

Riemann–Hilbert Problem 1. *The function $\Phi_1 : \mathbb{C} \setminus \mathbb{R} \rightarrow \mathbb{C}^{1 \times 2}$, $\Phi_1(z) = \Phi_1(z; x, t)$, satisfies*

$$\Phi_1^+(s) = \Phi_1^-(s) \begin{bmatrix} 1 - |R_1(s)|^2 & -\overline{R_1(s)} e^{2isx + 8is^3 t} \\ R_1(s) e^{-2isx - 8is^3 t} & 1 \end{bmatrix}, \quad s \in \mathbb{R},$$

$$\Phi_1^+(s) = \Phi_1^-(s) \begin{bmatrix} 1 & 0 \\ -\frac{c(z_j)}{s - z_j} e^{-2iz_j x - 8iz_j^3 t} & 1 \end{bmatrix}, \quad s \in \Sigma_j,$$

$$\Phi_1^+(s) = \Phi_1^-(s) \begin{bmatrix} 1 & -\frac{c(z_j)}{s + z_j} e^{-2iz_j x - 8iz_j^3 t} \\ 0 & 1 \end{bmatrix}, \quad s \in -\Sigma_j,$$

$$\Phi_1(z) = \begin{bmatrix} 1 & 1 \end{bmatrix} + O(z^{-1}), \quad z \in \mathbb{C} \setminus \mathbb{R},$$

with the symmetry condition

$$\Phi_1(-z) = \Phi_1(z) \sigma_1, \quad z \in \mathbb{C} \setminus \Gamma, \quad \Gamma = \mathbb{R} \cup \bigcup_j (\Sigma_j \cup -\Sigma_j).$$



Riemann–Hilbert Problem 2. *The function $\Phi_2 : \mathbb{C} \setminus \mathbb{R} \rightarrow \mathbb{C}^{1 \times 2}$, $\Phi_2(z) = \Phi_2(z; x, t)$ satisfies*

$$\Phi_2^+(s) = \Phi_2^-(s) \begin{bmatrix} 1 - |R_r(s)|^2 & -R_r(-s)e^{-2i\lambda(s)x - 8i\varphi(s)t} \\ R_r(s)e^{2i\lambda(s)x + 8i\varphi(s)t} & 1 \end{bmatrix}, \quad s^2 > c^2,$$

$$\Phi_2^+(s) = \Phi_2^-(s) \begin{bmatrix} 1 & -R_r(-s)e^{-2i\lambda^-(s)x - 8i\varphi^-(s)t} \\ 0 & 1 \end{bmatrix} \sigma_1 \begin{bmatrix} 1 & 0 \\ R_r(s)e^{2i\lambda^+(s)x + 8i\varphi^+(s)t} & 1 \end{bmatrix}, \quad -c \leq s \leq c,$$

$$\Phi_2^+(s) = \Phi_2^-(s) \begin{bmatrix} 1 & 0 \\ -\frac{C(z_j)}{s - z_j} e^{2i\lambda(z_j)x + 8i\varphi(z_j)t} & 1 \end{bmatrix}, \quad s \in \Sigma_j,$$

$$\Phi_2^+(s) = \Phi_2^-(s) \begin{bmatrix} 1 & -\frac{C(z_j)}{s + z_j} e^{2i\lambda(z_j)x + 8i\varphi(z_j)t} \\ 0 & 1 \end{bmatrix}, \quad s \in -\Sigma_j,$$

$$\Phi_2(z) = \begin{bmatrix} 1 & 1 \end{bmatrix} + O(z^{-1}), \quad z \rightarrow \infty, \quad \varphi(s) = \lambda^3(s) + 3/2c^2 \lambda(s),$$

with the symmetry condition

$$\Phi_2(-z) = \Phi_2(z)\sigma_1, \quad z \in \mathbb{C} \setminus \mathbb{R}.$$

