

Incomplete list of other possible approaches

- “Classical” numerics with exponential integrators. This requires discretization in both space and time.

C Klein. Fourth order time-stepping for low dispersion Korteweg–de Vries and nonlinear Schroedinger equations. Electronic Transactions on Numerical Analysis, 29:116–135, 2008

- Compute the spectral data (bands and Dirichlet spectrum) and solve the Dubrovin equations. This requires discretization in both space and time.

B A Dubrovin. Integrable Systems and Riemann Surfaces Lecture Notes. <http://people.sis> 2009

- Compute the spectral data (bands and Dirichlet spectrum), compute the periods of the basis of differentials and evaluate the theta function formula.

B Deconinck, M Heil, A Bobenko, M van Hoeij, and M Schmies. Computing Riemann Theta Functions. Mathematics of Computation, 73(247):1417–1442, 2004

B Deconinck and J N Kutz. Computing spectra of linear operators using the Floquet-Fourier-Hill method. Journal of Computational Physics, 219:296–321, 2006

`Approxfun.jl`, `Chebfun`

A R Osborne. Nonlinear Fourier Analysis for the Korteweg-de Vries Equation I : An Algorithm for the Direct Scattering Transform. Journal of Computational Physics, 313:284–313, 1991

S Wahls, S Chimmalgi, and P J Prins. FNFT: A Software Library for Computing Nonlinear Fourier Transforms. Journal of Open Source Software, 3(23):597, mar 2018

- Others?



Example 1.a: Cosine initial data

$$q_t + qq_x + \delta^2 q_{xxx} = 0, \quad x \in [0, 2\pi), \quad t > 0,$$

$$q(x, t) = q(x + 2\pi, t),$$

$$q(x, 0) = \cos(x).$$

$$\delta = 0.1$$

```
15x2 Array{Float64,2}:
-0.49353  0.367011
 0.457722  0.941054
 1.36853  1.50714
 2.41864  2.43406
 3.73614  3.73699
 5.35378  5.35381
 7.27178  7.27178
 9.48824  9.48824
12.0021  12.0021
14.8129  14.8129
17.9204  17.9204
21.3243  21.3243
25.0245  25.0245
29.021  29.021
33.3137  33.3137
```

```
15-element Array{Float64,1}:
 0.8605406708802891
 0.483331460213977
 0.13860686561876623
 0.015420561819371681
 0.0008517721582448345
 2.924721513775097e-5
 6.911844128154598e-7
 1.1957725476463565e-8
 1.5856649326906336e-10
 1.5720758028692217e-12
 7.460698725481052e-14
 1.2434497875801753e-13
 4.583000645652646e-13
 3.694822225952521e-13
 8.881784197001252e-13
```

