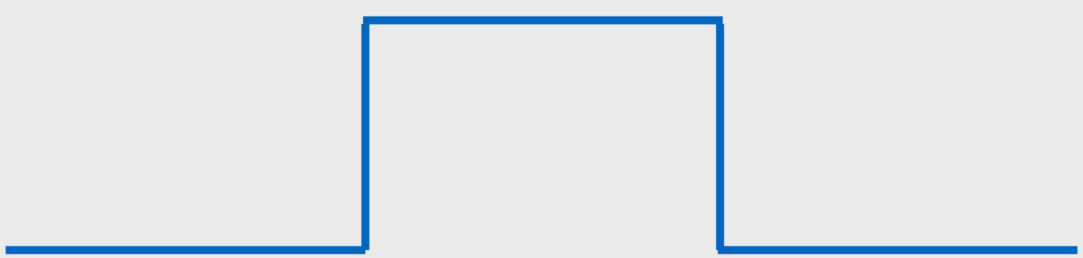
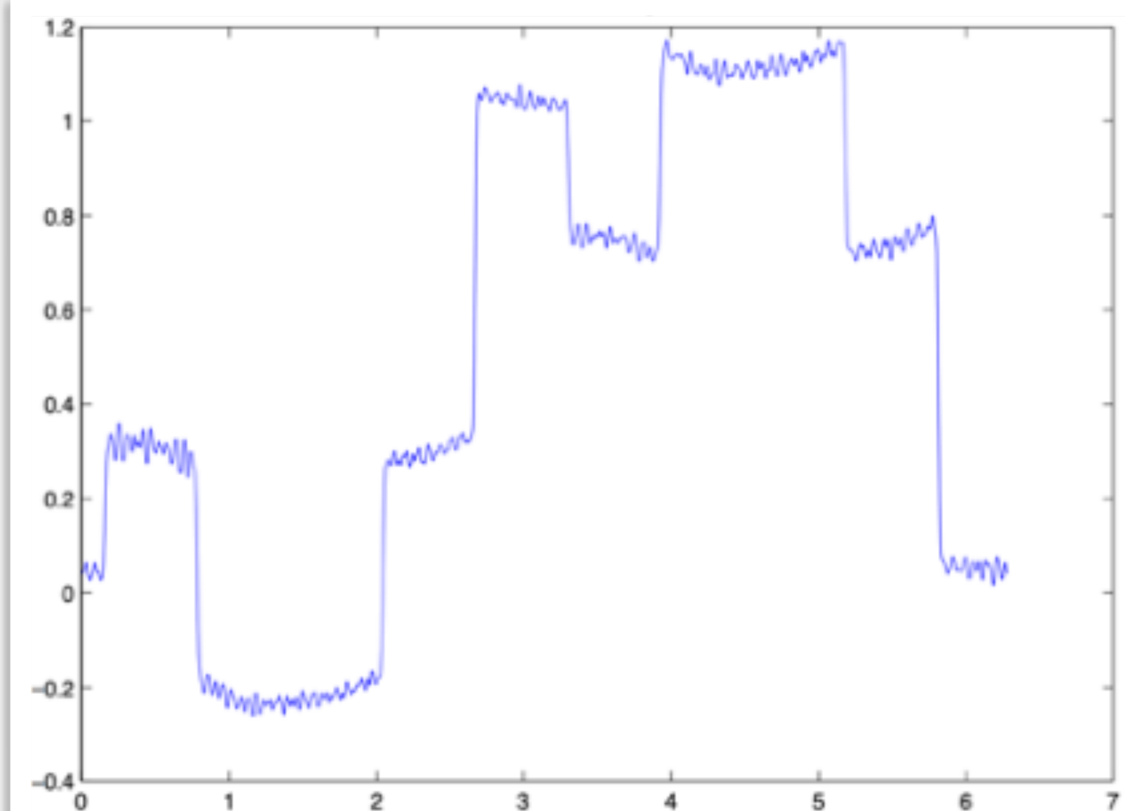
One motivation: Dispersive quantization



$t = 0$



$t = .1\pi$

G Chen and P J. Olver. Numerical simulation of nonlinear dispersive quantization. Discrete & Continuous Dynamical Systems - A, 34(3):991–1008, 2014

Fixed-time theory

Consider $t = 0$

$$\mathscr{L}(0) = -\partial_x - q(\diamond, 0)$$

Then it is well-known that the Bloch spectrum, for sufficiently regular periodic potentials,

$$\sigma_{\mathrm{B}}(q) := \left\{ \lambda \in \mathbb{C} : \text{ there exists a solution } \psi(\diamond; \lambda) \text{ of } \mathscr{L}\psi = \lambda\psi \text{ such that } \sup_{x\in\mathbb{R}} |\psi(x;\lambda)| < \infty \right\}$$

is a countable union of real intervals

$$\sigma_{\mathrm{B}}(q) = \bigcup_{k=1}^{g+1} [\alpha_k, \beta_k], \quad \text{where } g \in \mathbb{Z}_{>0} \ (\beta_{g+1} = \infty) \text{ or } g = \infty.$$

with

$$\alpha_k < \beta_k < \alpha_{k+1}, \quad k = 1, 2, \ldots$$

S P Novikov, S V Manakov, L P Pitaevskii, and V E Zakharov. Theory of Solitons. Constants Bureau, New York, 1984