Where is the hyperelliptic Riemann surface?

I have yet to mention any Riemann surface theory in this discussion. One needs this theory to:

- Show that one can “move" the Dirichlet eigenvalues (poles) to the band ends.
- Compute the constants that appear on the gaps when poles are moved (Abel map).
- Compute the functions $\theta_j(x,t)$ (linear system involving the period matrix).

A choice of basis of holomorphic differentials

Classically, for hyperelliptic surfaces, one uses the basis

$$\frac{\lambda^\ell}{w}, \quad \ell = 0, 1, 2, \ldots, g-1$$

where

$$w^2 = (\lambda - \alpha_{g+1}) \prod_{j=1}^{g} [(\lambda - \alpha_j)(\lambda - \beta_j)].$$

But this results in terribly ill-conditioned period matrices. It is much better to use something like

$$\frac{1}{w} \prod_{\substack{j=1 \\ j \neq \ell}}^{g} (\lambda - \beta_j), \quad \ell = 1, 2, \ldots, g.$$