isospectral deformation of the Schrödinger operator

$$\mathscr{L}(t)\psi = -\psi_{xx} - q(x,t)\psi.$$

The “spectrum" of $\mathscr{L}$ is constant in time.

The forward scattering transform amounts to determining linearly independent solutions $\psi_\pm(x,t,\lambda)$ of the Lax pair, $\mathscr{L}(t)\psi_\pm = \lambda\psi_\pm$, that are:

1. analytic on the complement of the spectrum of $\mathscr{L}$,
2. are uniquely determined by jump/residue conditions on the spectrum, and
3. satisfy $\psi_\pm(x,t,\lambda) = \mathrm{e}^{\pm \mathrm{i}\sqrt{\lambda}x \pm 4\mathrm{i}\sqrt{\lambda}^3 t}(1+o(1))$, $\lambda \to \infty$.

The KdV equation with decaying data

w/ Bernard Deconinck, Sheehan Olver