

# The inverse scattering transform

General references for the inverse scattering transform are:

M J Ablowitz and H Segur. Solitons and the Inverse Scattering Transform. SIAM, Philadelphia, PA, 1981

S P Novikov, S V Manakov, L P Pitaevskii, and V E Zakharov. Theory of Solitons. Consultants Bureau, New York, 1984

M J Ablowitz and P A Clarkson. Solitons, Nonlinear Evolution Equations and Inverse Scattering. Cambridge University Press, 1991



# The KdV equation

The entirety of this talk will concern the KdV equation

$$q_t + 6qq_x + q_{xxx} = 0, \quad q(x, 0) = q_0(x), \quad (x, t) \in \mathbb{R} \times (0, \infty).$$

We will initially assume that  $q_0$  decays rapidly but that will be relaxed later.

The IST for the KdV equation is derived by considering the Lax pair

$$\mathcal{L}(t)\psi = -\psi_{xx} - q(x, t)\psi = \lambda\psi, \quad \psi_t = M(\lambda, q)\psi.$$

The specific form of  $M(\lambda, q)$  is not important for this talk.

The existence of the Lax pair is really the statement that if  $q(x, t)$  solves the KdV equation then one can find simultaneous solutions  $\psi_{\pm}(x, t, \lambda)$  of these two equations.

