Some notes on complexity

Consider computing a genus g solution. Suppose ϵ is the tolerance parameter.

Initialization:

- We first have to compute the periods of the basis of differentials: $O(g^2 \text{polylog}(\epsilon))$ complexity
- Setting up the Riemann–Hilbert problem requires solving 3 $g \times g$ linear systems — $O(g^3)$ in the current code.

Solve at (x, t):

- We adaptively choose the number of collocation nodes per contour (i.e., gap) based on the known analyticity of the true solution.
- We use a block-diagonal preconditioner — $O(g\,\text{polylog}(\epsilon))$ to apply.
- We solve with GMRES using this preconditioner in $O(\text{polylog}(\epsilon))$ iterations, a total cost of $O(g^2 \text{polylog}(\epsilon))$

Some of the issues under the rug

The majority of the numerical complications here arise in the high-genus regime:

- Need to work with a numerically well-conditioned basis of differentials (and then normalize).
- Solve large linear systems iterative (find a good preconditioner).
- Deal with numerical instabilities intrinsic to the Abel map.

Some unresolved issues:

- Estimates for the truncation of the jump contours?
- Limit of the period matrix?
- Stability of the discretized singular integral operators?
- Zabusky-Kruskal problem?
- ...