The inverse scattering transform

The inverse scattering transform is a method to "solve" an integrable evolution partial differential equation (PDE). The resulting methods share commonalities with the Fourier transform method for solving linear, constant coefficient PDEs:

- The initial condition is transformed to a function of a "spectral" variable

$$q_0(x) \mapsto \hat{q}_0(k).$$

- The time evolution of this spectral function is trivial

$$\hat{q}_0(k) \mapsto \hat{q}_0(k)e^{-i\omega(k)t}.$$

- The transform is invertible

$$\hat{q}_0(k)e^{-i\omega(k)t} \mapsto q(x,t).$$

The inverse scattering transform

General references for the inverse scattering transform are:

M J Ablowitz and H Segur. Solitons and the Inverse Scattering Transform. SIAM, Philadelphia, PA, 1981

S P Novikov, S V Manakov, L P Pitaevskii, and V E Zakharov. Theory of Solitons. Constants Bureau, New York, 1984

M J Ablowitz and P A Clarkson. Solitons, Nonlinear Evolution Equations and Inverse Scattering. Cambridge University Press, 1991