$$\text{p.v.}\int_{\mathbb{R}} r_{j,-\alpha}(s)\mathrm{d}s = \begin{cases} -4\pi e^{-|\alpha|}L^{(1)}_{|j|-1}(2|\alpha|\sigma) & j\alpha > 0, \\ -2\pi|j| & \alpha = 0, \\ 0 & j\alpha < 0, \end{cases}$$

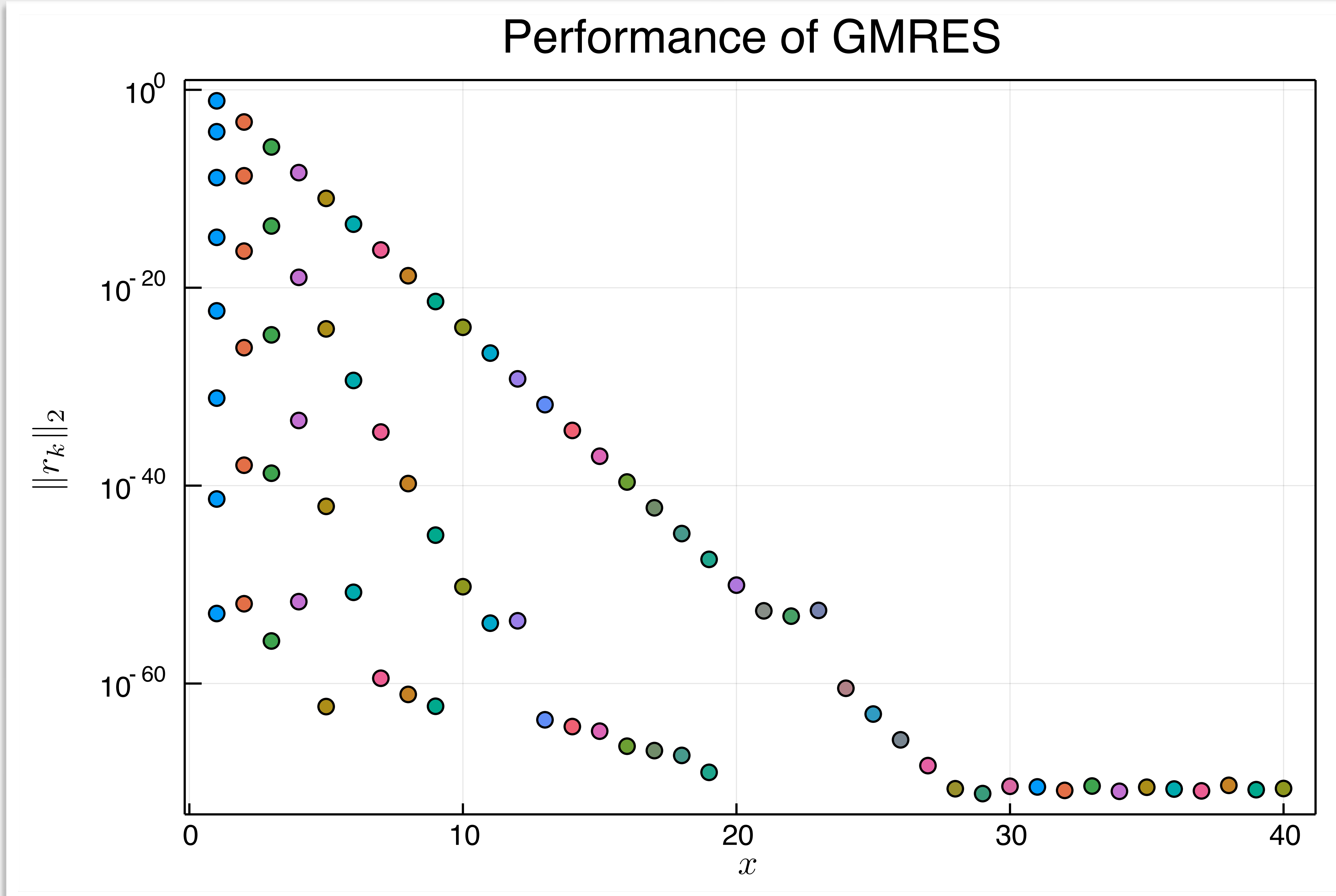
With this formula we can compute $L^2(\mathbb{R})$ inner products.

Apply the approximate operator exactly to the basis
Compute inner products exactly $\implies$ apply GMRES

To do this in high precision, the only tool that is needed is a high-precision enabled FFT.

This also necessary since we will need ≈ 4000 basis functions to adequately approximate the reflection coefficient —- 8000×8000 dense `BigFloat` matrix. And this is supposing we knew a priori which α's to choose.

At $t = 0$



T T. Scattering and inverse scattering for the AKNS system: A rational function approach. Studies in Applied Mathematics, page sapm.12434, aug 2021