Other work

- Schrödinger scattering theory for such potentials:

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- Existence of solutions using GLM:

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T Kappeler. Solutions of the Korteweg-deVries equation with steplike initial data. *J. Differ. Equ.*, 63(3):306–331, jul 1986

- Riemann–Hilbert approach to rarefaction:

K Andreiev, I Egorova, T L Lange, and G Teschl. Rarefaction waves of the Korteweg–de Vries equation via nonlinear steepest descent. *J. Differ. Equ.*, 261(10):5371–5410, 2016

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- DSWs and long-time asymptotics:

M J Ablowitz and D E Baldwin. Dispersive shock wave interactions and asymptotics. *Phys. Rev. E*, 87(2):022906, feb 2013

- Riemann–Hilbert approach to dispersive shock waves:

I Egorova, Z Gladka, V Kotlyarov, and G Teschl. Long-time asymptotics for the Korteweg–de Vries equation with step-like initial data. *Nonlinearity*, 26(7):1839–1864, jul 2013

Riemann–Hilbert Problem 1. *The function* $\Phi_1 : \mathbb{C} \setminus \mathbb{R} \to \mathbb{C}^{1\times 2}$, $\Phi_1(z) = \Phi_1(z; x, t)$, *satisfies*

$$\begin{aligned}
\Phi_1^+(s) &= \Phi_1^-(s) \begin{bmatrix} 1 - |R_1(s)|^2 & -\overline{R_1}(s) e^{2isx + 8is^3 t} \\ R_1(s) e^{-2isx - 8is^3 t} & 1 \end{bmatrix}, \quad s \in \mathbb{R}, \\
\Phi_1^+(s) &= \Phi_1^-(s) \begin{bmatrix} 1 & 0 \\ -\frac{c(z_j)}{s - z_j} e^{-2iz_j x - 8iz_j^3 t} & 1 \end{bmatrix}, \quad s \in \Sigma_j, \\
\Phi_1^+(s) &= \Phi_1^-(s) \begin{bmatrix} 1 & -\frac{c(z_j)}{s + z_j} e^{-2iz_j x - 8iz_j^3 t} \\ 0 & 1 \end{bmatrix}, \quad s \in -\Sigma_j, \\
\Phi_1(z) &= \begin{bmatrix} 1 & 1 \end{bmatrix} + O(z^{-1}), \quad z \in \mathbb{C} \setminus \mathbb{R},
\end{aligned}$$

with the symmetry condition

$$\Phi_1(-z) = \Phi_1(z)\sigma_1, \quad z \in \mathbb{C} \setminus \Gamma, \quad \Gamma = \mathbb{R} \cup \bigcup_j (\Sigma_j \cup -\Sigma_j).$$