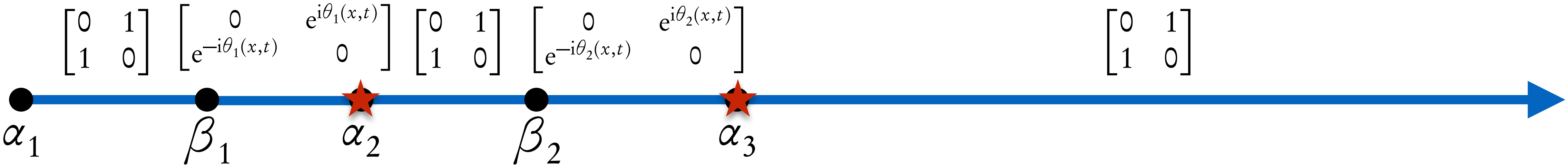
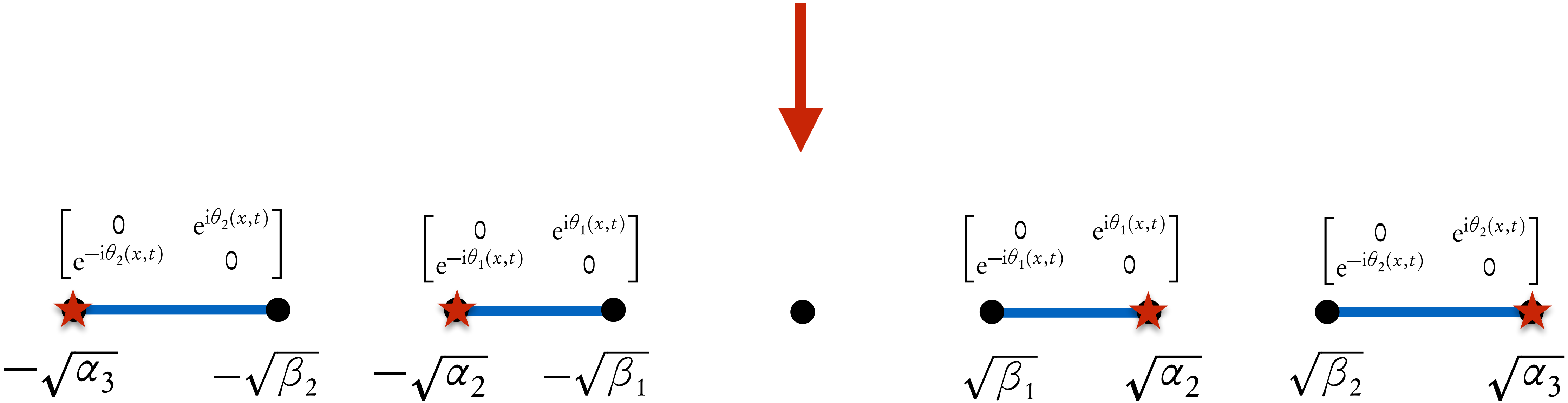
An example

$$N(z) = \begin{cases} M(z^2) & \operatorname{Im} z > 0, \\ M(z^2)\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} & \operatorname{Im} z < 0. \end{cases}$$



$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
$\begin{bmatrix} 0 & e^{i\theta_1(x,t)} \\ e^{-i\theta_1(x,t)} & 0 \end{bmatrix}$
$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
$\begin{bmatrix} 0 & e^{i\theta_2(x,t)} \\ e^{-i\theta_2(x,t)} & 0 \end{bmatrix}$
$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
α_1
β_1
α_2
β_2
α_3

Suppose $\alpha_1 = 0$



$\begin{bmatrix} 0 & e^{i\theta_2(x,t)} \\ e^{-i\theta_2(x,t)} & 0 \end{bmatrix}$
$\begin{bmatrix} 0 & e^{i\theta_1(x,t)} \\ e^{-i\theta_1(x,t)} & 0 \end{bmatrix}$
$\begin{bmatrix} 0 & e^{i\theta_1(x,t)} \\ e^{-i\theta_1(x,t)} & 0 \end{bmatrix}$
$\begin{bmatrix} 0 & e^{i\theta_2(x,t)} \\ e^{-i\theta_2(x,t)} & 0 \end{bmatrix}$
$-\sqrt{\alpha_3}$
$-\sqrt{\beta_2}$
$-\sqrt{\alpha_2}$
$-\sqrt{\beta_1}$
$\sqrt{\beta_1}$
$\sqrt{\alpha_2}$
$\sqrt{\beta_2}$
$\sqrt{\alpha_3}$

The jump matrix $J(z)$ is piecewise constant as a function of z.

T T and B Deconinck. A Riemann–Hilbert problem for the finite-genus solutions of the KdV equation and its numerical solution. Physica D, 251:1–18, 2013

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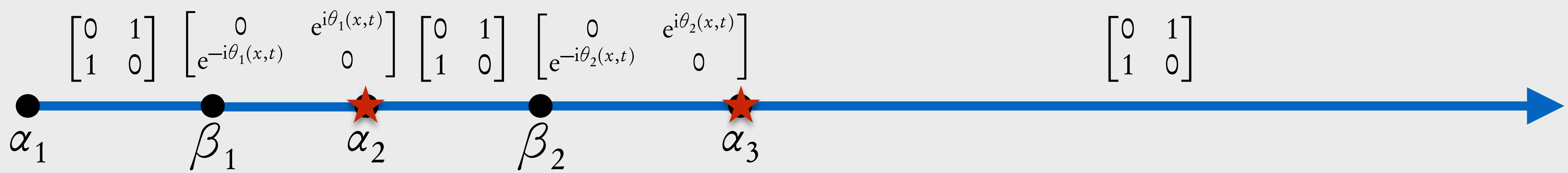

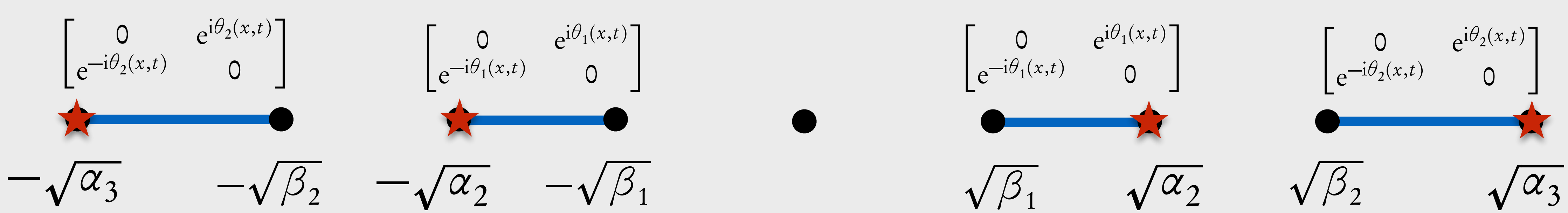

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A singular integral equation

We expect that

$$N(z) = \begin{bmatrix} 1 & 1 \end{bmatrix} + \frac{1}{2\pi \mathrm{i}} \int_\Gamma \frac{U(z')}{z' - z} \mathrm{d}z' = \begin{bmatrix} 1 & 1 \end{bmatrix} + \mathscr{C}_\Gamma U(z).$$

Owing to the Plemelj lemma, U will then satisfy

$$\begin{aligned} U(z) - \mathscr{C}_\Gamma^- U(z)(J(z) - I) &= \begin{bmatrix} 1 & 1 \end{bmatrix}(J(z) - I), \\ \mathscr{C}_\Gamma^- U(z) &:= \lim_{\epsilon \downarrow 0} \mathscr{C}_\Gamma U(z - \mathrm{i}\epsilon) \end{aligned}$$