Steepest descent

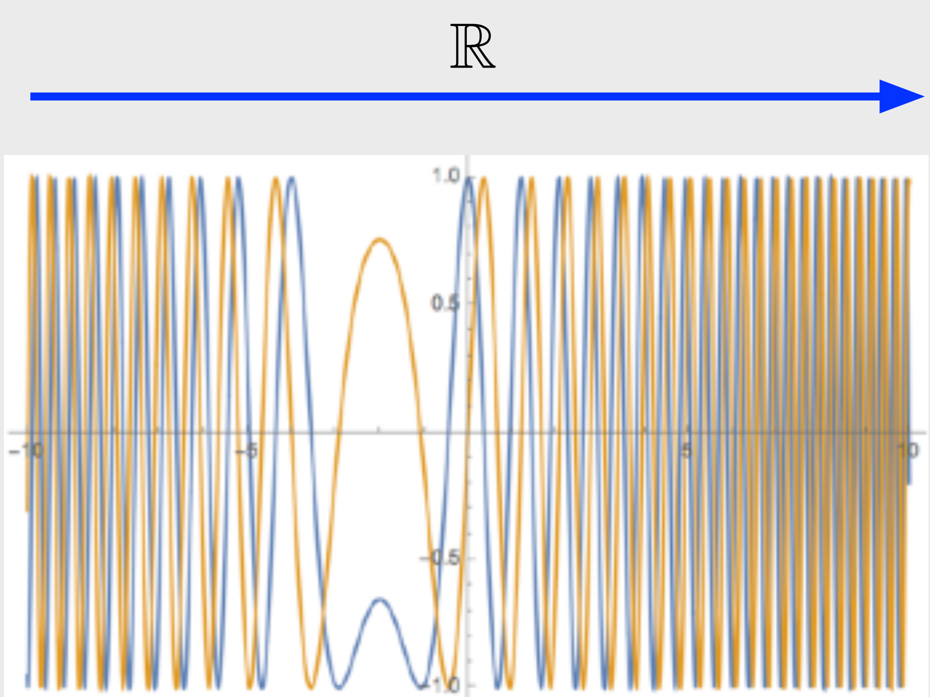
To illustrate the complications that arise in solving this RH problem for all x and t, consider solving the linear equation

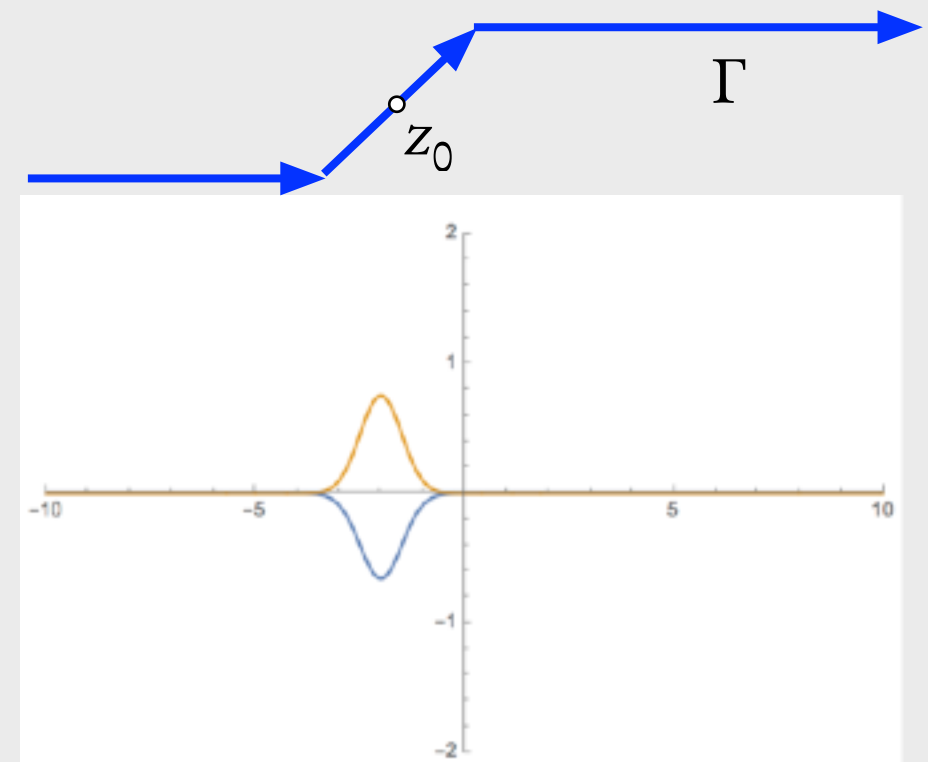
$$-\mathrm{i}q_t + q_{xx} = 0, \quad q(x,0) = q_0(x), \quad q(x,t) = \frac{1}{2\pi}\int_{\mathbb{R}} \mathrm{e}^{\mathrm{i}sx + \mathrm{i}s^2 t}\hat{q}_0(s)\mathrm{d}s.$$

$\mathrm{e}^{\mathrm{i}sx+\mathrm{i}s^2t}$ for $s \in \mathbb{R}$

$$z_0 = -\frac{x}{2t}$$

$\mathrm{e}^{\mathrm{i}sx+\mathrm{i}s^2t}$ for $s \in \Gamma$





Steepest descent

Integrals

Steepest descent for integrals is a contour-deformation-based method to derive asymptotic expansions.

Quadrature routines can be used along deformed contours for numerical computations.

Riemann–Hilbert problems

Nonlinear steepest descent (due to Deift & Zhou) is the extension of the method for integrals to RH problems.

Numerical methods can be used along these deformed contours for accurate evaluation (Numerical nonlinear steepest descent).