Fixed-time theory

Define the monodromy matrix

$$T(\lambda) = \begin{bmatrix} c(x_0+L;\lambda) & s(x_0+L;\lambda) \\ c_x(x_0+L;\lambda) & s_x(x_0+L;\lambda) \end{bmatrix}.$$

Then

$$\psi_\pm(x;\lambda) = c(x;\lambda) + \frac{\pm\sqrt{\Delta^2(\lambda)-1} + \frac{1}{2}(T_{22}(\lambda)-T_{11}(\lambda))}{T_{12}(\lambda)} s(x;\lambda),$$

$$\Delta(\lambda) = \frac{1}{2}\mathrm{Tr}\,T(\lambda).$$

For x fixed, knowing $\psi_\pm$ as a function of λ is enough to determine $q(x,0)$.

B A Dubrovin. Integrable Systems and Riemann Surfaces Lecture Notes. http://people.sissa.it/~dubrovin/r
2009

Fixed-time theory

$$\psi_{\pm}(x;\lambda) = c(x;\lambda) + \frac{\pm\sqrt{\Delta^2(\lambda)-1} + \frac{1}{2}(T_{22}(\lambda) - T_{11}(\lambda))}{T_{12}(\lambda)} s(x;\lambda)$$

Since T is an entire function of λ, the singularities of $\psi_{\pm}$ (possibly) occur at when $\Delta^2 = 1$ and $T_{12}(\lambda) = 0$.

From this, one sees that the Bloch spectrum, combined with the Dirichlet spectrum (and a little more), is enough to uniquely specify $\psi_{\pm}$.