Fixed-time theory

$$\psi_\pm(x;\lambda) = c(x;\lambda) + \frac{\pm\sqrt{\Delta^2(\lambda)-1} + \frac{1}{2}(T_{22}(\lambda) - T_{11}(\lambda))}{T_{12}(\lambda)} s(x;\lambda)$$

Since T is an entire function of λ, the singularities of $\psi_\pm$ (possibly) occur at when $\Delta^2 = 1$ and $T_{12}(\lambda) = 0$.

From this, one sees that the Bloch spectrum, combined with the Dirichlet spectrum (and a little more), is enough to uniquely specify $\psi_\pm$.

Time dependence

It can then be shown that the Bloch spectrum and Dirichlet spectrum are independent of time if $q(x,t)$ evolves according the the KdV equation.

The behavior at infinity is then

$$\psi_\pm(x,t;\lambda) = \mathrm{e}^{\pm \mathrm{i}(x-x_0)\sqrt{\lambda} \pm 4\mathrm{i}t\lambda^{3/2}}(1+o(1)).$$

This is enough information to set up a RH problem for $\psi_\pm(x,t;\lambda)$.