MATHEMATICA, TENSOR NETWORKS, MERA AND ENTANGLEMENT

TOWARDS HOLOGRAPHY FROM THE FIELD THEORY SIDE

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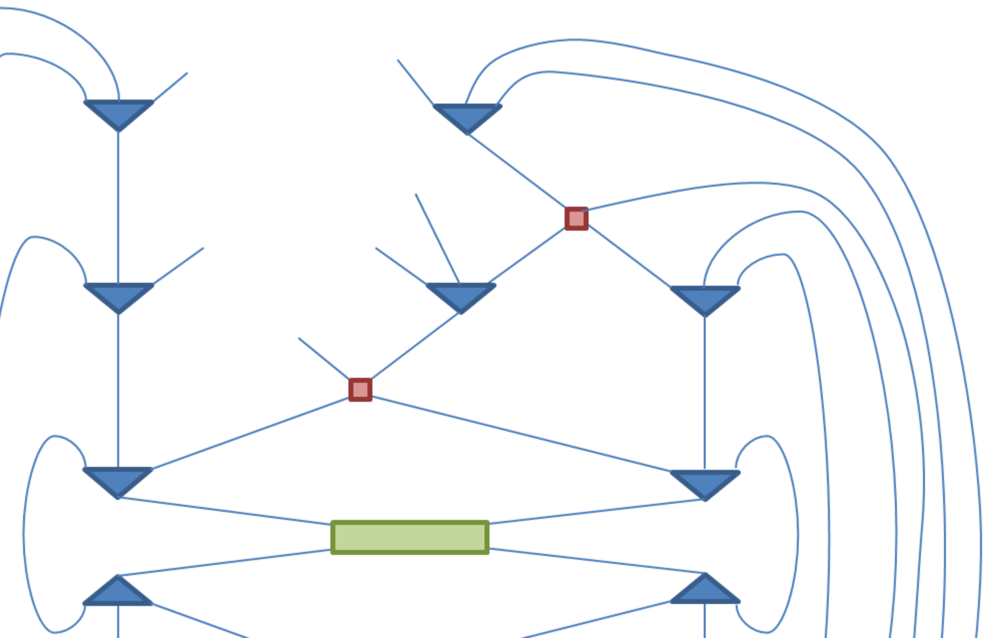
Mathematica School, PI, August 2015



OUTLINE

MERA, entanglement and AdS/MERA

- Tensor networks and contractions
- Entanglement on a slice of AdS?
- Goal: do MERA computations, and make AdS/MERA quantitative
    - →get simple MERA example + routines and do exercises

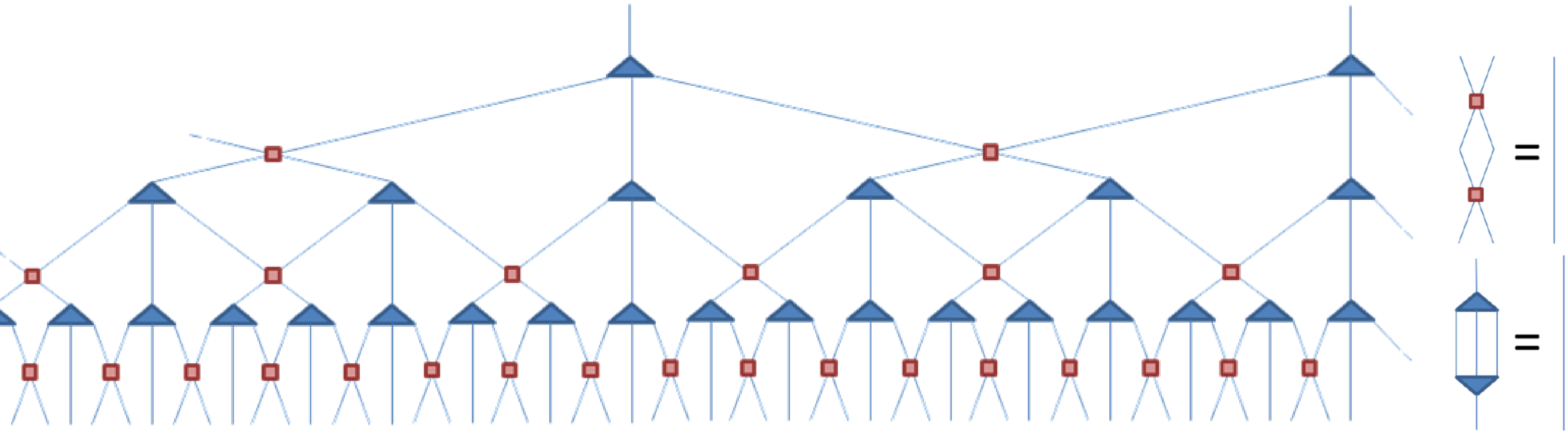
Frequently used *Mathematica*

- Making and using packages
- NDSolve and/or spectral methods
- Transforming functions/coordinate transformations

MULTISCALE ENTANGLEMENT RENORMALISATION ANSATZ (MERA)

MPS correlations/entanglement requires larger χ

Choose different *ansatz* to incorporate RG flow:



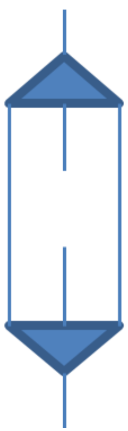
Disentanglers and coarse grainers (ternary)

Extra advantage: scale invariance is very natural!

G. Vidal, Entanglement Renormalization (2007)

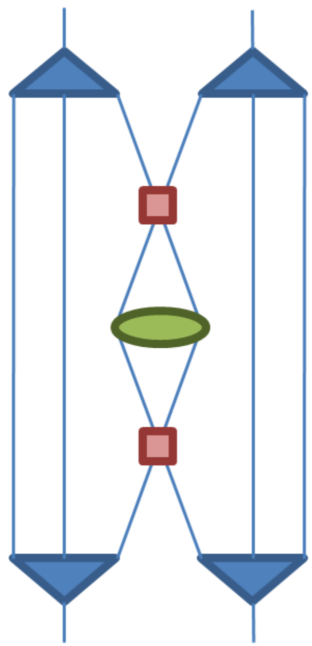
FUN FACT: TENSOR CONTRACTIONS NP COMPLETE

Algorithm depends crucially on \`efficiently contractible'

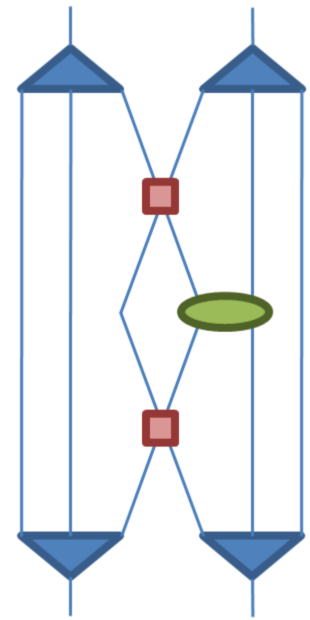


χ^6

$\Sigma_{i,j} w^{a}_{ibj} w^{c*}_{idj}$



χ^6



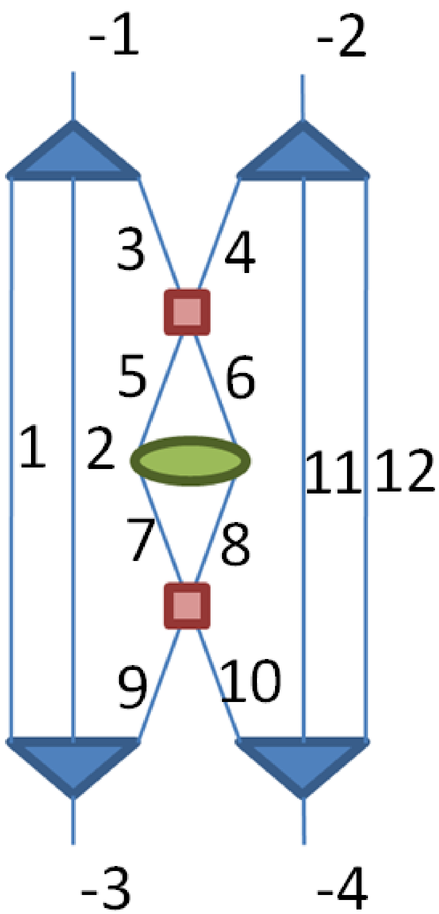
χ^8

Much harder for 2 dimensions (i.e. χ^{16} or χ^{23})

R.N.C. Pfeifer, J. Haegeman and F. Verstraete, Faster identification of optimal contraction sequences for tensor networks (2013)

NCON FUNCTION

Idea: contract sequentially, contracting two tensors at a time:

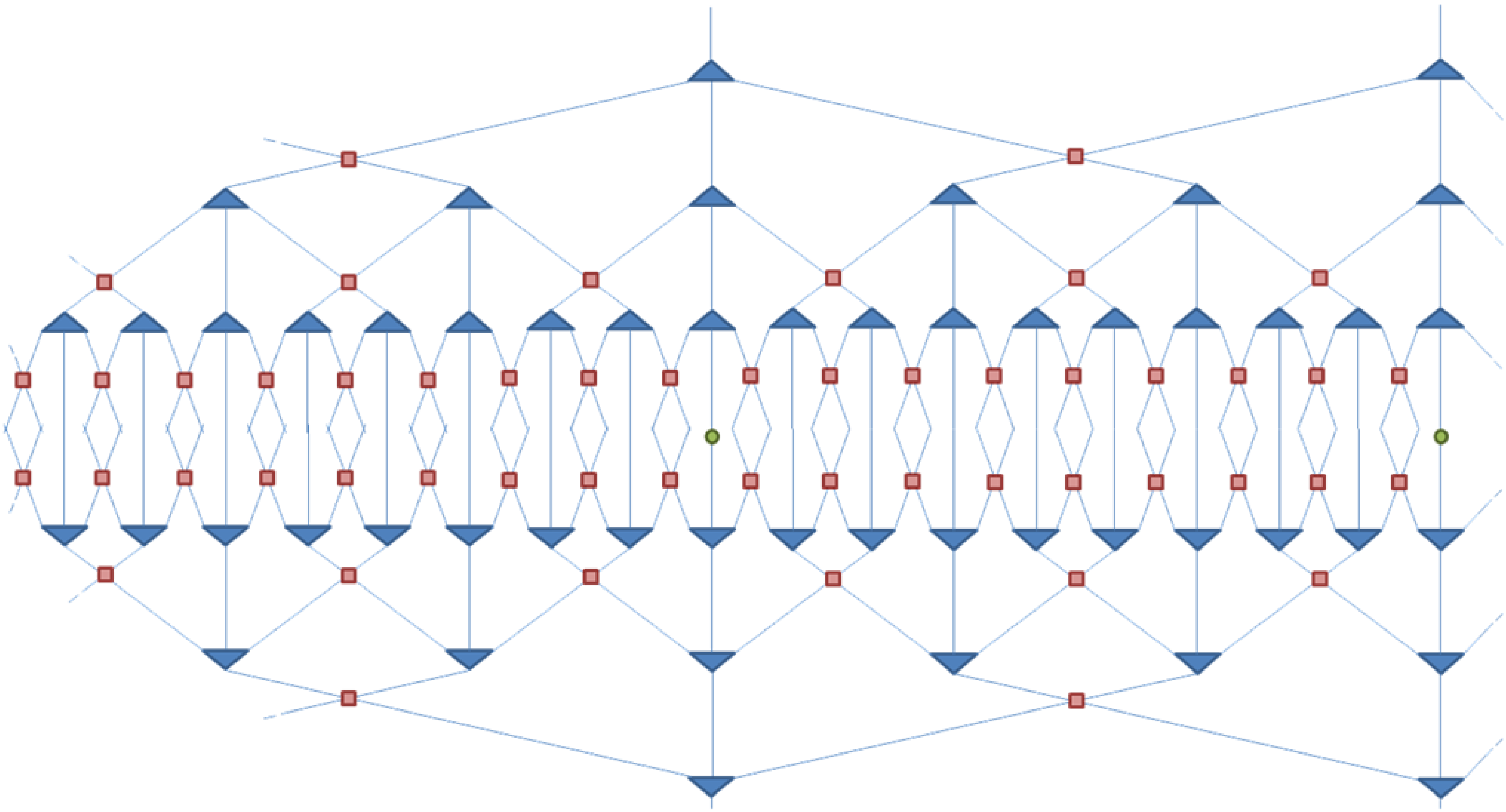


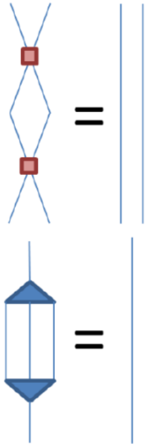

```
ncon[{w,w*,w,w*,u,u*,o},
{{-1,1,2,3},
 {-3,1,2,9},
 {-2,4,11,12},
 {-4,10,11,12},
 {3,4,5,6},
 {7,8,9,10},
 {5,6,7,8}},
??
]
```


R. N. C. Pfeifer, G. Evenbly, S. Singh, G. Vidal, NCON: A tensor network contractor for MATLAB (2014)

EXAMPLE: CORRELATORS IN MERA

Choose operators at smart locations



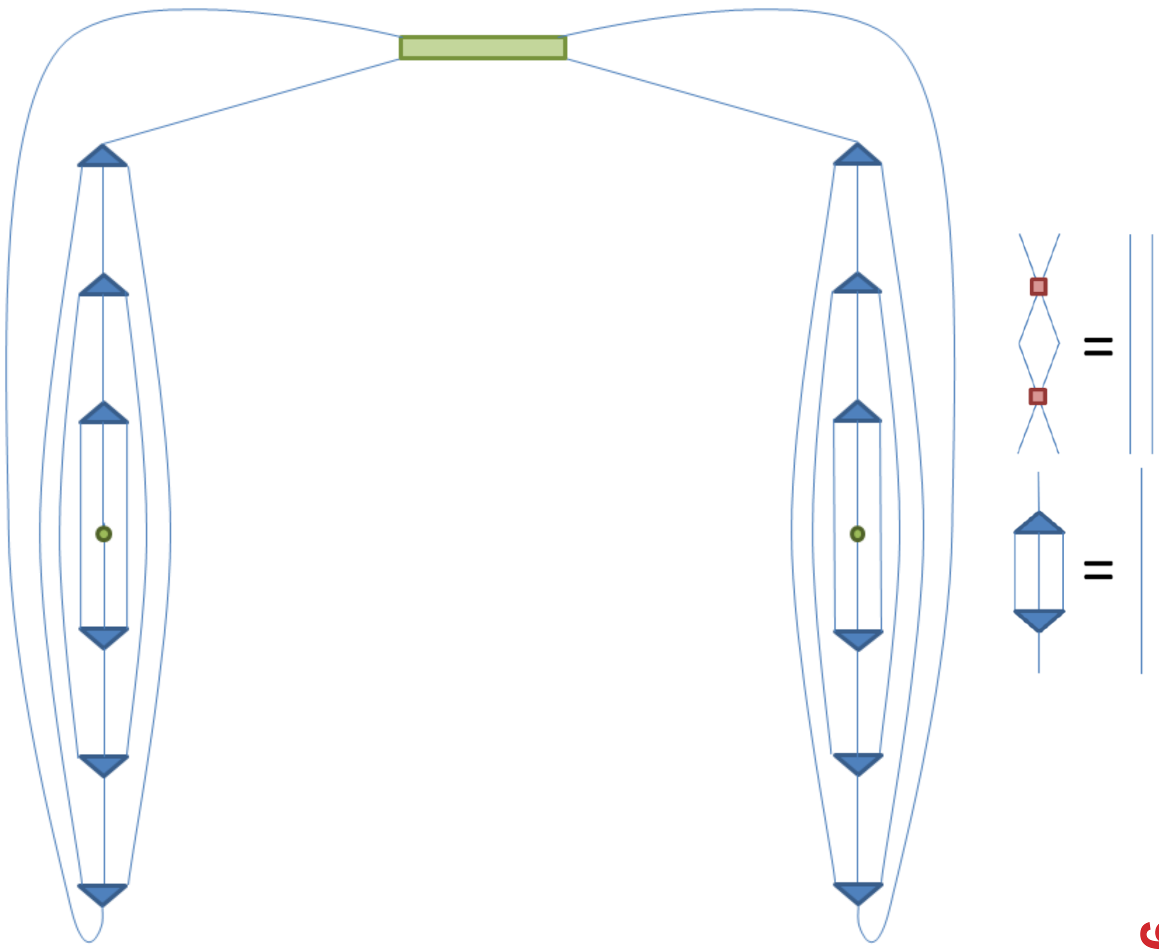


Simplify ☺

R.N.C. Pfeifer, G. Evenbly and G. Vidal, Entanglement renormalization, scale invariance, and quantum criticality (2009)

EXAMPLE: CORRELATORS IN MERA

Add reduced density matrix (green)



ENTANGLEMENT ENTROPY

Reduced density matrix: $\rho_{red,L} = \mathrm{tr}(\rho)$

Obtain mixed state with probabilities: $p_\rho = \mathrm{eig}(\rho_{red,L})$

- **Has entropy:** $S_{EE} = -\sum_i p_{\rho,i} \log(p_{\rho,i}) = \frac{c}{3} \log(L) + \mathcal{O}(1)$
- **I.e. ground state → excited state!**

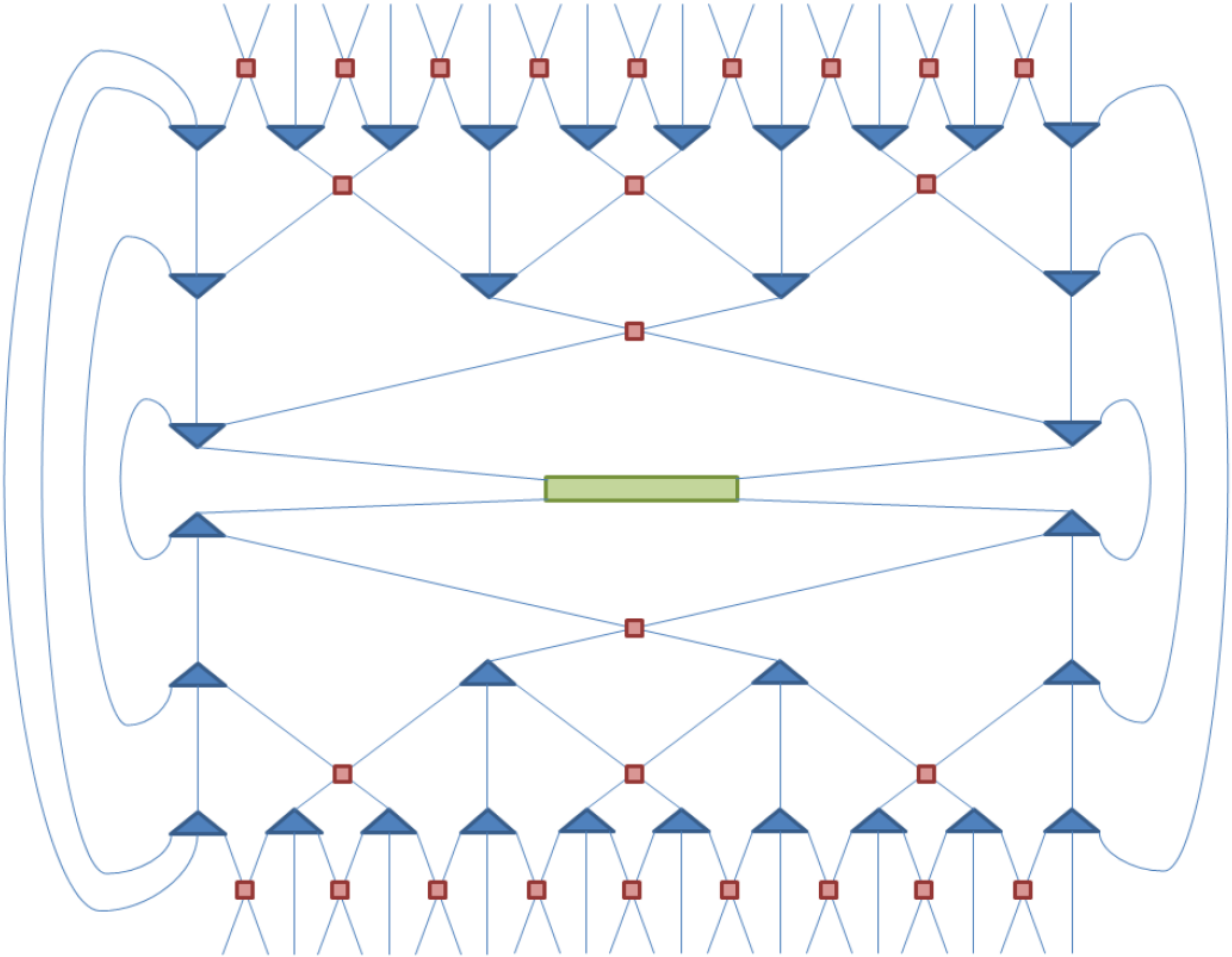
Ising model: $H_{\mathrm{Ising}} = -\sum_r \left(\lambda \sigma_z^{[r]} + \sigma_x^{[r]} \sigma_x^{[r+1]}\right)$

- **Energy:** $e_0 = -\frac{2}{L \sin(\pi/2L)} \approx -\frac{4}{\pi} - \frac{\pi}{6L^2}$
- **Central charge 1/2**

P. Calabrese and J. Cardy, Entanglement entropy and quantum field theory (2004)

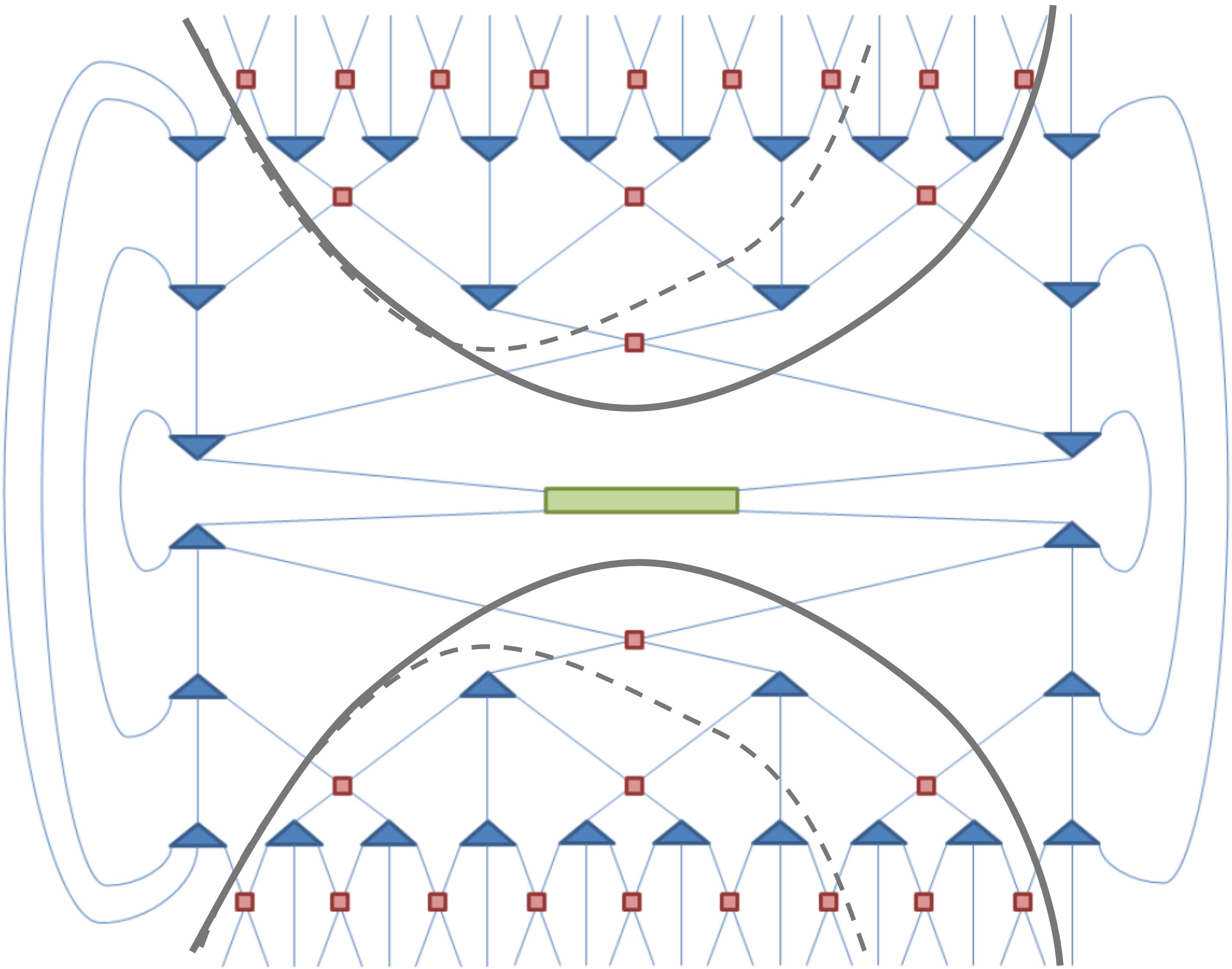
$$\rho_{red,L} = \mathrm{tr}(\rho)$$

REDUCED DENSITY MATRIX IN MERA



$$\rho_{red,L} = \mathrm{tr}(\rho)$$

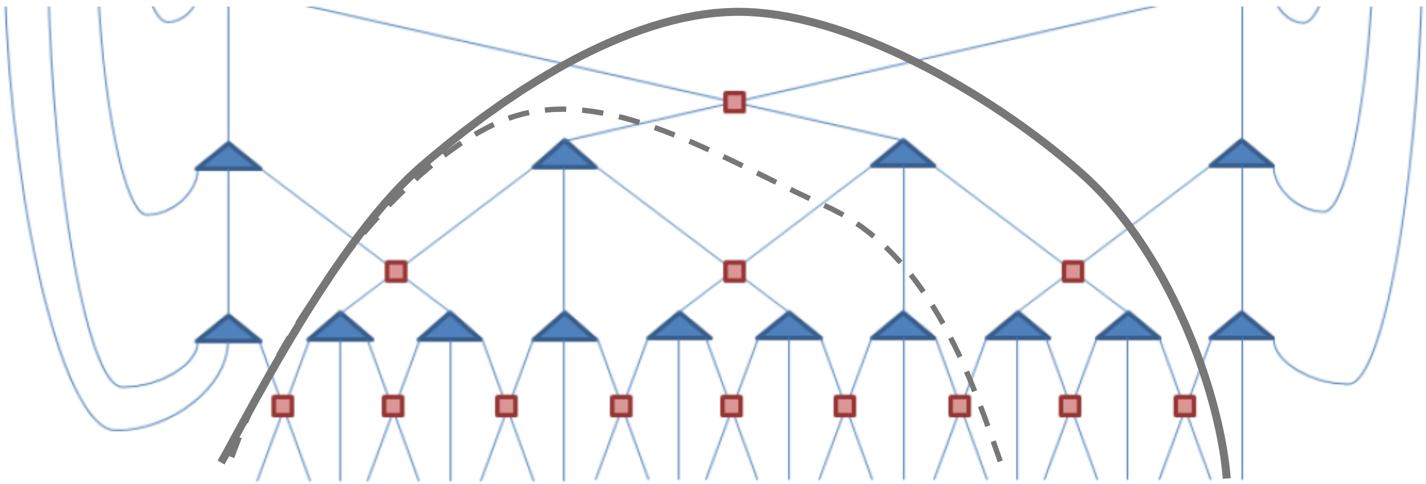
`GEODESIC' IN ADS SPACETIME (SWINGLE)



Brian Swingle, Entanglement Renormalization and Holography (2009)

$\rho_{red,L} = \mathrm{tr}(\rho)$

LOG(L) SCALING



Important: MERA has $S_{EE} \lesssim \log(\chi)\log(L)$

Most local Hamiltonians obey this (but $S_{EE} \sim \sqrt{L}$ **possible)**

R. Movassagh, P.W. Shor, Power law violation of the area law in quantum spin chains (2014)

GRAVITY AS AN EMERGENT FORCE, ADS/MERA

Field theory without gravity ⟺ string theory with gravity

Holographic: gravity has one extra dimension, RG scale

→ Propose connection between MERA and gravity

Caveat: gravity `emerges' only for specific field theories
(large N, strong coupling)

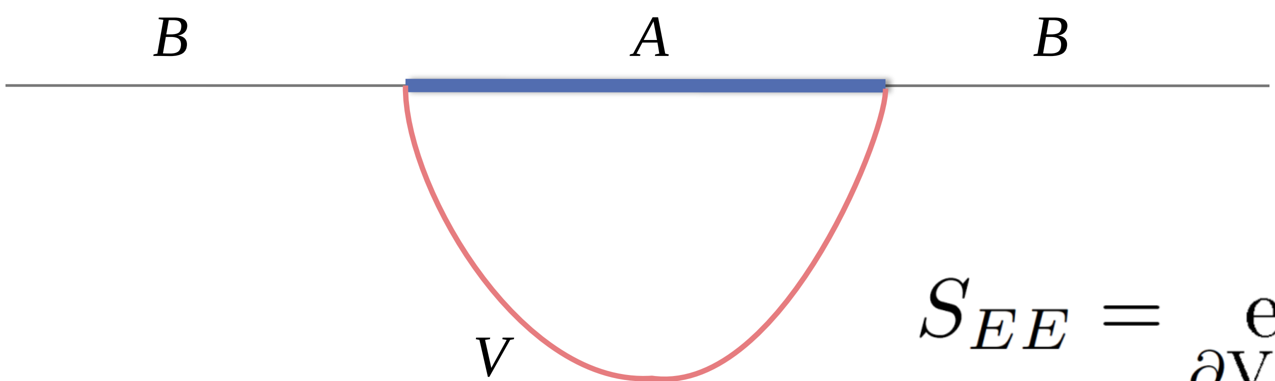
Brian Swingle, Entanglement Renormalization and Holography (2009)

ENTANGLEMENT = GEOMETRY (PICTURE)

Entanglement: trace out part of space → mixed state (entropy!)

Remarkable statement (Ryu+Takayanagi):

entanglement entropy = area extremal surface in AdS

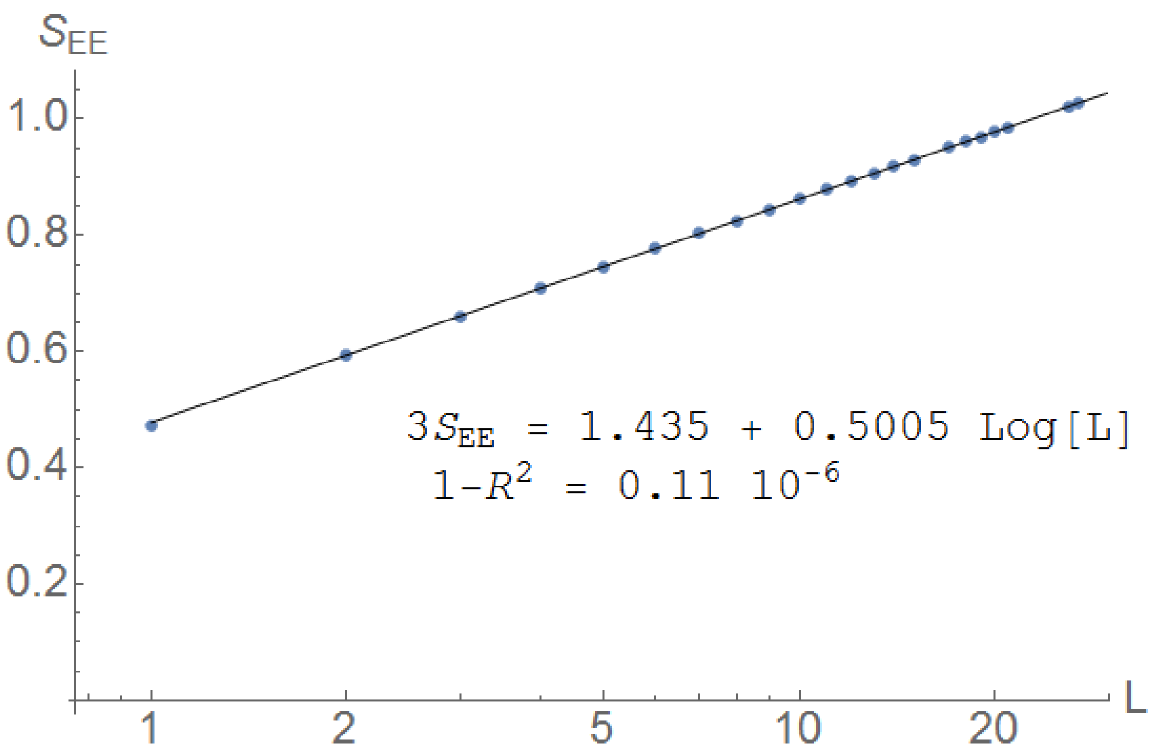


$$S_{EE} = \underset{\partial V = \partial A}{\text{ext}} \frac{A_V}{4G_N}$$

S. Ryu, T. Takayanagi, Holographic derivation of entanglement entropy from the anti–de sitter space/conformal field theory correspondence (2006

EQUIVALENT REDUCED DENSITY MATRIX

Resulting entanglement entropies:

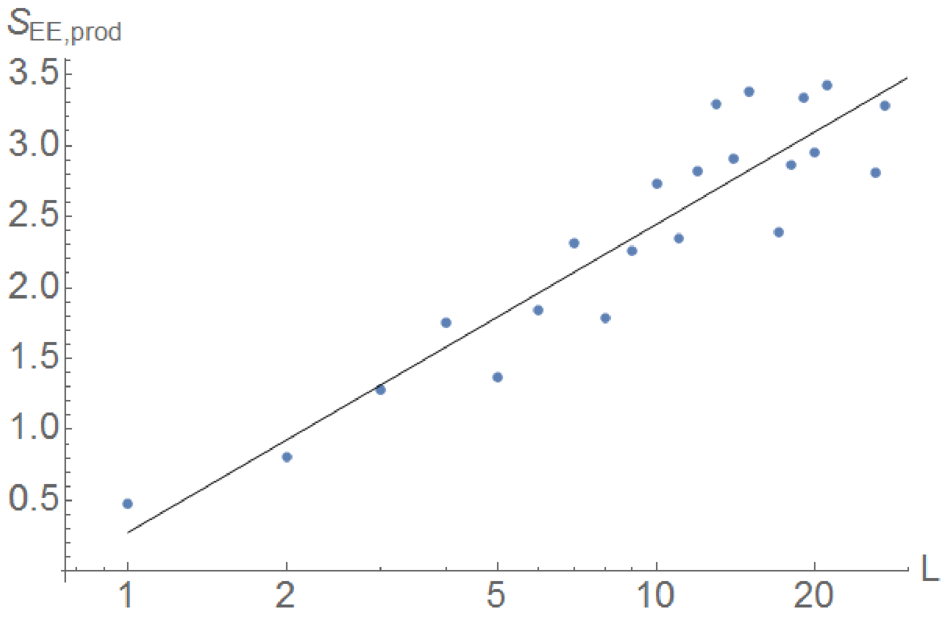


QUESTION: WHAT GIVES A GEOMETRIC PICTURE?

Reduced density matrix gives a local `slice' in `AdS'

Hard to formalise: legs do not in general decouple

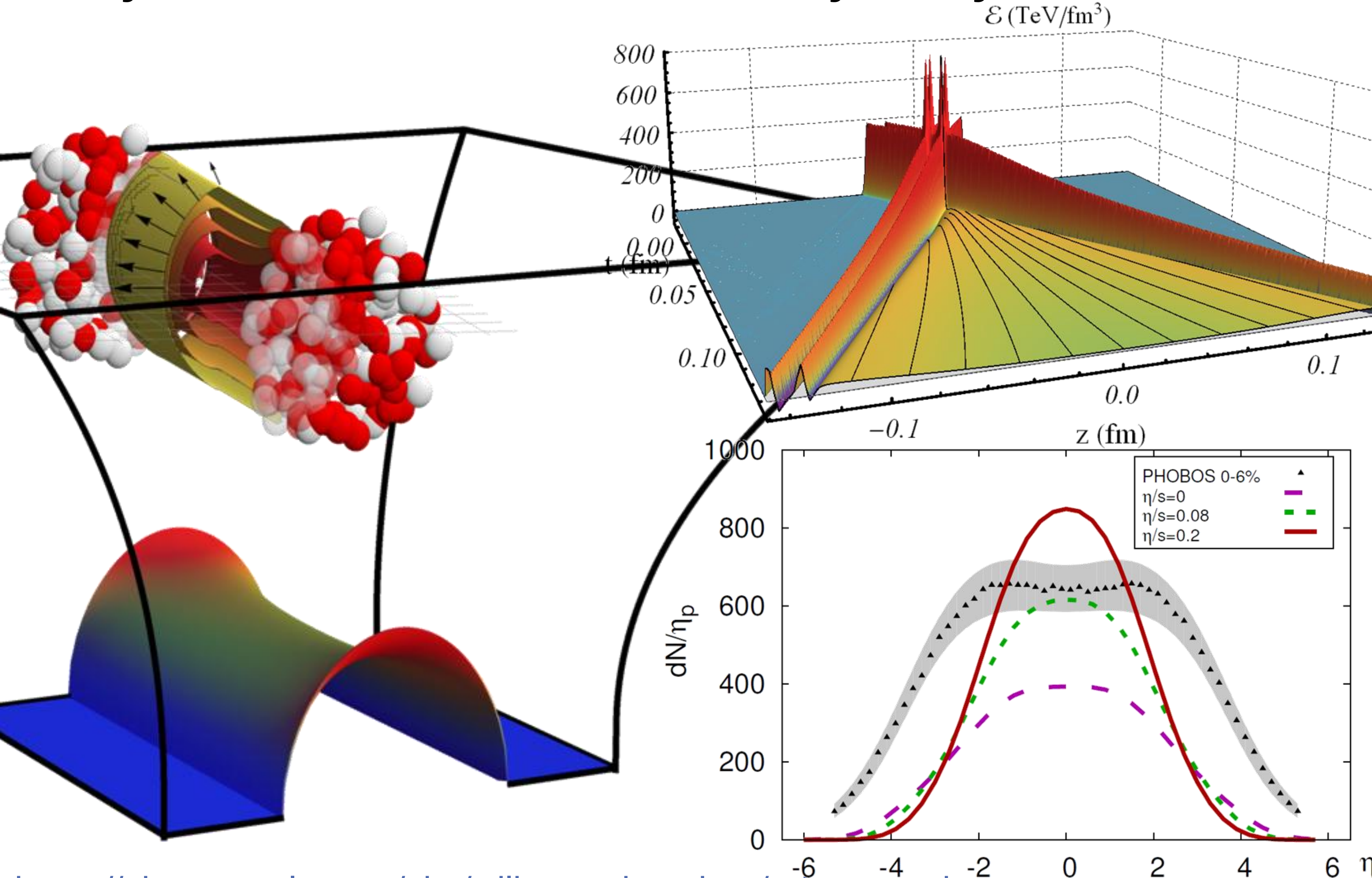
- Do they decouple with large *c*?
- If so, then entanglement entropy = sum over entropy/leg
- What about dual of Ising model?



$3S_{EE} = 0.8247 + 2.828\ \mathrm{Log}[L]$
$R^2 = 0.69$

SOME OTHER WORK

Dynamics in Anti-de Sitter to study heavy ion collisions



https://sites.google.com/site/wilkevanderschee/ads-numerics

REFERENCES FOR LEARNING NUMERICS

Characteristic formulation:

- Chesler-Yaffe: 1309.1439
- Casalderrey, Heller, Mateos, WS, Triana: 1407.1849, 1304.5172
- Balasubramanian, Herzog: 1312.4953

ADM formulation

- Heller, Janik, Witaszczyk : 1203.0755
- Bantilan, Gubser, Pretorius: 1201.2132

Elliptic Einstein-DeTurck: Donos, Gauntlett: 1409.6875

Reviews/books:

- Grandclément, Novak: livingreviews.org (see also Winicour)
- Casalderrey, Liu, Matoes, Rajagopal, Wiedemann: 1101.0618 (and book!)
- Boyd: Chebyshev and Fourier spectral methods