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Riemann–Hilbert Problem 2. *The function $\Phi_2 : \mathbb{C} \setminus \mathbb{R} \rightarrow \mathbb{C}^{1 \times 2}$, $\Phi_2(z) = \Phi_2(z; x, t)$ satisfies*

$$\Phi_2^+(s) = \Phi_2^-(s) \begin{bmatrix} 1 - |R_r(s)|^2 & -R_r(-s)e^{-2i\lambda(s)x - 8i\varphi(s)t} \\ R_r(s)e^{2i\lambda(s)x + 8i\varphi(s)t} & 1 \end{bmatrix}, \quad s^2 > c^2,$$

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$$\Phi_2(z) = \begin{bmatrix} 1 & 1 \end{bmatrix} + O(z^{-1}), \quad z \rightarrow \infty, \quad \varphi(s) = \lambda^3(s) + 3/2c^2 \lambda(s),$$

with the symmetry condition

$$\Phi_2(-z) = \Phi_2(z)\sigma_1, \quad z \in \mathbb{C} \setminus \mathbb{R}.$$

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Deformations

We have not implemented all the deformations that are required to compute $q(x, t)$ for all (x, t) .

Two deformations allow us to compute the solution for all x and small time, $0 < t < 2$.

More to come, including $t \rightarrow \infty$!

All deformations are performed assuming $u_0 \in L^1(\mathbb{R}, e^{\delta|x|} dx)$ for some $\delta > 0$.

