A Supersymmetry Primer

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I provide a pedagogical introduction to supersymmetry. The level of discussion is aimed at readers who are familiar with the Standard Model and quantum field theory, but who have had little or no prior exposure to supersymmetry. Topics covered include: motivations for supersymmetry, the construction of supersymmetric Lagrangians, supersymmetry-breaking interactions, the Minimal Supersymmetric Standard Model (MSSM), R-parity and its consequences, the origins of supersymmetry breaking, the mass spectrum of the MSSM, decays of supersymmetric particles, experimental signals for supersymmetry, and some extensions of the minimal framework.

1.1. Introduction

The Standard Model of high-energy physics, augmented by neutrino masses, provides a remarkably successful description of presently known phenomena. The experimental frontier has advanced into the TeV range with no unambiguous hints of additional structure. Still, it seems clear that the Standard Model is a work in progress and will have to be extended to describe physics at higher energies. Certainly, a new framework will be required at the reduced Planck scale $M_{\rm P} = (8\pi G_{\rm Newton})^{-1/2} = 2.4 \times 10^{18}$ GeV, where quantum gravitational effects become important. Based only on a proper respect for the power of Nature to surprise us, it seems nearly as obvious that new physics exists in the 16 orders of magnitude in energy between the presently explored territory near the electroweak scale, M_W, and the Planck scale.

The mere fact that the ratio $M_{\rm P}/M_W$ is so huge is already a powerful clue to the character of physics beyond the Standard Model, because of the infamous "hierarchy problem".[1] This is not really a difficulty with the Standard Model itself, but rather a disturbing sensitivity of the Higgs potential to new physics in almost any imaginable extension of the Standard Model. The electrically neutral part of the Standard Model Higgs field is a complex scalar H with a classical potential

$$V = m_H^2 |H|^2 + \lambda |H|^4 \,. \tag{1.1}$$

The Standard Model requires a non-vanishing vacuum expectation value (VEV) for H at the minimum of the potential. This will occur if $\lambda > 0$ and $m_H^2 < 0$, resulting in $\langle H \rangle = \sqrt{-m_H^2/2\lambda}$. Since we know experimentally that $\langle H \rangle$ is approximately 174 GeV, from measurements of the properties of the weak interactions, it must be that m_H^2 is very roughly of order $-(100$ GeV$)^2$. The problem is that m_H^2 receives enormous quantum corrections from the virtual effects of every particle that couples, directly or indirectly, to the Higgs field.

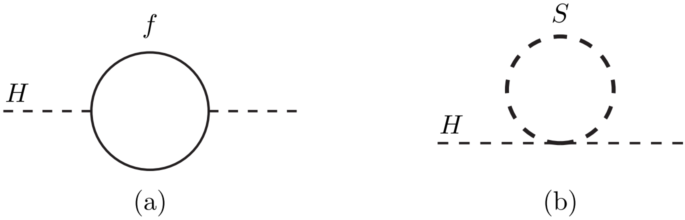

Fig. 1.1. One-loop quantum corrections to the Higgs squared mass parameter m_H^2, due to (a) a Dirac fermion f, and (b) a scalar S.

For example, in Figure 1.1a we have a correction to m_H^2 from a loop containing a Dirac fermion f with mass m_f. If the Higgs field couples to f with a term in the Lagrangian $-\lambda_f H \overline{f} f$, then the Feynman diagram in Figure 1.1a yields a correction

$$\Delta m_H^2 \;=\; -\frac{|\lambda_f|^2}{8\pi^2}\Lambda_{\rm UV}^2 + \ldots . \tag{1.2}$$

Here $\Lambda_{\rm UV}$ is an ultraviolet momentum cutoff used to regulate the loop integral; it should be interpreted as at least the energy scale at which new physics enters to alter the high-energy behavior of the theory. The ellipses represent terms proportional to m_f^2, which grow at most logarithmically with $\Lambda_{\rm UV}$ (and actually differ for the real and imaginary parts of H). Each of the leptons and quarks of the Standard Model can play the role of f;

for quarks, eq. (1.2) should be multiplied by 3 to account for color. The largest correction comes when f is the top quark with $\lambda_f \approx 1$. The problem is that if $\Lambda_{\rm UV}$ is of order $M_{\rm P}$, say, then this quantum correction to m_H^2 is some 30 orders of magnitude larger than the required value of $m_H^2 \sim -(100$ GeV$)^2$. This is only directly a problem for corrections to the Higgs scalar boson squared mass, because quantum corrections to fermion and gauge boson masses do not have the direct quadratic sensitivity to $\Lambda_{\rm UV}$ found in eq. (1.2). However, the quarks and leptons and the electroweak gauge bosons Z^0, $W^\pm$ of the Standard Model all obtain masses from $\langle H \rangle$, so that the entire mass spectrum of the Standard Model is directly or indirectly sensitive to the cutoff $\Lambda_{\rm UV}$.

One could imagine that the solution is to simply pick a $\Lambda_{\rm UV}$ that is not too large. But then one still must concoct some new physics at the scale $\Lambda_{\rm UV}$ that not only alters the propagators in the loop, but actually cuts off the loop integral. This is not easy to do in a theory whose Lagrangian does not contain more than two derivatives, and higher-derivative theories generally suffer from a failure of either unitarity or causality.[2] In string theories, loop integrals are nevertheless cut off at high Euclidean momentum p by factors $e^{-p^2/\Lambda_{\rm UV}^2}$. However, then $\Lambda_{\rm UV}$ is a string scale that is usually[a] thought to be not very far below $M_{\rm P}$. Furthermore, there are contributions similar to eq. (1.2) from the virtual effects of any arbitrarily heavy particles that might exist, and these involve the masses of the heavy particles, not just the cutoff.

For example, suppose there exists a heavy complex scalar particle S with mass m_S that couples to the Higgs with a Lagrangian term $-\lambda_S |H|^2 |S|^2$. Then the Feynman diagram in Figure 1.1b gives a correction

$$\Delta m_H^2 \;=\; \frac{\lambda_S}{16\pi^2}\left[\Lambda_{\rm UV}^2 - 2m_S^2 \ln(\Lambda_{\rm UV}/m_S) + \ldots\right]. \tag{1.3}$$

If one rejects the possibility of a physical interpretation of $\Lambda_{\rm UV}$ and uses dimensional regularization on the loop integral instead of a momentum cutoff, then there will be no $\Lambda_{\rm UV}^2$ piece. However, even then the term proportional to m_S^2 cannot be eliminated without the physically unjustifiable tuning of a counter-term specifically for that purpose. So m_H^2 is sensitive to the masses of the *heaviest* particles that H couples to; if m_S is very large, its effects on the Standard Model do not decouple, but instead make it difficult to understand why m_H^2 is so small.

[a]Some recent attacks on the hierarchy problem, not reviewed here, are based on the proposition that the ultimate cutoff scale is actually close to the electroweak scale, rather than the apparent Planck scale.

This problem arises even if there is no direct coupling between the Standard Model Higgs boson and the unknown heavy particles. For example, suppose there exists a heavy fermion F that, unlike the quarks and leptons of the Standard Model, has vector-like quantum numbers and therefore gets a large mass m_F without coupling to the Higgs field. [In other words, an arbitrarily large mass term of the form $m_F\overline{F}F$ is not forbidden by any symmetry, including weak isospin $SU(2)_L$.] In that case, no diagram like Figure 1.1a exists for F. Nevertheless there will be a correction to m_H^2 as long as F shares some gauge interactions with the Standard Model Higgs field; these may be the familiar electroweak interactions, or some unknown gauge forces that are broken at a very high energy scale inaccessible to experiment. In either case, the two-loop Feynman diagrams in Figure 1.2 yield a correction

$$\Delta m_H^2 \;=\; C_H T_F \left(\frac{g^2}{16\pi^2}\right)^2 \left[a\Lambda_{\rm UV}^2 + 24 m_F^2 \ln(\Lambda_{\rm UV}/m_F) + \ldots\right], \quad (1.4)$$

where C_H and T_F are group theory factors[b] of order 1, and g is the appropriate gauge coupling. The coefficient a depends on the method used to cut off the momentum integrals. It does not arise at all if one uses dimensional regularization, but the m_F^2 contribution is always present with the given coefficient. The numerical factor $(g^2/16\pi^2)^2$ may be quite small (of order 10^{-5} for electroweak interactions), but the important point is that these contributions to Δm_H^2 are sensitive both to the largest masses and to the ultraviolet cutoff in the theory, presumably of order $M_{\rm P}$. The "natural" squared mass of a fundamental Higgs scalar, including quantum corrections, therefore seems to be more like $M_{\rm P}^2$ than the experimentally favored value! Even very indirect contributions from Feynman diagrams with three

Fig. 1.2. Two-loop corrections to the Higgs squared mass parameter involving a heavy fermion F that couples only indirectly to the Standard Model Higgs through gauge interactions.

[b]Specifically, C_H is the quadratic Casimir invariant of H, and T_F is the Dynkin index of F in a normalization such that $T_F = 1$ for a Dirac fermion (or two Weyl fermions) in a fundamental representation of $SU(n)$.

or more loops can give unacceptably large contributions to Δm_H^2. The argument above applies not just for heavy particles, but for arbitrary high-scale physical phenomena such as condensates or additional compactified dimensions.

It could be that there is no fundamental Higgs boson, as in technicolor models, top-quark condensate models, and models in which the Higgs boson is composite. Or it could be that the ultimate ultraviolet cutoff scale is much lower than the Planck scale. These ideas are certainly worth exploring, although they often present difficulties in their simplest forms. But, if the Higgs boson is a fundamental particle, and there really is physics far above the electroweak scale, then we have two remaining options: either we must make the rather bizarre assumption that there do not exist *any* high-mass particles or effects that couple (even indirectly or extremely weakly) to the Higgs scalar field, or else some striking cancellation is needed between the various contributions to Δm_H^2.

The systematic cancellation of the dangerous contributions to Δm_H^2 can only be brought about by the type of conspiracy that is better known to physicists as a symmetry. Comparing eqs. (1.2) and (1.3) strongly suggests that the new symmetry ought to relate fermions and bosons, because of the relative minus sign between fermion loop and boson loop contributions to Δm_H^2. (Note that λ_S must be positive if the scalar potential is to be bounded from below.) If each of the quarks and leptons of the Standard Model is accompanied by two complex scalars with $\lambda_S = |\lambda_f|^2$, then the $\Lambda_{\rm UV}^2$ contributions of Figures 1.1a and 1.1b will neatly cancel.[3] Clearly, more restrictions on the theory will be necessary to ensure that this success persists to higher orders, so that, for example, the contributions in Figure 1.2 and eq. (1.4) from a very heavy fermion are canceled by the two-loop effects of some very heavy bosons. Fortunately, the cancellation of all such contributions to scalar masses is not only possible, but is actually unavoidable, once we merely assume that there exists a symmetry relating fermions and bosons, called a *supersymmetry*.

A supersymmetry transformation turns a bosonic state into a fermionic state, and vice versa. The operator Q that generates such transformations must be an anticommuting spinor, with

$$Q|\text{Boson}\rangle = |\text{Fermion}\rangle, \qquad\qquad Q|\text{Fermion}\rangle = |\text{Boson}\rangle. \tag{1.5}$$

Spinors are intrinsically complex objects, so $Q^\dagger$ (the hermitian conjugate of Q) is also a symmetry generator. Because Q and $Q^\dagger$ are fermionic operators, they carry spin angular momentum 1/2, so it is clear that supersymmetry must be a spacetime symmetry. The possible forms for such symmetries in an interacting quantum field theory are highly restricted by the Haag-Lopuszanski-Sohnius extension of the Coleman-Mandula theorem.[4] For realistic theories that, like the Standard Model, have chiral fermions (i.e., fermions whose left- and right-handed pieces transform differently under the gauge group) and thus the possibility of parity-violating interactions, this theorem implies that the generators Q and $Q^\dagger$ must satisfy an algebra of anticommutation and commutation relations with the schematic form

$$\{Q, Q^\dagger\} = P^\mu, \tag{1.6}$$

$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0, \tag{1.7}$$

$$[P^\mu, Q] = [P^\mu, Q^\dagger] = 0, \tag{1.8}$$

where P^μ is the four-momentum generator of spacetime translations. Here we have ruthlessly suppressed the spinor indices on Q and $Q^\dagger$; after developing some notation we will, in section 1.3.1, derive the precise version of eqs. (1.6)-(1.8) with indices restored. In the meantime, we simply note that the appearance of P^μ on the right-hand side of eq. (1.6) is unsurprising, since it transforms under Lorentz boosts and rotations as a spin-1 object while Q and $Q^\dagger$ on the left-hand side each transform as spin-1/2 objects.

The single-particle states of a supersymmetric theory fall into irreducible representations of the supersymmetry algebra, called *supermultiplets*. Each supermultiplet contains both fermion and boson states, which are commonly known as *superpartners* of each other. By definition, if $|\Omega\rangle$ and $|\Omega'\rangle$ are members of the same supermultiplet, then $|\Omega'\rangle$ is proportional to some combination of Q and $Q^\dagger$ operators acting on $|\Omega\rangle$, up to a spacetime translation or rotation. The squared-mass operator $-P^2$ commutes with the operators Q, $Q^\dagger$, and with all spacetime rotation and translation operators, so it follows immediately that particles inhabiting the same irreducible supermultiplet must have equal eigenvalues of $-P^2$, and therefore equal masses.

The supersymmetry generators $Q, Q^\dagger$ also commute with the generators of gauge transformations. Therefore particles in the same supermultiplet must also be in the same representation of the gauge group, and so must have the same electric charges, weak isospin, and color degrees of freedom.

Each supermultiplet contains an equal number of fermion and boson degrees of freedom. To prove this, consider the operator $(-1)^{2s}$ where s is the spin angular momentum. By the spin-statistics theorem, this operator has eigenvalue $+1$ acting on a bosonic state and eigenvalue -1 acting on a fermionic state. Any fermionic operator will turn a bosonic state into a fermionic state and vice versa. Therefore $(-1)^{2s}$ must anticommute with every fermionic operator in the theory, and in particular with Q and $Q^\dagger$. Now, within a given supermultiplet, consider the subspace of states $|i\rangle$ with the same eigenvalue p^μ of the four-momentum operator P^μ. In view of eq. (1.8), any combination of Q or $Q^\dagger$ acting on $|i\rangle$ must give another state $|i'\rangle$ with the same four-momentum eigenvalue. Therefore one has a completeness relation $\sum_i |i\rangle\langle i| = 1$ within this subspace of states. Now one can take a trace over all such states of the operator $(-1)^{2s}P^\mu$ (including each spin helicity state separately):

$$\begin{aligned}
\sum_i \langle i|(-1)^{2s}P^\mu|i\rangle &= \sum_i \langle i|(-1)^{2s}QQ^\dagger|i\rangle + \sum_i \langle i|(-1)^{2s}Q^\dagger Q|i\rangle \\
&= \sum_i \langle i|(-1)^{2s}QQ^\dagger|i\rangle + \sum_i\sum_j \langle i|(-1)^{2s}Q^\dagger|j\rangle\langle j|Q|i\rangle \\
&= \sum_i \langle i|(-1)^{2s}QQ^\dagger|i\rangle + \sum_j \langle j|Q(-1)^{2s}Q^\dagger|j\rangle \\
&= \sum_i \langle i|(-1)^{2s}QQ^\dagger|i\rangle - \sum_j \langle j|(-1)^{2s}QQ^\dagger|j\rangle \\
&= 0. \qquad (1.9)
\end{aligned}$$

The first equality follows from the supersymmetry algebra relation eq. (1.6); the second and third from use of the completeness relation; and the fourth from the fact that $(-1)^{2s}$ must anticommute with Q. Now $\sum_i\langle i|(-1)^{2s}P^\mu|i\rangle = p^\mu \ \mathrm{Tr}[(-1)^{2s}]$ is just proportional to the number of bosonic degrees of freedom n_B minus the number of fermionic degrees of freedom n_F in the trace, so that

$$n_B = n_F \qquad (1.10)$$

must hold for a given $p^\mu \neq 0$ in each supermultiplet.

The simplest possibility for a supermultiplet consistent with eq. (1.10) has a single Weyl fermion (with two spin helicity states, so $n_F = 2$) and two real scalars (each with $n_B = 1$). It is natural to assemble the two real scalar degrees of freedom into a complex scalar field; as we will see below this provides for convenient formulations of the supersymmetry algebra, Feynman rules, supersymmetry-violating effects, etc. This combination of a two-component Weyl fermion and a complex scalar field is called a *chiral* or *matter* or *scalar* supermultiplet.

The next-simplest possibility for a supermultiplet contains a spin-1 vector boson. If the theory is to be renormalizable, this must be a gauge boson that is massless, at least before the gauge symmetry is spontaneously broken. A massless spin-1 boson has two helicity states, so the number of bosonic degrees of freedom is $n_B = 2$. Its superpartner is therefore a massless spin-1/2 Weyl fermion, again with two helicity states, so $n_F = 2$. (If one tried to use a massless spin-3/2 fermion instead, the theory would not be renormalizable.) Gauge bosons must transform as the adjoint representation of the gauge group, so their fermionic partners, called *gauginos*, must also. Since the adjoint representation of a gauge group is always its own conjugate, the gaugino fermions must have the same gauge transformation properties for left-handed and for right-handed components. Such a combination of spin-1/2 gauginos and spin-1 gauge bosons is called a *gauge* or *vector* supermultiplet.

If we include gravity, then the spin-2 graviton (with 2 helicity states, so $n_B = 2$) has a spin-3/2 superpartner called the gravitino. The gravitino would be massless if supersymmetry were unbroken, and so it has $n_F = 2$ helicity states.

There are other possible combinations of particles with spins that can satisfy eq. (1.10). However, these are always reducible to combinations[c] of chiral and gauge supermultiplets if they have renormalizable interactions, except in certain theories with "extended" supersymmetry. Theories with extended supersymmetry have more than one distinct copy of the supersymmetry generators $Q, Q^\dagger$. Such models are mathematically amusing, but evidently do not have any phenomenological prospects. The reason is that extended supersymmetry in four-dimensional field theories cannot allow for chiral fermions or parity violation as observed in the Standard Model. So we will not discuss such possibilities further, although extended supersymmetry in higher-dimensional field theories might describe the real world if the extra dimensions are compactified in an appropriate way, and extended supersymmetry in four dimensions provides interesting toy models. The ordinary, non-extended, phenomenologically viable type of supersymmetric model is sometimes called $N = 1$ supersymmetry, with N referring to the number of supersymmetries (the number of distinct copies of $Q, Q^\dagger$).

In a supersymmetric extension of the Standard Model,[5–7] each of the known fundamental particles is therefore in either a chiral or gauge super-

[c]For example, if a gauge symmetry were to spontaneously break without breaking supersymmetry, then a massless vector supermultiplet would "eat" a chiral supermultiplet, resulting in a massive vector supermultiplet with physical degrees of freedom consisting of a massive vector ($n_B = 3$), a massive Dirac fermion formed from the gaugino and the chiral fermion ($n_F = 4$), and a real scalar ($n_B = 1$).

multiplet, and must have a superpartner with spin differing by 1/2 unit. The first step in understanding the exciting phenomenological consequences of this prediction is to decide exactly how the known particles fit into supermultiplets, and to give them appropriate names. A crucial observation here is that only chiral supermultiplets can contain fermions whose left-handed parts transform differently under the gauge group than their right-handed parts. All of the Standard Model fermions (the known quarks and leptons) have this property, so they must be members of chiral supermultiplets.[d] The names for the spin-0 partners of the quarks and leptons are constructed by prepending an "s", for scalar. So, generically they are called *squarks* and *sleptons* (short for "scalar quark" and "scalar lepton"), or sometimes *sfermions.* The left-handed and right-handed pieces of the quarks and leptons are separate two-component Weyl fermions with different gauge transformation properties in the Standard Model, so each must have its own complex scalar partner. The symbols for the squarks and sleptons are the same as for the corresponding fermion, but with a tilde ($\widetilde{\ }$) used to denote the superpartner of a Standard Model particle. For example, the superpartners of the left-handed and right-handed parts of the electron Dirac field are called left- and right-handed selectrons, and are denoted $\widetilde{e}_L$ and $\widetilde{e}_R$. It is important to keep in mind that the "handedness" here does not refer to the helicity of the selectrons (they are spin-0 particles) but to that of their superpartners. A similar nomenclature applies for smuons and staus: $\widetilde{\mu}_L$, $\widetilde{\mu}_R$, $\widetilde{\tau}_L$, $\widetilde{\tau}_R$. The Standard Model neutrinos (neglecting their very small masses) are always left-handed, so the sneutrinos are denoted generically by $\widetilde{\nu}$, with a possible subscript indicating which lepton flavor they carry: $\widetilde{\nu}_e$, $\widetilde{\nu}_\mu$, $\widetilde{\nu}_\tau$. Finally, a complete list of the squarks is $\widetilde{q}_L$, $\widetilde{q}_R$ with $q = u, d, s, c, b, t$. The gauge interactions of each of these squark and slepton fields are the same as for the corresponding Standard Model fermions; for instance, the left-handed squarks $\widetilde{u}_L$ and $\widetilde{d}_L$ couple to the W boson, while $\widetilde{u}_R$ and $\widetilde{d}_R$ do not.

It seems clear that the Higgs scalar boson must reside in a chiral supermultiplet, since it has spin 0. Actually, it turns out that just one chiral supermultiplet is not enough. One reason for this is that if there were only one Higgs chiral supermultiplet, the electroweak gauge symmetry would suffer a gauge anomaly, and would be inconsistent as a quantum theory. This is because the conditions for cancellation of gauge anomalies include

[d]In particular, one cannot attempt to make a spin-1/2 neutrino be the superpartner of the spin-1 photon; the neutrino is in a doublet, and the photon is neutral, under weak isospin.

$\mathrm{Tr}[T_3^2 Y] = \mathrm{Tr}[Y^3] = 0$, where T_3 and Y are the third component of weak isospin and the weak hypercharge, respectively, in a normalization where the ordinary electric charge is $Q_{\rm EM} = T_3 + Y$. The traces run over all of the left-handed Weyl fermionic degrees of freedom in the theory. In the Standard Model, these conditions are already satisfied, somewhat miraculously, by the known quarks and leptons. Now, a fermionic partner of a Higgs chiral supermultiplet must be a weak isodoublet with weak hypercharge $Y = 1/2$ or $Y = -1/2$. In either case alone, such a fermion will make a non-zero contribution to the traces and spoil the anomaly cancellation. This can be avoided if there are two Higgs supermultiplets, one with each of $Y = \pm 1/2$, so that the total contribution to the anomaly traces from the two fermionic members of the Higgs chiral supermultiplets vanishes by cancellation. As we will see in section 1.5.1, both of these are also necessary for another completely different reason: because of the structure of supersymmetric theories, only a $Y = 1/2$ Higgs chiral supermultiplet can have the Yukawa couplings necessary to give masses to charge $+2/3$ up-type quarks (up, charm, top), and only a $Y = -1/2$ Higgs can have the Yukawa couplings necessary to give masses to charge $-1/3$ down-type quarks (down, strange, bottom) and to the charged leptons. We will call the $SU(2)_L$-doublet complex scalar fields with $Y = 1/2$ and $Y = -1/2$ by the names H_u and H_d, respectively.[e] The weak isospin components of H_u with $T_3 = (1/2, -1/2)$ have electric charges 1, 0 respectively, and are denoted (H_u^+, H_u^0). Similarly, the $SU(2)_L$-doublet complex scalar H_d has $T_3 = (1/2, -1/2)$ components (H_d^0, H_d^-). The neutral scalar that corresponds to the physical Standard Model Higgs boson is in a linear combination of H_u^0 and H_d^0; we will discuss this further in section 1.7.1. The generic nomenclature for a spin-1/2 superpartner is to append "-ino" to the name of the Standard Model particle, so the fermionic partners of the Higgs scalars are called higgsinos. They are denoted by $\widetilde{H}_u$, $\widetilde{H}_d$ for the $SU(2)_L$-doublet left-handed Weyl spinor fields, with weak isospin components $\widetilde{H}_u^+$, $\widetilde{H}_u^0$ and $\widetilde{H}_d^0$, $\widetilde{H}_d^-$.

We have now found all of the chiral supermultiplets of a minimal phenomenologically viable extension of the Standard Model. They are summarized in Table 1.1, classified according to their transformation properties under the Standard Model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, which combines u_L, d_L and ν, e_L degrees of freedom into $SU(2)_L$ doublets. Here we follow a standard convention, that all chiral supermultiplets are defined

[e] Other notations in the literature have H_1, H_2 or $H, \overline{H}$ instead of H_u, H_d. The notation used here has the virtue of making it easy to remember which Higgs VEVs gives masses to which type of quarks.

Table 1.1. Chiral supermultiplets in the Minimal Supersymmetric Standard Model. The spin-0 fields are complex scalars, and the spin-1/2 fields are left-handed two-component Weyl fermions.

Names		spin 0	spin 1/2	$SU(3)_C$, $SU(2)_L$, $U(1)_Y$
squarks, quarks	Q	$(\widetilde{u}_L \ \ \widetilde{d}_L)$	$(u_L \ \ d_L)$	$(\,\mathbf{3},\ \mathbf{2}\,,\ \frac{1}{6})$
($\times 3$ families)	$\overline{u}$	$\widetilde{u}^*_R$	$u^\dagger_R$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	$\overline{d}$	$\widetilde{d}^*_R$	$d^\dagger_R$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons, leptons	L	$(\widetilde{\nu}\ \widetilde{e}_L)$	$(\nu \ \ e_L)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$
($\times 3$ families)	$\overline{e}$	$\widetilde{e}^*_R$	$e^\dagger_R$	$(\,\mathbf{1},\ \mathbf{1},\ 1)$
Higgs, higgsinos	H_u	$(H^+_u \ \ H^0_u)$	$(\widetilde{H}^+_u \ \ \widetilde{H}^0_u)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ +\frac{1}{2})$
	H_d	$(H^0_d \ \ H^-_d)$	$(\widetilde{H}^0_d \ \ \widetilde{H}^-_d)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$

in terms of left-handed Weyl spinors, so that the *conjugates* of the right-handed quarks and leptons (and their superpartners) appear in Table 1.1. This protocol for defining chiral supermultiplets turns out to be very useful for constructing supersymmetric Lagrangians, as we will see in section 1.3. It is also useful to have a symbol for each of the chiral supermultiplets as a whole; these are indicated in the second column of Table 1.1. Thus, for example, Q stands for the $SU(2)_L$-doublet chiral supermultiplet containing $\widetilde{u}_L, u_L$ (with weak isospin component $T_3 = 1/2$), and $\widetilde{d}_L, d_L$ (with $T_3 = -1/2$), while $\overline{u}$ stands for the $SU(2)_L$-singlet supermultiplet containing $\widetilde{u}^*_R, u^\dagger_R$. There are three families for each of the quark and lepton supermultiplets, Table 1.1 lists the first-family representatives. A family index $i = 1, 2, 3$ can be affixed to the chiral supermultiplet names (Q_i, $\overline{u}_i, \ldots$) when needed, for example $(\overline{e}_1, \overline{e}_2, \overline{e}_3) = (\overline{e}, \overline{\mu}, \overline{\tau})$. The bar on $\overline{u}$, $\overline{d}$, $\overline{e}$ fields is part of the name, and does not denote any kind of conjugation.

The Higgs chiral supermultiplet H_d (containing H^0_d, H^-_d, $\widetilde{H}^0_d$, $\widetilde{H}^-_d$) has exactly the same Standard Model gauge quantum numbers as the left-handed sleptons and leptons L_i, for example ($\widetilde{\nu}$, $\widetilde{e}_L$, ν, e_L). Naively, one might therefore suppose that we could have been more economical in our assignment by taking a neutrino and a Higgs scalar to be superpartners, instead of putting them in separate supermultiplets. This would amount to the proposal that the Higgs boson and a sneutrino should be the same particle. This attempt played a key role in some of the first attempts to connect supersymmetry to phenomenology,[5] but it is now known to not work. Even ignoring the anomaly cancellation problem mentioned above, many insoluble phenomenological problems would result, including lepton-number non-conservation and a mass for at least one of the neutrinos in

Table 1.2. Gauge supermultiplets in the Minimal Supersymmetric Standard Model.

Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\,\mathbf{8},\ \mathbf{1}\,,\ 0)$
winos, W bosons	$\widetilde{W}^{\pm}\ \ \widetilde{W}^{0}$	$W^{\pm}\ \ W^{0}$	$(\,\mathbf{1},\ \mathbf{3}\,,\ 0)$
bino, B boson	$\widetilde{B}^{0}$	B^{0}	$(\,\mathbf{1},\ \mathbf{1}\,,\ 0)$

gross violation of experimental bounds. Therefore, all of the superpartners of Standard Model particles are really new particles, and cannot be identified with some other Standard Model state.

The vector bosons of the Standard Model clearly must reside in gauge supermultiplets. Their fermionic superpartners are generically referred to as gauginos. The $SU(3)_C$ color gauge interactions of QCD are mediated by the gluon, whose spin-1/2 color-octet supersymmetric partner is the gluino. As usual, a tilde is used to denote the supersymmetric partner of a Standard Model state, so the symbols for the gluon and gluino are g and $\widetilde{g}$ respectively. The electroweak gauge symmetry $SU(2)_L \times U(1)_Y$ is associated with spin-1 gauge bosons W^+, W^0, W^- and B^0, with spin-1/2 superpartners $\widetilde{W}^+, \widetilde{W}^0, \widetilde{W}^-$ and $\widetilde{B}^0$, called *winos* and *bino*. After electroweak symmetry breaking, the W^0, B^0 gauge eigenstates mix to give mass eigenstates Z^0 and γ. The corresponding gaugino mixtures of $\widetilde{W}^0$ and $\widetilde{B}^0$ are called zino ($\widetilde{Z}^0$) and photino ($\widetilde{\gamma}$); if supersymmetry were unbroken, they would be mass eigenstates with masses m_Z and 0. Table 1.2 summarizes the gauge supermultiplets of a minimal supersymmetric extension of the Standard Model.

The chiral and gauge supermultiplets in Tables 1.1 and 1.2 make up the particle content of the Minimal Supersymmetric Standard Model (MSSM). The most obvious and interesting feature of this theory is that none of the superpartners of the Standard Model particles has been discovered as of this writing. If supersymmetry were unbroken, then there would have to be selectrons $\widetilde{e}_L$ and $\widetilde{e}_R$ with masses exactly equal to $m_e = 0.511...$ MeV. A similar statement applies to each of the other sleptons and squarks, and there would also have to be a massless gluino and photino. These particles would have been extraordinarily easy to detect long ago. Clearly, therefore, *supersymmetry is a broken symmetry* in the vacuum state chosen by Nature.

An important clue as to the nature of supersymmetry breaking can be obtained by returning to the motivation provided by the hierarchy problem.

Supersymmetry forced us to introduce two complex scalar fields for each Standard Model Dirac fermion, which is just what is needed to enable a cancellation of the quadratically divergent ($\Lambda_{\rm UV}^2$) pieces of eqs. (1.2) and (1.3). This sort of cancellation also requires that the associated dimensionless couplings should be related (for example $\lambda_S = |\lambda_f|^2$). The necessary relationships between couplings indeed occur in unbroken supersymmetry, as we will see in section 1.3. In fact, unbroken supersymmetry guarantees that the quadratic divergences in scalar squared masses must vanish to all orders in perturbation theory.[f] Now, if broken supersymmetry is still to provide a solution to the hierarchy problem even in the presence of supersymmetry breaking, then the relationships between dimensionless couplings that hold in an unbroken supersymmetric theory must be maintained. Otherwise, there would be quadratically divergent radiative corrections to the Higgs scalar masses of the form

$$\Delta m_H^2 = \frac{1}{8\pi^2}(\lambda_S - |\lambda_f|^2)\Lambda_{\rm UV}^2 + \ldots . \tag{1.11}$$

We are therefore led to consider "soft" supersymmetry breaking. This means that the effective Lagrangian of the MSSM can be written in the form

$$\mathcal{L} = \mathcal{L}_{\rm SUSY} + \mathcal{L}_{\rm soft}, \tag{1.12}$$

where $\mathcal{L}_{\rm SUSY}$ contains all of the gauge and Yukawa interactions and preserves supersymmetry invariance, and $\mathcal{L}_{\rm soft}$ violates supersymmetry but contains only mass terms and coupling parameters with *positive* mass dimension. Without further justification, soft supersymmetry breaking might seem like a rather arbitrary requirement. Fortunately, we will see in section 1.6 that theoretical models for supersymmetry breaking do indeed yield effective Lagrangians with just such terms for $\mathcal{L}_{\rm soft}$. If the largest mass scale associated with the soft terms is denoted $m_{\rm soft}$, then the additional nonsupersymmetric corrections to the Higgs scalar squared mass must vanish in the $m_{\rm soft} \to 0$ limit, so by dimensional analysis they cannot be proportional to $\Lambda_{\rm UV}^2$. More generally, these models maintain the cancellation of quadratically divergent terms in the radiative corrections of all scalar masses, to all orders in perturbation theory. The corrections also cannot

[f] A simple way to understand this is to recall that unbroken supersymmetry requires the degeneracy of scalar and fermion masses. Radiative corrections to fermion masses are known to diverge at most logarithmically in any renormalizable field theory, so the same must be true for scalar masses in unbroken supersymmetry.

go like $\Delta m_H^2 \sim m_{\rm soft}\Lambda_{\rm UV}$, because in general the loop momentum integrals always diverge either quadratically or logarithmically, not linearly, as $\Lambda_{\rm UV} \to \infty$. So they must be of the form

$$\Delta m_H^2 = m_{\rm soft}^2 \left[\frac{\lambda}{16\pi^2} \ln(\Lambda_{\rm UV}/m_{\rm soft}) + \ldots\right]. \tag{1.13}$$

Here λ is schematic for various dimensionless couplings, and the ellipses stand both for terms that are independent of $\Lambda_{\rm UV}$ and for higher loop corrections (which depend on $\Lambda_{\rm UV}$ through powers of logarithms).

Because the mass splittings between the known Standard Model particles and their superpartners are just determined by the parameters $m_{\rm soft}$ appearing in $\mathcal{L}_{\rm soft}$, eq. (1.13) tells us that the superpartner masses cannot be too huge. Otherwise, we would lose our successful cure for the hierarchy problem, since the $m_{\rm soft}^2$ corrections to the Higgs scalar squared mass parameter would be unnaturally large compared to the square of the electroweak breaking scale of 174 GeV. The top and bottom squarks and the winos and bino give especially large contributions to $\Delta m_{H_u}^2$ and $\Delta m_{H_d}^2$, but the gluino mass and all the other squark and slepton masses also feed in indirectly, through radiative corrections to the top and bottom squark masses. Furthermore, in most viable models of supersymmetry breaking that are not unduly contrived, the superpartner masses do not differ from each other by more than about an order of magnitude. Using $\Lambda_{\rm UV} \sim M_{\rm P}$ and $\lambda \sim 1$ in eq. (1.13), one finds that $m_{\rm soft}$, and therefore the masses of at least the lightest few superpartners, should be at the most about 1 TeV or so, in order for the MSSM scalar potential to provide a Higgs VEV resulting in $m_W, m_Z = 80.4, 91.2$ GeV without miraculous cancellations. This is the best reason for the optimism among many theorists that supersymmetry will be discovered at the Fermilab Tevatron or the CERN Large Hadron Collider, and can be studied at a future e^+e^- linear collider.

However, it should be noted that the hierarchy problem was *not* the historical motivation for the development of supersymmetry in the early 1970's. The supersymmetry algebra and supersymmetric field theories were originally concocted independently in various disguises[8–11] bearing little resemblance to the MSSM. It is quite impressive that a theory developed for quite different reasons, including purely aesthetic ones, can later be found to provide a solution for the hierarchy problem.

One might also wonder whether there is any good reason why all of the superpartners of the Standard Model particles should be heavy enough to have avoided discovery so far. There is. All of the particles in the

MSSM that have been found so far have something in common; they would necessarily be massless in the absence of electroweak symmetry breaking. In particular, the masses of the $W^{\pm}, Z^0$ bosons and all quarks and leptons are equal to dimensionless coupling constants times the Higgs VEV ~ 174 GeV, while the photon and gluon are required to be massless by electromagnetic and QCD gauge invariance. Conversely, all of the undiscovered particles in the MSSM have exactly the opposite property; each of them can have a Lagrangian mass term in the absence of electroweak symmetry breaking. For the squarks, sleptons, and Higgs scalars this follows from a general property of complex scalar fields that a mass term $m^2|\phi|^2$ is always allowed by all gauge symmetries. For the higgsinos and gauginos, it follows from the fact that they are fermions in a real representation of the gauge group. So, from the point of view of the MSSM, the discovery of the top quark in 1995 marked a quite natural milestone; the already-discovered particles are precisely those that had to be light, based on the principle of electroweak gauge symmetry. There is a single exception: one neutral Higgs scalar boson should be lighter than about 135 GeV if the minimal version of supersymmetry is correct, for reasons to be discussed in section 1.7.1. In non-minimal models that do not have extreme fine tuning of parameters, and that remain perturbative up to the scale of apparent gauge coupling unification, the lightest Higgs scalar boson can have a mass up to about 150 GeV.

An important feature of the MSSM is that the superpartners listed in Tables 1.1 and 1.2 are not necessarily the mass eigenstates of the theory. This is because after electroweak symmetry breaking and supersymmetry breaking effects are included, there can be mixing between the electroweak gauginos and the higgsinos, and within the various sets of squarks and sleptons and Higgs scalars that have the same electric charge. The lone exception is the gluino, which is a color octet fermion and therefore does not have the appropriate quantum numbers to mix with any other particle. The masses and mixings of the superpartners are obviously of paramount importance to experimentalists. It is perhaps slightly less obvious that these phenomenological issues are all quite directly related to one central question that is also the focus of much of the theoretical work in supersymmetry: "How is supersymmetry broken?" The reason for this is that most of what we do not already know about the MSSM has to do with $\mathcal{L}_{\text{soft}}$. The structure of supersymmetric Lagrangians allows little arbitrariness, as we will see in section 1.3. In fact, all of the dimensionless couplings and all but one mass term in the supersymmetric part of the MSSM Lagrangian correspond directly to parameters in the ordinary Standard Model that have already been measured by experiment. For example, we will find out that the supersymmetric coupling of a gluino to a squark and a quark is determined by the QCD coupling constant α_S. In contrast, the supersymmetry-breaking part of the Lagrangian contains many unknown parameters and, apparently, a considerable amount of arbitrariness. Each of the mass splittings between Standard Model particles and their superpartners correspond to terms in the MSSM Lagrangian that are purely supersymmetry-breaking in their origin and effect. These soft supersymmetry-breaking terms can also introduce a large number of mixing angles and CP-violating phases not found in the Standard Model. Fortunately, as we will see in section 1.5.4, there is already strong evidence that the supersymmetry-breaking terms in the MSSM are actually not arbitrary at all. Furthermore, the additional parameters will be measured and constrained as the superpartners are detected. From a theoretical perspective, the challenge is to explain all of these parameters with a predictive model for supersymmetry breaking.

The rest of the discussion is organized as follows. Section 1.2 provides a list of important notations. In section 1.3, we will learn how to construct Lagrangians for supersymmetric field theories. Soft supersymmetry-breaking couplings are described in section 1.4. In section 1.5, we will apply the preceding general results to the special case of the MSSM, introduce the concept of R-parity, and emphasize the importance of the structure of the soft terms. Section 1.6 outlines some considerations for understanding the origin of supersymmetry breaking, and the consequences of various proposals. In section 1.7, we will study the mass and mixing angle patterns of the new particles predicted by the MSSM. Their decay modes are considered in section 1.8. The discussion will be lacking in historical accuracy or perspective; the reader is encouraged to consult the many outstanding books,[12–24] review articles[25–48] and the reprint volume,[49] which contain a much more consistent guide to the original literature.

1.2. Interlude: Notations and Conventions

This section specifies my notations and conventions. Four-vector indices are represented by letters from the middle of the Greek alphabet $\mu, \nu, \rho, \ldots = 0, 1, 2, 3$. The contravariant four-vector position and momentum of a particle are

$$x^\mu = (t, \vec{x}), \qquad\qquad p^\mu = (E, \vec{p}), \tag{1.14}$$

while the four-vector derivative is

$$\partial_\mu = (\partial/\partial t, \vec{\nabla}). \tag{1.15}$$

The spacetime metric is

$$\eta_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1), \tag{1.16}$$

so that $p^2 = -m^2$ for an on-shell particle of mass m.

It is overwhelmingly convenient to employ two-component Weyl spinor notation for fermions, rather than four-component Dirac or Majorana spinors. The Lagrangian of the Standard Model (and any supersymmetric extension of it) violates parity; each Dirac fermion has left-handed and right-handed parts with completely different electroweak gauge interactions. If one used four-component spinor notation instead, then there would be clumsy left- and right-handed projection operators

$$P_L = (1 - \gamma_5)/2, \qquad\qquad P_R = (1 + \gamma_5)/2 \tag{1.17}$$

all over the place. The two-component Weyl fermion notation has the advantage of treating fermionic degrees of freedom with different gauge quantum numbers separately from the start, as Nature intended for us to do. But an even better reason for using two-component notation here is that in supersymmetric models the minimal building blocks of matter are chiral supermultiplets, each of which contains a single two-component Weyl fermion. Since two-component fermion notation may be unfamiliar to some readers, I now specify my conventions by showing how they correspond to the four-component spinor language. A four-component Dirac fermion Ψ_D with mass M is described by the Lagrangian

$$\mathcal{L}_{\mathrm{Dirac}} = i\overline{\Psi}_D \gamma^\mu \partial_\mu \Psi_D - M\overline{\Psi}_D \Psi_D \,. \tag{1.18}$$

For our purposes it is convenient to use the specific representation of the 4×4 gamma matrices given in 2×2 blocks by

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}, \qquad\qquad \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{1.19}$$

where

$$\begin{aligned} \sigma^0 = \overline{\sigma}^0 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, & \sigma^1 = -\overline{\sigma}^1 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\ \sigma^2 = -\overline{\sigma}^2 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, & \sigma^3 = -\overline{\sigma}^3 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \end{aligned} \tag{1.20}$$

In this representation, a four-component Dirac spinor is written in terms of 2 two-component, complex, anticommuting objects ξ_α and $(\chi^\dagger)^{\dot\alpha} \equiv \chi^{\dagger\dot\alpha}$ with two distinct types of spinor indices $\alpha = 1, 2$ and $\dot\alpha = 1, 2$:

$$\Psi_D = \begin{pmatrix} \xi_\alpha \\ \chi^{\dagger\dot\alpha} \end{pmatrix}. \tag{1.21}$$

It follows that

$$\overline{\Psi}_D = \Psi_D^\dagger \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = (\,\chi^\alpha \quad \xi^\dagger_{\dot\alpha}\,)\ . \tag{1.22}$$

Undotted (dotted) indices from the beginning of the Greek alphabet are used for the first (last) two components of a Dirac spinor. The field ξ is called a "left-handed Weyl spinor" and $\chi^\dagger$ is a "right-handed Weyl spinor". The names fit, because

$$P_L\Psi_D = \begin{pmatrix} \xi_\alpha \\ 0 \end{pmatrix}, \qquad\qquad P_R\Psi_D = \begin{pmatrix} 0 \\ \chi^{\dagger\dot\alpha} \end{pmatrix}\ . \tag{1.23}$$

The Hermitian conjugate of any left-handed Weyl spinor is a right-handed Weyl spinor:

$$\psi^\dagger_{\dot\alpha} \equiv (\psi_\alpha)^\dagger = (\psi^\dagger)_{\dot\alpha}\ , \tag{1.24}$$

and vice versa:

$$(\psi^{\dagger\dot\alpha})^\dagger = \psi^\alpha. \tag{1.25}$$

Therefore, any particular fermionic degrees of freedom can be described equally well using a left-handed Weyl spinor (with an undotted index) or by a right-handed one (with a dotted index). By convention, all names of fermion fields are chosen so that left-handed Weyl spinors do not carry daggers and right-handed Weyl spinors do carry daggers, as in eq. (1.21).

The heights of the dotted and undotted spinor indices are important; for example, comparing eqs. (1.18)-(1.22), we observe that the matrices $(\sigma^\mu)_{\alpha\dot\alpha}$ and $(\overline{\sigma}^\mu)^{\dot\alpha\alpha}$ defined by eq. (1.20) carry indices with the heights as indicated. The spinor indices are raised and lowered using the antisymmetric symbol $\epsilon^{12} = -\epsilon^{21} = \epsilon_{21} = -\epsilon_{12} = 1$, $\epsilon_{11} = \epsilon_{22} = \epsilon^{11} = \epsilon^{22} = 0$, according to

$$\xi_\alpha = \epsilon_{\alpha\beta}\xi^\beta, \qquad \xi^\alpha = \epsilon^{\alpha\beta}\xi_\beta, \qquad \chi^\dagger_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\chi^{\dagger\dot\beta}, \qquad \chi^{\dagger\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\chi^\dagger_{\dot\beta}\ . \tag{1.26}$$

This is consistent since $\epsilon_{\alpha\beta}\epsilon^{\beta\gamma} = \epsilon^{\gamma\beta}\epsilon_{\beta\alpha} = \delta^\gamma_\alpha$ and $\epsilon_{\dot\alpha\dot\beta}\epsilon^{\dot\beta\dot\gamma} = \epsilon^{\dot\gamma\dot\beta}\epsilon_{\dot\beta\dot\alpha} = \delta^{\dot\gamma}_{\dot\alpha}$.

As a convention, repeated spinor indices contracted like

$${}^\alpha{}_\alpha \qquad\qquad \text{or} \qquad\qquad {}_{\dot\alpha}{}^{\dot\alpha} \tag{1.27}$$

can be suppressed. In particular,

$$\xi\chi \equiv \xi^\alpha \chi_\alpha = \xi^\alpha \epsilon_{\alpha\beta} \chi^\beta = -\chi^\beta \epsilon_{\alpha\beta} \xi^\alpha = \chi^\beta \epsilon_{\beta\alpha} \xi^\alpha = \chi^\beta \xi_\beta \equiv \chi\xi \tag{1.28}$$

with, conveniently, no minus sign in the end. [A minus sign appeared in eq. (1.28) from exchanging the order of anticommuting spinors, but it disappeared due to the antisymmetry of the ϵ symbol.] Likewise, $\xi^\dagger\chi^\dagger$ and $\chi^\dagger\xi^\dagger$ are equivalent abbreviations for $\chi^\dagger_{\dot\alpha}\xi^{\dagger\dot\alpha} = \xi^\dagger_{\dot\alpha}\chi^{\dagger\dot\alpha}$, and in fact this is the complex conjugate of $\xi\chi$:

$$\xi^\dagger\chi^\dagger = \chi^\dagger\xi^\dagger = (\xi\chi)^*. \tag{1.29}$$

In a similar way, one can check that

$$\xi^\dagger\overline{\sigma}^\mu\chi = -\chi\sigma^\mu\xi^\dagger = (\chi^\dagger\overline{\sigma}^\mu\xi)^* = -(\xi\sigma^\mu\chi^\dagger)^* \tag{1.30}$$

stands for $\xi^\dagger_{\dot\alpha}(\overline{\sigma}^\mu)^{\dot\alpha\alpha}\chi_\alpha$, etc. The anti-commuting spinors here are taken to be classical fields; for quantum fields the complex conjugation in the last two equations would be replaced by Hermitian conjugation in the Hilbert space operator sense.

Some other identities that will be useful below include:

$$\xi\sigma^\mu\overline{\sigma}^\nu\chi \;=\; \chi\sigma^\nu\overline{\sigma}^\mu\xi \;=\; (\chi^\dagger\overline{\sigma}^\nu\sigma^\mu\xi^\dagger)^* = (\xi^\dagger\overline{\sigma}^\mu\sigma^\nu\chi^\dagger)^*, \tag{1.31}$$

and the Fierz rearrangement identity:

$$\chi_\alpha\,(\xi\eta) = -\xi_\alpha\,(\eta\chi) - \eta_\alpha\,(\chi\xi), \tag{1.32}$$

and the reduction identities

$$\begin{aligned}
\sigma^\mu_{\alpha\dot\alpha}\,\overline{\sigma}^{\dot\beta\beta}_\mu \;&=\; -2\delta_\alpha^\beta\delta_{\dot\alpha}^{\dot\beta}, && (1.33)\\
\sigma^\mu_{\alpha\dot\alpha}\,\sigma_{\mu\beta\dot\beta} \;&=\; -2\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}, && (1.34)\\
\overline{\sigma}^{\mu\dot\alpha\alpha}\,\overline{\sigma}^{\dot\beta\beta}_\mu \;&=\; -2\epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta}, && (1.35)\\
\left[\sigma^\mu\overline{\sigma}^\nu + \sigma^\nu\overline{\sigma}^\mu\right]_\alpha{}^\beta \;&=\; -2\eta^{\mu\nu}\delta_\alpha^\beta, && (1.36)\\
\left[\overline{\sigma}^\mu\sigma^\nu + \overline{\sigma}^\nu\sigma^\mu\right]^{\dot\beta}{}_{\dot\alpha} \;&=\; -2\eta^{\mu\nu}\delta_{\dot\alpha}^{\dot\beta}, && (1.37)\\
\overline{\sigma}^\mu\sigma^\nu\overline{\sigma}^\rho \;&=\; -\eta^{\mu\nu}\overline{\sigma}^\rho - \eta^{\nu\rho}\overline{\sigma}^\mu + \eta^{\mu\rho}\overline{\sigma}^\nu + i\epsilon^{\mu\nu\rho\kappa}\overline{\sigma}_\kappa, && (1.38)\\
\sigma^\mu\overline{\sigma}^\nu\sigma^\rho \;&=\; -\eta^{\mu\nu}\sigma^\rho - \eta^{\nu\rho}\sigma^\mu + \eta^{\mu\rho}\sigma^\nu - i\epsilon^{\mu\nu\rho\kappa}\sigma_\kappa, && (1.39)
\end{aligned}$$

where $\epsilon^{\mu\nu\rho\kappa}$ is the totally antisymmetric tensor with $\epsilon^{0123} = +1$.

With these conventions, the Dirac Lagrangian eq. (1.18) can now be rewritten:

$$\mathcal{L}_{\text{Dirac}} \;=\; i\xi^\dagger\overline{\sigma}^\mu\partial_\mu\xi + i\chi^\dagger\overline{\sigma}^\mu\partial_\mu\chi - M(\xi\chi + \xi^\dagger\chi^\dagger) \tag{1.40}$$

where we have dropped a total derivative piece $-i\partial_\mu(\chi^\dagger\overline{\sigma}^\mu\chi)$, which does not affect the action.

A four-component Majorana spinor can be obtained from the Dirac spinor of eq. (1.22) by imposing the constraint $\chi = \xi$, so that

$$\Psi_{\rm M} = \begin{pmatrix} \xi_\alpha \\ \xi^{\dagger\dot{\alpha}} \end{pmatrix}, \qquad\qquad \overline{\Psi}_{\rm M} = \begin{pmatrix} \xi^\alpha & \xi^\dagger_{\dot{\alpha}} \end{pmatrix}. \tag{1.41}$$

The four-component spinor form of the Lagrangian for a Majorana fermion with mass M,

$$\mathcal{L}_{\rm Majorana} = \frac{i}{2}\overline{\Psi}_{\rm M}\gamma^\mu\partial_\mu\Psi_{\rm M} - \frac{1}{2}M\overline{\Psi}_{\rm M}\Psi_{\rm M} \tag{1.42}$$

can therefore be rewritten as

$$\mathcal{L}_{\rm Majorana} = i\xi^\dagger\overline{\sigma}^\mu\partial_\mu\xi - \frac{1}{2}M(\xi\xi + \xi^\dagger\xi^\dagger) \tag{1.43}$$

in the more economical two-component Weyl spinor representation. Note that even though ξ_α is anticommuting, $\xi\xi$ and its complex conjugate $\xi^\dagger\xi^\dagger$ do not vanish, because of the suppressed ϵ symbol, see eq. (1.28). Explicitly, $\xi\xi = \epsilon^{\alpha\beta}\xi_\beta\xi_\alpha = \xi_2\xi_1 - \xi_1\xi_2 = 2\xi_2\xi_1$.

More generally, any theory involving spin-1/2 fermions can always be written in terms of a collection of left-handed Weyl spinors ψ_i with

$$\mathcal{L} = i\psi^{\dagger i}\overline{\sigma}^\mu\partial_\mu\psi_i + \ldots \tag{1.44}$$

where the ellipses represent possible mass terms, gauge interactions, and Yukawa interactions with scalar fields. Here the index i runs over the appropriate gauge and flavor indices of the fermions; it is raised or lowered by Hermitian conjugation. Gauge interactions are obtained by promoting the ordinary derivative to a gauge-covariant derivative:

$$\mathcal{L} = i\psi^{\dagger i}\overline{\sigma}^\mu D_\mu\psi_i + \ldots \tag{1.45}$$

with

$$D_\mu\psi_i = \partial_\mu\psi_i - ig_a A^a_\mu T^{aj}_i\psi_j, \tag{1.46}$$

where g_a is the gauge coupling corresponding to the Hermitian Lie algebra generator matrix T^a with vector field A^a_μ.

There is a different ψ_i for the left-handed piece and for the hermitian conjugate of the right-handed piece of a Dirac fermion. Given any expression involving bilinears of four-component spinors

$$\Psi_i = \begin{pmatrix} \xi_i \\ \chi^\dagger_i \end{pmatrix}, \tag{1.47}$$

labeled by a flavor or gauge-representation index i, one can translate into two-component Weyl spinor language (or vice versa) using the dictionary:

$$\overline{\Psi}_i P_L \Psi_j = \chi_i \xi_j, \qquad \overline{\Psi}_i P_R \Psi_j = \xi_i^\dagger \chi_j^\dagger, \tag{1.48}$$

$$\overline{\Psi}_i \gamma^\mu P_L \Psi_j = \xi_i^\dagger \overline{\sigma}^\mu \xi_j, \qquad \overline{\Psi}_i \gamma^\mu P_R \Psi_j = \chi_i \sigma^\mu \chi_j^\dagger \tag{1.49}$$

etc.

Let us now see how the Standard Model quarks and leptons are described in this notation. The complete list of left-handed Weyl spinors can be given names corresponding to the chiral supermultiplets in Table 1.1:

$$Q_i = \begin{pmatrix} u \\ d \end{pmatrix}, \ \begin{pmatrix} c \\ s \end{pmatrix}, \ \begin{pmatrix} t \\ b \end{pmatrix}, \tag{1.50}$$

$$\overline{u}_i = \ \overline{u}, \ \overline{c}, \ \overline{t}, \tag{1.51}$$

$$\overline{d}_i = \ \overline{d}, \ \overline{s}, \ \overline{b} \tag{1.52}$$

$$L_i = \begin{pmatrix} \nu_e \\ e \end{pmatrix}, \ \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \ \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}, \tag{1.53}$$

$$\overline{e}_i = \ \overline{e}, \ \overline{\mu}, \ \overline{\tau}. \tag{1.54}$$

Here $i = 1, 2, 3$ is a family index. The bars on these fields are part of the names of the fields, and do *not* denote any kind of conjugation. Rather, the unbarred fields are the left-handed pieces of a Dirac spinor, while the barred fields are the names given to the conjugates of the right-handed piece of a Dirac spinor. For example, e is the same thing as e_L in Table 1.1, and $\overline{e}$ is the same as $e_R^\dagger$. Together they form a Dirac spinor:

$$\begin{pmatrix} e \\ \overline{e}^\dagger \end{pmatrix} \equiv \begin{pmatrix} e_L \\ e_R \end{pmatrix}, \tag{1.55}$$

and similarly for all of the other quark and charged lepton Dirac spinors. (The neutrinos of the Standard Model are not part of a Dirac spinor, at least in the approximation that they are massless.) The fields Q_i and L_i are weak isodoublets, which always go together when one is constructing interactions invariant under the full Standard Model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. Suppressing all color and weak isospin indices, the kinetic and gauge part of the Standard Model fermion Lagrangian density is then

$$\begin{aligned} \mathcal{L} = {} & iQ^{\dagger i} \overline{\sigma}^\mu D_\mu Q_i + i\overline{u}_i^\dagger \overline{\sigma}^\mu D_\mu \overline{u}^i + i\overline{d}_i^\dagger \overline{\sigma}^\mu D_\mu \overline{d}^i \\ & + iL^{\dagger i} \overline{\sigma}^\mu D_\mu L_i + i\overline{e}_i^\dagger \overline{\sigma}^\mu D_\mu \overline{e}^i \end{aligned} \tag{1.56}$$

with the family index i summed over, and D_μ the appropriate Standard Model covariant derivative. For example,

$$D_\mu \begin{pmatrix} \nu_e \\ e \end{pmatrix} = \left[\partial_\mu - igW^a_\mu(\tau^a/2) - ig'Y_L B_\mu\right] \begin{pmatrix} \nu_e \\ e \end{pmatrix} \tag{1.57}$$

$$D_\mu \overline{e} = \left[\partial_\mu - ig'Y_{\overline{e}} B_\mu\right] \overline{e} \tag{1.58}$$

with τ^a $(a = 1, 2, 3)$ equal to the Pauli matrices, $Y_L = -1/2$ and $Y_{\overline{e}} = +1$. The gauge eigenstate weak bosons are related to the mass eigenstates by

$$W^\pm_\mu = (W^1_\mu \mp iW^2_\mu)/\sqrt{2}, \tag{1.59}$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}. \tag{1.60}$$

Similar expressions hold for the other quark and lepton gauge eigenstates, with $Y_Q = 1/6$, $Y_{\overline{u}} = -2/3$, and $Y_{\overline{d}} = 1/3$. The quarks also have a term in the covariant derivative corresponding to gluon interactions proportional to g_3 (with $\alpha_S = g_3^2/4\pi$) with generators $T^a = \lambda^a/2$ for Q, and in the complex conjugate representation $T^a = -(\lambda^a)^*/2$ for $\overline{u}$ and $\overline{d}$, where λ^a are the Gell-Mann matrices.

1.3. Supersymmetric Lagrangians

In this section we will describe the construction of supersymmetric Lagrangians. Our aim is to arrive at a recipe that will allow us to write down the allowed interactions and mass terms of a general supersymmetric theory, so that later we can apply the results to the special case of the MSSM. We will not use the superfield[50] language, which is often more elegant and efficient for those who know it, but might seem rather cabalistic to some. Our approach is therefore intended to be complementary to the superspace and superfield derivations given in other works. We begin by considering the simplest example of a supersymmetric theory in four dimensions.

1.3.1. *The simplest supersymmetric model: A free chiral supermultiplet*

The minimum fermion content of a field theory in four dimensions consists of a single left-handed two-component Weyl fermion ψ. Since this is an intrinsically complex object, it seems sensible to choose as its superpartner a complex scalar field ϕ. The simplest action we can write down for these fields just consists of kinetic energy terms for each:

$$S = \int d^4x \; (\mathcal{L}_{\rm scalar} + \mathcal{L}_{\rm fermion}) \,, \tag{1.61}$$

$$\mathcal{L}_{\rm scalar} = -\partial^\mu \phi^* \partial_\mu \phi, \qquad\qquad \mathcal{L}_{\rm fermion} = i\psi^\dagger \overline{\sigma}^\mu \partial_\mu \psi. \tag{1.62}$$

This is called the massless, non-interacting *Wess-Zumino model*,[10] and it corresponds to a single chiral supermultiplet as discussed in the Introduction.

A supersymmetry transformation should turn the scalar boson field ϕ into something involving the fermion field ψ_α. The simplest possibility for the transformation of the scalar field is

$$\delta\phi = \epsilon\psi, \qquad\qquad \delta\phi^* = \epsilon^\dagger \psi^\dagger, \tag{1.63}$$

where ϵ^α is an infinitesimal, anticommuting, two-component Weyl fermion object parameterizing the supersymmetry transformation. Until section 1.6.5, we will be discussing global supersymmetry, which means that ϵ^α is a constant, satisfying $\partial_\mu \epsilon^\alpha = 0$. Since ψ has dimensions of $[\text{mass}]^{3/2}$ and ϕ has dimensions of [mass], it must be that ϵ has dimensions of $[\text{mass}]^{-1/2}$. Using eq. (1.63), we find that the scalar part of the Lagrangian transforms as

$$\delta\mathcal{L}_{\rm scalar} \;=\; -\epsilon\partial^\mu\psi \,\partial_\mu\phi^* - \epsilon^\dagger \partial^\mu \psi^\dagger \,\partial_\mu \phi. \tag{1.64}$$

We would like for this to be canceled by $\delta\mathcal{L}_{\rm fermion}$, at least up to a total derivative, so that the action will be invariant under the supersymmetry transformation. Comparing eq. (1.64) with $\mathcal{L}_{\rm fermion}$, we see that for this to have any chance of happening, $\delta\psi$ should be linear in $\epsilon^\dagger$ and in ϕ, and should contain one spacetime derivative. Up to a multiplicative constant, there is only one possibility to try:

$$\delta\psi_\alpha \;=\; -i(\sigma^\mu\epsilon^\dagger)_\alpha \,\partial_\mu\phi, \qquad\qquad \delta\psi^\dagger_{\dot\alpha} \;=\; i(\epsilon\sigma^\mu)_{\dot\alpha}\,\partial_\mu\phi^*. \tag{1.65}$$

With this guess, one immediately obtains

$$\delta\mathcal{L}_{\rm fermion} = -\epsilon\sigma^\mu\overline{\sigma}^\nu\partial_\nu\psi\,\partial_\mu\phi^* + \psi^\dagger\overline{\sigma}^\nu\sigma^\mu\epsilon^\dagger\,\partial_\mu\partial_\nu\phi\,. \tag{1.66}$$

This can be put in a slightly more useful form by employing the Pauli matrix identities eqs. (1.36), (1.37) and using the fact that partial derivatives commute ($\partial_\mu\partial_\nu = \partial_\nu\partial_\mu$). Equation (1.66) then becomes

$$\begin{aligned}\delta\mathcal{L}_{\rm fermion} = {} & \epsilon\partial^\mu\psi\,\partial_\mu\phi^* + \epsilon^\dagger\partial^\mu\psi^\dagger\,\partial_\mu\phi \\ & -\partial_\mu\left(\epsilon\sigma^\nu\overline{\sigma}^\mu\psi\,\partial_\nu\phi^* + \epsilon\psi\,\partial^\mu\phi^* + \epsilon^\dagger\psi^\dagger\,\partial^\mu\phi\right).\end{aligned} \tag{1.67}$$

The first two terms here just cancel against $\delta\mathcal{L}_{\text{scalar}}$, while the remaining contribution is a total derivative. So we arrive at

$$\delta S = \int d^4x \;\; (\delta\mathcal{L}_{\text{scalar}} + \delta\mathcal{L}_{\text{fermion}}) = 0, \tag{1.68}$$

justifying our guess of the numerical multiplicative factor made in eq. (1.65).

We are not quite finished in showing that the theory described by eq. (1.61) is supersymmetric. We must also show that the supersymmetry algebra closes; in other words, that the commutator of two supersymmetry transformations parameterized by two different spinors ϵ_1 and ϵ_2 is another symmetry of the theory. Using eq. (1.65) in eq. (1.63), one finds

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})\phi \;\equiv\; \delta_{\epsilon_2}(\delta_{\epsilon_1}\phi) - \delta_{\epsilon_1}(\delta_{\epsilon_2}\phi) \;=\; i(-\epsilon_1\sigma^\mu\epsilon_2^\dagger + \epsilon_2\sigma^\mu\epsilon_1^\dagger)\,\partial_\mu\phi. \tag{1.69}$$

This is a remarkable result; in words, we have found that the commutator of two supersymmetry transformations gives us back the derivative of the original field. Since ∂_μ corresponds to the generator of spacetime translations P_μ, eq. (1.69) implies the form of the supersymmetry algebra that was foreshadowed in eq. (1.6) of the Introduction. (We will make this statement more explicit before the end of this section.)

All of this will be for nothing if we do not find the same result for the fermion ψ. Using eq. (1.63) in eq. (1.65), we get

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})\psi_\alpha \;=\; -i(\sigma^\mu\epsilon_1^\dagger)_\alpha\,\epsilon_2\partial_\mu\psi + i(\sigma^\mu\epsilon_2^\dagger)_\alpha\,\epsilon_1\partial_\mu\psi. \tag{1.70}$$

This can be put into a more useful form by applying the Fierz identity eq. (1.32) with $\chi = \sigma^\mu\epsilon_1^\dagger$, $\xi = \epsilon_2$, $\eta = \partial_\mu\psi$, and again with $\chi = \sigma^\mu\epsilon_2^\dagger$, $\xi = \epsilon_1$, $\eta = \partial_\mu\psi$, followed in each case by an application of the identity eq. (1.30). The result is

$$\begin{aligned}(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})\psi_\alpha &= i(-\epsilon_1\sigma^\mu\epsilon_2^\dagger + \epsilon_2\sigma^\mu\epsilon_1^\dagger)\,\partial_\mu\psi_\alpha \\ &\quad + i\epsilon_{1\alpha}\,\epsilon_2^\dagger\overline{\sigma}^\mu\partial_\mu\psi - i\epsilon_{2\alpha}\,\epsilon_1^\dagger\overline{\sigma}^\mu\partial_\mu\psi.\end{aligned} \tag{1.71}$$

The last two terms in (1.71) vanish on-shell; that is, if the equation of motion $\overline{\sigma}^\mu\partial_\mu\psi = 0$ following from the action is enforced. The remaining piece is exactly the same spacetime translation that we found for the scalar field.

The fact that the supersymmetry algebra only closes on-shell (when the classical equations of motion are satisfied) might be somewhat worrisome, since we would like the symmetry to hold even quantum mechanically. This can be fixed by a trick. We invent a new complex scalar field F, which does not have a kinetic term. Such fields are called *auxiliary*, and they are really just book-keeping devices that allow the symmetry algebra to close off-shell. The Lagrangian density for F and its complex conjugate is simply

$$\mathcal{L}_{\text{auxiliary}} = F^* F\,. \tag{1.72}$$

The dimensions of F are [mass]^2, unlike an ordinary scalar field, which has dimensions of [mass]. Equation (1.72) implies the not-very-exciting equations of motion $F = F^* = 0$. However, we can use the auxiliary fields to our advantage by including them in the supersymmetry transformation rules. In view of eq. (1.71), a plausible thing to do is to make F transform into a multiple of the equation of motion for ψ:

$$\delta F = -i\epsilon^\dagger \overline{\sigma}^\mu \partial_\mu \psi, \qquad\qquad \delta F^* = i\partial_\mu \psi^\dagger \overline{\sigma}^\mu \epsilon. \tag{1.73}$$

Once again we have chosen the overall factor on the right-hand sides by virtue of foresight. Now the auxiliary part of the Lagrangian density transforms as

$$\delta\mathcal{L}_{\text{auxiliary}} = -i\epsilon^\dagger \overline{\sigma}^\mu \partial_\mu \psi\, F^* + i\partial_\mu \psi^\dagger \overline{\sigma}^\mu \epsilon\, F, \tag{1.74}$$

which vanishes on-shell, but not for arbitrary off-shell field configurations. Now, by adding an extra term to the transformation law for ψ and $\psi^\dagger$:

$$\delta\psi_\alpha = -i(\sigma^\mu \epsilon^\dagger)_\alpha\, \partial_\mu \phi + \epsilon_\alpha F, \qquad \delta\psi^\dagger_{\dot\alpha} = i(\epsilon\sigma^\mu)_{\dot\alpha}\, \partial_\mu \phi^* + \epsilon^\dagger_{\dot\alpha} F^*, \tag{1.75}$$

one obtains an additional contribution to $\delta\mathcal{L}_{\text{fermion}}$, which just cancels with $\delta\mathcal{L}_{\text{auxiliary}}$, up to a total derivative term. So our "modified" theory with $\mathcal{L} = \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{auxiliary}}$ is still invariant under supersymmetry transformations. Proceeding as before, one now obtains for each of the fields $X = \phi, \phi^*, \psi, \psi^\dagger, F, F^*$,

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})X = i(-\epsilon_1\sigma^\mu\epsilon_2^\dagger + \epsilon_2\sigma^\mu\epsilon_1^\dagger)\,\partial_\mu X \tag{1.76}$$

using eqs. (1.63), (1.73), and (1.75), but now without resorting to any of the equations of motion. So we have succeeded in showing that supersymmetry is a valid symmetry of the Lagrangian off-shell.

In retrospect, one can see why we needed to introduce the auxiliary field F in order to get the supersymmetry algebra to work off-shell. On-shell, the complex scalar field ϕ has two real propagating degrees of freedom, matching the two spin polarization states of ψ. Off-shell, however, the Weyl fermion ψ is a complex two-component object, so it has four real degrees of freedom. (Going on-shell eliminates half of the propagating degrees of freedom for ψ, because the Lagrangian is linear in time derivatives, so that

Table 1.3. Counting of real degrees of freedom in the Wess-Zumino model.

	ϕ	ψ	F
on-shell ($n_B = n_F = 2$)	2	2	0
off-shell ($n_B = n_F = 4$)	2	4	2

the canonical momenta can be reexpressed in terms of the configuration variables without time derivatives and are not independent phase space coordinates.) To make the numbers of bosonic and fermionic degrees of freedom match off-shell as well as on-shell, we had to introduce two more real scalar degrees of freedom in the complex field F, which are eliminated when one goes on-shell. This counting is summarized in Table 1.3. The auxiliary field formulation is especially useful when discussing spontaneous supersymmetry breaking, as we will see in section 1.6.

Invariance of the action under a symmetry transformation always implies the existence of a conserved current, and supersymmetry is no exception. The *supercurrent* J^μ_α is an anticommuting four-vector. It also carries a spinor index, as befits the current associated with a symmetry with fermionic generators.[51] By the usual Noether procedure, one finds for the supercurrent (and its hermitian conjugate) in terms of the variations of the fields $X = \phi, \phi^*, \psi, \psi^\dagger, F, F^*$:

$$\epsilon J^\mu + \epsilon^\dagger J^{\dagger\mu} \equiv \sum_X \delta X \, \frac{\delta \mathcal{L}}{\delta(\partial_\mu X)} - K^\mu, \tag{1.77}$$

where K^μ is an object whose divergence is the variation of the Lagrangian density under the supersymmetry transformation, $\delta\mathcal{L} = \partial_\mu K^\mu$. Note that K^μ is not unique; one can always replace K^μ by $K^\mu + k^\mu$, where k^μ is any vector satisfying $\partial_\mu k^\mu = 0$, for example $k^\mu = \partial^\mu \partial_\nu a^\nu - \partial_\nu \partial^\nu a^\mu$. A little work reveals that, up to the ambiguity just mentioned,

$$J^\mu_\alpha = (\sigma^\nu \overline{\sigma}^\mu \psi)_\alpha \, \partial_\nu \phi^*, \qquad\qquad J^{\dagger\mu}_{\dot\alpha} = (\psi^\dagger \overline{\sigma}^\mu \sigma^\nu)_{\dot\alpha} \, \partial_\nu \phi. \tag{1.78}$$

The supercurrent and its hermitian conjugate are separately conserved:

$$\partial_\mu J^\mu_\alpha = 0, \qquad\qquad \partial_\mu J^{\dagger\mu}_{\dot\alpha} = 0, \tag{1.79}$$

as can be verified by use of the equations of motion. From these currents one constructs the conserved charges

$$Q_\alpha = \sqrt{2} \int d^3\vec{x} \, J^0_\alpha, \qquad\qquad Q^\dagger_{\dot\alpha} = \sqrt{2} \int d^3\vec{x} \, J^{\dagger 0}_{\dot\alpha}, \tag{1.80}$$

which are the generators of supersymmetry transformations. (The factor of $\sqrt{2}$ normalization is included to agree with an arbitrary historical convention.) As quantum mechanical operators, they satisfy

$$\left[\epsilon Q + \epsilon^\dagger Q^\dagger, X\right] = -i\sqrt{2}\,\delta X \tag{1.81}$$

for any field X, up to terms that vanish on-shell. This can be verified explicitly by using the canonical equal-time commutation and anticommutation relations

$$[\phi(\vec{x}), \pi(\vec{y})] = [\phi^*(\vec{x}), \pi^*(\vec{y})] = i\delta^{(3)}(\vec{x}-\vec{y}), \tag{1.82}$$

$$\{\psi_\alpha(\vec{x}), \psi^\dagger_{\dot{\alpha}}(\vec{y})\} = (\sigma^0)_{\alpha\dot{\alpha}}\,\delta^{(3)}(\vec{x}-\vec{y}) \tag{1.83}$$

derived from the free field theory Lagrangian eq. (1.61). Here $\pi = \partial_0\phi^*$ and $\pi^* = \partial_0\phi$ are the momenta conjugate to ϕ and ϕ^* respectively.

Using eq. (1.81), the content of eq. (1.76) can be expressed in terms of canonical commutators as

$$\Big[\epsilon_2 Q + \epsilon_2^\dagger Q^\dagger,\ \big[\epsilon_1 Q + \epsilon_1^\dagger Q^\dagger,\ X\big]\Big] - \Big[\epsilon_1 Q + \epsilon_1^\dagger Q^\dagger,\ \big[\epsilon_2 Q + \epsilon_2^\dagger Q^\dagger,\ X\big]\Big]$$
$$= 2(\epsilon_1\sigma^\mu\epsilon_2^\dagger - \epsilon_2\sigma^\mu\epsilon_1^\dagger)\, i\partial_\mu X, \tag{1.84}$$

up to terms that vanish on-shell. The spacetime momentum operator is $P^\mu = (H, \vec{P})$, where H is the Hamiltonian and $\vec{P}$ is the three-momentum operator, given in terms of the canonical fields by

$$H = \int d^3\vec{x}\left[\pi^*\pi + (\vec{\nabla}\phi^*)\cdot(\vec{\nabla}\phi) + i\psi^\dagger\vec{\sigma}\cdot\vec{\nabla}\psi\right], \tag{1.85}$$

$$\vec{P} = -\int d^3\vec{x}\left(\pi\vec{\nabla}\phi + \pi^*\vec{\nabla}\phi^* + i\psi^\dagger\overline{\sigma}^0\vec{\nabla}\psi\right). \tag{1.86}$$

It generates spacetime translations on the fields X according to

$$[P^\mu, X] = i\partial^\mu X. \tag{1.87}$$

Rearranging the terms in eq. (1.84) using the Jacobi identity, we therefore have

$$\Big[\big[\epsilon_2 Q + \epsilon_2^\dagger Q^\dagger,\ \epsilon_1 Q + \epsilon_1^\dagger Q^\dagger\big],\ X\Big] = 2(\epsilon_1\sigma_\mu\epsilon_2^\dagger - \epsilon_2\sigma_\mu\epsilon_1^\dagger)\,[P^\mu, X], \tag{1.88}$$

for any X, up to terms that vanish on-shell, so it must be that

$$\big[\epsilon_2 Q + \epsilon_2^\dagger Q^\dagger,\ \epsilon_1 Q + \epsilon_1^\dagger Q^\dagger\big] = 2(\epsilon_1\sigma_\mu\epsilon_2^\dagger - \epsilon_2\sigma_\mu\epsilon_1^\dagger)\,P^\mu. \tag{1.89}$$

Now by expanding out eq. (1.89), one obtains the precise form of the supersymmetry algebra relations

$$\{Q_\alpha, Q^\dagger_{\dot{\alpha}}\} = -2\sigma^\mu_{\alpha\dot{\alpha}}P_\mu, \tag{1.90}$$

$$\{Q_\alpha, Q_\beta\} = 0, \qquad \{Q^\dagger_{\dot{\alpha}}, Q^\dagger_{\dot{\beta}}\} = 0, \tag{1.91}$$

as promised in the Introduction. [The commutator in eq. (1.89) turns into anticommutators in eqs. (1.90) and (1.91) in the process of extracting the anticommuting spinors ϵ_1 and ϵ_2.] The results

$$[Q_\alpha, P^\mu] = 0, \qquad [Q^\dagger_{\dot\alpha}, P^\mu] = 0 \tag{1.92}$$

follow immediately from eq. (1.87) and the fact that the supersymmetry transformations are global (independent of position in spacetime). This demonstration of the supersymmetry algebra in terms of the canonical generators Q and $Q^\dagger$ requires the use of the Hamiltonian equations of motion, but the symmetry itself is valid off-shell at the level of the Lagrangian, as we have already shown.

1.3.2. *Interactions of chiral supermultiplets*

In a realistic theory like the MSSM, there are many chiral supermultiplets, with both gauge and non-gauge interactions. In this subsection, our task is to construct the most general possible theory of masses and non-gauge interactions for particles that live in chiral supermultiplets. In the MSSM these are the quarks, squarks, leptons, sleptons, Higgs scalars and higgsino fermions. We will find that the form of the non-gauge couplings, including mass terms, is highly restricted by the requirement that the action is invariant under supersymmetry transformations. (Gauge interactions will be dealt with in the following subsections.)

Our starting point is the Lagrangian density for a collection of free chiral supermultiplets labeled by an index i, which runs over all gauge and flavor degrees of freedom. Since we will want to construct an interacting theory with supersymmetry closing off-shell, each supermultiplet contains a complex scalar ϕ_i and a left-handed Weyl fermion ψ_i as physical degrees of freedom, plus a complex auxiliary field F_i, which does not propagate. The results of the previous subsection tell us that the free part of the Lagrangian is

$$\mathcal{L}_{\text{free}} = -\partial^\mu \phi^{*i} \partial_\mu \phi_i + i\psi^{\dagger i} \overline{\sigma}^\mu \partial_\mu \psi_i + F^{*i} F_i, \tag{1.93}$$

where we sum over repeated indices i (not to be confused with the suppressed spinor indices), with the convention that fields ϕ_i and ψ_i always carry lowered indices, while their conjugates always carry raised indices. It is invariant under the supersymmetry transformation

$$\delta\phi_i = \epsilon\psi_i, \qquad \delta\phi^{*i} = \epsilon^\dagger \psi^{\dagger i}, \tag{1.94}$$

$$\delta(\psi_i)_\alpha = -i(\sigma^\mu\epsilon^\dagger)_\alpha\, \partial_\mu\phi_i + \epsilon_\alpha F_i, \qquad \delta(\psi^{\dagger i})_{\dot\alpha} = i(\epsilon\sigma^\mu)_{\dot\alpha}\, \partial_\mu\phi^{*i} + \epsilon^\dagger_{\dot\alpha} F^{*i}, \tag{1.95}$$

$$\delta F_i = -i\epsilon^\dagger \overline{\sigma}^\mu \partial_\mu \psi_i, \qquad \delta F^{*i} = i\partial_\mu \psi^{\dagger i} \overline{\sigma}^\mu \epsilon\,. \tag{1.96}$$

We will now find the most general set of renormalizable interactions for these fields that is consistent with supersymmetry. We do this working in the field theory before integrating out the auxiliary fields. To begin, note that in order to be renormalizable by power counting, each term must have field content with total mass dimension ≤ 4. So, the only candidate terms are:

$$\mathcal{L}_{\text{int}} = \left(-\frac{1}{2}W^{ij}\psi_i\psi_j + W^i F_i + x^{ij}F_i F_j\right) + \text{c.c.} - U, \tag{1.97}$$

where W^{ij}, W^i, x^{ij}, and U are polynomials in the scalar fields ϕ_i, ϕ^{*i}, with degrees 1, 2, 0, and 4, respectively. [Terms of the form $F^{*i}F_j$ are already included in eq. (1.93), with the coefficient fixed by the transformation rules (1.94)-(1.96).]

We must now require that $\mathcal{L}_{\text{int}}$ is invariant under the supersymmetry transformations, since $\mathcal{L}_{\text{free}}$ was already invariant by itself. This immediately requires that the candidate term $U(\phi_i, \phi^{*i})$ must vanish. If there were such a term, then under a supersymmetry transformation eq. (1.94) it would transform into another function of the scalar fields only, multiplied by $\epsilon\psi_i$ or $\epsilon^\dagger\psi^{\dagger i}$, and with no spacetime derivatives or F_i, F^{*i} fields. It is easy to see from eqs. (1.94)-(1.97) that nothing of this form can possibly be canceled by the supersymmetry transformation of any other term in the Lagrangian. Similarly, the dimensionless coupling x^{ij} must be zero, because its supersymmetry transformation likewise cannot possibly be canceled by any other term. So, we are left with

$$\mathcal{L}_{\text{int}} = \left(-\frac{1}{2}W^{ij}\psi_i\psi_j + W^i F_i\right) + \text{c.c.} \tag{1.98}$$

as the only possibilities. At this point, we are not assuming that W^{ij} and W^i are related to each other in any way. However, soon we will find out that they *are* related, which is why we have chosen to use the same letter for them. Notice that eq. (1.28) tells us that W^{ij} is symmetric under $i \leftrightarrow j$.

It is easiest to divide the variation of $\mathcal{L}_{\text{int}}$ into several parts, which must cancel separately. First, we consider the part that contains four spinors:

$$\delta\mathcal{L}_{\text{int}}|_{4-\text{spinor}} = \left[-\frac{1}{2}\frac{\delta W^{ij}}{\delta\phi_k}(\epsilon\psi_k)(\psi_i\psi_j) - \frac{1}{2}\frac{\delta W^{ij}}{\delta\phi^{*k}}(\epsilon^\dagger\psi^{\dagger k})(\psi_i\psi_j)\right] + \text{c.c.} \tag{1.99}$$

The term proportional to $(\epsilon\psi_k)(\psi_i\psi_j)$ cannot cancel against any other term. Fortunately, however, the Fierz identity eq. (1.32) implies

$$(\epsilon\psi_i)(\psi_j\psi_k) + (\epsilon\psi_j)(\psi_k\psi_i) + (\epsilon\psi_k)(\psi_i\psi_j) = 0, \tag{1.100}$$

so this contribution to $\delta\mathcal{L}_{\rm int}$ vanishes identically if and only if $\delta W^{ij}/\delta\phi_k$ is totally symmetric under interchange of i, j, k. There is no such identity available for the term proportional to $(\epsilon^\dagger \psi^{\dagger k})(\psi_i\psi_j)$. Since that term cannot cancel with any other, requiring it to be absent just tells us that W^{ij} cannot contain ϕ^{*k}. In other words, W^{ij} is analytic (or holomorphic) in the complex fields ϕ_k.

Combining what we have learned so far, we can write

$$W^{ij} = M^{ij} + y^{ijk}\phi_k \tag{1.101}$$

where M^{ij} is a symmetric mass matrix for the fermion fields, and y^{ijk} is a Yukawa coupling of a scalar ϕ_k and two fermions $\psi_i\psi_j$ that must be totally symmetric under interchange of i, j, k. It is therefore possible, and it turns out to be convenient, to write

$$W^{ij} = \frac{\delta^2}{\delta\phi_i\delta\phi_j} W \tag{1.102}$$

where we have introduced a useful object

$$W = \frac{1}{2}M^{ij}\phi_i\phi_j + \frac{1}{6}y^{ijk}\phi_i\phi_j\phi_k, \tag{1.103}$$

called the *superpotential*. This is not a scalar potential in the ordinary sense; in fact, it is not even real. It is instead an analytic function of the scalar fields ϕ_i treated as complex variables.

Continuing on our vaunted quest, we next consider the parts of $\delta\mathcal{L}_{\rm int}$ that contain a spacetime derivative:

$$\delta\mathcal{L}_{\rm int}|_\partial = \left(iW^{ij}\partial_\mu\phi_j\, \psi_i\sigma^\mu\epsilon^\dagger + iW^i\,\partial_\mu\psi_i\sigma^\mu\epsilon^\dagger\right) + \text{c.c.} \tag{1.104}$$

Here we have used the identity eq. (1.30) on the second term, which came from $(\delta F_i)W^i$. Now we can use eq. (1.102) to observe that

$$W^{ij}\partial_\mu\phi_j = \partial_\mu\left(\frac{\delta W}{\delta\phi_i}\right). \tag{1.105}$$

Therefore, eq. (1.104) will be a total derivative if

$$W^i = \frac{\delta W}{\delta\phi_i} = M^{ij}\phi_j + \frac{1}{2}y^{ijk}\phi_j\phi_k\,, \tag{1.106}$$

which explains why we chose its name as we did. The remaining terms in $\delta\mathcal{L}_{\rm int}$ are all linear in F_i or F^{*i}, and it is easy to show that they cancel, given the results for W^i and W^{ij} that we have already found.

Actually, we can include a linear term in the superpotential without disturbing the validity of the previous discussion at all:

$$W = L^i \phi_i + \frac{1}{2} M^{ij} \phi_i \phi_j + \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k. \tag{1.107}$$

Here L^i are parameters with dimensions of [mass]^2, which affect only the scalar potential part of the Lagrangian. Such linear terms are only allowed when ϕ_i is a gauge singlet, and there are no such gauge singlet chiral supermultiplets in the MSSM with minimal field content. I will therefore omit this term from the remaining discussion of this section. However, this type of term does play an important role in the discussion of spontaneous supersymmetry breaking, as we will see in section 1.6.1.

To recap, we have found that the most general non-gauge interactions for chiral supermultiplets are determined by a single analytic function of the complex scalar fields, the superpotential W. The auxiliary fields F_i and F^{*i} can be eliminated using their classical equations of motion. The part of $\mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}$ that contains the auxiliary fields is $F_i F^{*i} + W^i F_i + W_i^* F^{*i}$, leading to the equations of motion

$$F_i = -W_i^*, \qquad\qquad F^{*i} = -W^i\, . \tag{1.108}$$

Thus the auxiliary fields are expressible algebraically (without any derivatives) in terms of the scalar fields.

After making the replacement[g] eq. (1.108) in $\mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}$, we obtain the Lagrangian density

$$\mathcal{L} = -\partial^\mu \phi^{*i} \partial_\mu \phi_i + i\psi^{\dagger i} \overline{\sigma}^\mu \partial_\mu \psi_i - \frac{1}{2} \left(W^{ij} \psi_i \psi_j + W_{ij}^* \psi^{\dagger i} \psi^{\dagger j} \right) - W^i W_i^*. \tag{1.109}$$

Now that the non-propagating fields F_i, F^{*i} have been eliminated, it follows from eq. (1.109) that the scalar potential for the theory is just given in terms of the superpotential by

$$\begin{aligned} V(\phi, \phi^*) = W^k W_k^* = F^{*k} F_k = {}& M_{ik}^* M^{kj} \phi^{*i} \phi_j \\ &+ \frac{1}{2} M^{in} y_{jkn}^* \phi_i \phi^{*j} \phi^{*k} + \frac{1}{2} M_{in}^* y^{jkn} \phi^{*i} \phi_j \phi_k + \frac{1}{4} y^{ijn} y_{kln}^* \phi_i \phi_j \phi^{*k} \phi^{*l}\, . \end{aligned} \tag{1.110}$$

This scalar potential is automatically bounded from below; in fact, since it is a sum of squares of absolute values (of the W^k), it is always non-negative. If we substitute the general form for the superpotential eq. (1.103) into

[g]Since F_i and F^{*i} appear only quadratically in the action, the result of instead doing a functional integral over them at the quantum level has precisely the same effect.

eq. (1.109), we obtain for the full Lagrangian density

$$\mathcal{L} = -\partial^\mu \phi^{*i} \partial_\mu \phi_i - V(\phi, \phi^*) + i\psi^{\dagger i} \overline{\sigma}^\mu \partial_\mu \psi_i - \frac{1}{2} M^{ij} \psi_i \psi_j - \frac{1}{2} M^*_{ij} \psi^{\dagger i} \psi^{\dagger j}$$
$$-\frac{1}{2} y^{ijk} \phi_i \psi_j \psi_k - \frac{1}{2} y^*_{ijk} \phi^{*i} \psi^{\dagger j} \psi^{\dagger k}. \quad (1.111)$$

Now we can compare the masses of the fermions and scalars by looking at the linearized equations of motion:

$$\partial^\mu \partial_\mu \phi_i = M^*_{ik} M^{kj} \phi_j + \ldots, \quad (1.112)$$
$$i\overline{\sigma}^\mu \partial_\mu \psi_i = M^*_{ij} \psi^{\dagger j} + \ldots, \qquad i\sigma^\mu \partial_\mu \psi^{\dagger i} = M^{ij} \psi_j + \ldots. \quad (1.113)$$

One can eliminate ψ in terms of $\psi^\dagger$ and vice versa in eq. (1.113), obtaining [after use of the identities eqs. (1.36) and (1.37)]:

$$\partial^\mu \partial_\mu \psi_i = M^*_{ik} M^{kj} \psi_j + \ldots, \qquad \partial^\mu \partial_\mu \psi^{\dagger j} = \psi^{\dagger i} M^*_{ik} M^{kj} + \ldots. \quad (1.114)$$

Therefore, the fermions and the bosons satisfy the same wave equation with exactly the same squared-mass matrix with real non-negative eigenvalues, namely $(M^2)_i{}^j = M^*_{ik} M^{kj}$. It follows that diagonalizing this matrix by redefining the fields with a unitary matrix gives a collection of chiral supermultiplets, each of which contains a mass-degenerate complex scalar and Weyl fermion, in agreement with the general argument in the Introduction.

1.3.3. *Lagrangians for gauge supermultiplets*

The propagating degrees of freedom in a gauge supermultiplet are a massless gauge boson field A^a_μ and a two-component Weyl fermion gaugino λ^a. The index a here runs over the adjoint representation of the gauge group ($a = 1, \ldots, 8$ for $SU(3)_C$ color gluons and gluinos; $a = 1, 2, 3$ for $SU(2)_L$ weak isospin; $a = 1$ for $U(1)_Y$ weak hypercharge). The gauge transformations of the vector supermultiplet fields are

$$\delta_{\text{gauge}} A^a_\mu = \partial_\mu \Lambda^a + g f^{abc} A^b_\mu \Lambda^c, \quad (1.115)$$
$$\delta_{\text{gauge}} \lambda^a = g f^{abc} \lambda^b \Lambda^c, \quad (1.116)$$

where Λ^a is an infinitesimal gauge transformation parameter, g is the gauge coupling, and f^{abc} are the totally antisymmetric structure constants that define the gauge group. The special case of an Abelian group is obtained by just setting $f^{abc} = 0$; the corresponding gaugino is a gauge singlet in that case. The conventions are such that for QED, $A^\mu = (V, \vec{A})$ where V and $\vec{A}$ are the usual electric potential and vector potential, with electric and magnetic fields given by $\vec{E} = -\vec{\nabla} V - \partial_0 \vec{A}$ and $\vec{B} = \vec{\nabla} \times \vec{A}$.

Table 1.4. Counting of real degrees of freedom for each gauge supermultiplet.

	A_μ	λ	D
on-shell ($n_B = n_F = 2$)	2	2	0
off-shell ($n_B = n_F = 4$)	3	4	1

The on-shell degrees of freedom for A^a_μ and λ^a_α amount to two bosonic and two fermionic helicity states (for each a), as required by supersymmetry. However, off-shell λ^a_α consists of two complex, or four real, fermionic degrees of freedom, while A^a_μ only has three real bosonic degrees of freedom; one degree of freedom is removed by the inhomogeneous gauge transformation eq. (1.115). So, we will need one real bosonic auxiliary field, traditionally called D^a, in order for supersymmetry to be consistent off-shell. This field also transforms as an adjoint of the gauge group [i.e., like eq. (1.116) with λ^a replaced by D^a] and satisfies $(D^a)^* = D^a$. Like the chiral auxiliary fields F_i, the gauge auxiliary field D^a has dimensions of [mass]^2 and no kinetic term, so it can be eliminated on-shell using its algebraic equation of motion. The counting of degrees of freedom is summarized in Table 1.4.

Therefore, the Lagrangian density for a gauge supermultiplet ought to be

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + i\lambda^{\dagger a}\overline{\sigma}^\mu D_\mu \lambda^a + \frac{1}{2}D^a D^a, \tag{1.117}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \tag{1.118}$$

is the usual Yang-Mills field strength, and

$$D_\mu \lambda^a = \partial_\mu \lambda^a + g f^{abc} A^b_\mu \lambda^c \tag{1.119}$$

is the covariant derivative of the gaugino field. To check that eq. (1.117) is really supersymmetric, one must specify the supersymmetry transformations of the fields. The forms of these follow from the requirements that they should be linear in the infinitesimal parameters $\epsilon, \epsilon^\dagger$ with dimensions of [mass]$^{-1/2}$, that δA^a_μ is real, and that δD^a should be real and proportional to the field equations for the gaugino, in analogy with the role of the auxiliary field F in the chiral supermultiplet case. Thus one can guess, up to multiplicative factors, that[h]

$$\delta A^a_\mu = -\frac{1}{\sqrt{2}}\left(\epsilon^\dagger\overline{\sigma}_\mu\lambda^a + \lambda^{\dagger a}\overline{\sigma}_\mu\epsilon\right), \tag{1.120}$$

$$\delta\lambda^a_\alpha = \frac{i}{2\sqrt{2}}(\sigma^\mu\overline{\sigma}^\nu\epsilon)_\alpha\, F^a_{\mu\nu} + \frac{1}{\sqrt{2}}\epsilon_\alpha\, D^a, \tag{1.121}$$

$$\delta D^a = \frac{i}{\sqrt{2}}\left(-\epsilon^\dagger\overline{\sigma}^\mu D_\mu\lambda^a + D_\mu\lambda^{\dagger a}\overline{\sigma}^\mu\epsilon\right). \tag{1.122}$$

The factors of $\sqrt{2}$ are chosen so that the action obtained by integrating $\mathcal{L}_{\rm gauge}$ is indeed invariant, and the phase of λ^a is chosen for future convenience in treating the MSSM.

It is now a little bit tedious, but straightforward, to also check that

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})X = i(-\epsilon_1\sigma^\mu\epsilon_2^\dagger + \epsilon_2\sigma^\mu\epsilon_1^\dagger)D_\mu X \tag{1.123}$$

for X equal to any of the gauge-covariant fields $F^a_{\mu\nu}$, λ^a, $\lambda^{\dagger a}$, D^a, as well as for arbitrary covariant derivatives acting on them. This ensures that the supersymmetry algebra eqs. (1.90)-(1.91) is realized on gauge-invariant combinations of fields in gauge supermultiplets, as they were on the chiral supermultiplets [compare eq. (1.76)]. This check requires the use of identities eqs. (1.31), (1.33) and (1.38). If we had not included the auxiliary field D^a, then the supersymmetry algebra eq. (1.123) would hold only after using the equations of motion for λ^a and $\lambda^{\dagger a}$. The auxiliary fields satisfies a trivial equation of motion $D^a = 0$, but this is modified if one couples the gauge supermultiplets to chiral supermultiplets, as we now do.

1.3.4. *Supersymmetric gauge interactions*

Now we are ready to consider a general Lagrangian density for a supersymmetric theory with both chiral and gauge supermultiplets. Suppose that the chiral supermultiplets transform under the gauge group in a representation with hermitian matrices $(T^a)_i{}^j$ satisfying $[T^a, T^b] = if^{abc}T^c$. [For example, if the gauge group is $SU(2)$, then $f^{abc} = \epsilon^{abc}$, and the T^a are $1/2$ times the Pauli matrices for a chiral supermultiplet transforming in the

[h]The supersymmetry transformations eqs. (1.120)-(1.122) are non-linear for non-Abelian gauge symmetries, since there are gauge fields in the covariant derivatives acting on the gaugino fields and in the field strength $F^a_{\mu\nu}$. By adding even more auxiliary fields besides D^a, one can make the supersymmetry transformations linear in the fields. The version here, in which those extra auxiliary fields have been removed by gauge transformations, is called "Wess-Zumino gauge".[52]

fundamental representation.] Since supersymmetry and gauge transformations commute, the scalar, fermion, and auxiliary fields must be in the same representation of the gauge group, so

$$\delta_{\text{gauge}} X_i = ig\Lambda^a (T^a X)_i \tag{1.124}$$

for $X_i = \phi_i, \psi_i, F_i$. To have a gauge-invariant Lagrangian, we now need to replace the ordinary derivatives in eq. (1.93) with covariant derivatives:

$$\partial_\mu \phi_i \rightarrow D_\mu \phi_i = \partial_\mu \phi_i - igA^a_\mu (T^a \phi)_i \tag{1.125}$$

$$\partial_\mu \phi^{*i} \rightarrow D_\mu \phi^{*i} = \partial_\mu \phi^{*i} + igA^a_\mu (\phi^* T^a)^i \tag{1.126}$$

$$\partial_\mu \psi_i \rightarrow D_\mu \psi_i = \partial_\mu \psi_i - igA^a_\mu (T^a \psi)_i. \tag{1.127}$$

Naively, this simple procedure achieves the goal of coupling the vector bosons in the gauge supermultiplet to the scalars and fermions in the chiral supermultiplets. However, we also have to consider whether there are any other interactions allowed by gauge invariance and involving the gaugino and D^a fields, which might have to be included to make a supersymmetric Lagrangian. Since A^a_μ couples to ϕ_i and ψ_i, it makes sense that λ^a and D^a should as well.

In fact, there are three such possible interaction terms that are renormalizable (of field mass dimension ≤ 4), namely

$$(\phi^* T^a \psi)\lambda^a, \qquad \lambda^{\dagger a}(\psi^\dagger T^a \phi), \qquad \text{and} \qquad (\phi^* T^a \phi) D^a. \tag{1.128}$$

Now one can add them, with unknown dimensionless coupling coefficients, to the Lagrangians for the chiral and gauge supermultiplets, and demand that the whole mess be real and invariant under supersymmetry, up to a total derivative. Not surprisingly, this is possible only if the supersymmetry transformation laws for the matter fields are modified to include gauge-covariant rather than ordinary derivatives. Also, it is necessary to include one strategically chosen extra term in δF_i, so:

$$\delta\phi_i = \epsilon\psi_i \tag{1.129}$$

$$\delta\psi_{i\alpha} = -i(\sigma^\mu \epsilon^\dagger)_\alpha \, D_\mu \phi_i + \epsilon_\alpha F_i \tag{1.130}$$

$$\delta F_i = -i\epsilon^\dagger \overline{\sigma}^\mu D_\mu \psi_i \; + \; \sqrt{2} g (T^a \phi)_i \, \epsilon^\dagger \lambda^{\dagger a}. \tag{1.131}$$

After some algebra one can now fix the coefficients for the terms in eq. (1.128), with the result that the full Lagrangian density for a renormalizable supersymmetric theory is

$$\begin{aligned} \mathcal{L} = \; & \mathcal{L}_{\text{chiral}} + \mathcal{L}_{\text{gauge}} \\ & - \sqrt{2} g (\phi^* T^a \psi)\lambda^a - \sqrt{2} g \lambda^{\dagger a} (\psi^\dagger T^a \phi) + g(\phi^* T^a \phi) D^a. \end{aligned} \tag{1.132}$$

Here $\mathcal{L}_{\text{chiral}}$ means the chiral supermultiplet Lagrangian found in section 1.3.2 [e.g., eq. (1.109) or (1.111)], but with ordinary derivatives replaced everywhere by gauge-covariant derivatives, and $\mathcal{L}_{\text{gauge}}$ was given in eq. (1.117). To prove that eq. (1.132) is invariant under the supersymmetry transformations, one must use the identity

$$W^i(T^a\phi)_i = 0. \tag{1.133}$$

This is precisely the condition that must be satisfied anyway in order for the superpotential, and thus $\mathcal{L}_{\text{chiral}}$, to be gauge invariant, since the left side is proportional to $\delta_{\text{gauge}}W$.

The second line in eq. (1.132) consists of interactions whose strengths are fixed to be gauge couplings by the requirements of supersymmetry, even though they are not gauge interactions from the point of view of an ordinary field theory. The first two terms are a direct coupling of gauginos to matter fields; this can be thought of as the "supersymmetrization" of the usual gauge boson couplings to matter fields. The last term combines with the $D^aD^a/2$ term in $\mathcal{L}_{\text{gauge}}$ to provide an equation of motion

$$D^a = -g(\phi^* T^a \phi). \tag{1.134}$$

Thus, like the auxiliary fields F_i and F^{*i}, the D^a are expressible purely algebraically in terms of the scalar fields. Replacing the auxiliary fields in eq. (1.132) using eq. (1.134), one finds that the complete scalar potential is (recall that $\mathcal{L}$ contains $-V$):

$$V(\phi,\phi^*) = F^{*i}F_i + \frac{1}{2}\sum_a D^aD^a = W_i^*W^i + \frac{1}{2}\sum_a g_a^2(\phi^*T^a\phi)^2. \tag{1.135}$$

The two types of terms in this expression are called "F-term" and "D-term" contributions, respectively. In the second term in eq. (1.135), we have now written an explicit sum $\sum_a$ to cover the case that the gauge group has several distinct factors with different gauge couplings g_a. [For instance, in the MSSM the three factors $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ have different gauge couplings g_3, g and g'.] Since $V(\phi,\phi^*)$ is a sum of squares, it is always greater than or equal to zero for every field configuration. It is an interesting and unique feature of supersymmetric theories that the scalar potential is completely determined by the *other* interactions in the theory. The F-terms are fixed by Yukawa couplings and fermion mass terms, and the D-terms are fixed by the gauge interactions.

By using Noether's procedure [see eq. (1.77)], one finds the conserved supercurrent

$$J^\mu_\alpha = (\sigma^\nu \overline{\sigma}^\mu \psi_i)_\alpha \, D_\nu \phi^{*i} + i(\sigma^\mu \psi^{\dagger i})_\alpha \, W_i^* \\ - \frac{1}{2\sqrt{2}}(\sigma^\nu \overline{\sigma}^\rho \sigma^\mu \lambda^{\dagger a})_\alpha \, F^a_{\nu\rho} + \frac{i}{\sqrt{2}} g_a \phi^* T^a \phi \, (\sigma^\mu \lambda^{\dagger a})_\alpha, \qquad (1.136)$$

generalizing the expression given in eq. (1.78) for the Wess-Zumino model. This result will be useful when we discuss certain aspects of spontaneous supersymmetry breaking in section 1.6.5.

1.3.5. *Summary: How to build a supersymmetric model*

In a renormalizable supersymmetric field theory, the interactions and masses of all particles are determined just by their gauge transformation properties and by the superpotential W. By construction, we found that W had to be an analytic function of the complex scalar fields ϕ_i, which are always defined to transform under supersymmetry into left-handed Weyl fermions. We should mention that in an equivalent language, W is said to be a function of chiral *superfields*.[50] A superfield is a single object that contains as components all of the bosonic, fermionic, and auxiliary fields within the corresponding supermultiplet, for example $\Phi_i \supset (\phi_i, \psi_i, F_i)$. (This is analogous to the way in which one often describes a weak isospin doublet or color triplet by a multicomponent field.) The gauge quantum numbers and the mass dimension of a chiral superfield are the same as that of its scalar component. In the superfield formulation, one writes instead of eq. (1.107)

$$W = L^i \Phi_i + \frac{1}{2} M^{ij} \Phi_i \Phi_j + \frac{1}{6} y^{ijk} \Phi_i \Phi_j \Phi_k, \qquad (1.137)$$

which implies exactly the same physics. The derivation of all of our preceding results can be obtained somewhat more elegantly using superfield methods, which have the advantage of making invariance under supersymmetry transformations manifest by defining the Lagrangian in terms of integrals over a "superspace" with fermionic as well as ordinary commuting coordinates. We have avoided this extra layer of notation on purpose, in favor of the more pedestrian, but more familiar and accessible, component field approach. The latter is at least more appropriate for making contact with phenomenology in a universe with supersymmetry breaking. The only (occasional) use we will make of superfield notation is the purely cosmetic one of following the common practice of specifying superpotentials like eq. (1.137) rather than (1.107). The specification of the superpotential is really a code for the terms that it implies in the Lagrangian, so the reader may feel free to think of the superpotential either as a function of the scalar fields ϕ_i or as the same function of the superfields Φ_i.

Given the supermultiplet content of the theory, the form of the superpotential is restricted by the requirement of gauge invariance [see eq. (1.133)]. In any given theory, only a subset of the parameters L^i, M^{ij}, and y^{ijk} are allowed to be non-zero. The parameter L^i is only allowed if Φ_i is a gauge singlet. (There are no such chiral supermultiplets in the MSSM with the minimal field content.) The entries of the mass matrix M^{ij} can only be non-zero for i and j such that the supermultiplets Φ_i and Φ_j transform under the gauge group in representations that are conjugates of each other. (In the MSSM there is only one such term, as we will see.) Likewise, the Yukawa couplings y^{ijk} can only be non-zero when Φ_i, Φ_j, and Φ_k transform in representations that can combine to form a singlet.

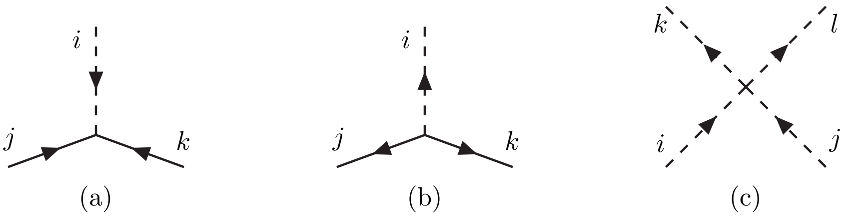

Fig. 1.3. The dimensionless non-gauge interaction vertices in a supersymmetric theory: (a) scalar-fermion-fermion Yukawa interaction y^{ijk}, (b) the complex conjugate interaction interaction y_{ijk}, and (c) quartic scalar interaction $y^{ijn}y^*_{kln}$.

The interactions implied by the superpotential eq. (1.137) (with $L^i = 0$) were listed in eqs. (1.111), (1.111), and are shown[i] in Figures 1.3 and 1.4. Those in Figure 1.3 are all determined by the dimensionless parameters y^{ijk}. The Yukawa interaction in Figure 1.3a corresponds to the next-to-last term in eq. (1.111). For each particular Yukawa coupling of $\phi_i\psi_j\psi_k$ with strength y^{ijk}, there must be equal couplings of $\phi_j\psi_i\psi_k$ and $\phi_k\psi_i\psi_j$, since y^{ijk} is completely symmetric under interchange of any two of its indices as shown in section 1.3.2. The arrows on the fermion and scalar lines point in the direction for propagation of ϕ and ψ and opposite the direction of propagation of ϕ^* and $\psi^\dagger$. Thus there is also a vertex corresponding to

[i]Here, the auxiliary fields have been eliminated using their equations of motion ("integrated out"). One could instead give Feynman rules that include the auxiliary fields, or directly in terms of superfields on superspace, although this is usually less useful in practical phenomenological applications.

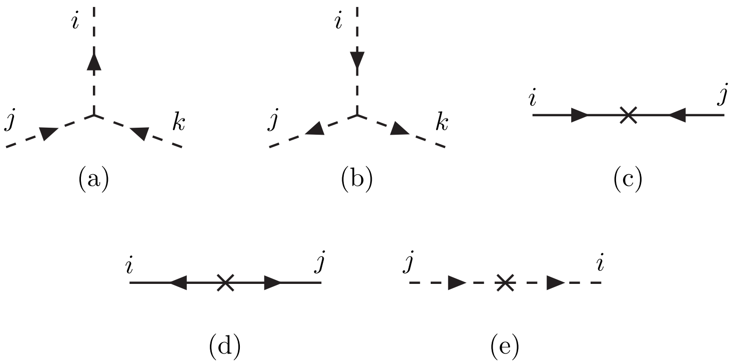

Fig. 1.4. Supersymmetric dimensionful couplings: (a) (scalar)^3 interaction vertex $M^*_{in} y^{jkn}$ and (b) the conjugate interaction $M^{in} y^*_{jkn}$, (c) fermion mass term M^{ij} and (d) conjugate fermion mass term M^*_{ij}, and (e) scalar squared-mass term $M^*_{ik} M^{kj}$.

the one in Figure 1.3a but with all arrows reversed, corresponding to the complex conjugate [the last term in eq. (1.111)]. It is shown in Figure 1.3b. There is also a dimensionless coupling for $\phi_i \phi_j \phi^{*k} \phi^{*l}$, with strength $y^{ijn} y^*_{kln}$, as required by supersymmetry [see the last term in eq. (1.111)]. The relationship between the Yukawa interactions in Figures 1.3a,b and the scalar interaction of Figure 1.3c is exactly of the special type needed to cancel the quadratic divergences in quantum corrections to scalar masses, as discussed in the Introduction [compare Figure 1.1, and eq. (1.11)].

Figure 1.4 shows the only interactions corresponding to renormalizable and supersymmetric vertices with coupling dimensions of [mass] and [mass]^2. First, there are (scalar)^3 couplings in Figure 1.4a,b, which are entirely determined by the superpotential mass parameters M^{ij} and Yukawa couplings y^{ijk}, as indicated by the second and third terms in eq. (1.111). The propagators of the fermions and scalars in the theory are constructed in the usual way using the fermion mass M^{ij} and scalar squared mass $M^*_{ik} M^{kj}$. The fermion mass terms M^{ij} and M_{ij} each lead to a chirality-changing insertion in the fermion propagator; note the directions of the arrows in Figure 1.4c,d. There is no such arrow-reversal for a scalar propagator in a theory with exact supersymmetry; as depicted in Figure 1.4e, if one treats the scalar squared-mass term as an insertion in the propagator, the arrow direction is preserved.

Figure 1.5 shows the gauge interactions in a supersymmetric theory. Figures 1.5a, b, c occur only when the gauge group is non-Abelian, for example for $SU(3)_C$ color and $SU(2)_L$ weak isospin in the MSSM. Figures 1.5a

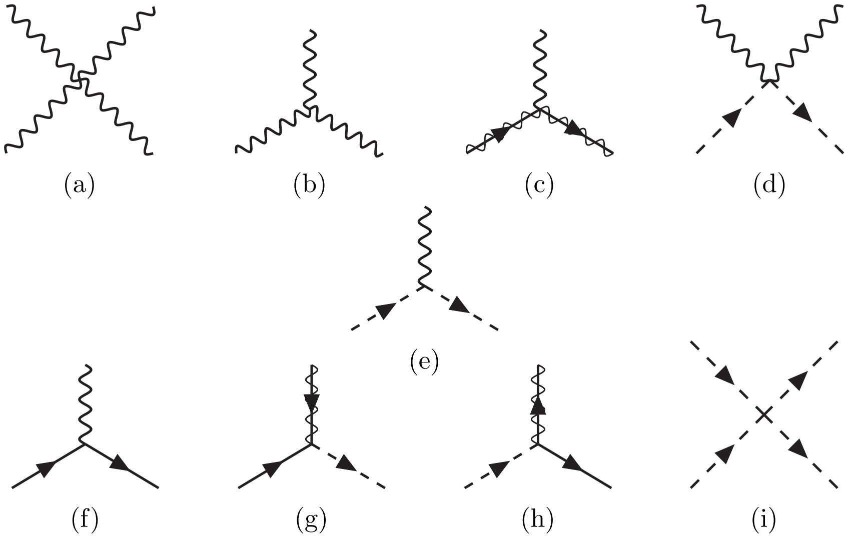

Fig. 1.5. Supersymmetric gauge interaction vertices.

and 1.5b are the interactions of gauge bosons, which derive from the first term in eq. (1.117). In the MSSM these are exactly the same as the well-known QCD gluon and electroweak gauge boson vertices of the Standard Model. (We do not show the interactions of ghost fields, which are necessary only for consistent loop amplitudes.) Figures 1.5c,d,e,f are just the standard interactions between gauge bosons and fermion and scalar fields that must occur in any gauge theory because of the form of the covariant derivative; they come from eqs. (1.119) and (1.125)-(1.127) inserted in the kinetic part of the Lagrangian. Figure 1.5c shows the coupling of a gaugino to a gauge boson; the gaugino line in a Feynman diagram is traditionally drawn as a solid fermion line superimposed on a wavy line. In Figure 1.5g we have the coupling of a gaugino to a chiral fermion and a complex scalar [the first term in the second line of eq. (1.132)]. One can think of this as the "supersymmetrization" of Figure 1.5e or 1.5f; any of these three vertices may be obtained from any other (up to a factor of $\sqrt{2}$) by replacing two of the particles by their supersymmetric partners. There is also an interaction in Figure 1.5h which is just like Figure 1.5g but with all arrows reversed, corresponding to the complex conjugate term in the Lagrangian [the second term in the second line in eq. (1.132)]. Finally in Figure 1.5i we have a scalar quartic interaction vertex [the last term in eq. (1.135)], which is also determined by the gauge coupling.

The results of this section can be used as a recipe for constructing the supersymmetric interactions for any model. In the case of the MSSM, we already know the gauge group, particle content and the gauge transformation properties, so it only remains to decide on the superpotential. This we will do in section 1.5.1.

1.4. Soft Supersymmetry Breaking Interactions

A realistic phenomenological model must contain supersymmetry breaking. From a theoretical perspective, we expect that supersymmetry, if it exists at all, should be an exact symmetry that is broken spontaneously. In other words, the underlying model should have a Lagrangian density that is invariant under supersymmetry, but a vacuum state that is not. In this way, supersymmetry is hidden at low energies in a manner analogous to the fate of the electroweak symmetry in the ordinary Standard Model.

Many models of spontaneous symmetry breaking have indeed been proposed and we will mention the basic ideas of some of them in section 1.6. These always involve extending the MSSM to include new particles and interactions at very high mass scales, and there is no consensus on exactly how this should be done. However, from a practical point of view, it is extremely useful to simply parameterize our ignorance of these issues by just introducing extra terms that break supersymmetry explicitly in the effective MSSM Lagrangian. As was argued in the Introduction, the supersymmetry-breaking couplings should be soft (of positive mass dimension) in order to be able to naturally maintain a hierarchy between the electroweak scale and the Planck (or any other very large) mass scale. This means in particular that dimensionless supersymmetry-breaking couplings should be absent.

The possible soft supersymmetry-breaking terms in the Lagrangian of a general theory are

$$\begin{aligned}\mathcal{L}_{\text{soft}} = &-\left(\frac{1}{2}M_a\,\lambda^a\lambda^a + \frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + t^i\phi_i\right) + \text{c.c.}\\ &-(m^2)^i_j\phi^{j*}\phi_i, \end{aligned} \tag{1.138}$$

$$\mathcal{L}_{\text{maybe soft}} = -\frac{1}{2}c_i^{jk}\phi^{*i}\phi_j\phi_k + \text{c.c.} \tag{1.139}$$

They consist of gaugino masses M_a for each gauge group, scalar squared-mass terms $(m^2)_i^j$ and b^{ij}, and (scalar)^3 couplings a^{ijk} and c_i^{jk}, and "tadpole" couplings t^i. The last of these can only occur if ϕ_i is a gauge singlet, and so is absent from the MSSM. One might wonder why we have not included possible soft mass terms for the chiral supermultiplet fermions, like $\mathcal{L} = -\frac{1}{2}m^{ij}\psi_i\psi_j + \text{c.c.}$ Including such terms would be redundant; they can always be absorbed into a redefinition of the superpotential and the terms $(m^2)^i_j$ and c^{jk}_i.

It has been shown rigorously that a softly broken supersymmetric theory with $\mathcal{L}_{\text{soft}}$ as given by eq. (1.138) is indeed free of quadratic divergences in quantum corrections to scalar masses, to all orders in perturbation theory.[53] The situation is slightly more subtle if one tries to include the non-analytic (scalar)^3 couplings in $\mathcal{L}_{\text{maybe soft}}$. If any of the chiral supermultiplets in the theory are singlets under all gauge symmetries, then non-zero c^{jk}_i terms can lead to quadratic divergences, despite the fact that they are formally soft. Now, this constraint need not apply to the MSSM, which does not have any gauge-singlet chiral supermultiplets. Nevertheless, the possibility of c^{jk}_i terms is nearly always neglected. The real reason for this is that it is difficult to construct models of spontaneous supersymmetry breaking in which the c^{jk}_i are not negligibly small. In the special case of a theory that has chiral supermultiplets that are singlets or in the adjoint representation of a simple factor of the gauge group, then there are also possible soft supersymmetry-breaking Dirac mass terms between the corresponding fermions ψ_a and the gauginos:[54–59]

$$\mathcal{L} = -M^a_{\text{Dirac}}\lambda^a\psi_a + \text{c.c.} \tag{1.140}$$

This is not relevant for the MSSM with minimal field content, which does not have adjoint representation chiral supermultiplets. Therefore, equation (1.138) is usually taken to be the general form of the soft supersymmetry-breaking Lagrangian. For some interesting exceptions, see Refs. 54–65.

The terms in $\mathcal{L}_{\text{soft}}$ clearly do break supersymmetry, because they involve only scalars and gauginos and not their respective superpartners. In fact, the soft terms in $\mathcal{L}_{\text{soft}}$ are capable of giving masses to all of the scalars and gauginos in a theory, even if the gauge bosons and fermions in chiral supermultiplets are massless (or relatively light). The gaugino masses M_a are always allowed by gauge symmetry. The $(m^2)^i_j$ terms are allowed for i, j such that ϕ_i, ϕ^{j*} transform in complex conjugate representations of each other under all gauge symmetries; in particular this is true of course when $i = j$, so every scalar is eligible to get a mass in this way if supersymmetry is broken. The remaining soft terms may or may not be allowed by the symmetries. The a^{ijk}, b^{ij}, and t^i terms have the same form as the y^{ijk}, M^{ij}, and L^i terms in the superpotential [compare eq. (1.138) to eq. (1.107)

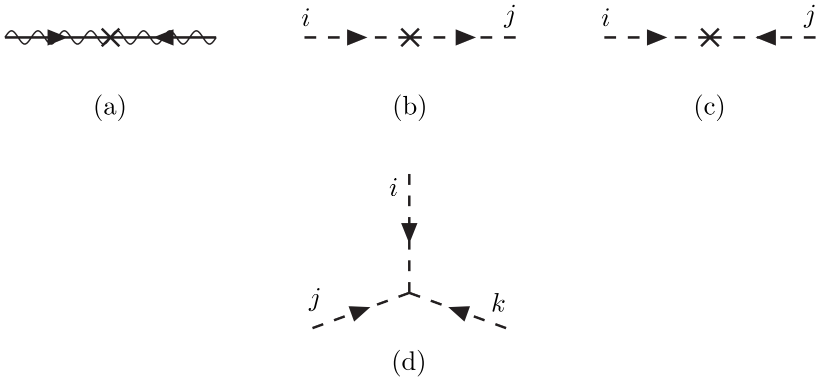

Fig. 1.6. Soft supersymmetry-breaking terms: (a) Gaugino mass M_a; (b) non-analytic scalar squared mass $(m^2)^i_j$; (c) analytic scalar squared mass b^{ij}; and (d) scalar cubic coupling a^{ijk}.

or eq. (1.137)], so they will each be allowed by gauge invariance if and only if a corresponding superpotential term is allowed.

The Feynman diagram interactions corresponding to the allowed soft terms in eq. (1.138) are shown in Figure 1.6. For each of the interactions in Figures 1.6a,c,d there is another with all arrows reversed, corresponding to the complex conjugate term in the Lagrangian. We will apply these general results to the specific case of the MSSM in the next section.

1.5. The Minimal Supersymmetric Standard Model

In sections 1.3 and 1.4, we have found a general recipe for constructing Lagrangians for softly broken supersymmetric theories. We are now ready to apply these general results to the MSSM. The particle content for the MSSM was described in the Introduction. In this section we will complete the model by specifying the superpotential and the soft supersymmetry-breaking terms.

1.5.1. *The superpotential and supersymmetric interactions*

The superpotential for the MSSM is

$$W_{\text{MSSM}} = \overline{u}\mathbf{y_u}QH_u - \overline{d}\mathbf{y_d}QH_d - \overline{e}\mathbf{y_e}LH_d + \mu H_u H_d\,. \qquad (1.141)$$

The objects H_u, H_d, Q, L, $\overline{u}$, $\overline{d}$, $\overline{e}$ appearing here are chiral superfields corresponding to the chiral supermultiplets in Table 1.1. (Alternatively, they can be just thought of as the corresponding scalar fields, as was done in section 1.3, but we prefer not to put the tildes on Q, L, $\overline{u}$, $\overline{d}$, $\overline{e}$ in order to reduce clutter.) The dimensionless Yukawa coupling parameters $\mathbf{y_u}, \mathbf{y_d}, \mathbf{y_e}$ are $3{\times}3$ matrices in family space. All of the gauge [$SU(3)_C$ color and $SU(2)_L$ weak isospin] and family indices in eq. (1.141) are suppressed. The "μ term", as it is traditionally called, can be written out as $\mu(H_u)_\alpha(H_d)_\beta\epsilon^{\alpha\beta}$, where $\epsilon^{\alpha\beta}$ is used to tie together $SU(2)_L$ weak isospin indices $\alpha, \beta = 1, 2$ in a gauge-invariant way. Likewise, the term $\overline{u}\mathbf{y_u}QH_u$ can be written out as $\overline{u}^{ia}\,(\mathbf{y_u})_i{}^j\,Q_{j\alpha a}\,(H_u)_\beta\epsilon^{\alpha\beta}$, where $i = 1, 2, 3$ is a family index, and $a = 1, 2, 3$ is a color index which is lowered (raised) in the $\mathbf{3}$ ($\overline{\mathbf{3}}$) representation of $SU(3)_C$.

The μ term in eq. (1.141) is the supersymmetric version of the Higgs boson mass in the Standard Model. It is unique, because terms $H_u^* H_u$ or $H_d^* H_d$ are forbidden in the superpotential, which must be analytic in the chiral superfields (or equivalently in the scalar fields) treated as complex variables, as shown in section 1.3.2. We can also see from the form of eq. (1.141) why both H_u and H_d are needed in order to give Yukawa couplings, and thus masses, to all of the quarks and leptons. Since the superpotential must be analytic, the $\overline{u}QH_u$ Yukawa terms cannot be replaced by something like $\overline{u}QH_d^*$. Similarly, the $\overline{d}QH_d$ and $\overline{e}LH_d$ terms cannot be replaced by something like $\overline{d}QH_u^*$ and $\overline{e}LH_u^*$. The analogous Yukawa couplings would be allowed in a general non-supersymmetric two Higgs doublet model, but are forbidden by the structure of supersymmetry. So we need both H_u and H_d, even without invoking the argument based on anomaly cancellation mentioned in the Introduction.

The Yukawa matrices determine the current masses and CKM mixing angles of the ordinary quarks and leptons, after the neutral scalar components of H_u and H_d get VEVs. Since the top quark, bottom quark and tau lepton are the heaviest fermions in the Standard Model, it is often useful to make an approximation that only the $(3, 3)$ family components of each of $\mathbf{y_u}$, $\mathbf{y_d}$ and $\mathbf{y_e}$ are important:

$$\mathbf{y_u} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix}, \quad \mathbf{y_d} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}, \quad \mathbf{y_e} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix}. \tag{1.142}$$

In this limit, only the third family and Higgs fields contribute to the MSSM superpotential. It is instructive to write the superpotential in terms of the separate $SU(2)_L$ weak isospin components $[Q_3 = (t\, b)$, $L_3 = (\nu_\tau\, \tau)$, $H_u = (H_u^+\, H_u^0)$, $H_d = (H_d^0\, H_d^-)$, $\overline{u}_3 = \overline{t}$, $\overline{d}_3 = \overline{b}$, $\overline{e}_3 = \overline{\tau}]$, so:

$$W_{\rm MSSM} \approx y_t(\overline{t}tH_u^0 - \overline{t}bH_u^+) - y_b(\overline{b}tH_d^- - \overline{b}bH_d^0) - y_\tau(\overline{\tau}\nu_\tau H_d^- - \overline{\tau}\tau H_d^0) \\ +\mu(H_u^+ H_d^- - H_u^0 H_d^0). \tag{1.143}$$

The minus signs inside the parentheses appear because of the antisymmetry of the $\epsilon^{\alpha\beta}$ symbol used to tie up the $SU(2)_L$ indices. The other minus signs in eq. (1.141) were chosen so that the terms $y_t\overline{t}tH_u^0$, $y_b\overline{b}bH_d^0$, and $y_\tau\overline{\tau}\tau H_d^0$, which will become the top, bottom and tau masses when H_u^0 and H_d^0 get VEVs, each have overall positive signs in eq. (1.143).

Since the Yukawa interactions y^{ijk} in a general supersymmetric theory must be completely symmetric under interchange of i, j, k, we know that $\mathbf{y_u}$, $\mathbf{y_d}$ and $\mathbf{y_e}$ imply not only Higgs-quark-quark and Higgs-lepton-lepton couplings as in the Standard Model, but also squark-Higgsino-quark and slepton-Higgsino-lepton interactions. To illustrate this, Figures 1.7a,b,c show some of the interactions involving the top-quark Yukawa coupling y_t.

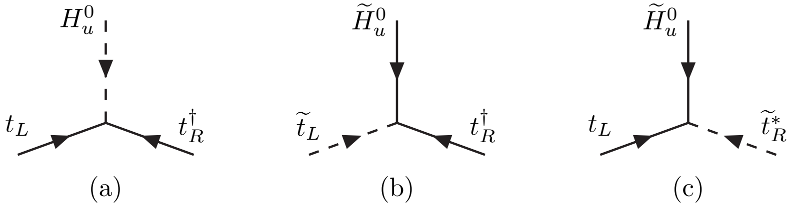

Fig. 1.7. The top-quark Yukawa coupling (a) and its "supersymmetrizations" (b), (c), all of strength y_t.

Figure 1.7a is the Standard Model-like coupling of the top quark to the neutral complex scalar Higgs boson, which follows from the first term in eq. (1.143). For variety, we have used t_L and $t_R^\dagger$ in place of their synonyms t and $\overline{t}$ (see the discussion near the end of section 1.2). In Figure 1.7b, we have the coupling of the left-handed top squark $\widetilde{t}_L$ to the neutral higgsino field $\widetilde{H}_u^0$ and right-handed top quark, while in Figure 1.7c the right-handed top anti-squark field (known either as $\widetilde{\overline{t}}$ or $\widetilde{t}_R^*$ depending on taste) couples to $\widetilde{H}_u^0$ and t_L. For each of the three interactions, there is another with $H_u^0 \to H_u^+$ and $t_L \to -b_L$ (with tildes where appropriate), corresponding to the second part of the first term in eq. (1.143). All of these interactions are required by supersymmetry to have the same strength y_t. These couplings are dimensionless and can be modified by the introduction of soft supersymmetry breaking only through finite (and small) radiative correc-

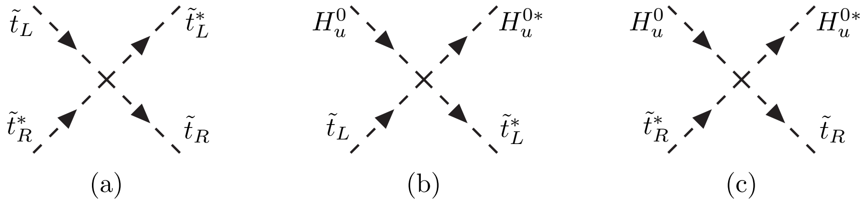

Fig. 1.8. Some of the (scalar)^4 interactions with strength proportional to y_t^2.

tions, so this equality of interaction strengths is also a prediction of softly broken supersymmetry. A useful mnemonic is that each of Figures 1.7a,b,c can be obtained from any of the others by changing two of the particles into their superpartners.

There are also scalar quartic interactions with strength proportional to y_t^2, as can be seen from Figure 1.3c or the last term in eq. (1.111). Three of them are shown in Figure 1.8. Using eq. (1.111) and eq. (1.143), one can see that there are five more, which can be obtained by replacing $\widetilde{t}_L \to \widetilde{b}_L$ and/or $H_u^0 \to H_u^+$ in each vertex. This illustrates the remarkable economy of supersymmetry; there are many interactions determined by only a single parameter. In a similar way, the existence of all the other quark and lepton Yukawa couplings in the superpotential eq. (1.141) leads not only to Higgs-quark-quark and Higgs-lepton-lepton Lagrangian terms as in the ordinary Standard Model, but also to squark-higgsino-quark and slepton-higgsino-lepton terms, and scalar quartic couplings [(squark)^4, (slepton)^4, (squark)^2(slepton)^2, (squark)^2(Higgs)^2, and (slepton)^2(Higgs)^2]. If needed, these can all be obtained in terms of the Yukawa matrices $\mathbf{y_u}$, $\mathbf{y_d}$, and $\mathbf{y_e}$ as outlined above.

However, the dimensionless interactions determined by the superpotential are usually not the most important ones of direct interest for phenomenology. This is because the Yukawa couplings are already known to be very small, except for those of the third family (top, bottom, tau). Instead, production and decay processes for superpartners in the MSSM are typically dominated by the supersymmetric interactions of gauge-coupling strength, as we will explore in more detail in sections 1.8. The couplings of the Standard Model gauge bosons (photon, $W^\pm$, Z^0 and gluons) to the MSSM particles are determined completely by the gauge invariance of the kinetic terms in the Lagrangian. The gauginos also couple to (squark, quark) and (slepton, lepton) and (Higgs, higgsino) pairs as illustrated in the general case in Figure 1.5g,h and the first two terms in the second line in eq. (1.132). For instance, each of the squark-quark-gluino couplings is

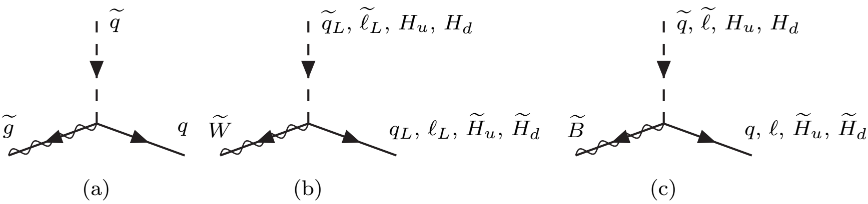

Fig. 1.9. Couplings of the gluino, wino, and bino to MSSM (scalar, fermion) pairs.

given by $\sqrt{2}g_3(\widetilde{q}T^a q\widetilde{g} + \text{c.c.})$ where $T^a = \lambda^a/2$ $(a = 1\ldots8)$ are the matrix generators for $SU(3)_C$. The Feynman diagram for this interaction is shown in Figure 1.9a. In Figures 1.9b,c we show in a similar way the couplings of (squark, quark), (lepton, slepton) and (Higgs, higgsino) pairs to the winos and bino, with strengths proportional to the electroweak gauge couplings g and g' respectively. For each of these diagrams, there is another with all arrows reversed. Note that the winos only couple to the left-handed squarks and sleptons, and the (lepton, slepton) and (Higgs, higgsino) pairs of course do not couple to the gluino. The bino coupling to each (scalar, fermion) pair is also proportional to the weak hypercharge Y as given in Table 1.1. The interactions shown in Figure 1.9 provide, for example, for decays $\widetilde{q} \to q\widetilde{g}$ and $\widetilde{q} \to \widetilde{W}q'$ and $\widetilde{q} \to \widetilde{B}q$ when the final states are kinematically allowed to be on-shell. However, a complication is that the $\widetilde{W}$ and $\widetilde{B}$ states are not mass eigenstates, because of splitting and mixing due to electroweak symmetry breaking, as we will see in section 1.7.2.

There are also various scalar quartic interactions in the MSSM that are uniquely determined by gauge invariance and supersymmetry, according to the last term in eq. (1.135), as illustrated in Figure 1.5i. Among them are (Higgs)^4 terms proportional to g^2 and g'^2 in the scalar potential. These are the direct generalization of the last term in the Standard Model Higgs potential, eq. (1.1), to the case of the MSSM. We will have occasion to identify them explicitly when we discuss the minimization of the MSSM Higgs potential in section 1.7.1.

The dimensionful couplings in the supersymmetric part of the MSSM Lagrangian are all dependent on μ. Using the general result of eq. (1.111), μ provides for higgsino fermion mass terms

$$-\mathcal{L}_{\text{higgsino mass}} = \mu(\widetilde{H}_u^+\widetilde{H}_d^- - \widetilde{H}_u^0\widetilde{H}_d^0) + \text{c.c.}, \tag{1.144}$$

as well as Higgs squared-mass terms in the scalar potential

$$-\mathcal{L}_{\text{supersymmetric Higgs mass}} = |\mu|^2\big(|H_u^0|^2 + |H_u^+|^2 + |H_d^0|^2 + |H_d^-|^2\big). \tag{1.145}$$

Since eq. (1.145) is non-negative with a minimum at $H_u^0 = H_d^0 = 0$, we cannot understand electroweak symmetry breaking without including a negative supersymmetry-breaking squared-mass soft term for the Higgs scalars. An explicit treatment of the Higgs scalar potential will therefore have to wait until we have introduced the soft terms for the MSSM. However, we can already see a puzzle: we expect that μ should be roughly of order 10^2 or 10^3 GeV, in order to allow a Higgs VEV of order 174 GeV without too much miraculous cancellation between $|\mu|^2$ and the negative soft squared-mass terms that we have not written down yet. But why should $|\mu|^2$ be so small compared to, say, $M_{\rm P}^2$, and in particular why should it be roughly of the same order as $m^2_{\rm soft}$? The scalar potential of the MSSM seems to depend on two types of dimensionful parameters that are conceptually quite distinct, namely the supersymmetry-respecting mass μ and the supersymmetry-breaking soft mass terms. Yet the observed value for the electroweak breaking scale suggests that without miraculous cancellations, both of these apparently unrelated mass scales should be within an order of magnitude or so of 100 GeV. This puzzle is called "the μ problem". Several different solutions to the μ problem have been proposed, involving extensions of the MSSM of varying intricacy. They all work in roughly the same way; the μ term is required or assumed to be absent at tree-level before symmetry breaking, and then it arises from the VEV(s) of some new field(s). These VEVs are in turn determined by minimizing a potential that depends on soft supersymmetry-breaking terms. In this way, the value of the effective parameter μ is no longer conceptually distinct from the mechanism of supersymmetry breaking; if we can explain why $m_{\rm soft} \ll M_{\rm P}$, we will also be able to understand why μ is of the same order. Some other attractive solutions for the μ problem are proposed in Refs. 66–68. From the point of view of the MSSM, however, we can just treat μ as an independent parameter.

The μ-term and the Yukawa couplings in the superpotential eq. (1.141) combine to yield (scalar)^3 couplings [see the second and third terms on the right-hand side of eq. (1.111)] of the form

$$\mathcal{L}_{\text{supersymmetric (scalar)}^3} = \mu^* (\widetilde{\overline{u}} \mathbf{y_u} \widetilde{u} H_d^{0*} + \widetilde{\overline{d}} \mathbf{y_d} \widetilde{d} H_u^{0*} + \widetilde{\overline{e}} \mathbf{y_e} \widetilde{e} H_u^{0*} + \widetilde{\overline{u}} \mathbf{y_u} \widetilde{d} H_d^{-*} + \widetilde{\overline{d}} \mathbf{y_d} \widetilde{u} H_u^{+*} + \widetilde{\overline{e}} \mathbf{y_e} \widetilde{\nu} H_u^{+*}) + \text{c.c.} \qquad (1.146)$$

Figure 1.10 shows some of these couplings, proportional to $\mu^* y_t$, $\mu^* y_b$, and $\mu^* y_\tau$ respectively. These play an important role in determining the mixing of top squarks, bottom squarks, and tau sleptons, as we will see in section 1.7.4.

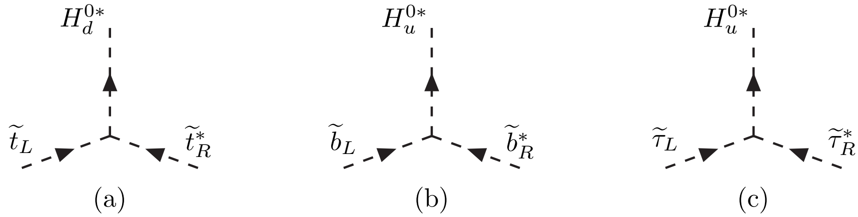

Fig. 1.10. Some of the supersymmetric (scalar)^3 couplings proportional to $\mu^* y_t$, $\mu^* y_b$, and $\mu^* y_\tau$. When H_u^0 and H_d^0 get VEVs, these contribute to (a) $\widetilde{t}_L, \widetilde{t}_R$ mixing, (b) $\widetilde{b}_L, \widetilde{b}_R$ mixing, and (c) $\widetilde{\tau}_L, \widetilde{\tau}_R$ mixing.

1.5.2. *R-parity (also known as matter parity) and its consequences*

The superpotential eq. (1.141) is minimal in the sense that it is sufficient to produce a phenomenologically viable model. However, there are other terms that one can write that are gauge-invariant and analytic in the chiral superfields, but are not included in the MSSM because they violate either baryon number (B) or total lepton number (L). The most general gauge-invariant and renormalizable superpotential would include not only eq. (1.141), but also the terms

$$W_{\Delta \mathrm{L}=1} = \frac{1}{2}\lambda^{ijk} L_i L_j \overline{e}_k + \lambda'^{ijk} L_i Q_j \overline{d}_k + \mu'^i L_i H_u \tag{1.147}$$

$$W_{\Delta \mathrm{B}=1} = \frac{1}{2}\lambda''^{ijk} \overline{u}_i \overline{d}_j \overline{d}_k \tag{1.148}$$

where family indices $i = 1, 2, 3$ have been restored. The chiral supermultiplets carry baryon number assignments $\mathrm{B} = +1/3$ for Q_i; $\mathrm{B} = -1/3$ for $\overline{u}_i, \overline{d}_i$; and $\mathrm{B} = 0$ for all others. The total lepton number assignments are $\mathrm{L} = +1$ for L_i, $\mathrm{L} = -1$ for $\overline{e}_i$, and $\mathrm{L} = 0$ for all others. Therefore, the terms in eq. (1.147) violate total lepton number by 1 unit (as well as the individual lepton flavors) and those in eq. (1.148) violate baryon number by 1 unit.

The possible existence of such terms might seem rather disturbing, since corresponding B- and L-violating processes have not been seen experimentally. The most obvious experimental constraint comes from the non-observation of proton decay, which would violate both B and L by 1 unit. If both λ' and λ'' couplings were present and unsuppressed, then the lifetime of the proton would be extremely short. For example, Feynman

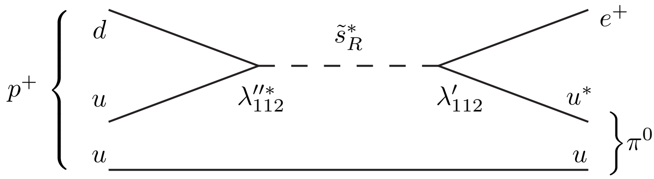

Fig. 1.11. Squarks would mediate disastrously rapid proton decay if R-parity were violated by both $\Delta\mathrm{B} = 1$ and $\Delta\mathrm{L} = 1$ interactions. This example shows $p \to e^+\pi^0$ mediated by a strange (or bottom) squark.

diagrams like the one in Figure 1.11[j] would lead to $p^+ \to e^+\pi^0$ (shown) or e^+K^0 or $\mu^+\pi^0$ or μ^+K^0 or $\nu\pi^+$ or νK^+ etc. depending on which components of λ' and λ'' are largest.[k] As a rough estimate based on dimensional analysis, for example,

$$\Gamma_{p\to e^+\pi^0} \sim m^5_{\rm proton} \sum_{i=2,3} |\lambda'^{11i}\lambda''^{11i}|^2/m^4_{\tilde{d}_i}, \tag{1.149}$$

which would be a tiny fraction of a second if the couplings were of order unity and the squarks have masses of order 1 TeV. In contrast, the decay time of the proton into lepton+meson final states is known experimentally to be in excess of 10^{32} years. Therefore, at least one of λ'^{ijk} or λ''^{11k} for each of $i = 1, 2$; $j = 1, 2$; $k = 2, 3$ must be extremely small. Many other processes also give strong constraints on the violation of lepton and baryon numbers.[69,70]

One could simply try to take B and L conservation as a postulate in the MSSM. However, this is clearly a step backward from the situation in the Standard Model, where the conservation of these quantum numbers is *not* assumed, but is rather a pleasantly "accidental" consequence of the fact that there are no possible renormalizable Lagrangian terms that violate B or L. Furthermore, there is a quite general obstacle to treating B and L as fundamental symmetries of Nature, since they are known to be necessarily violated by non-perturbative electroweak effects[71] (even though those effects are calculably negligible for experiments at ordinary energies). Therefore, in the MSSM one adds a new symmetry, which has the effect

[j]In this diagram and others below, the arrows on propagators are often omitted for simplicity, and external fermion label refer to physical particle states rather than 2-component fermion fields.

[k]The coupling λ'' must be antisymmetric in its last two flavor indices, since the color indices are combined antisymmetrically. That is why the squark in Figure 1.11 can be $\widetilde{s}$ or $\widetilde{b}$, but not $\widetilde{d}$, for u, d quarks in the proton.

of eliminating the possibility of B and L violating terms in the renormalizable superpotential, while allowing the good terms in eq. (1.141). This new symmetry is called "R-parity"[7] or equivalently "matter parity".[72]

Matter parity is a multiplicatively conserved quantum number defined as

$$P_M = (-1)^{3(\mathrm{B-L})} \tag{1.150}$$

for each particle in the theory. It is easy to check that the quark and lepton supermultiplets all have $P_M = -1$, while the Higgs supermultiplets H_u and H_d have $P_M = +1$. The gauge bosons and gauginos of course do not carry baryon number or lepton number, so they are assigned matter parity $P_M = +1$. The symmetry principle to be enforced is that a candidate term in the Lagrangian (or in the superpotential) is allowed only if the product of P_M for all of the fields in it is $+1$. It is easy to see that each of the terms in eqs. (1.147) and (1.148) is thus forbidden, while the good and necessary terms in eq. (1.141) are allowed. This discrete symmetry commutes with supersymmetry, as all members of a given supermultiplet have the same matter parity. The advantage of matter parity is that it can in principle be an *exact* and fundamental symmetry, which B and L themselves cannot, since they are known to be violated by non-perturbative electroweak effects. So even with exact matter parity conservation in the MSSM, one expects that baryon number and total lepton number violation can occur in tiny amounts, due to non-renormalizable terms in the Lagrangian. However, the MSSM does not have renormalizable interactions that violate B or L, with the standard assumption of matter parity conservation.

It is often useful to recast matter parity in terms of R-parity, defined for each particle as

$$P_R = (-1)^{3(\mathrm{B-L})+2s} \tag{1.151}$$

where s is the spin of the particle. Now, matter parity conservation and R-parity conservation are precisely equivalent, since the product of $(-1)^{2s}$ for the particles involved in any interaction vertex in a theory that conserves angular momentum is always equal to $+1$. However, particles within the same supermultiplet do not have the same R-parity. In general, symmetries with the property that fields within the same supermultiplet have different transformations are called R symmetries; they do not commute with supersymmetry. Continuous $U(1)$ R symmetries are often encountered in the model-building literature; they should not be confused with R-parity, which is a discrete Z_2 symmetry. In fact, the matter parity version of R-parity makes clear that there is really nothing intrinsically "R" about it; in other words it secretly does commute with supersymmetry, so its name is somewhat suboptimal. Nevertheless, the R-parity assignment is very useful for phenomenology because all of the Standard Model particles and the Higgs bosons have even R-parity ($P_R = +1$), while all of the squarks, sleptons, gauginos, and higgsinos have odd R-parity ($P_R = -1$).

The R-parity odd particles are known as "supersymmetric particles" or "sparticles" for short, and they are distinguished by a tilde (see Tables 1.1 and 1.2). If R-parity is exactly conserved, then there can be no mixing between the sparticles and the $P_R = +1$ particles. Furthermore, every interaction vertex in the theory contains an even number of $P_R = -1$ sparticles. This has three extremely important phenomenological consequences:

- The lightest sparticle with $P_R = -1$, called the "lightest supersymmetric particle" or LSP, must be absolutely stable. If the LSP is electrically neutral, it interacts only weakly with ordinary matter, and so can make an attractive candidate[73] for the non-baryonic dark matter that seems to be required by cosmology.
- Each sparticle other than the LSP must eventually decay into a state that contains an odd number of LSPs (usually just one).
- In collider experiments, sparticles can only be produced in even numbers (usually two-at-a-time).

We *define* the MSSM to conserve R-parity or equivalently matter parity. While this decision seems to be well-motivated phenomenologically by proton decay constraints and the hope that the LSP will provide a good dark matter candidate, it might appear somewhat artificial from a theoretical point of view. After all, the MSSM would not suffer any internal inconsistency if we did not impose matter parity conservation. Furthermore, it is fair to ask why matter parity should be exactly conserved, given that the discrete symmetries in the Standard Model (ordinary parity P, charge conjugation C, time reversal T, etc.) are all known to be inexact symmetries. Fortunately, it *is* sensible to formulate matter parity as a discrete symmetry that is exactly conserved. In general, exactly conserved, or "gauged" discrete symmetries[74] can exist provided that they satisfy certain anomaly cancellation conditions[75] (much like continuous gauged symmetries). One particularly attractive way this could occur is if B$-$L is a continuous gauge symmetry that is spontaneously broken at some very high energy scale. A continuous $U(1)_{\mathrm{B-L}}$ forbids the renormalizable terms that violate B and L,[76,77] but this gauge symmetry must be spontaneously broken, since there is no corresponding massless vector boson. However, if gauged $U(1)_{\mathrm{B-L}}$ is only broken by scalar VEVs (or other order parameters) that carry even integer values of 3(B−L), then P_M will automatically survive as an exactly conserved discrete remnant subgroup.[77] A variety of extensions of the MSSM in which exact R-parity conservation is guaranteed in just this way have been proposed (see for example[77,78]).

It may also be possible to have gauged discrete symmetries that do not owe their exact conservation to an underlying continuous gauged symmetry, but rather to some other structure such as can occur in string theory. It is also possible that R-parity is broken, or is replaced by some alternative discrete symmetry.

1.5.3. *Soft supersymmetry breaking in the MSSM*

To complete the description of the MSSM, we need to specify the soft supersymmetry breaking terms. In section 1.4, we learned how to write down the most general set of such terms in any supersymmetric theory. Applying this recipe to the MSSM, we have:

$$\begin{aligned} \mathcal{L}_{\rm soft}^{\rm MSSM} = & -\frac{1}{2}\left(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B} + \text{c.c.}\right) \\ & - \left(\widetilde{\overline{u}}\,\mathbf{a_u}\,\widetilde{Q}H_u - \widetilde{\overline{d}}\,\mathbf{a_d}\,\widetilde{Q}H_d - \widetilde{\overline{e}}\,\mathbf{a_e}\,\widetilde{L}H_d + \text{c.c.}\right) \\ & -\widetilde{Q}^\dagger\,\mathbf{m^2_Q}\,\widetilde{Q} - \widetilde{L}^\dagger\,\mathbf{m^2_L}\,\widetilde{L} - \widetilde{\overline{u}}\,\mathbf{m^2_{\overline{u}}}\,\widetilde{\overline{u}}^\dagger - \widetilde{\overline{d}}\,\mathbf{m^2_{\overline{d}}}\,\widetilde{\overline{d}}^\dagger - \widetilde{\overline{e}}\,\mathbf{m^2_{\overline{e}}}\,\widetilde{\overline{e}}^\dagger \\ & - m^2_{H_u}H_u^*H_u - m^2_{H_d}H_d^*H_d - (bH_uH_d + \text{c.c.})\,. \end{aligned} \tag{1.152}$$

In eq. (1.152), M_3, M_2, and M_1 are the gluino, wino, and bino mass terms. Here, and from now on, we suppress the adjoint representation gauge indices on the wino and gluino fields, and the gauge indices on all of the chiral supermultiplet fields. The second line in eq. (1.152) contains the (scalar)^3 couplings [of the type a^{ijk} in eq. (1.138)]. Each of $\mathbf{a_u}$, $\mathbf{a_d}$, $\mathbf{a_e}$ is a complex 3×3 matrix in family space, with dimensions of [mass]. They are in one-to-one correspondence with the Yukawa couplings of the superpotential. The third line of eq. (1.152) consists of squark and slepton mass terms of the $(m^2)_i^j$ type in eq. (1.138). Each of $\mathbf{m^2_Q}$, $\mathbf{m^2_{\overline{u}}}$, $\mathbf{m^2_{\overline{d}}}$, $\mathbf{m^2_L}$, $\mathbf{m^2_{\overline{e}}}$ is a 3×3 matrix in family space that can have complex entries, but they must be hermitian so that the Lagrangian is real. (To avoid clutter, we do not put tildes on the $\mathbf{Q}$ in $\mathbf{m^2_Q}$, etc.) Finally, in the last line of eq. (1.152) we have supersymmetry-breaking contributions to the Higgs potential; $m^2_{H_u}$ and $m^2_{H_d}$ are squared-mass terms of the $(m^2)_i^j$ type, while b is the only squared-mass term of the type b^{ij} in eq. (1.138) that can occur in the MSSM.[1] As argued in the Introduction, we expect

$$M_1,\, M_2,\, M_3,\, \mathbf{a_u},\, \mathbf{a_d},\, \mathbf{a_e} \sim m_{\rm soft}, \tag{1.153}$$
$$\mathbf{m^2_Q},\, \mathbf{m^2_L},\, \mathbf{m^2_{\overline{u}}},\, \mathbf{m^2_{\overline{d}}},\, \mathbf{m^2_{\overline{e}}},\, m^2_{H_u},\, m^2_{H_d},\, b \sim m^2_{\rm soft}, \tag{1.154}$$

with a characteristic mass scale $m_{\rm soft}$ that is not much larger than 1000 GeV. The expression eq. (1.152) is the most general soft supersymmetry-breaking Lagrangian of the form eq. (1.138) that is compatible with gauge invariance and matter parity conservation in the MSSM.

Unlike the supersymmetry-preserving part of the Lagrangian, the above $\mathcal{L}^{\rm MSSM}_{\rm soft}$ introduces many new parameters that were not present in the ordinary Standard Model. A careful count[79] reveals that there are 105 masses, phases and mixing angles in the MSSM Lagrangian that cannot be rotated away by redefining the phases and flavor basis for the quark and lepton supermultiplets, and that have no counterpart in the ordinary Standard Model. Thus, in principle, supersymmetry *breaking* (as opposed to supersymmetry itself) appears to introduce a tremendous arbitrariness in the Lagrangian.

1.5.4. *Hints of an organizing principle*

Fortunately, there is already good experimental evidence that some powerful organizing principle must govern the soft supersymmetry breaking Lagrangian. This is because most of the new parameters in eq. (1.152) imply flavor mixing or CP violating processes of the types that are severely restricted by experiment.[80–105]

For example, suppose that $\mathbf{m^2_{\overline{e}}}$ is not diagonal in the basis $(\widetilde{e}_R, \widetilde{\mu}_R, \widetilde{\tau}_R)$ of sleptons whose superpartners are the right-handed parts of the Standard Model mass eigenstates e, μ, τ. In that case, slepton mixing occurs, so the individual lepton numbers will not be conserved, even for processes that only involve the sleptons as virtual particles. A particularly strong limit on this possibility comes from the experimental bound on the process $\mu \rightarrow e\gamma$, which could arise from the one-loop diagram shown in Figure 1.12a. The symbol "×" on the slepton line represents an insertion coming from $-(\mathbf{m^2_{\overline{e}}})_{21}\widetilde{\mu}^*_R\widetilde{e}_R$ in $\mathcal{L}^{\rm MSSM}_{\rm soft}$, and the slepton-bino vertices are determined by the weak hypercharge gauge coupling [see Figures 1.5g,h and eq. (1.132)].

[1]The parameter called b here is often seen elsewhere as $B\mu$ or m^2_{12} or m^2_3.

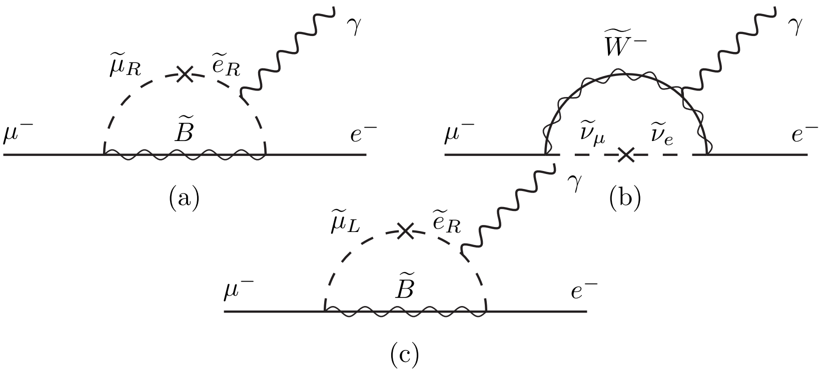

Fig. 1.12. Some of the diagrams that contribute to the process $\mu^- \to e^-\gamma$ in models with lepton flavor-violating soft supersymmetry breaking parameters (indicated by $\times$). Diagrams (a), (b), and (c) contribute to constraints on the off-diagonal elements of $\mathbf{m^2_{\bar{e}}}$, $\mathbf{m^2_L}$, and $\mathbf{a_e}$, respectively.

The result of calculating this diagram gives,[82,85] approximately,

$$\mathrm{Br}(\mu \to e\gamma) = \left(\frac{|m^2_{\tilde{\mu}^*_R \tilde{e}_R}|}{m^2_{\tilde{\ell}_R}}\right)^2 \left(\frac{100\,\mathrm{GeV}}{m_{\tilde{\ell}_R}}\right)^4 10^{-6} \times \begin{cases} 15 & \text{for } m_{\tilde{B}} \ll m_{\tilde{\ell}_R}, \\ 5.6 & \text{for } m_{\tilde{B}} = 0.5 m_{\tilde{\ell}_R}, \\ 1.4 & \text{for } m_{\tilde{B}} = m_{\tilde{\ell}_R}, \\ 0.13 & \text{for } m_{\tilde{B}} = 2 m_{\tilde{\ell}_R}, \end{cases} \tag{1.155}$$

where it is assumed for simplicity that both $\tilde{e}_R$ and $\tilde{\mu}_R$ are nearly mass eigenstates with almost degenerate squared masses $m^2_{\tilde{\ell}_R}$, that $m^2_{\tilde{\mu}^*_R \tilde{e}_R} \equiv (\mathbf{m^2_{\bar{e}}})_{21} = [(\mathbf{m^2_{\bar{e}}})_{12}]^*$ can be treated as a perturbation, and that the bino $\widetilde{B}$ is nearly a mass eigenstate. This result is to be compared to the present experimental upper limit $\mathrm{Br}(\mu \to e\gamma)_{\mathrm{exp}} < 1.2 \times 10^{-11}$ from.[106] So, if the right-handed slepton squared-mass matrix $\mathbf{m^2_{\bar{e}}}$ were "random", with all entries of comparable size, then the prediction for $\mathrm{Br}(\mu \to e\gamma)$ would be too large even if the sleptons and bino masses were at 1 TeV. For lighter superpartners, the constraint on $\tilde{\mu}_R, \tilde{e}_R$ squared-mass mixing becomes correspondingly more severe. There are also contributions to $\mu \to e\gamma$ that depend on the off-diagonal elements of the left-handed slepton squared-mass matrix $\mathbf{m^2_L}$, coming from the diagram shown in Figure 1.12b involving the charged wino and the sneutrinos, as well as diagrams just like Figure 1.12a but with left-handed sleptons and either $\widetilde{B}$ or $\widetilde{W}^0$ exchanged. Therefore, the slepton squared-mass matrices must not have significant mixings for $\widetilde{e}_L, \widetilde{\mu}_L$ either.

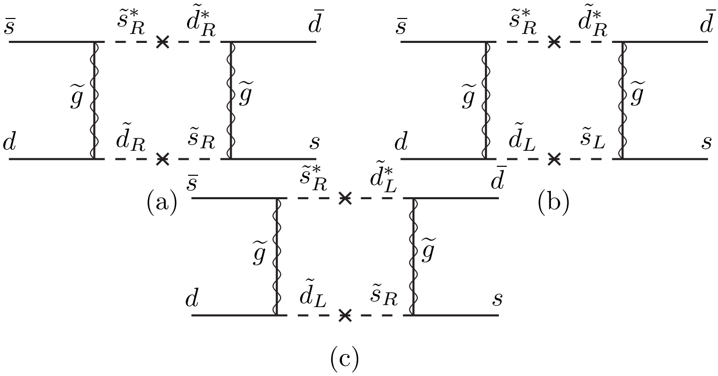

Fig. 1.13. Some of the diagrams that contribute to $K^0 \leftrightarrow \overline{K}^0$ mixing in models with strangeness-violating soft supersymmetry breaking parameters (indicated by $\times$). These diagrams contribute to constraints on the off-diagonal elements of (a) $\mathbf{m^2_{\overline{d}}}$, (b) the combination of $\mathbf{m^2_{\overline{d}}}$ and $\mathbf{m^2_Q}$, and (c) $\mathbf{a_d}$.

Furthermore, after the Higgs scalars get VEVs, the $\mathbf{a_e}$ matrix could imply squared-mass terms that mix left-handed and right-handed sleptons with different lepton flavors. For example, $\mathcal{L}^{\rm MSSM}_{\rm soft}$ contains $\widetilde{\bar{e}}\mathbf{a_e}\widetilde{L}H_d + $ c.c. which implies terms $-\langle H_d^0\rangle(\mathbf{a_e})_{12}\widetilde{e}^*_R\widetilde{\mu}_L - \langle H_d^0\rangle(\mathbf{a_e})_{21}\widetilde{\mu}^*_R\widetilde{e}_L + $ c.c. These also contribute to $\mu \to e\gamma$, as illustrated in Figure 1.12c. So the magnitudes of $(\mathbf{a_e})_{12}$ and $(\mathbf{a_e})_{21}$ are also constrained by experiment to be small, but in a way that is more strongly dependent on other model parameters.[85] Similarly, $(\mathbf{a_e})_{13}, (\mathbf{a_e})_{31}$ and $(\mathbf{a_e})_{23}, (\mathbf{a_e})_{32}$ are constrained, although more weakly,[86] by the experimental limits on $\mathrm{Br}(\tau \to e\gamma)$ and $\mathrm{Br}(\tau \to \mu\gamma)$.

There are also important experimental constraints on the squark squared-mass matrices. The strongest of these come from the neutral kaon system. The effective Hamiltonian for $K^0 \leftrightarrow \overline{K}^0$ mixing gets contributions from the diagrams in Figure 1.13, among others, if $\mathcal{L}^{\rm MSSM}_{\rm soft}$ contains terms that mix down squarks and strange squarks. The gluino-squark-quark vertices in Figure 1.13 are all fixed by supersymmetry to be of QCD interaction strength. (There are similar diagrams in which the bino and winos are exchanged, which can be important depending on the relative sizes of the gaugino masses.) For example, suppose that there is a non-zero right-handed down-squark squared-mass mixing $(\mathbf{m^2_{\overline{d}}})_{21}$ in the basis corresponding to the quark mass eigenstates. Assuming that the supersymmetric correction to $\Delta m_K \equiv m_{K_L} - m_{K_S}$ following from Figure 1.13a and others does not exceed, in absolute value, the experimental value 3.5×10^{-12} MeV, Ref. 95 obtains:

$$\frac{|\mathrm{Re}[(m^2_{\tilde{s}^*_R \tilde{d}_R})^2]|^{1/2}}{m^2_{\tilde{q}}} < \left(\frac{m_{\tilde{q}}}{500\,\mathrm{GeV}}\right) \times \begin{cases} 0.02 \text{ for } m_{\tilde{g}} = 0.5 m_{\tilde{q}}, \\ 0.05 \text{ for } m_{\tilde{g}} = m_{\tilde{q}}, \\ 0.11 \text{ for } m_{\tilde{g}} = 2 m_{\tilde{q}}. \end{cases} \tag{1.156}$$

Here nearly degenerate squarks with mass $m_{\tilde{q}}$ are assumed for simplicity, with $m^2_{\tilde{s}^*_R \tilde{d}_R} = (\mathbf{m^2_d})_{21}$ treated as a perturbation. The same limit applies when $m^2_{\tilde{s}^*_R \tilde{d}_R}$ is replaced by $m^2_{\tilde{s}^*_L \tilde{d}_L} = (\mathbf{m^2_Q})_{21}$, in a basis corresponding to the down-type quark mass eigenstates. An even more striking limit applies to the combination of both types of flavor mixing when they are comparable in size, from diagrams including Figure 1.13b. The numerical constraint is:[95]

$$\frac{|\mathrm{Re}[m^2_{\tilde{s}^*_R \tilde{d}_R} m^2_{\tilde{s}^*_L \tilde{d}_L}]|^{1/2}}{m^2_{\tilde{q}}} < \left(\frac{m_{\tilde{q}}}{500\,\mathrm{GeV}}\right) \times \begin{cases} 0.0008 \text{ for } m_{\tilde{g}} = 0.5 m_{\tilde{q}}, \\ 0.0010 \text{ for } m_{\tilde{g}} = m_{\tilde{q}}, \\ 0.0013 \text{ for } m_{\tilde{g}} = 2 m_{\tilde{q}}. \end{cases} \tag{1.157}$$

An off-diagonal contribution from $\mathbf{a_d}$ would cause flavor mixing between left-handed and right-handed squarks, just as discussed above for sleptons, resulting in a strong constraint from diagrams like Figure 1.13c. More generally, limits on Δm_K and ϵ and ϵ'/ϵ appearing in the neutral kaon effective Hamiltonian severely restrict the amounts of $\widetilde{d}_{L,R}$, $\widetilde{s}_{L,R}$ squark mixings (separately and in various combinations), and associated CP-violating complex phases, that one can tolerate in the soft squared masses.

Weaker, but still interesting, constraints come from the $D^0, \overline{D}^0$ system, which limits the amounts of $\widetilde{u}, \widetilde{c}$ mixings from $\mathbf{m^2_{\overline{u}}}$, $\mathbf{m^2_Q}$ and $\mathbf{a_u}$. The $B^0_d, \overline{B}^0_d$ and $B^0_s, \overline{B}^0_s$ systems similarly limit the amounts of $\widetilde{d}, \widetilde{b}$ and $\widetilde{s}, \widetilde{b}$ squark mixings from soft supersymmetry-breaking sources. More constraints follow from rare $\Delta F = 1$ meson decays, notably those involving the parton-level processes $b \to s\gamma$ and $b \to s\ell^+\ell^-$ and $c \to u\ell^+\ell^-$ and $s \to de^+e^-$ and $s \to d\nu\bar{\nu}$, all of which can be mediated by flavor mixing in soft supersymmetry breaking. There are also strict constraints on CP-violating phases in the gaugino masses and (scalar)^3 soft couplings following from limits on the electric dipole moments of the neutron and electron.[83] Detailed limits can be found in the literature,[80–105] but the essential lesson from experiment is that the soft supersymmetry-breaking Lagrangian cannot be arbitrary or random.

All of these potentially dangerous flavor-changing and CP-violating effects in the MSSM can be evaded if one assumes (or can explain!) that supersymmetry breaking is suitably "universal". Consider an idealized limit in which the squark and slepton squared-mass matrices are flavor-blind, each proportional to the 3×3 identity matrix in family space:

$$\mathbf{m^2_Q} = m^2_Q \mathbf{1}, \quad \mathbf{m^2_{\overline{u}}} = m^2_{\overline{u}} \mathbf{1}, \quad \mathbf{m^2_{\overline{d}}} = m^2_{\overline{d}} \mathbf{1}, \quad \mathbf{m^2_L} = m^2_L \mathbf{1}, \quad \mathbf{m^2_{\overline{e}}} = m^2_{\overline{e}} \mathbf{1}. \tag{1.158}$$

Then all squark and slepton mixing angles are rendered trivial, because squarks and sleptons with the same electroweak quantum numbers will be degenerate in mass and can be rotated into each other at will. Supersymmetric contributions to flavor-changing neutral current processes will therefore be very small in such an idealized limit, up to mixing induced by $\mathbf{a_u}$, $\mathbf{a_d}$, $\mathbf{a_e}$. Making the further assumption that the (scalar)^3 couplings are each proportional to the corresponding Yukawa coupling matrix,

$$\mathbf{a_u} = A_{u0}\, \mathbf{y_u}, \qquad \mathbf{a_d} = A_{d0}\, \mathbf{y_d}, \qquad \mathbf{a_e} = A_{e0}\, \mathbf{y_e}, \tag{1.159}$$

will ensure that only the squarks and sleptons of the third family can have large (scalar)^3 couplings. Finally, one can avoid disastrously large CP-violating effects by assuming that the soft parameters do not introduce new complex phases. This is automatic for $m^2_{H_u}$ and $m^2_{H_d}$, and for m^2_Q, $m^2_{\overline{u}}$, etc. if eq. (1.158) is assumed; if they were not real numbers, the Lagrangian would not be real. One can also fix μ in the superpotential and b in eq. (1.152) to be real, by appropriate phase rotations of fermion and scalar components of the H_u and H_d supermultiplets. If one then assumes that

$$\arg(M_1),\ \arg(M_2),\ \arg(M_3),\ \arg(A_{u0}),\ \arg(A_{d0}),\ \arg(A_{e0}) = 0 \text{ or } \pi, \tag{1.160}$$

then the only CP-violating phase in the theory will be the usual CKM phase found in the ordinary Yukawa couplings. Together, the conditions eqs. (1.158)-(1.160) make up a rather weak version of what is often called the hypothesis of *soft supersymmetry-breaking universality.* The MSSM with these flavor- and CP-preserving relations imposed has far fewer parameters than the most general case. Besides the usual Standard Model gauge and Yukawa coupling parameters, there are 3 independent real gaugino masses, only 5 real squark and slepton squared mass parameters, 3 real scalar cubic coupling parameters, and 4 Higgs mass parameters (one of which can be traded for the known electroweak breaking scale).

It must be mentioned in passing that there are at least three other possible types of explanations for the suppression of flavor violation in the MSSM that could replace the universality hypothesis of eqs. (1.158)-(1.160). They can be referred to as the "irrelevancy", "alignment", and "R-symmetry" hypotheses for the soft masses. The "irrelevancy" idea is that the sparticles masses are *extremely* heavy, so that their contributions to flavor-changing and CP-violating diagrams like Figures 1.13a,b are suppressed, as can be seen for example in eqs. (1.155)-(1.157). In practice, however, the degree of suppression needed typically requires $m_{\rm soft}$ much larger than 1 TeV for at least some of the scalar masses; this seems to go directly against the motivation for supersymmetry as a cure for the hierarchy problem as discussed in the Introduction. Nevertheless, it has been argued that this is a sensible possibility.[107,108] The "alignment" idea is that the squark squared-mass matrices do not have the flavor-blindness indicated in eq. (1.158), but are arranged in flavor space to be aligned with the relevant Yukawa matrices in just such a way as to avoid large flavor-changing effects.[56,109] The alignment models typically require rather special flavor symmetries. The third possibility is that the theory is (approximately) invariant under a continuous $U(1)_R$ symmetry.[62] This requires that the MSSM is supplemented, as in,[59] by additional chiral supermultiplets in the adjoint representations of $SU(3)_c$, $SU(2)_L$, and $U(1)_Y$, as well as an additional pair of Higgs chiral supermultiplets. The gaugino masses in this theory are purely Dirac, of the type in eq. (1.140), and the couplings $\mathbf{a_u}$, $\mathbf{a_d}$, and $\mathbf{a_e}$ are absent. This implies a very efficient suppression of flavor-changing effects,[62,63] even if the squark and slepton mass eigenstates are light, non-degenerate, and have large mixings in the basis determined by the Standard Model quark and lepton mass eigenstates. This can lead to unique and intriguing collider signatures.[62,65] However, we will not consider these possibilities further here.

The soft-breaking universality relations eqs. (1.158)-(1.160), or stronger (more special) versions of them, can be presumed to be the result of some specific model for the origin of supersymmetry breaking, although there is considerable disagreement among theorists as to what the specific model should actually be. In any case, they are indicative of an assumed underlying simplicity or symmetry of the Lagrangian at some very high energy scale Q_0. If we used this Lagrangian to compute masses and cross-sections and decay rates for experiments at ordinary energies near the electroweak scale, the results would involve large logarithms of order $\ln(Q_0/m_Z)$ coming from loop diagrams. As is usual in quantum field theory, the large logarithms can be conveniently resummed using renormalization group (RG) equations, by treating the couplings and masses appearing in the Lagrangian as running parameters. Therefore, eqs. (1.158)-(1.160) should be interpreted as boundary conditions on the running soft parameters at the scale Q_0, which is likely very far removed from direct experimental probes. We must then RG-evolve all of the soft parameters, the superpotential parameters, and the gauge couplings down to the electroweak scale or comparable scales where humans perform experiments.

At the electroweak scale, eqs. (1.158) and (1.159) will no longer hold, even if they were exactly true at the input scale Q_0. However, to a good approximation, key flavor- and CP-conserving properties remain. This is because, as we will see in section 1.5.5 below, RG corrections due to gauge interactions will respect the form of eqs. (1.158) and (1.159), while RG corrections due to Yukawa interactions are quite small except for couplings involving the top, bottom, and tau flavors. Therefore, the (scalar)^3 couplings and scalar squared-mass mixings should be quite negligible for the squarks and sleptons of the first two families. Furthermore, RG evolution does not introduce new CP-violating phases. Therefore, if universality can be arranged to hold at the input scale, supersymmetric contributions to flavor-changing and CP-violating observables can be acceptably small in comparison to present limits (although quite possibly measurable in future experiments).

One good reason to be optimistic that such a program can succeed is the celebrated apparent unification of gauge couplings in the MSSM.[110] The 1-loop RG equations for the Standard Model gauge couplings g_1, g_2, g_3 are

$$\beta_{g_a} \equiv \frac{d}{dt} g_a = \frac{1}{16\pi^2} b_a g_a^3,$$
$$(b_1, b_2, b_3) = \begin{cases} (41/10,\ -19/6,\ -7) & \text{Standard Model} \\ (33/5,\ 1,\ -3) & \text{MSSM} \end{cases} \qquad (1.161)$$

where $t = \ln(Q/Q_0)$, with Q the RG scale. The MSSM coefficients are larger because of the extra MSSM particles in loops. The normalization for g_1 here is chosen to agree with the canonical covariant derivative for grand unification of the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ into $SU(5)$ or $SO(10)$. Thus in terms of the conventional electroweak gauge couplings g and g' with $e = g \sin\theta_W = g' \cos\theta_W$, one has $g_2 = g$ and $g_1 = \sqrt{5/3}\, g'$. The quantities $\alpha_a = g_a^2/4\pi$ have the nice property that their reciprocals run linearly with RG scale at one-loop order:

$$\frac{d}{dt}\alpha_a^{-1} = -\frac{b_a}{2\pi} \qquad (a = 1, 2, 3). \qquad (1.162)$$

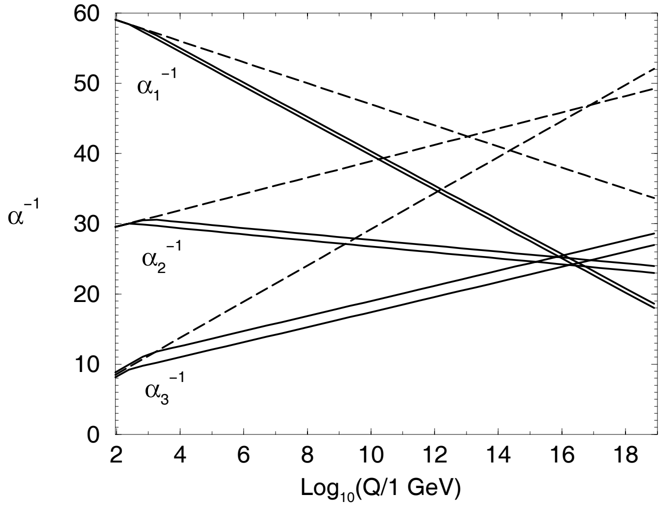

Fig. 1.14. RG evolution of the inverse gauge couplings $\alpha_a^{-1}(Q)$ in the Standard Model (dashed lines) and the MSSM (solid lines). In the MSSM case, the sparticle mass thresholds are varied between 250 GeV and 1 TeV, and $\alpha_3(m_Z)$ between 0.113 and 0.123. Two-loop effects are included.

Figure 1.14 compares the RG evolution of the α_a^{-1}, including two-loop effects, in the Standard Model (dashed lines) and the MSSM (solid lines). Unlike the Standard Model, the MSSM includes just the right particle content to ensure that the gauge couplings can unify, at a scale $M_U \sim 2 \times 10^{16}$ GeV. While the apparent unification of gauge couplings at M_U might be just an accident, it may also be taken as a strong hint in favor of a grand unified theory (GUT) or superstring models, both of which can naturally accommodate gauge coupling unification below M_{P}. Furthermore, if this hint is taken seriously, then we can reasonably expect to be able to apply a similar RG analysis to the other MSSM couplings and soft masses as well. The next section discusses the form of the necessary RG equations.

1.5.5. *Renormalization group equations for the MSSM*

In order to translate a set of predictions at an input scale into physically meaningful quantities that describe physics near the electroweak scale, it is necessary to evolve the gauge couplings, superpotential parameters, and soft terms using their renormalization group (RG) equations. This ensures that the loop expansions for calculations of observables will not suffer from very large logarithms.

As a technical aside, some care is required in choosing regularization and renormalization procedures in supersymmetry. The most popular regularization method for computations of radiative corrections within the Standard Model is dimensional regularization (DREG), in which the number of spacetime dimensions is continued to $d = 4 - 2\epsilon$. Unfortunately, DREG introduces a spurious violation of supersymmetry, because it has a mismatch between the numbers of gauge boson degrees of freedom and the gaugino degrees of freedom off-shell. This mismatch is only 2ϵ, but can be multiplied by factors up to $1/\epsilon^n$ in an n-loop calculation. In DREG, supersymmetric relations between dimensionless coupling constants ("supersymmetric Ward identities") are therefore not explicitly respected by radiative corrections involving the finite parts of one-loop graphs and by the divergent parts of two-loop graphs. Instead, one may use the slightly different scheme known as regularization by dimensional reduction, or DRED, which does respect supersymmetry.[111] In the DRED method, all momentum integrals are still performed in $d = 4 - 2\epsilon$ dimensions, but the vector index μ on the gauge boson fields A^a_μ now runs over all 4 dimensions to maintain the match with the gaugino degrees of freedom. Running couplings are then renormalized using DRED with modified minimal subtraction ($\overline{\text{DR}}$) rather than the usual DREG with modified minimal subtraction ($\overline{\text{MS}}$). In particular, the boundary conditions at the input scale should presumably be applied in a supersymmetry-preserving scheme like $\overline{\text{DR}}$. One loop β-functions are always the same in these two schemes, but it is important to realize that the $\overline{\text{MS}}$ scheme does violate supersymmetry, so that $\overline{\text{DR}}$ is preferred[m] from that point of view. (The NSVZ scheme[116] also respects supersymmetry and has some very useful properties, but with a less obvious connection to calculations of physical observables. It is also possible, but not always very practical, to work consistently within the $\overline{\text{MS}}$ scheme, as long as one translates all $\overline{\text{DR}}$ couplings and masses into their $\overline{\text{MS}}$ counterparts.[117–119])

A general and powerful result known as the *supersymmetric nonrenormalization theorem*[120] governs the form of the renormalization group equations for supersymmetric theories. This theorem implies that the logarithmically divergent contributions to a particular process can always be written in terms of wave-function renormalizations, without any coupling

[m]Even the DRED scheme may not provide a supersymmetric regulator, because of either ambiguities or inconsistencies (depending on the precise method) appearing at five-loop order at the latest.[112] Fortunately, this does not seem to cause practical difficulties.[113,114] See also Ref. 115 for an interesting proposal that avoids doing violence to the number of spacetime dimensions.

vertex renormalization.[n] It can be proved most easily using superfield techniques. For the parameters appearing in the superpotential eq. (1.107), the implication is that

$$\beta_{y^{ijk}} \equiv \frac{d}{dt} y^{ijk} = \gamma_n^i y^{njk} + \gamma_n^j y^{ink} + \gamma_n^k y^{ijn}, \tag{1.163}$$

$$\beta_{M^{ij}} \equiv \frac{d}{dt} M^{ij} = \gamma_n^i M^{nj} + \gamma_n^j M^{in}, \tag{1.164}$$

$$\beta_{L^i} \equiv \frac{d}{dt} L^i = \gamma_n^i L^n, \tag{1.165}$$

where the γ_j^i are anomalous dimension matrices associated with the superfields, which generally have to be calculated in a perturbative loop expansion. [Recall $t = \ln(Q/Q_0)$, where Q is the renormalization scale, and Q_0 is a reference scale.] The anomalous dimensions and RG equations for softly broken supersymmetry are now known up to 3-loop order, with some partial 4-loop results; they have been given in Refs. 121–126. There are also relations, good to all orders in perturbation theory, that give the RG equations for soft supersymmetry couplings in terms of those for the supersymmetric couplings.[116,127] Here we will only use the 1-loop approximation, for simplicity.

In general, at 1-loop order,

$$\gamma_j^i = \frac{1}{16\pi^2}\left[\frac{1}{2} y^{imn} y^*_{jmn} - 2g_a^2 C_a(i)\delta_j^i\right], \tag{1.166}$$

where $C_a(i)$ are the quadratic Casimir group theory invariants for the superfield Φ_i, defined in terms of the Lie algebra generators T^a by

$$(T^a T^a)_i{}^j = C_a(i)\delta_i^j \tag{1.167}$$

with gauge couplings g_a. Explicitly, for the MSSM supermultiplets:

$$C_3(i) = \begin{cases} 4/3 \text{ for } \Phi_i = Q, \overline{u}, \overline{d}, \\ 0 \quad\;\;\, \text{for } \Phi_i = L, \overline{e}, H_u, H_d, \end{cases} \tag{1.168}$$

$$C_2(i) = \begin{cases} 3/4 \text{ for } \Phi_i = Q, L, H_u, H_d, \\ 0 \quad\;\;\, \text{for } \Phi_i = \overline{u}, \overline{d}, \overline{e}, \end{cases} \tag{1.169}$$

$$C_1(i) = 3Y_i^2/5 \text{ for each } \Phi_i \text{ with weak hypercharge } Y_i. \tag{1.170}$$

[n] Actually, there *is* vertex renormalization working in a supersymmetric gauge theory in which auxiliary fields have been integrated out, but the sum of divergent contributions for a process always has the form of wave-function renormalization. This is related to the fact that the anomalous dimensions of the superfields differ, by gauge-fixing dependent terms, from the anomalous dimensions of the fermion and boson component fields.[31]

For the one-loop renormalization of gauge couplings, one has in general

$$\beta_{g_a} = \frac{d}{dt} g_a = \frac{1}{16\pi^2} g_a^3 \Big[\sum_i I_a(i) - 3C_a(G)\Big], \tag{1.171}$$

where $C_a(G)$ is the quadratic Casimir invariant of the group [0 for $U(1)$, and N for $SU(N)$], and $I_a(i)$ is the Dynkin index of the chiral supermultiplet ϕ_i [normalized to 1/2 for each fundamental representation of $SU(N)$ and to $3Y_i^2/5$ for $U(1)_Y$]. Equation (1.161) is a special case of this.

The 1-loop renormalization group equations for the general soft supersymmetry breaking Lagrangian parameters appearing in eq. (1.138) are:

$$\beta_{M_a} \equiv \frac{d}{dt} M_a = \frac{1}{16\pi^2} g_a^2 \Big[2 \sum_n I_a(n) - 6C_a(G)\Big] M_a, \tag{1.172}$$

$$\begin{aligned} \beta_{a^{ijk}} \equiv \frac{d}{dt} a^{ijk} = \frac{1}{16\pi^2} \Big[&\frac{1}{2} a^{ijp} y^*_{pmn} y^{kmn} + y^{ijp} y^*_{pmn} a^{mnp} \\ &+ g_a^2 C_a(i)(4M_a y^{ijk} - 2a^{ijk})\Big] + (i \leftrightarrow k) + (j \leftrightarrow k), \end{aligned} \tag{1.173}$$

$$\begin{aligned} \beta_{b^{ij}} \equiv \frac{d}{dt} b^{ij} = \frac{1}{16\pi^2} \Big[&\frac{1}{2} b^{ip} y^*_{pmn} y^{jmn} + \frac{1}{2} y^{ijp} y^*_{pmn} b^{mn} + M^{ip} y^*_{pmn} a^{mnj} \\ &+ g_a^2 C_a(i)(4M_a M^{ij} - 2b^{ij})\Big] + (i \leftrightarrow j), \end{aligned} \tag{1.174}$$

$$\begin{aligned} \beta_{t^i} \equiv \frac{d}{dt} t^i = \frac{1}{16\pi^2} \Big[&\frac{1}{2} y^{imn} y^*_{mnp} t^p + a^{imn} y^*_{mnp} L^p \\ &+ M^{ip} y^*_{pmn} b^{mn}\Big], \end{aligned} \tag{1.175}$$

$$\begin{aligned} \beta_{(m^2)_i^j} \equiv \frac{d}{dt} (m^2)_i^j = \frac{1}{16\pi^2} \Big[&\frac{1}{2} y^*_{ipq} y^{pqn} (m^2)_n^j + \frac{1}{2} y^{jpq} y^*_{pqn} (m^2)_i^n \\ &+ 2 y^*_{ipq} y^{jpr} (m^2)_r^q + a^*_{ipq} a^{jpq} \\ &- 8 g_a^2 C_a(i) |M_a|^2 \delta_i^j + 2 g_a^2 (T^a)_i{}^j \mathrm{Tr}(T^a m^2)\Big]. \end{aligned} \tag{1.176}$$

Applying the above results to the special case of the MSSM, we will use the approximation that only the third-family Yukawa couplings are significant, as in eq. (1.142). Then the Higgs and third-family superfield anomalous dimensions are diagonal matrices, and from eq. (1.166) they are, at 1-loop order:

$$\gamma_{H_u} = \frac{1}{16\pi^2} \left[3 y_t^* y_t - \frac{3}{2} g_2^2 - \frac{3}{10} g_1^2\right], \tag{1.177}$$

$$\gamma_{H_d} = \frac{1}{16\pi^2}\left[3y_b^* y_b + y_\tau^* y_\tau - \frac{3}{2}g_2^2 - \frac{3}{10}g_1^2\right], \tag{1.178}$$

$$\gamma_{Q_3} = \frac{1}{16\pi^2}\left[y_t^* y_t + y_b^* y_b - \frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{1}{30}g_1^2\right], \tag{1.179}$$

$$\gamma_{\overline{u}_3} = \frac{1}{16\pi^2}\left[2y_t^* y_t - \frac{8}{3}g_3^2 - \frac{8}{15}g_1^2\right], \tag{1.180}$$

$$\gamma_{\overline{d}_3} = \frac{1}{16\pi^2}\left[2y_b^* y_b - \frac{8}{3}g_3^2 - \frac{2}{15}g_1^2\right], \tag{1.181}$$

$$\gamma_{L_3} = \frac{1}{16\pi^2}\left[y_\tau^* y_\tau - \frac{3}{2}g_2^2 - \frac{3}{10}g_1^2\right], \tag{1.182}$$

$$\gamma_{\overline{e}_3} = \frac{1}{16\pi^2}\left[2y_\tau^* y_\tau - \frac{6}{5}g_1^2\right]. \tag{1.183}$$

[The first and second family anomalous dimensions in the approximation of eq. (1.142) follow by setting y_t, y_b, and y_τ to 0 in the above.] Putting these into eqs. (1.163), (1.164) gives the running of the superpotential parameters with renormalization scale:

$$\beta_{y_t} \equiv \frac{d}{dt}y_t = \frac{y_t}{16\pi^2}\left[6y_t^* y_t + y_b^* y_b - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right], \tag{1.184}$$

$$\beta_{y_b} \equiv \frac{d}{dt}y_b = \frac{y_b}{16\pi^2}\left[6y_b^* y_b + y_t^* y_t + y_\tau^* y_\tau - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\right], \tag{1.185}$$

$$\beta_{y_\tau} \equiv \frac{d}{dt}y_\tau = \frac{y_\tau}{16\pi^2}\left[4y_\tau^* y_\tau + 3y_b^* y_b - 3g_2^2 - \frac{9}{5}g_1^2\right], \tag{1.186}$$

$$\beta_{\mu} \equiv \frac{d}{dt}\mu = \frac{\mu}{16\pi^2}\left[3y_t^* y_t + 3y_b^* y_b + y_\tau^* y_\tau - 3g_2^2 - \frac{3}{5}g_1^2\right]. \tag{1.187}$$

The one-loop RG equations for the gauge couplings g_1, g_2, and g_3 were already listed in eq. (1.161). The presence of soft supersymmetry breaking does not affect eqs. (1.161) and (1.184)-(1.187). As a result of the supersymmetric non-renormalization theorem, the β-functions for each supersymmetric parameter are proportional to the parameter itself. One consequence of this is that once we have a theory that can explain why μ is of order 10^2 or 10^3 GeV at tree-level, we do not have to worry about μ being made very large by radiative corrections involving the masses of some very heavy unknown particles; all such RG corrections to μ will be directly proportional to μ itself and to some combinations of dimensionless couplings.

The one-loop RG equations for the three gaugino mass parameters in the MSSM are determined by the same quantities b_a^{MSSM} that appear in the gauge coupling RG eqs. (1.161):

$$\beta_{M_a} \equiv \frac{d}{dt} M_a = \frac{1}{8\pi^2} b_a g_a^2 M_a \qquad (b_a = 33/5,\ 1,\ -3) \tag{1.188}$$

for $a = 1, 2, 3$. It follows that the three ratios M_a/g_a^2 are each constant (RG scale independent) up to small two-loop corrections. Since the gauge couplings are observed to unify at $Q = M_U = 2 \times 10^{16}$ GeV, it is a popular assumption that the gaugino masses also unify[o] near that scale, with a value called $m_{1/2}$. If so, then it follows that

$$\frac{M_1}{g_1^2} = \frac{M_2}{g_2^2} = \frac{M_3}{g_3^2} = \frac{m_{1/2}}{g_U^2} \tag{1.189}$$

at any RG scale, up to small (and known) two-loop effects and possibly much larger (and not so known) threshold effects near M_U. Here g_U is the unified gauge coupling at $Q = M_U$. The hypothesis of eq. (1.189) is particularly powerful because the gaugino mass parameters feed strongly into the RG equations for all of the other soft terms, as we are about to see.

Next we consider the 1-loop RG equations for the analytic soft parameters $\mathbf{a_u}$, $\mathbf{a_d}$, $\mathbf{a_e}$. In models obeying eq. (1.159), these matrices start off proportional to the corresponding Yukawa couplings at the input scale. The RG evolution respects this property. With the approximation of eq. (1.142), one can therefore also write, at any RG scale,

$$\mathbf{a_u} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a_t \end{pmatrix}, \quad \mathbf{a_d} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a_b \end{pmatrix}, \quad \mathbf{a_e} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a_\tau \end{pmatrix}, \tag{1.190}$$

which defines[p] running parameters a_t, a_b, and a_τ. In this approximation, the RG equations for these parameters and b are

$$16\pi^2 \frac{d}{dt} a_t = a_t \Big[18 y_t^* y_t + y_b^* y_b - \frac{16}{3} g_3^2 - 3 g_2^2 - \frac{13}{15} g_1^2 \Big] + 2 a_b y_b^* y_t \\ + y_t \Big[\frac{32}{3} g_3^2 M_3 + 6 g_2^2 M_2 + \frac{26}{15} g_1^2 M_1 \Big], \tag{1.191}$$

[o] In GUT models, it is automatic that the gauge couplings and gaugino masses are unified at all scales $Q \geq M_U$, because in the unified theory the gauginos all live in the same representation of the unified gauge group. In many superstring models, this can also be a good approximation.

[p] Rescaled soft parameters $A_t = a_t/y_t$, $A_b = a_b/y_b$, and $A_\tau = a_\tau/y_\tau$ are commonly used in the literature. We do not follow this notation, because it cannot be generalized beyond the approximation of eqs. (1.142), (1.190) without introducing horrible complications such as non-polynomial RG equations, and because a_t, a_b and a_τ are the couplings that actually appear in the Lagrangian anyway.

$$16\pi^2 \frac{d}{dt} a_b = a_b \Big[18 y_b^* y_b + y_t^* y_t + y_\tau^* y_\tau - \frac{16}{3} g_3^2 - 3 g_2^2 - \frac{7}{15} g_1^2 \Big] + 2 a_t y_t^* y_b$$
$$+ 2 a_\tau y_\tau^* y_b + y_b \Big[\frac{32}{3} g_3^2 M_3 + 6 g_2^2 M_2 + \frac{14}{15} g_1^2 M_1 \Big], \qquad (1.192)$$

$$16\pi^2 \frac{d}{dt} a_\tau = a_\tau \Big[12 y_\tau^* y_\tau + 3 y_b^* y_b - 3 g_2^2 - \frac{9}{5} g_1^2 \Big] + 6 a_b y_b^* y_\tau$$
$$+ y_\tau \Big[6 g_2^2 M_2 + \frac{18}{5} g_1^2 M_1 \Big], \qquad (1.193)$$

$$16\pi^2 \frac{d}{dt} b = b \Big[3 y_t^* y_t + 3 y_b^* y_b + y_\tau^* y_\tau - 3 g_2^2 - \frac{3}{5} g_1^2 \Big]$$
$$+ \mu \Big[6 a_t y_t^* + 6 a_b y_b^* + 2 a_\tau y_\tau^* + 6 g_2^2 M_2 + \frac{6}{5} g_1^2 M_1 \Big]. \qquad (1.194)$$

The β-function for each of these soft parameters is *not* proportional to the parameter itself, because couplings that violate supersymmetry are not protected by the supersymmetric non-renormalization theorem. So, even if a_t, a_b, a_τ and b vanish at the input scale, the RG corrections proportional to gaugino masses appearing in eqs. (1.191)-(1.194) ensure that they will not vanish at the electroweak scale.

Next let us consider the RG equations for the scalar squared masses in the MSSM. In the approximation of eqs. (1.142) and (1.190), the squarks and sleptons of the first two families have only gauge interactions. This means that if the scalar squared masses satisfy a boundary condition like eq. (1.158) at an input RG scale, then when renormalized to any other RG scale, they will still be almost diagonal, with the approximate form

$$\mathbf{m_Q^2} \approx \begin{pmatrix} m^2_{Q_1} & 0 & 0 \\ 0 & m^2_{Q_1} & 0 \\ 0 & 0 & m^2_{Q_3} \end{pmatrix}, \quad \mathbf{m^2_{\overline{u}}} \approx \begin{pmatrix} m^2_{\overline{u}_1} & 0 & 0 \\ 0 & m^2_{\overline{u}_1} & 0 \\ 0 & 0 & m^2_{\overline{u}_3} \end{pmatrix}, \qquad (1.195)$$

etc. The first and second family squarks and sleptons with given gauge quantum numbers remain very nearly degenerate, but the third-family squarks and sleptons feel the effects of the larger Yukawa couplings and so their squared masses get renormalized differently. The one-loop RG equations for the first and second family squark and slepton squared masses are

$$16\pi^2 \frac{d}{dt} m^2_{\phi_i} \;=\; -\sum_{a=1,2,3} 8 C_a(i) g_a^2 |M_a|^2 + \frac{6}{5} Y_i g_1^2 S \qquad (1.196)$$

for each scalar ϕ_i, where the $\sum_a$ is over the three gauge groups $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$, with Casimir invariants $C_a(i)$ as in eqs. (1.168)-(1.170), and M_a are the corresponding running gaugino mass parameters.

Also,

$$S \equiv \mathrm{Tr}[Y_j m^2_{\phi_j}] = m^2_{H_u} - m^2_{H_d} + \mathrm{Tr}[\mathbf{m^2_Q} - \mathbf{m^2_L} - 2\mathbf{m^2_{\overline{u}}} + \mathbf{m^2_{\overline{d}}} + \mathbf{m^2_{\overline{e}}}]. \quad (1.197)$$

An important feature of eq. (1.196) is that the terms on the right-hand sides proportional to gaugino squared masses are negative, so[q] the scalar squared-mass parameters grow as they are RG-evolved from the input scale down to the electroweak scale. Even if the scalars have zero or very small masses at the input scale, they can obtain large positive squared masses at the electroweak scale, thanks to the effects of the gaugino masses.

The RG equations for the squared-mass parameters of the Higgs scalars and third-family squarks and sleptons get the same gauge contributions as in eq. (1.196), but they also have contributions due to the large Yukawa ($y_{t,b,\tau}$) and soft ($a_{t,b,\tau}$) couplings. At one-loop order, these only appear in three combinations:

$$X_t = 2|y_t|^2(m^2_{H_u} + m^2_{Q_3} + m^2_{\overline{u}_3}) + 2|a_t|^2, \quad (1.198)$$

$$X_b = 2|y_b|^2(m^2_{H_d} + m^2_{Q_3} + m^2_{\overline{d}_3}) + 2|a_b|^2, \quad (1.199)$$

$$X_\tau = 2|y_\tau|^2(m^2_{H_d} + m^2_{L_3} + m^2_{\overline{e}_3}) + 2|a_\tau|^2. \quad (1.200)$$

In terms of these quantities, the RG equations for the soft Higgs squared-mass parameters $m^2_{H_u}$ and $m^2_{H_d}$ are

$$16\pi^2 \frac{d}{dt} m^2_{H_u} = 3X_t - 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2 + \frac{3}{5}g_1^2 S, \quad (1.201)$$

$$16\pi^2 \frac{d}{dt} m^2_{H_d} = 3X_b + X_\tau - 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2 - \frac{3}{5}g_1^2 S. \quad (1.202)$$

Note that X_t, X_b, and X_τ are generally positive, so their effect is to decrease the Higgs masses as one evolves the RG equations down from the input scale to the electroweak scale. If y_t is the largest of the Yukawa couplings, as suggested by the experimental fact that the top quark is heavy, then X_t will typically be much larger than X_b and X_τ. This can cause the RG-evolved $m^2_{H_u}$ to run negative near the electroweak scale, helping to destabilize the point $H_u = H_d = 0$ and so provoking a Higgs VEV (for a linear combination of H_u and H_d, as we will see in section 1.7.1), which is just what we want.[r] Thus a large top Yukawa coupling favors the breakdown of the electroweak

[q]The contributions proportional to S are relatively small in most known realistic models.
[r]One should think of "$m^2_{H_u}$" as a parameter unto itself, and not as the square of some mythical real number m_{H_u}. So there is nothing strange about having $m^2_{H_u} < 0$. However, strictly speaking $m^2_{H_u} < 0$ is neither necessary nor sufficient for electroweak symmetry breaking; see section 1.7.1.

symmetry breaking because it induces negative radiative corrections to the Higgs squared mass.

The third-family squark and slepton squared-mass parameters also get contributions that depend on X_t, X_b and X_τ. Their RG equations are given by

$$16\pi^2 \frac{d}{dt} m_{Q_3}^2 = X_t + X_b - \frac{32}{3} g_3^2 |M_3|^2 - 6 g_2^2 |M_2|^2 - \frac{2}{15} g_1^2 |M_1|^2 + \frac{1}{5} g_1^2 S, \tag{1.203}$$

$$16\pi^2 \frac{d}{dt} m_{\overline{u}_3}^2 = 2X_t - \frac{32}{3} g_3^2 |M_3|^2 - \frac{32}{15} g_1^2 |M_1|^2 - \frac{4}{5} g_1^2 S, \tag{1.204}$$

$$16\pi^2 \frac{d}{dt} m_{\overline{d}_3}^2 = 2X_b - \frac{32}{3} g_3^2 |M_3|^2 - \frac{8}{15} g_1^2 |M_1|^2 + \frac{2}{5} g_1^2 S, \tag{1.205}$$

$$16\pi^2 \frac{d}{dt} m_{L_3}^2 = X_\tau - 6 g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2 - \frac{3}{5} g_1^2 S, \tag{1.206}$$

$$16\pi^2 \frac{d}{dt} m_{\overline{e}_3}^2 = 2X_\tau - \frac{24}{5} g_1^2 |M_1|^2 + \frac{6}{5} g_1^2 S. \tag{1.207}$$

In eqs. (1.201)-(1.207), the terms proportional to $|M_3|^2$, $|M_2|^2$, $|M_1|^2$, and S are just the same ones as in eq. (1.196). Note that the terms proportional to X_t and X_b appear with smaller numerical coefficients in the $m_{Q_3}^2$, $m_{\overline{u}_3}^2$, $m_{\overline{d}_3}^2$ RG equations than they did for the Higgs scalars, and they do not appear at all in the $m_{L_3}^2$ and $m_{\overline{e}_3}^2$ RG equations. Furthermore, the third-family squark squared masses get a large positive contribution proportional to $|M_3|^2$ from the RG evolution, which the Higgs scalars do not get. These facts make it plausible that the Higgs scalars in the MSSM get VEVs, while the squarks and sleptons, having large positive squared mass, do not.

An examination of the RG equations (1.191)-(1.194), (1.196), and (1.201)-(1.207) reveals that if the gaugino mass parameters M_1, M_2, and M_3 are non-zero at the input scale, then all of the other soft terms will be generated too. This implies that models in which gaugino masses dominate over all other effects in the soft supersymmetry breaking Lagrangian at the input scale can be viable. On the other hand, if the gaugino masses were to vanish at tree-level, then they would not get any contributions to their masses at one-loop order; in that case the gauginos would be extremely light and the model would not be phenomenologically acceptable.

Viable models for the origin of supersymmetry breaking typically make predictions for the MSSM soft terms that are refinements of eqs. (1.158)-(1.160). These predictions can then be used as boundary conditions for the RG equations listed above. In the next section we will study the ideas that go into making such predictions, before turning to their implications for the MSSM spectrum in section 1.7.

1.6. Origins of Supersymmetry Breaking

1.6.1. *General considerations for spontaneous supersymmetry breaking*

In the MSSM, supersymmetry breaking is simply introduced explicitly. However, we have seen that the soft parameters cannot be arbitrary. In order to understand how patterns like eqs. (1.158), (1.159) and (1.160) can emerge, it is necessary to consider models in which supersymmetry is spontaneously broken. By definition, this means that the vacuum state $|0\rangle$ is not invariant under supersymmetry transformations, so $Q_\alpha|0\rangle \neq 0$ and $Q^\dagger_{\dot\alpha}|0\rangle \neq 0$. Now, in global supersymmetry, the Hamiltonian operator H is related to the supersymmetry generators through the algebra eq. (1.90):

$$H = P^0 = \frac{1}{4}(Q_1 Q_1^\dagger + Q_1^\dagger Q_1 + Q_2 Q_2^\dagger + Q_2^\dagger Q_2). \tag{1.208}$$

If supersymmetry is unbroken in the vacuum state, it follows that $H|0\rangle = 0$ and the vacuum has zero energy. Conversely, if supersymmetry is spontaneously broken in the vacuum state, then the vacuum must have positive energy, since

$$\langle 0|H|0\rangle = \frac{1}{4}\Big(\|Q_1^\dagger|0\rangle\|^2 + \|Q_1|0\rangle\|^2 + \|Q_2^\dagger|0\rangle\|^2 + \|Q_2|0\rangle\|^2\Big) > 0 \tag{1.209}$$

if the Hilbert space is to have positive norm. If spacetime-dependent effects and fermion condensates can be neglected, then $\langle 0|H|0\rangle = \langle 0|V|0\rangle$, where V is the scalar potential in eq. (1.135). Therefore, supersymmetry will be spontaneously broken if the expectation value of F_i and/or D^a does not vanish in the vacuum state.

If any state exists in which all F_i and D^a vanish, then it will have zero energy, implying that supersymmetry is not spontaneously broken in the true ground state. Conversely, one way to guarantee spontaneous supersymmetry breaking is to look for models in which the equations $F_i = 0$ and $D^a = 0$ cannot all be simultaneously satisfied for *any* values of the fields. Then the true ground state necessarily has broken supersymmetry, as does the vacuum state we live in (if it is different).

However, another possibility is that the vacuum state in which we live is not the true ground state (which may preserve supersymmetry), but is instead a higher energy metastable supersymmetry-breaking state with lifetime at least of order the present age of the universe.[128–130] Finite temperature effects can indeed cause the early universe to prefer the metastable supersymmetry-breaking local minimum of the potential over the supersymmetry-breaking global minimum.[131]

Regardless of whether the vacuum state is stable or metastable, the spontaneous breaking of a global symmetry always implies a massless Nambu-Goldstone mode with the same quantum numbers as the broken symmetry generator. In the case of global supersymmetry, the broken generator is the fermionic charge Q_α, so the Nambu-Goldstone particle ought to be a massless neutral Weyl fermion, called the *goldstino*. To prove it, consider a general supersymmetric model with both gauge and chiral supermultiplets as in section 1.3. The fermionic degrees of freedom consist of gauginos (λ^a) and chiral fermions (ψ_i). After some of the scalar fields in the theory obtain VEVs, the fermion mass matrix has the form:

$$\mathbf{m}_{\rm F} = \begin{pmatrix} 0 & \sqrt{2}g_b(\langle\phi^*\rangle T^b)^i \\ \sqrt{2}g_a(\langle\phi^*\rangle T^a)^j & \langle W^{ji}\rangle \end{pmatrix} \tag{1.210}$$

in the (λ^a, ψ_i) basis. [The off-diagonal entries in this matrix come from the first term in the second line of eq. (1.132), and the lower right entry can be seen in eq. (1.109).] Now observe that $\mathbf{m}_{\rm F}$ annihilates the vector

$$\widetilde{G} = \begin{pmatrix} \langle D^a\rangle/\sqrt{2} \\ \langle F_i\rangle \end{pmatrix}. \tag{1.211}$$

The first row of $\mathbf{m}_{\rm F}$ annihilates $\widetilde{G}$ by virtue of the requirement eq. (1.133) that the superpotential is gauge invariant, and the second row does so because of the condition $\langle\partial V/\partial\phi_i\rangle = 0$, which must be satisfied at a local minimum of the scalar potential. Equation (1.211) is therefore proportional to the goldstino wavefunction; it is non-trivial if and only if at least one of the auxiliary fields has a VEV, breaking supersymmetry. So we have proved that if global supersymmetry is spontaneously broken, then there must be a massless goldstino, and that its components among the various fermions in the theory are just proportional to the corresponding auxiliary field VEVs.

There is also a useful sum rule that governs the tree-level squared masses of particles in theories with spontaneously broken supersymmetry. For a general theory of the type discussed in section 1.3, the squared masses of the real scalar degrees of freedom are the eigenvalues of the matrix

$$\mathbf{m}^2_{\rm S} = \begin{pmatrix} W^*_{jk}W^{ik} + g_a^2[(T^a\phi)_j(\phi^*T^a)^i + T^{ai}_j D^a] & W^*_{ijk}W^k + g_a^2(T^a\phi)_i(T^a\phi)_j \\ W^{ijk}W^*_k + g_a^2(\phi^*T^a)^i(\phi^*T^a)^j & W^*_{ik}W^{jk} + g_a^2[(T^a\phi)_i(\phi^*T^a)^j + T^{aj}_i D^a] \end{pmatrix}, \tag{1.212}$$

since the quadratic part of the tree-level potential is

$$V = \begin{pmatrix} \phi^{*j} & \phi_j \end{pmatrix} \mathbf{m}_{\rm S}^2 \begin{pmatrix} \phi_i \\ \phi^{*i} \end{pmatrix}. \tag{1.213}$$

Here $W^{ijk} = \delta^3 W/\delta\phi_i\delta\phi_j\delta\phi_k$, and the scalar fields on the right-hand side of eq. (1.212) are understood to be replaced by their VEVs. It follows that the sum of the real scalar squared-mass eigenvalues is

$$\mathrm{Tr}(\mathbf{m}_{\rm S}^2) = 2W_{ik}^* W^{ik} + 2g_a^2[C_a(i)\phi^{*i}\phi_i + \mathrm{Tr}(T^a)D^a], \tag{1.214}$$

with the Casimir invariants $C_a(i)$ defined by eq. (1.167). Meanwhile, the squared masses of the two-component fermions are given by the eigenvalues of

$$\mathbf{m}_{\rm F}^\dagger \mathbf{m}_{\rm F} = \begin{pmatrix} 2g_a g_b(\phi^* T^a T^b \phi) & \sqrt{2} g_b (T^b\phi)_k W^{ik} \\ \sqrt{2} g_a (\phi^* T^a)^k W_{jk}^* & W_{jk}^* W^{ik} + 2g_a^2 (T^a\phi)_j (\phi^* T^a)^i \end{pmatrix}, \tag{1.215}$$

so the sum of the two-component fermion squared masses is

$$\mathrm{Tr}(\mathbf{m}_{\rm F}^\dagger \mathbf{m}_{\rm F}) = W_{ik}^* W^{ik} + 4g_a^2 C_a(i)\phi^{*i}\phi_i. \tag{1.216}$$

Finally, the vector squared masses are:

$$\mathbf{m}_{\rm V}^2 = g_a^2(\phi^*\{T^a, T^b\}\phi), \tag{1.217}$$

so

$$\mathrm{Tr}(\mathbf{m}_{\rm V}^2) = 2g_a^2 C_a(i)\phi^{*i}\phi_i. \tag{1.218}$$

It follows that the *supertrace* of the tree-level squared-mass eigenvalues, defined in general by a weighted sum over all particles with spin j:

$$\mathrm{STr}(m^2) \equiv \sum_j (-1)^{2j}(2j+1)\mathrm{Tr}(m_j^2), \tag{1.219}$$

satisfies the sum rule

$$\mathrm{STr}(m^2) = \mathrm{Tr}(\mathbf{m}_{\rm S}^2) - 2\mathrm{Tr}(\mathbf{m}_{\rm F}^\dagger \mathbf{m}_{\rm F}) + 3\mathrm{Tr}(\mathbf{m}_{\rm V}^2) = 2g_a^2 \mathrm{Tr}(T^a)D^a = 0. \tag{1.220}$$

The last equality assumes that the traces of the $U(1)$ charges over the chiral superfields are 0. This holds for $U(1)_Y$ in the MSSM, and more generally for any non-anomalous gauge symmetry. The sum rule eq. (1.220) is often a useful check on models of spontaneous supersymmetry breaking.

1.6.2. *Fayet-Iliopoulos (D-term) supersymmetry breaking*

Supersymmetry breaking with a non-zero D-term VEV can occur through the Fayet-Iliopoulos mechanism.[132] If the gauge symmetry includes a $U(1)$ factor, then one can introduce a term linear in the corresponding auxiliary field of the gauge supermultiplet:

$$\mathcal{L}_{\text{Fayet-Iliopoulos}} = -\kappa D \tag{1.221}$$

where κ is a constant with dimensions of [mass]^2. This term is gauge-invariant and supersymmetric by itself. [Note that for a $U(1)$ gauge symmetry, the supersymmetry transformation δD in eq. (1.122) is a total derivative.] If we include it in the Lagrangian, then D may be forced to get a non-zero VEV. To see this, consider the relevant part of the scalar potential from eqs. (1.117) and (1.132):

$$V = \kappa D - \frac{1}{2}D^2 - gD\sum_i q_i|\phi_i|^2. \tag{1.222}$$

Here the q_i are the charges of the scalar fields ϕ_i under the $U(1)$ gauge group in question. The presence of the Fayet-Iliopoulos term modifies the equation of motion eq. (1.134) to

$$D = \kappa - g\sum_i q_i|\phi_i|^2. \tag{1.223}$$

Now suppose that the scalar fields ϕ_i that are charged under the $U(1)$ all have non-zero superpotential masses m_i. (Gauge invariance then requires that they come in pairs with opposite charges.) Then the potential will have the form

$$V = \sum_i |m_i|^2|\phi_i|^2 + \frac{1}{2}(\kappa - g\sum_i q_i|\phi_i|^2)^2. \tag{1.224}$$

Since this cannot vanish, supersymmetry must be broken; one can check that the minimum always occurs for non-zero D. For the simplest case in which $|m_i|^2 > gq_i\kappa$ for each i, the minimum is realized for all $\phi_i = 0$ and $D = \kappa$, with the $U(1)$ gauge symmetry unbroken. As further evidence that supersymmetry has indeed been spontaneously broken, note that the scalars then have squared masses $|m_i|^2 - gq_i\kappa$, while their fermion partners have squared masses $|m_i|^2$. The gaugino remains massless, as can be understood from the fact that it is the goldstino, as argued on general grounds in section 1.6.1.

For non-Abelian gauge groups, the analog of eq. (1.221) would not be gauge-invariant and is therefore not allowed, so only $U(1)$ D-terms can drive spontaneous symmetry breaking. In the MSSM, one might imagine that the D term for $U(1)_Y$ has a Fayet-Iliopoulos term as the principal source of supersymmetry breaking. Unfortunately, this cannot work, because the squarks and sleptons do not have superpotential mass terms. So, at least some of them would just get non-zero VEVs in order to make eq. (1.223) vanish. That would break color and/or electromagnetism, but not supersymmetry. Therefore, a Fayet-Iliopoulos term for $U(1)_Y$ must be subdominant compared to other sources of supersymmetry breaking in the MSSM, if not absent altogether. One could instead attempt to trigger supersymmetry breaking with a Fayet-Iliopoulos term for some other $U(1)$ gauge symmetry, which is as yet unknown because it is spontaneously broken at a very high mass scale or because it does not couple to the Standard Model particles. However, if this is the dominant source for supersymmetry breaking, it proves difficult to give appropriate masses to all of the MSSM particles, especially the gauginos. In any case, we will not discuss D-term breaking as the ultimate origin of supersymmetry violation any further (although it may not be ruled out[133]).

1.6.3. *O'Raifeartaigh (F-term) supersymmetry breaking*

Models where spontaneous supersymmetry breaking is ultimately due to a non-zero F-term VEV, called O'Raifeartaigh models,[134] have brighter phenomenological prospects. The idea is to pick a set of chiral supermultiplets $\Phi_i \supset (\phi_i, \psi_i, F_i)$ and a superpotential W in such a way that the equations $F_i = -\delta W^*/\delta\phi^{*i} = 0$ have no simultaneous solution. Then $V = \sum_i |F_i|^2$ will have to be positive at its minimum, ensuring that supersymmetry is broken.

The simplest example that does this has three chiral supermultiplets with

$$W = -k\Phi_1 + m\Phi_2\Phi_3 + \frac{y}{2}\Phi_1\Phi_3^2. \tag{1.225}$$

Note that W contains a linear term, with k having dimensions of [mass]^2. Such a term is allowed if the corresponding chiral supermultiplet is a gauge singlet. In fact, a linear term is necessary to achieve F-term breaking at tree-level in renormalizable theories,[s] since otherwise setting all $\phi_i = 0$ will always give a supersymmetric global minimum with all $F_i = 0$. Without loss of generality, we can choose k, m, and y to be real and positive (by a

[s]Non-polynomial superpotential terms, for example arising from non-perturbative effects, can avoid this requirement.

phase rotation of the fields). The scalar potential following from eq. (1.225) is

$$V = |F_1|^2 + |F_2|^2 + |F_3|^2, \tag{1.226}$$

$$F_1 = k - \frac{y}{2}\phi_3^{*2}, \qquad F_2 = -m\phi_3^*, \qquad F_3 = -m\phi_2^* - y\phi_1^*\phi_3^*. \tag{1.227}$$

Clearly, $F_1 = 0$ and $F_2 = 0$ are not compatible, so supersymmetry must indeed be broken. If $m^2 > yk$ (which we assume from now on), then the absolute minimum of the potential is at $\phi_2 = \phi_3 = 0$ with ϕ_1 undetermined, so $F_1 = k$ and $V = k^2$ at the minimum. The fact that ϕ_1 is undetermined is an example of a "flat direction" in the scalar potential; this is a common feature of supersymmetric models.[t]

If we presciently choose to expand V around $\phi_1 = 0$, the mass spectrum of the theory consists of 6 real scalars with tree-level squared masses

$$0,\ 0,\ m^2,\ m^2,\ m^2 - yk,\ m^2 + yk. \tag{1.228}$$

Meanwhile, there are 3 Weyl fermions with squared masses

$$0,\ m^2,\ m^2. \tag{1.229}$$

The non-degeneracy of scalars and fermions is a clear sign that supersymmetry has been spontaneously broken. [Note that the sum rule eq. (1.220) is indeed satisfied by these squared masses.] The 0 eigenvalues in eqs. (1.228) and (1.229) correspond to the complex scalar ϕ_1 and its fermionic partner ψ_1. However, ϕ_1 and ψ_1 have different reasons for being massless. The masslessness of ϕ_1 corresponds to the existence of the flat direction, since any value of ϕ_1 gives the same energy at tree-level. This flat direction is an accidental feature of the classical scalar potential, and in this case it is removed ("lifted") by quantum corrections. This can be seen by computing the Coleman-Weinberg one-loop effective potential.[136] A little calculation reveals that the global minimum is indeed fixed at $\phi_1 = \phi_2 = \phi_3 = 0$, with the complex scalar ϕ_1 receiving a small positive-definite squared mass equal to

$$m_{\phi_1}^2 = \frac{1}{16\pi^2} y^2 m^2 \left[\ln(1-r^2) - 1 + \frac{1}{2}\left(r + 1/r\right)\ln\left(\frac{1+r}{1-r}\right)\right], \tag{1.230}$$

where $r = yk/m^2$. [Equation (1.230) reduces to $m_{\phi_1}^2 = y^4k^2/48\pi^2 m^2$ in the limit $yk \ll m^2$.] In contrast, the Weyl fermion ψ_1 remains exactly massless, because it is the goldstino, as predicted in section 1.6.1.

[t] More generally, "flat directions" are non-compact lines and surfaces in the space of scalar fields along which the scalar potential vanishes. The classical scalar potential of the MSSM would have many flat directions if supersymmetry were not broken.[135]

The O'Raifeartaigh superpotential determines the mass scale of supersymmetry breaking $\sqrt{F_1}$ in terms of a dimensionful parameter k put in by hand. This appears somewhat artificial, since k will have to be tiny compared to $M_{\rm P}^2$ in order to give the right order of magnitude for the MSSM soft terms. We would like to have a mechanism that can instead generate such scales naturally. This can be done in models of dynamical supersymmetry breaking, in which the small (compared to $M_{\rm P}$) mass scales associated with supersymmetry breaking arise by dimensional transmutation. In other words, they generally feature a new asymptotically free non-Abelian gauge symmetry with a gauge coupling g that is perturbative at $M_{\rm P}$ and gets strong in the infrared at some smaller scale $\Lambda \sim e^{-8\pi^2/|b|g_0^2} M_{\rm P}$, where g_0 is the running gauge coupling at $M_{\rm P}$ with negative beta function $-|b|g^3/16\pi^2$. Just as in QCD, it is perfectly natural for Λ to be many orders of magnitude below the Planck scale. Supersymmetry breaking may then be best described in terms of the effective dynamics of the strongly coupled theory. Supersymmetry is still broken by the VEV of an F field, but it may be the auxiliary field of a composite chiral supermultiplet built out of fields that are charged under the new strongly coupled gauge group.

Constructing non-perturbative models that actually break supersymmetry in an acceptable way is not a simple business. It is particularly difficult if one requires that the supersymmetry-breaking vacuum state is the true ground state (classically, the global minimum of the potential). One can prove using the Witten index[137,138] that any strongly coupled gauge theory with only vector-like, massive matter cannot spontaneously break supersymmetry in its ground state. Furthermore, a theory that has a generic superpotential and spontaneously breaks supersymmetry in its ground state must[139] have a continuous $U(1)$ R-symmetry, a quite non-trivial requirement. (However, effective superpotentials generated by non-perturbative dynamics are often not generic, so this requirement can be evaded.[139]) Many models that spontaneously break supersymmetry in their ground states have been found; for reviews see Ref. 140.

However, as noted in section 1.6.1, the supersymmetry-breaking vacuum state in which we live may instead correspond to only a local minimum of the potential. It has recently been shown by Intriligator, Seiberg, and Shih[130] that even supersymmetric Yang-Mills theories with vector-like matter can have metastable vacuum states with non-vanishing F-terms that break supersymmetry, and lifetimes that can be arbitrarily long. (The simplest model that does this is just supersymmetric $SU(N_c)$ gauge theory, with N_f massive flavors of quark and antiquark supermultiplets, with

$N_c + 1 \leq N_f < 3N_c/2$.) The possibility of a metastable vacuum state simplifies model building and opens up many new possibilities.[130,141]

Finding the ultimate cause of supersymmetry breaking is one of the most important goals for the future. However, for many purposes, one can simply assume that an F-term has obtained a VEV, without worrying about the specific dynamics that caused it. For understanding collider phenomenology, the most immediate concern is usually the nature of the couplings of the F-term VEV to the MSSM fields. This is the subject we turn to next.

1.6.4. *The need for a separate supersymmetry-breaking sector*

It is now clear that spontaneous supersymmetry breaking (dynamical or not) requires us to extend the MSSM. The ultimate supersymmetry-breaking order parameter cannot belong to any of the MSSM supermultiplets; a D-term VEV for $U(1)_Y$ does not lead to an acceptable spectrum, and there is no candidate gauge singlet whose F-term could develop a VEV. Therefore one must ask what effects *are* responsible for spontaneous supersymmetry breaking, and how supersymmetry breakdown is "communicated" to the MSSM particles. It is very difficult to achieve the latter in a phenomenologically viable way working only with renormalizable interactions at tree-level, even if the model is extended to involve new supermultiplets including gauge singlets. First, on general grounds it would be problematic to give masses to the MSSM gauginos, because the results of section 1.3 inform us that renormalizable supersymmetry never has any (scalar)-(gaugino)-(gaugino) couplings that could turn into gaugino mass terms when the scalar gets a VEV. Second, at least some of the MSSM squarks and sleptons would have to be unacceptably light, and should have been discovered already. This can be understood from the existence of sum rules that can be obtained in the same way as eq. (1.220) when the restrictions imposed by flavor symmetries are taken into account. For example, in the limit in which lepton flavors are conserved, the selectron mass eigenstates $\tilde{e}_1$ and $\tilde{e}_2$ could in general be mixtures of $\tilde{e}_L$ and $\tilde{e}_R$. But if they do not mix with other scalars, then part of the sum rule decouples from the rest, and one obtains:

$$m^2_{\tilde{e}_1} + m^2_{\tilde{e}_2} = 2m^2_e, \tag{1.231}$$

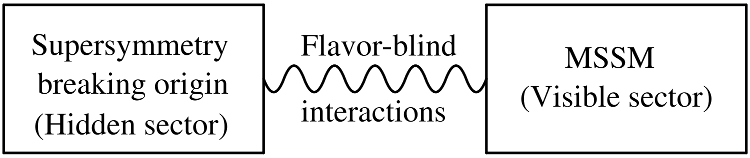

Fig. 1.15. The presumed schematic structure for supersymmetry breaking.

which is of course ruled out by experiment. Similar sum rules follow for each of the fermions of the Standard Model, at tree-level and in the limits in which the corresponding flavors are conserved. In principle, the sum rules can be evaded by introducing flavor-violating mixings, but it is very difficult to see how to make a viable model in this way. Even ignoring these problems, there is no obvious reason why the resulting MSSM soft supersymmetry-breaking terms in this type of model should satisfy flavor-blindness conditions like eqs. (1.158) or (1.159).

For these reasons, we expect that the MSSM soft terms arise indirectly or radiatively, rather than from tree-level renormalizable couplings to the supersymmetry-breaking order parameters. Supersymmetry breaking evidently occurs in a "hidden sector" of particles that have no (or only very small) direct couplings to the "visible sector" chiral supermultiplets of the MSSM. However, the two sectors do share some interactions that are responsible for mediating supersymmetry breaking from the hidden sector to the visible sector, resulting in the MSSM soft terms. (See Figure 1.15.) In this scenario, the tree-level squared mass sum rules need not hold, even approximately, for the physical masses of the visible sector fields, so that a phenomenologically viable superpartner mass spectrum is, in principle, achievable. As a bonus, if the mediating interactions are flavor-blind, then the soft terms appearing in the MSSM will automatically obey conditions like eqs. (1.158), (1.159) and (1.160).

There have been two main competing proposals for what the mediating interactions might be. The first (and historically the more popular) is that they are gravitational. More precisely, they are associated with the new physics, including gravity, that enters near the Planck scale. In this "gravity-mediated", or *Planck-scale-mediated supersymmetry breaking* (PMSB) scenario, if supersymmetry is broken in the hidden sector by a VEV $\langle F \rangle$, then the soft terms in the visible sector should be roughly

$$m_{\rm soft} \sim \langle F \rangle / M_{\rm P}, \tag{1.232}$$

by dimensional analysis. This is because we know that $m_{\rm soft}$ must vanish in the limit $\langle F\rangle \rightarrow 0$ where supersymmetry is unbroken, and also in the limit $M_{\rm P} \rightarrow \infty$ (corresponding to $G_{\rm Newton} \rightarrow 0$) in which gravity becomes irrelevant. For $m_{\rm soft}$ of order a few hundred GeV, one would therefore expect that the scale associated with the origin of supersymmetry breaking in the hidden sector should be roughly $\sqrt{\langle F\rangle} \sim 10^{10}$ or 10^{11} GeV. Another possibility is that the supersymmetry breaking order parameter is a gaugino condensate $\langle 0|\lambda^a\lambda^b|0\rangle = \delta^{ab}\Lambda^3 \neq 0$. If the composite field $\lambda^a\lambda^b$ is part of an auxiliary field F for some (perhaps composite) chiral superfield, then by dimensional analysis we expect supersymmetry breaking soft terms of order

$$m_{\rm soft} \sim \Lambda^3/M_{\rm P}^2, \tag{1.233}$$

with, effectively, $\langle F\rangle \sim \Lambda^3/M_{\rm P}$. In that case, the scale associated with dynamical supersymmetry breaking should be more like $\Lambda \sim 10^{13}$ GeV.

A second possibility is that the flavor-blind mediating interactions for supersymmetry breaking are the ordinary electroweak and QCD gauge interactions. In this *gauge-mediated supersymmetry breaking* (GMSB) scenario, the MSSM soft terms come from loop diagrams involving some *messenger* particles. The messengers are new chiral supermultiplets that couple to a supersymmetry-breaking VEV $\langle F\rangle$, and also have $SU(3)_C \times SU(2)_L \times U(1)_Y$ interactions, which provide the necessary connection to the MSSM. Then, using dimensional analysis, one estimates for the MSSM soft terms

$$m_{\rm soft} \sim \frac{\alpha_a}{4\pi}\frac{\langle F\rangle}{M_{\rm mess}} \tag{1.234}$$

where the $\alpha_a/4\pi$ is a loop factor for Feynman diagrams involving gauge interactions, and $M_{\rm mess}$ is a characteristic scale of the masses of the messenger fields. So if $M_{\rm mess}$ and $\sqrt{\langle F\rangle}$ are roughly comparable, then the scale of supersymmetry breaking can be as low as about $\sqrt{\langle F\rangle} \sim 10^4$ GeV (much lower than in the gravity-mediated case!) to give $m_{\rm soft}$ of the right order of magnitude.

1.6.5. *The goldstino and the gravitino*

As shown in section 1.6.1, the spontaneous breaking of global supersymmetry implies the existence of a massless Weyl fermion, the goldstino. The goldstino is the fermionic component of the supermultiplet whose auxiliary field obtains a VEV.

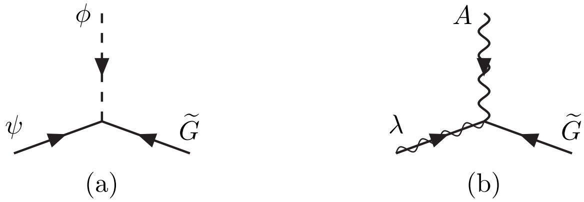

Fig. 1.16. Goldstino/gravitino $\tilde{G}$ interactions with superpartner pairs (ϕ, ψ) and (λ, A).

We can derive an important property of the goldstino by considering the form of the conserved supercurrent eq. (1.136). Suppose for simplicity[u] that the only non-vanishing auxiliary field VEV is $\langle F \rangle$ with goldstino superpartner $\widetilde{G}$. Then the supercurrent conservation equation tells us that

$$0 = \partial_\mu J^\mu_\alpha = -i\langle F\rangle(\sigma^\mu \partial_\mu \widetilde{G}^\dagger)_\alpha + \partial_\mu j^\mu_\alpha + \ldots \qquad (1.235)$$

where j^μ_α is the part of the supercurrent that involves all of the other supermultiplets, and the ellipses represent other contributions of the goldstino supermultiplet to $\partial_\mu J^\mu_\alpha$, which we can ignore. [The first term in eq. (1.235) comes from the second term in eq. (1.136), using the equation of motion $F_i = -W_i^*$ for the goldstino's auxiliary field.] This equation of motion for the goldstino field allows us to write an effective Lagrangian

$$\mathcal{L}_{\text{goldstino}} = i\widetilde{G}^\dagger \overline{\sigma}^\mu \partial_\mu \widetilde{G} - \frac{1}{\langle F\rangle}(\widetilde{G}\partial_\mu j^\mu + \text{c.c.}), \qquad (1.236)$$

which describes the interactions of the goldstino with all of the other fermion-boson pairs.[142] In particular, since $j^\mu_\alpha = (\sigma^\nu \overline{\sigma}^\mu \psi_i)_\alpha \partial_\nu \phi^{*i} - \sigma^\nu \overline{\sigma}^\rho \sigma^\mu \lambda^{\dagger a} F^a_{\nu\rho}/2\sqrt{2} + \ldots$, there are goldstino-scalar-chiral fermion and goldstino-gaugino-gauge boson vertices as shown in Figure 1.16. Since this derivation depends only on supercurrent conservation, eq. (1.236) holds independently of the details of how supersymmetry breaking is communicated from $\langle F\rangle$ to the MSSM sector fields (ϕ_i, ψ_i) and (λ^a, A^a). It may appear strange at first that the interaction couplings in eq. (1.236) get larger in the limit $\langle F\rangle$ goes to zero. However, the interaction term $\widetilde{G}\partial_\mu j^\mu$ contains two derivatives, which turn out to always give a kinematic factor proportional to the squared-mass difference of the superpartners when they are on-shell, i.e. $m^2_\phi - m^2_\psi$ and $m^2_\lambda - m^2_A$ for Figures 1.16a and 1.16b respectively. These can be non-zero only by virtue of supersymmetry breaking, so they must

[u]More generally, if supersymmetry is spontaneously broken by VEVs for several auxiliary fields F_i and D^a, then one should make the replacement $\langle F\rangle \to (\sum_i |\langle F_i\rangle|^2 + \frac{1}{2}\sum_a \langle D^a\rangle^2)^{1/2}$ everywhere in the following.

also vanish as $\langle F\rangle \to 0$, and the interaction is well-defined in that limit. Nevertheless, for fixed values of $m_\phi^2 - m_\psi^2$ and $m_\lambda^2 - m_A^2$, the interaction term in eq. (1.236) can be phenomenologically important if $\langle F\rangle$ is not too large.[142–145]

The above remarks apply to the breaking of global supersymmetry. However, taking into account gravity, supersymmetry must be promoted to a local symmetry. This means that the spinor parameter ϵ^α, which first appeared in section 1.3.1, is no longer a constant, but can vary from point to point in spacetime. The resulting locally supersymmetric theory is called *supergravity*.[146,147] It necessarily unifies the spacetime symmetries of ordinary general relativity with local supersymmetry transformations. In supergravity, the spin-2 graviton has a spin-3/2 fermion superpartner called the gravitino, which we will denote $\widetilde{\Psi}_\mu^\alpha$. The gravitino has odd R-parity ($P_R = -1$), as can be seen from the definition eq. (1.151). It carries both a vector index (μ) and a spinor index (α), and transforms inhomogeneously under local supersymmetry transformations:

$$\delta\widetilde{\Psi}_\mu^\alpha \;=\; \partial_\mu \epsilon^\alpha + \ldots . \tag{1.237}$$

Thus the gravitino should be thought of as the "gauge" field of local supersymmetry transformations [compare eq. (1.115)]. As long as supersymmetry is unbroken, the graviton and the gravitino are both massless, each with two spin helicity states. Once supersymmetry is spontaneously broken, the gravitino acquires a mass by absorbing ("eating") the goldstino, which becomes its longitudinal (helicity $\pm 1/2$) components. This is called the *super-Higgs* mechanism, and it is analogous to the ordinary Higgs mechanism for gauge theories, by which the $W^\pm$ and Z^0 gauge bosons in the Standard Model gain mass by absorbing the Nambu-Goldstone bosons associated with the spontaneously broken electroweak gauge invariance. The massive spin-3/2 gravitino now has four helicity states, of which two were originally assigned to the would-be goldstino. The gravitino mass is traditionally called $m_{3/2}$, and in the case of F-term breaking it can be estimated as[148]

$$m_{3/2} \;\sim\; \langle F\rangle / M_{\mathrm{P}}. \tag{1.238}$$

This follows simply from dimensional analysis, since $m_{3/2}$ must vanish in the limits that supersymmetry is restored ($\langle F\rangle \to 0$) and that gravity is turned off ($M_P \to \infty$). Equation (1.238) implies very different expectations for the mass of the gravitino in gravity-mediated and in gauge-mediated models, because they usually make very different predictions for $\langle F\rangle$.

In the Planck-scale-mediated supersymmetry breaking case, the gravitino mass is comparable to the masses of the MSSM sparticles [compare eqs. (1.232) and (1.238)]. Therefore $m_{3/2}$ is expected to be at least of order 100 GeV or so. Its interactions will be of gravitational strength, so the gravitino will not play any role in collider physics, but it can be important in cosmology.[149] If it is the LSP, then it is stable and its primordial density could easily exceed the critical density, causing the universe to become matter-dominated too early. Even if it is not the LSP, the gravitino can cause problems unless its density is diluted by inflation at late times, or it decays sufficiently rapidly.

In contrast, gauge-mediated supersymmetry breaking models predict that the gravitino is much lighter than the MSSM sparticles as long as $M_{\rm mess} \ll M_{\rm P}$. This can be seen by comparing eqs. (1.234) and (1.238). The gravitino is almost certainly the LSP in this case, and all of the MSSM sparticles will eventually decay into final states that include it. Naively, one might expect that these decays are extremely slow. However, this is not necessarily true, because the gravitino inherits the non-gravitational interactions of the goldstino it has absorbed. This means that the gravitino, or more precisely its longitudinal (goldstino) components, can play an important role in collider physics experiments. The mass of the gravitino can generally be ignored for kinematic purposes, as can its transverse (helicity $\pm 3/2$) components, which really do have only gravitational interactions. Therefore in collider phenomenology discussions one may interchangeably use the same symbol $\widetilde{G}$ for the goldstino and for the gravitino of which it is the longitudinal (helicity $\pm 1/2$) part. By using the effective Lagrangian eq. (1.236), one can compute that the decay rate of any sparticle $\widetilde{X}$ into its Standard Model partner X plus a goldstino/gravitino $\widetilde{G}$ is

$$\Gamma(\widetilde{X} \to X\widetilde{G}) = \frac{m_{\widetilde{X}}^5}{16\pi\langle F\rangle^2}\left(1 - m_X^2/m_{\widetilde{X}}^2\right)^4 . \tag{1.239}$$

This corresponds to either Figure 1.16a or 1.16b, with $(\widetilde{X}, X) = (\phi, \psi)$ or (λ, A) respectively. One factor $(1 - m_X^2/m_{\widetilde{X}}^2)^2$ came from the derivatives in the interaction term in eq. (1.236) evaluated for on-shell final states, and another such factor comes from the kinematic phase space integral with $m_{3/2} \ll m_{\widetilde{X}}, m_X$.

If the supermultiplet containing the goldstino and $\langle F\rangle$ has canonically normalized kinetic terms, and the tree-level vacuum energy is required to vanish, then the estimate eq. (1.238) is sharpened to

$$m_{3/2} = \langle F\rangle/\sqrt{3}M_{\rm P}. \tag{1.240}$$

In that case, one can rewrite eq. (1.239) as

$$\Gamma(\widetilde{X} \to X\widetilde{G}) = \frac{m_{\widetilde{X}}^5}{48\pi M_{\rm P}^2 m_{3/2}^2}\left(1 - m_X^2/m_{\widetilde{X}}^2\right)^4, \tag{1.241}$$

and this is how the formula is sometimes presented, although it is less general since it assumes eq. (1.240). The decay width is larger for smaller $\langle F\rangle$, or equivalently for smaller $m_{3/2}$, if the other masses are fixed. If $\widetilde{X}$ is a mixture of superpartners of different Standard Model particles X, then each partial width in eq. (1.239) should be multiplied by a suppression factor equal to the square of the cosine of the appropriate mixing angle. If $m_{\widetilde{X}}$ is of order 100 GeV or more, and $\sqrt{\langle F\rangle} \lesssim$ few $\times 10^6$ GeV [corresponding to $m_{3/2}$ less than roughly 1 keV according to eq. (1.240)], then the decay $\widetilde{X} \to X\widetilde{G}$ can occur quickly enough to be observed in a modern collider detector. This implies some interesting phenomenological signatures, which we will discuss further in sections 1.8.5.

We now turn to a more systematic analysis of the way in which the MSSM soft terms arise.

1.6.6. *Planck-scale-mediated supersymmetry breaking models*

Consider the class of models defined by the feature that the spontaneous supersymmetry-breaking sector connects with our MSSM only (or dominantly) through gravitational-strength interactions.[150,151] This means that the supergravity effective Lagrangian contains non-renormalizable terms that communicate between the two sectors and are suppressed by powers of the Planck mass $M_{\rm P}$. These will include

$$\begin{aligned}\mathcal{L}_{\rm NR} = &-\frac{1}{M_{\rm P}}F\left(\frac{1}{2}f_a\lambda^a\lambda^a + \frac{1}{6}y'^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}\mu'^{ij}\phi_i\phi_j\right) + \text{c.c.}\\ &-\frac{1}{M_{\rm P}^2}FF^*\,k^i_j\phi_i\phi^{*j}\end{aligned} \tag{1.242}$$

where F is the auxiliary field for a chiral supermultiplet in the hidden sector, and ϕ_i and λ^a are the scalar and gaugino fields in the MSSM, and f^a, y'^{ijk}, and k^i_j are dimensionless constants. By themselves, the terms in eq. (1.242) are not supersymmetric, but it is possible to show that they are part of a non-renormalizable supersymmetric Lagrangian (see Appendix) that contains other terms that we may ignore. Now if one assumes that $\sqrt{\langle F\rangle} \sim 10^{10}$ or 10^{11} GeV, then $\mathcal{L}_{\rm NR}$ will give us nothing other than a

Lagrangian of the form $\mathcal{L}_{\rm soft}$ in eq. (1.138), with MSSM soft terms of order $m_{\rm soft} \sim \langle F \rangle / M_{\rm P} =$ a few hundred GeV.

Note that couplings of the form $\mathcal{L}_{\text{maybe soft}}$ in eq. (1.139) do not arise from eq. (1.242). They actually are expected to occur, but the largest term from which they could come is:

$$\mathcal{L} = -\frac{1}{M_{\rm P}^3} F F^* x_i^{jk} \phi^{*i} \phi_j \phi_k + \text{c.c.}, \tag{1.243}$$

so in this model framework they are of order $\langle F \rangle^2 / M_{\rm P}^3 \sim m_{\rm soft}^2 / M_{\rm P}$, and therefore negligible.

The parameters f_a, k_j^i, y'^{ijk} and μ'^{ij} in $\mathcal{L}_{\rm NR}$ are to be determined by the underlying theory. This is a difficult enterprise in general, but a dramatic simplification occurs if one assumes a "minimal" form for the normalization of kinetic terms and gauge interactions in the full, non-renormalizable supergravity Lagrangian (see Appendix). In that case, there is a common $f_a = f$ for the three gauginos; $k_j^i = k\delta_j^i$ is the same for all scalars; and the other couplings are proportional to the corresponding superpotential parameters, so that $y'^{ijk} = \alpha y^{ijk}$ and $\mu'^{ij} = \beta \mu^{ij}$ with universal dimensionless constants α and β. Then the soft terms in $\mathcal{L}_{\rm soft}^{\rm MSSM}$ are all determined by just four parameters:

$$m_{1/2} = f \frac{\langle F \rangle}{M_{\rm P}}, \quad m_0^2 = k \frac{|\langle F \rangle|^2}{M_{\rm P}^2}, \quad A_0 = \alpha \frac{\langle F \rangle}{M_{\rm P}}, \quad B_0 = \beta \frac{\langle F \rangle}{M_{\rm P}}. \tag{1.244}$$

In terms of these, the parameters appearing in eq. (1.152) are:

$$M_3 = M_2 = M_1 = m_{1/2}, \tag{1.245}$$

$$\mathbf{m_Q^2} = \mathbf{m_{\overline{u}}^2} = \mathbf{m_{\overline{d}}^2} = \mathbf{m_L^2} = \mathbf{m_{\overline{e}}^2} = m_0^2\, \mathbf{1}, \qquad m_{H_u}^2 = m_{H_d}^2 = m_0^2, \tag{1.246}$$

$$\mathbf{a_u} = A_0 \mathbf{y_u}, \qquad \mathbf{a_d} = A_0 \mathbf{y_d}, \qquad \mathbf{a_e} = A_0 \mathbf{y_e}, \tag{1.247}$$

$$b = B_0 \mu, \tag{1.248}$$

at a renormalization scale $Q \approx M_{\rm P}$. It is a matter of some controversy whether the assumptions going into this parameterization are well-motivated on purely theoretical grounds,[v] but from a phenomenological perspective they are clearly very nice. This framework successfully evades the most dangerous types of flavor changing and CP violation as discussed in section 1.5.4. In particular, eqs. (1.246) and (1.247) are just stronger versions of eqs. (1.158) and (1.159), respectively. If $m_{1/2}$, A_0 and B_0 all have the same complex phase, then eq. (1.160) will also be satisfied.

[v] The familiar flavor blindness of gravity expressed in Einstein's equivalence principle does not, by itself, tell us anything about the form of eq. (1.242), and in particular need not imply eqs. (1.245)-(1.247). (See Appendix.)

Equations (1.245)-(1.248) also have the virtue of being highly predictive. [Of course, eq. (1.248) is content-free unless one can relate B_0 to the other parameters in some non-trivial way.] As discussed in sections and 1.5.4 and 1.5.5, they should be applied as RG boundary conditions at the scale $M_{\rm P}$. The RG evolution of the soft parameters down to the electroweak scale will then allow us to predict the entire MSSM spectrum in terms of just five parameters $m_{1/2}$, m_0^2, A_0, B_0, and μ (plus the already-measured gauge and Yukawa couplings of the MSSM). A popular approximation is to start this RG running from the unification scale $M_U \approx 2 \times 10^{16}$ GeV instead of $M_{\rm P}$. The reason for this is more practical than principled; the apparent unification of gauge couplings gives us a strong hint that we know something about how the RG equations behave up to M_U, but unfortunately gives us little guidance about what to expect at scales between M_U and $M_{\rm P}$. The errors made in neglecting these effects are proportional to a loop suppression factor times $\ln(M_{\rm P}/M_U)$. These corrections hopefully can be partly absorbed into a redefinition of m_0^2, $m_{1/2}$, A_0 and B_0 at M_U, but in many cases can lead to other important effects.[152] The framework described in the above few paragraphs has been the subject of the bulk of phenomenological studies of supersymmetry. It is sometimes referred to as the *minimal supergravity* (MSUGRA) or *supergravity-inspired* scenario for the soft terms. A few examples of the many useful numerical RG studies of the MSSM spectrum that have been performed in this framework can be found in Ref. 153.

Particular models of gravity-mediated supersymmetry breaking can be even more predictive, relating some of the parameters $m_{1/2}$, m_0^2, A_0 and B_0 to each other and to the mass of the gravitino $m_{3/2}$. For example, three popular kinds of models for the soft terms are:

- Dilaton-dominated:[154] $m_0^2 = m_{3/2}^2, \quad m_{1/2} = -A_0 = \sqrt{3} m_{3/2}.$
- Polonyi:[155] $m_0^2 = m_{3/2}^2, \quad A_0 = (3-\sqrt{3}) m_{3/2}, \quad m_{1/2} = \mathcal{O}(m_{3/2}).$
- "No-scale":[156] $m_{1/2} \gg m_0, A_0, m_{3/2}.$

Dilaton domination arises in a particular limit of superstring theory. While it appears to be highly predictive, it can easily be generalized in other limits.[157] The Polonyi model has the advantage of being the simplest possible model for supersymmetry breaking in the hidden sector, but it is rather *ad hoc* and does not seem to have a special place in grander schemes like superstrings. The "no-scale" limit may appear in a low-energy limit of superstrings in which the gravitino mass scale is undetermined at tree-level (hence the name). It implies that the gaugino masses dominate over other sources of supersymmetry breaking near $M_{\rm P}$. As we saw in section 1.5.5, RG evolution feeds $m_{1/2}$ into the squark, slepton, and Higgs squared-mass parameters with sufficient magnitude to give acceptable phenomenology at the electroweak scale. More recent versions of the no-scale scenario, however, also can give significant A_0 and m_0^2 at $M_{\rm P}$. In many cases B_0 can also be predicted in terms of the other parameters, but this is quite sensitive to model assumptions. For phenomenological studies, $m_{1/2}$, m_0^2, A_0 and B_0 are usually just taken to be imperfect but convenient independent parameters of our ignorance of the supersymmetry breaking mechanism.

1.6.7. *Gauge-mediated supersymmetry breaking models*

In gauge-mediated supersymmetry breaking (GMSB) models,[158,159] the ordinary gauge interactions, rather than gravity, are responsible for the appearance of soft supersymmetry breaking in the MSSM. The basic idea is to introduce some new chiral supermultiplets, called messengers, that couple to the ultimate source of supersymmetry breaking, and also couple indirectly to the (s)quarks and (s)leptons and Higgs(inos) of the MSSM through the ordinary $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge boson and gaugino interactions. There is still gravitational communication between the MSSM and the source of supersymmetry breaking, of course, but that effect is now relatively unimportant compared to the gauge interaction effects.

In contrast to Planck-scale mediation, GMSB can be understood entirely in terms of loop effects in a renormalizable framework. In the simplest such model, the messenger fields are a set of left-handed chiral supermultiplets q, $\overline{q}$, ℓ, $\overline{\ell}$ transforming under $SU(3)_C \times SU(2)_L \times U(1)_Y$ as

$$q \sim (\mathbf{3}, \mathbf{1}, -\frac{1}{3}), \quad \overline{q} \sim (\overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3}), \quad \ell \sim (\mathbf{1}, \mathbf{2}, \frac{1}{2}), \quad \overline{\ell} \sim (\mathbf{1}, \mathbf{2}, -\frac{1}{2}). \quad (1.249)$$

These supermultiplets contain messenger quarks $\psi_q, \psi_{\overline{q}}$ and scalar quarks $q, \overline{q}$ and messenger leptons $\psi_\ell, \psi_{\overline{\ell}}$ and scalar leptons $\ell, \overline{\ell}$. All of these particles must get very large masses so as not to have been discovered already. Assume they do so by coupling to a gauge-singlet chiral supermultiplet S through a superpotential:

$$W_{\rm mess} = y_2 S \ell \overline{\ell} + y_3 S q \overline{q}. \quad (1.250)$$

The scalar component of S and its auxiliary (F-term) component are each supposed to acquire VEVs, denoted $\langle S \rangle$ and $\langle F_S \rangle$ respectively. This can be accomplished either by putting S into an O'Raifeartaigh-type model,[158] or by a dynamical mechanism.[159] Exactly how this happens is an interesting and important question, without a clear answer at present. Here, we will simply parameterize our ignorance of the precise mechanism of supersymmetry breaking by asserting that S participates in another part of the superpotential, call it W_{breaking}, which provides for the necessary spontaneous breaking of supersymmetry.

Let us now consider the mass spectrum of the messenger fermions and bosons. The fermionic messenger fields pair up to get mass terms:

$$\mathcal{L} = -y_2\langle S\rangle\psi_\ell\psi_{\overline{\ell}} - y_3\langle S\rangle\psi_q\psi_{\overline{q}} + \text{c.c.} \tag{1.251}$$

as in eq. (1.111). Meanwhile, their scalar messenger partners $\ell, \overline{\ell}$ and $q, \overline{q}$ have a scalar potential given by (neglecting D-term contributions, which do not affect the following discussion):

$$\begin{aligned} V = \left|\frac{\delta W_{\text{mess}}}{\delta\ell}\right|^2 + \left|\frac{\delta W_{\text{mess}}}{\delta\overline{\ell}}\right|^2 + \left|\frac{\delta W_{\text{mess}}}{\delta q}\right|^2 + \left|\frac{\delta W_{\text{mess}}}{\delta\overline{q}}\right|^2 \\ + \left|\frac{\delta}{\delta S}(W_{\text{mess}} + W_{\text{breaking}})\right|^2 \end{aligned} \tag{1.252}$$

as in eq. (1.111). Now, suppose that, at the minimum of the potential,

$$\langle S\rangle \neq 0, \tag{1.253}$$

$$\langle \delta W_{\text{breaking}}/\delta S\rangle = -\langle F_S^*\rangle \ \neq \ 0, \tag{1.254}$$

$$\langle \delta W_{\text{mess}}/\delta S\rangle = 0. \tag{1.255}$$

Replacing S and F_S by their VEVs, one finds quadratic mass terms in the potential for the messenger scalar leptons:

$$\begin{aligned} V = {} & |y_2\langle S\rangle|^2\big(|\ell|^2 + |\overline{\ell}|^2\big) + |y_3\langle S\rangle|^2\big(|q|^2 + |\overline{q}|^2\big) \\ & - \big(y_2\langle F_S\rangle\ell\overline{\ell} + y_3\langle F_S\rangle q\overline{q} + \text{c.c.}\big) \\ & + \text{quartic terms.} \end{aligned} \tag{1.256}$$

The first line in eq. (1.256) represents supersymmetric mass terms that go along with eq. (1.251), while the second line consists of soft supersymmetry-breaking masses. The complex scalar messengers $\ell, \overline{\ell}$ thus obtain a squared-mass matrix equal to:

$$\begin{pmatrix} |y_2\langle S\rangle|^2 & -y_2^*\langle F_S^*\rangle \\ -y_2\langle F_S\rangle & |y_2\langle S\rangle|^2 \end{pmatrix} \tag{1.257}$$

with squared mass eigenvalues $|y_2\langle S\rangle|^2 \pm |y_2\langle F_S\rangle|$. In just the same way, the scalars $q, \overline{q}$ get squared masses $|y_3\langle S\rangle|^2 \pm |y_3\langle F_S\rangle|$.

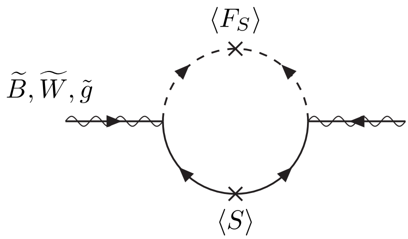

Fig. 1.17. Contributions to the MSSM gaugino masses in gauge-mediated supersymmetry breaking models come from one-loop graphs involving virtual messenger particles.

So far, we have found that the effect of supersymmetry breaking is to split each messenger supermultiplet pair apart:

$$\ell, \overline{\ell}: \quad m^2_{\text{fermions}} = |y_2 \langle S \rangle|^2 \,, \quad m^2_{\text{scalars}} = |y_2 \langle S \rangle|^2 \pm |y_2 \langle F_S \rangle| \,, \quad (1.258)$$

$$q, \overline{q}: \quad m^2_{\text{fermions}} = |y_3 \langle S \rangle|^2 \,, \quad m^2_{\text{scalars}} = |y_3 \langle S \rangle|^2 \pm |y_3 \langle F_S \rangle| \,. \quad (1.259)$$

The supersymmetry violation apparent in this messenger spectrum for $\langle F_S \rangle \neq 0$ is communicated to the MSSM sparticles through radiative corrections. The MSSM gauginos obtain masses from the 1-loop Feynman diagram shown in Figure 1.17. The scalar and fermion lines in the loop are messenger fields. Recall that the interaction vertices in Figure 1.17 are of gauge coupling strength even though they do not involve gauge bosons; compare Figure 1.5g. In this way, gauge-mediation provides that $q, \overline{q}$ messenger loops give masses to the gluino and the bino, and $\ell, \overline{\ell}$ messenger loops give masses to the wino and bino fields. Computing the 1-loop diagrams, one finds[159] that the resulting MSSM gaugino masses are given by

$$M_a = \frac{\alpha_a}{4\pi} \Lambda, \qquad (a = 1, 2, 3), \quad (1.260)$$

in the normalization for α_a discussed in section 1.5.4, where we have introduced a mass parameter

$$\Lambda \equiv \langle F_S \rangle / \langle S \rangle \,. \quad (1.261)$$

(Note that if $\langle F_S \rangle$ were 0, then $\Lambda = 0$ and the messenger scalars would be degenerate with their fermionic superpartners and there would be no contribution to the MSSM gaugino masses.) In contrast, the corresponding MSSM gauge bosons cannot get a corresponding mass shift, since they are protected by gauge invariance. So supersymmetry breaking has been successfully communicated to the MSSM ("visible sector"). To a good approximation, eq. (1.260) holds for the running gaugino masses at an RG scale Q_0 corresponding to the average characteristic mass of the heavy

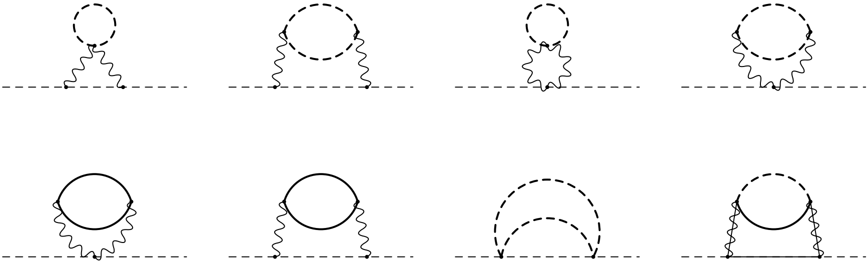

Fig. 1.18. MSSM scalar squared masses in gauge-mediated supersymmetry breaking models arise in leading order from these two-loop Feynman graphs. The heavy dashed lines are messenger scalars, the solid lines are messenger fermions, the wavy lines are ordinary Standard Model gauge bosons, and the solid lines with wavy lines superimposed are the MSSM gauginos.

messenger particles, roughly of order $M_{\text{mess}} \sim y_I \langle S \rangle$ for $I = 2, 3$. The running mass parameters can then be RG-evolved down to the electroweak scale to predict the physical masses to be measured by future experiments.

The scalars of the MSSM do not get any radiative corrections to their masses at one-loop order. The leading contribution to their masses comes from the two-loop graphs shown in Figure 1.18, with the messenger fermions (heavy solid lines) and messenger scalars (heavy dashed lines) and ordinary gauge bosons and gauginos running around the loops. By computing these graphs, one finds that each MSSM scalar ϕ_i gets a squared mass given by:

$$m^2_{\phi_i} = 2\Lambda^2 \left[\left(\frac{\alpha_3}{4\pi}\right)^2 C_3(i) + \left(\frac{\alpha_2}{4\pi}\right)^2 C_2(i) + \left(\frac{\alpha_1}{4\pi}\right)^2 C_1(i) \right], \quad (1.262)$$

with the quadratic Casimir invariants $C_a(i)$ as in eqs. (1.167)-(1.170). The squared masses in eq. (1.262) are positive (fortunately!).

The terms $\mathbf{a_u}$, $\mathbf{a_d}$, $\mathbf{a_e}$ arise first at two-loop order, and are suppressed by an extra factor of $\alpha_a/4\pi$ compared to the gaugino masses. So, to a very good approximation one has, at the messenger scale,

$$\mathbf{a_u} = \mathbf{a_d} = \mathbf{a_e} = 0, \quad (1.263)$$

a significantly stronger condition than eq. (1.159). Again, eqs. (1.262) and (1.263) should be applied at an RG scale equal to the average mass of the messenger fields running in the loops. However, evolving the RG equations down to the electroweak scale generates non-zero $\mathbf{a_u}$, $\mathbf{a_d}$, and $\mathbf{a_e}$ proportional to the corresponding Yukawa matrices and the non-zero gaugino masses, as indicated in section 1.5.5. These will only be large for the third-family squarks and sleptons, in the approximation of eq. (1.142). The parameter b may also be taken to vanish near the messenger scale, but this is quite model-dependent, and in any case b will be non-zero when it is RG-evolved to the electroweak scale. In practice, b can be fixed in terms of the other parameters by the requirement of correct electroweak symmetry breaking, as discussed below in section 1.7.1.

Because the gaugino masses arise at *one*-loop order and the scalar squared-mass contributions appear at *two*-loop order, both eq. (1.260) and (1.262) correspond to the estimate eq. (1.234) for $m_{\rm soft}$, with $M_{\rm mess} \sim y_I \langle S \rangle$. Equations (1.260) and (1.262) hold in the limit of small $\langle F_S \rangle / y_I \langle S \rangle^2$, corresponding to mass splittings within each messenger supermultiplet that are small compared to the overall messenger mass scale. The sub-leading corrections in an expansion in $\langle F_S \rangle / y_I \langle S \rangle^2$ turn out[160] to be quite small unless there are very large messenger mass splittings.

The model we have described so far is often called the minimal model of gauge-mediated supersymmetry breaking. Let us now generalize it to a more complicated messenger sector. Suppose that $q, \overline{q}$ and $\ell, \overline{\ell}$ are replaced by a collection of messengers $\Phi_I, \overline{\Phi}_I$ with a superpotential

$$W_{\rm mess} = \sum_I y_I S \Phi_I \overline{\Phi}_I. \tag{1.264}$$

The bar is used to indicate that the left-handed chiral superfields $\overline{\Phi}_I$ transform as the complex conjugate representations of the left-handed chiral superfields Φ_I. Together they are said to form a "vector-like" (real) representation of the Standard Model gauge group. As before, the fermionic components of each pair Φ_I and $\overline{\Phi}_I$ pair up to get squared masses $|y_I \langle S \rangle|^2$ and their scalar partners mix to get squared masses $|y_I \langle S \rangle|^2 \pm |y_I \langle F_S \rangle|$. The MSSM gaugino mass parameters induced are now

$$M_a = \frac{\alpha_a}{4\pi} \Lambda \sum_I n_a(I) \qquad (a = 1, 2, 3) \tag{1.265}$$

where $n_a(I)$ is the Dynkin index for each $\Phi_I + \overline{\Phi}_I$, in a normalization where $n_3 = 1$ for a $\mathbf{3} + \overline{\mathbf{3}}$ of $SU(3)_C$ and $n_2 = 1$ for a pair of doublets of $SU(2)_L$. For $U(1)_Y$, one has $n_1 = 6Y^2/5$ for each messenger pair with weak hypercharges $\pm Y$. In computing n_1 one must remember to add up the contributions for each component of an $SU(3)_C$ or $SU(2)_L$ multiplet. So, for example, $(n_1, n_2, n_3) = (2/5, 0, 1)$ for $q + \overline{q}$ and $(n_1, n_2, n_3) = (3/5, 1, 0)$ for $\ell + \overline{\ell}$. Thus the total is $\sum_I (n_1, n_2, n_3) = (1, 1, 1)$ for the minimal model, so that eq. (1.265) is in agreement with eq. (1.260). On general group-theoretic grounds, n_2 and n_3 must be integers, and n_1 is always an integer multiple of $1/5$ if fractional electric charges are confined.

The MSSM scalar masses in this generalized gauge mediation framework are now:

$$m_{\phi_i}^2 = 2\Lambda^2 \Big[\Big(\frac{\alpha_3}{4\pi}\Big)^2 C_3(i) \sum_I n_3(I) + \Big(\frac{\alpha_2}{4\pi}\Big)^2 C_2(i) \sum_I n_2(I) + \Big(\frac{\alpha_1}{4\pi}\Big)^2 C_1(i) \sum_I n_1(I) \Big]. \tag{1.266}$$

In writing eqs. (1.265) and (1.266) as simple sums, we have implicitly assumed that the messengers are all approximately equal in mass, with

$$M_{\rm mess} \approx y_I \langle S \rangle. \tag{1.267}$$

Equation (1.266) is still not a bad approximation if the y_I are not very different from each other, because the dependence of the MSSM mass spectrum on the y_I is only logarithmic (due to RG running) for fixed Λ. However, if large hierarchies in the messenger masses are present, then the additive contributions to the gaugino masses and scalar squared masses from each individual messenger multiplet I should really instead be incorporated at the mass scale of that messenger multiplet. Then RG evolution is used to run these various contributions down to the electroweak or TeV scale; the individual messenger contributions to scalar and gaugino masses as indicated above can be thought of as threshold corrections to this RG running.

Messengers with masses far below the GUT scale will affect the running of gauge couplings and might therefore be expected to ruin the apparent unification shown in Figure 1.14. However, if the messengers come in complete multiplets of the $SU(5)$ global symmetry[w] that contains the Standard Model gauge group, and are not very different in mass, then approximate unification of gauge couplings will still occur when they are extrapolated up to the same scale M_U (but with a larger unified value for the gauge couplings at that scale). For this reason, a popular class of models is obtained by taking the messengers to consist of N_5 copies of the $\mathbf{5} + \overline{\mathbf{5}}$ of $SU(5)$, resulting in

$$\sum_I n_1(I) = \sum_I n_2(I) = \sum_I n_3(I) = N_5\,. \tag{1.268}$$

[w] This $SU(5)$ may or may not be promoted to a local gauge symmetry at the GUT scale. For our present purposes, it is used only as a classification scheme, since the global $SU(5)$ symmetry is only approximate in the effective theory at the (much lower) messenger mass scale where gauge mediation takes place.

Equations (1.265) and (1.266) then reduce to

$$M_a = \frac{\alpha_a}{4\pi}\Lambda N_5, \tag{1.269}$$

$$m^2_{\phi_i} = 2\Lambda^2 N_5 \sum_{a=1}^{3} C_a(i)\left(\frac{\alpha_a}{4\pi}\right)^2, \tag{1.270}$$

since now there are N_5 copies of the minimal messenger sector particles running around the loops. For example, the minimal model in eq. (1.249) corresponds to $N_5 = 1$. A single copy of $\mathbf{10} + \overline{\mathbf{10}}$ of $SU(5)$ has Dynkin indices $\sum_I n_a(I) = 3$, and so can be substituted for 3 copies of $\mathbf{5} + \overline{\mathbf{5}}$. (Other combinations of messenger multiplets can also preserve the apparent unification of gauge couplings.) Note that the gaugino masses scale like N_5, while the scalar masses scale like $\sqrt{N_5}$. This means that sleptons and squarks will tend to be lighter relative to the gauginos for larger values of N_5 in non-minimal models. However, if N_5 is too large, then the running gauge couplings will diverge before they can unify at M_U. For messenger masses of order 10^6 GeV or less, for example, one needs $N_5 \leq 4$.

There are many other possible generalizations of the basic gauge-mediation scenario as described above. An important general expectation in these models is that the strongly interacting sparticles (squarks, gluino) should be heavier than weakly interacting sparticles (sleptons, bino, winos), simply because of the hierarchy of gauge couplings $\alpha_3 > \alpha_2 > \alpha_1$. The common feature that makes all of these models attractive is that the masses of the squarks and sleptons depend only on their gauge quantum numbers, leading automatically to the degeneracy of squark and slepton masses needed for suppression of flavor-changing effects. But the most distinctive phenomenological prediction of gauge-mediated models may be the fact that the gravitino is the LSP. This can have crucial consequences for both cosmology and collider physics, as we will discuss further in sections 1.8.5.

1.6.8. *Extra-dimensional and anomaly-mediated supersymmetry breaking*

It is also possible to take the partitioning of the MSSM and supersymmetry breaking sectors shown in Figure 1.15 seriously as geography. This can be accomplished by assuming that there are extra spatial dimensions of the Kaluza-Klein or warped type,[161] so that a physical distance separates the

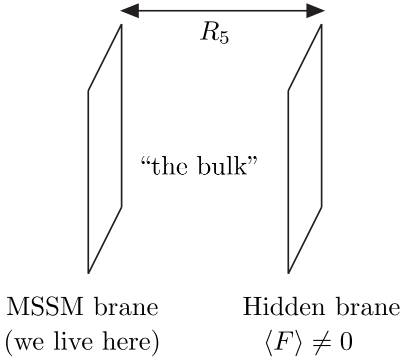

Fig. 1.19. The separation of the susy-breaking sector from the MSSM sector could take place along a hidden spatial dimension, as in the simple example shown here. The branes are 4-dimensional parallel spacetime hypersurfaces in a 5-dimensional spacetime.

visible and hidden[x] sectors. This general idea opens up numerous possibilities, which are hard to classify in a detailed way. For example, string theory suggests six such extra dimensions, with a staggeringly huge number of possible solutions.

Many of the more recently popular models used to explore this extra-dimensional mediated supersymmetry breaking (the acronym XMSB is tempting) use just one single hidden extra dimension with the MSSM chiral supermultiplets confined to one 4-dimensional spacetime brane and the supersymmetry-breaking sector confined to a parallel brane a distance R_5 away, separated by a 5-dimensional bulk, as in Figure 1.19. Using this as an illustration, the dangerous flavor-violating terms proportional to y'^{ijk} and k^i_j in eq. (1.242) are suppressed by factors like $e^{-R_5 M_5}$, where R_5 is the size of the 5th dimension and M_5 is the 5-dimensional fundamental (Planck) scale, and it is assumed that the MSSM chiral supermultiplets are confined to their brane. Therefore, it should be enough to require that $R_5 M_5 \gg 1$, in other words that the size of the 5th dimension (or, more generally, the volume of the compactified space) is relatively large in units of the fundamental length scale. Thus the suppression of flavor-violating effects does not require any fine-tuning or extreme hierarchies, because it is exponential.

One possibility is that the gauge supermultiplets of the MSSM propagate in the bulk, and so mediate supersymmetry breaking.[162–165] This

[x]The name "sequestered" is often used instead of "hidden" in this context.

mediation is direct for gauginos, with

$$M_a \sim \frac{\langle F \rangle}{M_5(R_5 M_5)}, \tag{1.271}$$

but is loop-suppressed for the soft terms involving scalars. This implies that in the simplest version of the idea, often called "gaugino mediation", soft supersymmetry breaking is dominated by the gaugino masses. The phenomenology is therefore quite similar to that of the "no-scale" boundary conditions mentioned in subsection 1.6.6 in the context of PMSB models. Scalar squared masses and the scalar cubic couplings come from renormalization group running down to the electroweak scale. It is useful to keep in mind that gaugino mass dominance is really the essential feature that defeats flavor violation, so it may well turn out to be more robust than any particular model that provides it.

It is also possible that the gauge supermultiplet fields are also confined to the MSSM brane, so that the transmission of supersymmetry breaking is due entirely to supergravity effects. This leads to anomaly-mediated supersymmetry breaking (AMSB),[166] so-named because the resulting MSSM soft terms can be understood in terms of the anomalous violation of a local superconformal invariance, an extension of scale invariance. In one formulation of supergravity,[147] Newton's constant (or equivalently, the Planck mass scale) is set by the VEV of a scalar field ϕ that is part of a non-dynamical chiral supermultiplet (called the "conformal compensator"). As a gauge fixing, this field obtains a VEV of $\langle \phi \rangle = 1$, spontaneously breaking the local superconformal invariance. Now, in the presence of spontaneous supersymmetry breaking $\langle F \rangle \neq 0$, for example on the hidden brane, the auxiliary field component also obtains a non-zero VEV, with

$$\langle F_\phi \rangle \sim \frac{\langle F \rangle}{M_\mathrm{P}} \sim m_{3/2}. \tag{1.272}$$

The non-dynamical conformal compensator field ϕ is taken to be dimensionless, so that F_ϕ has dimensions of [mass].

In the classical limit, there is still no supersymmetry breaking in the MSSM sector, due to the exponential suppression provided by the extra dimensions.[y] However, there is an anomalous violation of superconformal (scale) invariance manifested in the running of the couplings. This causes supersymmetry breaking to show up in the MSSM by virtue of the non-zero

[y] AMSB can also be realized without invoking extra dimensions. The suppression of flavor-violating MSSM soft terms can instead be achieved using a strongly-coupled conformal field theory near an infrared-stable fixed point.[167]

beta functions and anomalous dimensions of the MSSM brane couplings and fields. The resulting soft terms are[166] (using F_ϕ to denote its VEV from now on):

$$M_a = F_\phi \beta_{g_a}/g_a, \tag{1.273}$$

$$(m^2)^i_j = \frac{1}{2}|F_\phi|^2 \frac{d}{dt}\gamma^i_j = \frac{1}{2}|F_\phi|^2 \left[\beta_{g_a}\frac{\partial}{\partial g_a} + \beta_{y^{kmn}}\frac{\partial}{\partial y^{kmn}} + \beta_{y^*_{kmn}}\frac{\partial}{\partial y^*_{kmn}}\right]\gamma^i_j, \tag{1.274}$$

$$a^{ijk} = -F_\phi \beta_{y^{ijk}}, \tag{1.275}$$

where the anomalous dimensions γ^i_j are normalized as in eqs. (1.166) and (1.177)-(1.183). As in the GMSB scenario of the previous subsection, gaugino masses arise at one-loop order, but scalar squared masses arise at two-loop order. Also, these results are approximately flavor-blind for the first two families, because the non-trivial flavor structure derives only from the MSSM Yukawa couplings.

There are several unique features of the AMSB scenario. First, there is no need to specify at which renormalization scale eqs. (1.273)-(1.275) should be applied as boundary conditions. This is because they hold at every renormalization scale, exactly, to all orders in perturbation theory. In other words, eqs. (1.273)-(1.275) are not just boundary conditions for the renormalization group equations of the soft parameters, but solutions as well. (These AMSB renormalization group trajectories can also be found from this renormalization group invariance property alone,[168] without reference to the supergravity derivation.) In fact, even if there are heavy supermultiplets in the theory that have to be decoupled, the boundary conditions hold both above and below the arbitrary decoupling scale. This remarkable insensitivity to ultraviolet physics in AMSB ensures the absence of flavor violation in the low-energy MSSM soft terms. Another interesting prediction is that the gravitino mass $m_{3/2}$ in these models is actually much larger than the scale $m_{\rm soft}$ of the MSSM soft terms, since the latter are loop-suppressed compared to eq. (1.272).

There is only one unknown parameter, F_ϕ, among the MSSM soft terms in AMSB. Unfortunately, this exemplary falsifiability is marred by the fact that it is already falsified. The dominant contributions to the first-family squark and slepton squared masses are:

$$m^2_{\tilde{q}} = \frac{|F_\phi|^2}{(16\pi^2)^2}\left(8g_3^4 + \ldots\right), \tag{1.276}$$

$$m_{\tilde{e}_L}^2 = -\frac{|F_\phi|^2}{(16\pi^2)^2}\left(\frac{3}{2}g_2^4 + \frac{99}{50}g_1^4\right), \tag{1.277}$$

$$m_{\tilde{e}_R}^2 = -\frac{|F_\phi|^2}{(16\pi^2)^2}\frac{198}{25}g_1^4. \tag{1.278}$$

The squarks have large positive squared masses, but the sleptons have negative squared masses, so the AMSB model in its simplest form is not viable. These signs come directly from those of the beta functions of the strong and electroweak gauge interactions, as can be seen from the right side of eq. (1.274).

The characteristic ultraviolet insensitivity to physics at high mass scales also makes it somewhat non-trivial to modify the theory to escape this tachyonic slepton problem by deviating from the AMSB trajectory. There can be large deviations from AMSB provided by supergravity,[169] but then in general the flavor-blindness is also forfeit. One way to modify AMSB is to introduce additional supermultiplets that contain supersymmetry-breaking mass splittings that are large compared to their average mass.[170] Another way is to combine AMSB with gaugino mediation.[171] Some other proposals can be found in.[172] Finally, there is a perhaps less motivated approach in which a common parameter m_0^2 is added to all of the scalar squared masses at some scale, and chosen large enough to allow the sleptons to have positive squared masses above LEP bounds. This allows the phenomenology to be studied in a framework conveniently parameterized by just:

$$F_\phi,\ m_0^2,\ \tan\beta,\ \arg(\mu), \tag{1.279}$$

with $|\mu|$ and b determined by requiring correct electroweak symmetry breaking as described in the next section. (Some sources use $m_{3/2}$ or $M_{\rm aux}$ to denote F_ϕ.) The MSSM gaugino masses at the leading non-trivial order are unaffected by the *ad hoc* addition of m_0^2:

$$M_1 = \frac{F_\phi}{16\pi^2}\frac{33}{5}g_1^2, \tag{1.280}$$

$$M_2 = \frac{F_\phi}{16\pi^2}g_2^2, \tag{1.281}$$

$$M_3 = -\frac{F_\phi}{16\pi^2}3g_3^2. \tag{1.282}$$

This implies that $|M_2| \ll |M_1| \ll |M_3|$, so the lightest neutralino is actually mostly wino, with a lightest chargino that is only of order 200 MeV heavier, depending on the values of μ and $\tan\beta$. The decay $\widetilde{C}_1^\pm \to \widetilde{N}_1\pi^\pm$ produces a very soft pion, implying unique and difficult signatures in colliders.[173–177]

Another large general class of models breaks supersymmetry using the geometric or topological properties of the extra dimensions. In the Scherk-Schwarz mechanism,[178] the symmetry is broken by assuming different boundary conditions for the fermion and boson fields on the compactified space. In supersymmetric models where the size of the extra dimension is parameterized by a modulus (a massless or nearly massless excitation) called a radion, the F-term component of the radion chiral supermultiplet can obtain a VEV, which becomes a source for supersymmetry breaking in the MSSM. These two ideas turn out to be often related. Some of the variety of models proposed along these lines can be found in Ref. 179. These mechanisms can also be combined with gaugino-mediation and AMSB. It seems likely that the possibilities are not yet fully explored.

1.7. The Mass Spectrum of the MSSM

1.7.1. *Electroweak symmetry breaking and the Higgs bosons*

In the MSSM, the description of electroweak symmetry breaking is slightly complicated by the fact that there are two complex Higgs doublets $H_u = (H_u^+,\, H_u^0)$ and $H_d = (H_d^0,\, H_d^-)$ rather than just one in the ordinary Standard Model. The classical scalar potential for the Higgs scalar fields in the MSSM is given by

$$\begin{aligned} V = {} & (|\mu|^2 + m^2_{H_u})(|H_u^0|^2 + |H_u^+|^2) + (|\mu|^2 + m^2_{H_d})(|H_d^0|^2 + |H_d^-|^2) \\ & + [b\,(H_u^+ H_d^- - H_u^0 H_d^0) + \text{c.c.}] \\ & + \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2)^2 \\ & + \frac{1}{2} g^2 |H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2. \end{aligned} \tag{7.283}$$

The terms proportional to $|\mu|^2$ come from F-terms [see eq. (1.145)]. The terms proportional to g^2 and g'^2 are the D-term contributions, obtained from the general formula eq. (1.135) after some rearranging. Finally, the terms proportional to $m^2_{H_u}$, $m^2_{H_d}$ and b are just a rewriting of the last three terms of eq. (1.152). The full scalar potential of the theory also includes many terms involving the squark and slepton fields that we can ignore here, since they do not get VEVs because they have large positive squared masses.

We now have to demand that the minimum of this potential should break electroweak symmetry down to electromagnetism $SU(2)_L \times U(1)_Y \to U(1)_{\text{EM}}$, in accord with experiment. We can use the freedom to make gauge transformations to simplify this analysis. First, the freedom to make $SU(2)_L$ gauge transformations allows us to rotate away a possible VEV for one of the weak isospin components of one of the scalar fields, so without loss of generality we can take $H_u^+ = 0$ at the minimum of the potential. Then one can check that a minimum of the potential satisfying $\partial V/\partial H_u^+ = 0$ must also have $H_d^- = 0$. This is good, because it means that at the minimum of the potential electromagnetism is necessarily unbroken, since the charged components of the Higgs scalars cannot get VEVs. After setting $H_u^+ = H_d^- = 0$, we are left to consider the scalar potential

$$\begin{aligned} V = &(|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 - (b\, H_u^0 H_d^0 + \text{c.c.}) \\ &+ \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2. \end{aligned} \tag{7.284}$$

The only term in this potential that depends on the phases of the fields is the b-term. Therefore, a redefinition of the phase of H_u or H_d can absorb any phase in b, so we can take b to be real and positive. Then it is clear that a minimum of the potential V requires that $H_u^0 H_d^0$ is also real and positive, so $\langle H_u^0 \rangle$ and $\langle H_d^0 \rangle$ must have opposite phases. We can therefore use a $U(1)_Y$ gauge transformation to make them both be real and positive without loss of generality, since H_u and H_d have opposite weak hypercharges ($\pm 1/2$). It follows that CP cannot be spontaneously broken by the Higgs scalar potential, since the VEVs and b can be simultaneously chosen real, as a convention. This means that the Higgs scalar mass eigenstates can be assigned well-defined eigenvalues of CP, at least at tree-level. (CP-violating phases in other couplings can induce loop-suppressed CP violation in the Higgs sector, but do not change the fact that b, $\langle H_u^0 \rangle$, and $\langle H_d \rangle$ can always be chosen real and positive.)

In order for the MSSM scalar potential to be viable, we must first make sure that the potential is bounded from below for arbitrarily large values of the scalar fields, so that V will really have a minimum. (Recall from the discussion in sections 1.3.2 and 1.3.4 that scalar potentials in purely supersymmetric theories are automatically non-negative and so clearly bounded from below. But, now that we have introduced supersymmetry breaking, we must be careful.) The scalar quartic interactions in V will stabilize the potential for almost all arbitrarily large values of H_u^0 and H_d^0. However, for the special directions in field space $|H_u^0| = |H_d^0|$, the quartic contributions to V [the second line in eq. (7.284)] are identically zero. Such directions in field space are called D-flat directions, because along them the part of the scalar potential coming from D-terms vanishes. In order for the potential to be bounded from below, we need the quadratic part of the scalar potential to be positive along the D-flat directions. This requirement amounts to

$$2b < 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2. \tag{7.285}$$

Note that the b-term always favors electroweak symmetry breaking. Requiring that one linear combination of H_u^0 and H_d^0 has a negative squared mass near $H_u^0 = H_d^0 = 0$ gives

$$b^2 > (|\mu|^2 + m_{H_u}^2)(|\mu|^2 + m_{H_d}^2). \tag{7.286}$$

If this inequality is not satisfied, then $H_u^0 = H_d^0 = 0$ will be a stable minimum of the potential (or there will be no stable minimum at all), and electroweak symmetry breaking will not occur.

Interestingly, if $m_{H_u}^2 = m_{H_d}^2$ then the constraints eqs. (7.285) and (7.286) cannot both be satisfied. In models derived from the minimal supergravity or gauge-mediated boundary conditions, $m_{H_u}^2 = m_{H_d}^2$ is supposed to hold at tree level at the input scale, but the X_t contribution to the RG equation for $m_{H_u}^2$ [eq. (1.201)] naturally pushes it to negative or small values $m_{H_u}^2 < m_{H_d}^2$ at the electroweak scale. Unless this effect is significant, the parameter space in which the electroweak symmetry is broken would be quite small. So in these models electroweak symmetry breaking is actually driven by quantum corrections; this mechanism is therefore known as *radiative electroweak symmetry breaking*. Note that although a negative value for $|\mu|^2 + m_{H_u}^2$ will help eq. (7.286) to be satisfied, it is not strictly necessary. Furthermore, even if $m_{H_u}^2 < 0$, there may be no electroweak symmetry breaking if $|\mu|$ is too large or if b is too small. Still, the large negative contributions to $m_{H_u}^2$ from the RG equation are an important factor in ensuring that electroweak symmetry breaking can occur in models with simple boundary conditions for the soft terms. The realization that this works most naturally with a large top-quark Yukawa coupling provides additional motivation for these models.[151,180]

Having established the conditions necessary for H_u^0 and H_d^0 to get nonzero VEVs, we can now require that they are compatible with the observed phenomenology of electroweak symmetry breaking, $SU(2)_L \times U(1)_Y \to U(1)_{\text{EM}}$. Let us write

$$v_u = \langle H_u^0 \rangle, \qquad v_d = \langle H_d^0 \rangle. \tag{7.287}$$

These VEVs are related to the known mass of the Z^0 boson and the electroweak gauge couplings:

$$v_u^2 + v_d^2 = v^2 = 2m_Z^2/(g^2 + g'^2) \approx (174\,\text{GeV})^2. \tag{7.288}$$

The ratio of the VEVs is traditionally written as

$$\tan\beta \equiv v_u/v_d. \tag{7.289}$$

The value of $\tan\beta$ is not fixed by present experiments, but it depends on the Lagrangian parameters of the MSSM in a calculable way. Since $v_u = v\sin\beta$ and $v_d = v\cos\beta$ were taken to be real and positive by convention, we have $0 < \beta < \pi/2$, a requirement that will be sharpened below. Now one can write down the conditions $\partial V/\partial H_u^0 = \partial V/\partial H_d^0 = 0$ under which the potential eq. (7.284) will have a minimum satisfying eqs. (7.288) and (7.289):

$$m_{H_u}^2 + |\mu|^2 - b\cot\beta - (m_Z^2/2)\cos(2\beta) \;=\; 0, \tag{7.290}$$
$$m_{H_d}^2 + |\mu|^2 - b\tan\beta + (m_Z^2/2)\cos(2\beta) \;=\; 0. \tag{7.291}$$

It is easy to check that these equations indeed satisfy the necessary conditions eqs. (7.285) and (7.286). They allow us to eliminate two of the Lagrangian parameters b and $|\mu|$ in favor of $\tan\beta$, but do not determine the phase of μ. Taking $|\mu|^2$, b, $m_{H_u}^2$ and $m_{H_d}^2$ as input parameters, and m_Z^2 and $\tan\beta$ as output parameters obtained by solving these two equations, one obtains:

$$\sin(2\beta) = \frac{2b}{m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2}, \tag{7.292}$$
$$m_Z^2 = \frac{|m_{H_d}^2 - m_{H_u}^2|}{\sqrt{1-\sin^2(2\beta)}} - m_{H_u}^2 - m_{H_d}^2 - 2|\mu|^2. \tag{7.293}$$

(Note that $\sin(2\beta)$ is always positive. If $m_{H_u}^2 < m_{H_d}^2$, as is usually assumed, then $\cos(2\beta)$ is negative; otherwise it is positive.)

As an aside, eqs. (7.292) and (7.293) highlight the "μ problem" already mentioned in section 1.5.1. Without miraculous cancellations, all of the input parameters ought to be within an order of magnitude or two of m_Z^2. However, in the MSSM, μ is a supersymmetry-respecting parameter appearing in the superpotential, while b, $m_{H_u}^2$, $m_{H_d}^2$ are supersymmetry-breaking parameters. This has lead to a widespread belief that the MSSM must be extended at very high energies to include a mechanism that relates the effective value of μ to the supersymmetry-breaking mechanism in some way; see Refs. 66–68 for examples.

Even if the value of μ is set by soft supersymmetry breaking, the cancellation needed by eq. (7.293) is often remarkable when evaluated in specific model frameworks, after constraints from direct searches for the Higgs bosons and superpartners are taken into account. For example, expanding for large $\tan\beta$, eq. (7.293) becomes

$$m_Z^2 = -2(m_{H_u}^2 + |\mu|^2) + \frac{2}{\tan^2\beta}(m_{H_d}^2 - m_{H_u}^2) + \mathcal{O}(1/\tan^4\beta). \quad (7.294)$$

Typical viable solutions for the MSSM have $-m_{H_u}^2$ and $|\mu|^2$ each much larger than m_Z^2, so that significant cancellation is needed. In particular, large top squark squared masses, needed to avoid having the Higgs boson mass turn out too small [see eq. (7.307) below] compared to the direct search limits from LEP, will feed into $m_{H_u}^2$. The cancellation needed in the minimal model may therefore be at the several per cent level. It is impossible to objectively characterize whether this should be considered worrisome, but it could be taken as a weak hint in favor of non-minimal models.

The discussion above is based on the tree-level potential, and involves running renormalized Lagrangian parameters, which depend on the choice of renormalization scale. In practice, one must include radiative corrections at one-loop order, at least, in order to get numerically stable results. To do this, one can compute the loop corrections ΔV to the effective potential $V_{\text{eff}}(v_u, v_d) = V + \Delta V$ as a function of the VEVs. The impact of this is that the equations governing the VEVs of the full effective potential are obtained by simply replacing

$$m_{H_u}^2 \to m_{H_u}^2 + \frac{1}{2v_u}\frac{\partial(\Delta V)}{\partial v_u}, \qquad m_{H_d}^2 \to m_{H_d}^2 + \frac{1}{2v_d}\frac{\partial(\Delta V)}{\partial v_d} \quad (7.295)$$

in eqs. (7.290)-(7.293), treating v_u and v_d as real variables in the differentiation. The result for ΔV has now been obtained through two-loop order in the MSSM.[181] The most important corrections come from the one-loop diagrams involving the top squarks and top quark, and experience shows that the validity of the tree-level approximation and the convergence of perturbation theory are therefore improved by choosing a renormalization scale roughly of order the average of the top squark masses.

The Higgs scalar fields in the MSSM consist of two complex $SU(2)_L$-doublet, or eight real, scalar degrees of freedom. When the electroweak symmetry is broken, three of them are the would-be Nambu-Goldstone bosons G^0, $G^\pm$, which become the longitudinal modes of the Z^0 and $W^\pm$ massive vector bosons. The remaining five Higgs scalar mass eigenstates consist of two CP-even neutral scalars h^0 and H^0, one CP-odd neutral scalar A^0, and a charge $+1$ scalar H^+ and its conjugate charge -1 scalar H^-. (Here we define $G^- = G^{+*}$ and $H^- = H^{+*}$. Also, by convention, h^0

is lighter than H^0.) The gauge-eigenstate fields can be expressed in terms of the mass eigenstate fields as:

$$\begin{pmatrix} H_u^0 \\ H_d^0 \end{pmatrix} = \begin{pmatrix} v_u \\ v_d \end{pmatrix} + \frac{1}{\sqrt{2}} R_\alpha \begin{pmatrix} h^0 \\ H^0 \end{pmatrix} + \frac{i}{\sqrt{2}} R_{\beta_0} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} \tag{7.296}$$

$$\begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix} = R_{\beta_\pm} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} \tag{7.297}$$

where the orthogonal rotation matrices

$$R_\alpha = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}, \tag{7.298}$$

$$R_{\beta_0} = \begin{pmatrix} \sin\beta_0 & \cos\beta_0 \\ -\cos\beta_0 & \sin\beta_0 \end{pmatrix}, \qquad R_{\beta_\pm} = \begin{pmatrix} \sin\beta_\pm & \cos\beta_\pm \\ -\cos\beta_\pm & \sin\beta_\pm \end{pmatrix}, \tag{7.299}$$

are chosen so that the quadratic part of the potential has diagonal squared-masses:

$$\begin{aligned} V = {} & \frac{1}{2} m_{h^0}^2 (h^0)^2 + \frac{1}{2} m_{H^0}^2 (H^0)^2 + \frac{1}{2} m_{G^0}^2 (G^0)^2 + \frac{1}{2} m_{A^0}^2 (A^0)^2 \\ & + m_{G^\pm}^2 |G^+|^2 + m_{H^\pm}^2 |H^+|^2 + \ldots . \end{aligned} \tag{7.300}$$

Then, provided that v_u, v_d minimize the tree-level potential,[z] one finds that $\beta_0 = \beta_\pm = \beta$, and $m_{G^0}^2 = m_{G^\pm}^2 = 0$, and

$$m_{A^0}^2 = 2b/\sin(2\beta) \;=\; 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2 \,, \tag{7.301}$$

$$m_{h^0,H^0}^2 = \frac{1}{2}\Big(m_{A^0}^2 + m_Z^2 \mp \sqrt{(m_{A^0}^2 - m_Z^2)^2 + 4 m_Z^2 m_{A^0}^2 \sin^2(2\beta)} \Big), \tag{7.302}$$

$$m_{H^\pm}^2 = m_{A^0}^2 + m_W^2 . \tag{7.303}$$

The mixing angle α is determined, at tree-level, by

$$\frac{\sin 2\alpha}{\sin 2\beta} = -\left(\frac{m_{H^0}^2 + m_{h^0}^2}{m_{H^0}^2 - m_{h^0}^2} \right), \qquad \frac{\tan 2\alpha}{\tan 2\beta} = \left(\frac{m_{A^0}^2 + m_Z^2}{m_{A^0}^2 - m_Z^2} \right), \tag{7.304}$$

and is traditionally chosen to be negative; it follows that $-\pi/2 < \alpha < 0$ (provided $m_{A^0} > m_Z$). The Feynman rules for couplings of the mass eigenstate Higgs scalars to the Standard Model quarks and leptons and the electroweak vector bosons, as well as to the various sparticles, have been worked out in detail in Ref. 182, 183.

[z] It is often more useful to expand around VEVs v_u, v_d that do not minimize the tree-level potential, for example to minimize the loop-corrected effective potential instead. In that case, β, β_0, and $\beta_\pm$ are all slightly different.

The masses of A^0, H^0 and $H^\pm$ can in principle be arbitrarily large since they all grow with $b/\sin(2\beta)$. In contrast, the mass of h^0 is bounded above. From eq. (7.302), one finds at tree-level:[184]

$$m_{h^0} < m_Z|\cos(2\beta)|. \tag{7.305}$$

This corresponds to a shallow direction in the scalar potential, along the direction $(H_u^0 - v_u, H_d^0 - v_d) \propto (\cos\alpha, -\sin\alpha)$. The existence of this shallow direction can be traced to the fact that the quartic Higgs couplings are given by the square of the electroweak gauge couplings, via the D-term. A contour map of the potential, for a typical case with $\tan\beta \approx -\cot\alpha \approx 10$, is shown in Figure 1.20.If the tree-level inequality (7.305) were robust, the lightest Higgs boson of the MSSM would have been discovered at LEP2. However, the tree-level formula for the squared mass of h^0 is subject to quantum corrections that are relatively drastic. The largest such contributions typically come from top and stop loops, as shown[aa] in Figure 1.21. In

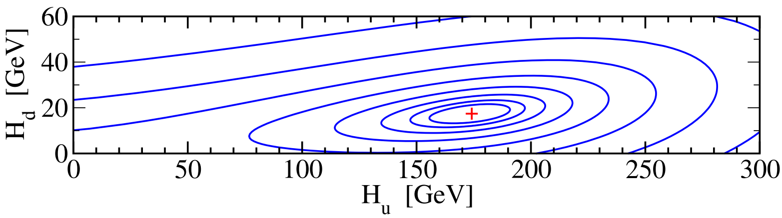

Fig. 1.20. A contour map of the Higgs potential, for a typical case with $\tan\beta \approx -\cot\alpha \approx 10$. The minimum of the potential is marked by $+$, and the contours are equally spaced equipotentials. Oscillations along the shallow direction, with $H_u^0/H_d^0 \approx 10$, correspond to the mass eigenstate h^0, while the orthogonal steeper direction corresponds to the mass eigenstate H^0.

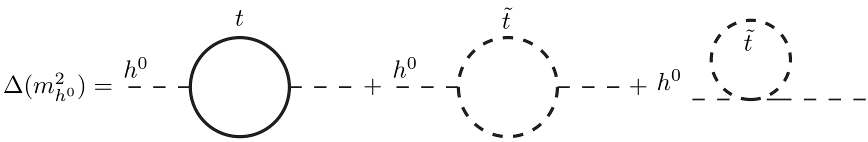

Fig. 1.21. Contributions to the MSSM lightest Higgs mass from top-quark and top-squark one-loop diagrams. Incomplete cancellation, due to soft supersymmetry breaking, leads to a large positive correction to $m^2_{h^0}$ in the limit of heavy top squarks.

[aa]In general, one-loop 1-particle-reducible tadpole diagrams should also be included. However, they just cancel against tree-level tadpoles, and so both can be omitted, if the VEVs v_u and v_d are taken at the minimum of the loop-corrected effective potential (see previous footnote).

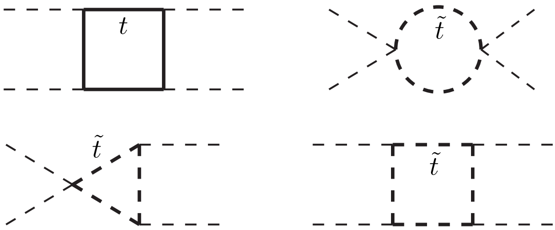

Fig. 1.22. Integrating out the top quark and top squarks yields large positive contributions to the quartic Higgs coupling in the low-energy effective theory, especially from these one-loop diagrams.

the simple limit of top squarks that have a small mixing in the gauge eigenstate basis and with masses $m_{\widetilde{t}_1}$, $m_{\widetilde{t}_2}$ much greater than the top quark mass m_t, one finds a large positive one-loop radiative correction to eq. (7.302):

$$\Delta(m^2_{h^0}) = \frac{3}{4\pi^2}\cos^2\!\alpha\ y_t^2 m_t^2 \ln\left(m_{\widetilde{t}_1} m_{\widetilde{t}_2}/m_t^2\right). \qquad (7.306)$$

This shows that m_{h^0} can exceed the LEP bounds.

An alternative way to understand the size of the radiative correction to the h^0 mass is to consider an effective theory in which the heavy top squarks and top quark have been integrated out. The quartic Higgs couplings in the low-energy effective theory get large positive contributions from the the one-loop diagrams of Figure 1.22. This increases the steepness of the Higgs potential, and can be used to obtain the same result for the enhanced h^0 mass.

An interesting case, often referred to as the "decoupling limit", occurs when $m_{A^0} \gg m_Z$. Then m_{h^0} can saturate the upper bounds just mentioned, with $m^2_{h^0} \approx m_Z^2\cos^2(2\beta)+$ loop corrections. The particles A^0, H^0, and $H^\pm$ will be much heavier and nearly degenerate, forming an isospin doublet that decouples from sufficiently low-energy experiments. The angle α is very nearly $\beta - \pi/2$, and h^0 has the same couplings to quarks and leptons and electroweak gauge bosons as would the physical Higgs boson of the ordinary Standard Model without supersymmetry. Indeed, model-building experiences have shown that it is not uncommon for h^0 to behave in a way nearly indistinguishable from a Standard Model-like Higgs boson, even if m_{A^0} is not too huge. However, it should be kept in mind that the couplings of h^0 might turn out to deviate significantly from those of a Standard Model Higgs boson.

Top-squark mixing (to be discussed in section 1.7.4) can result in a further large positive contribution to $m^2_{h^0}$. At one-loop order, and working in the decoupling limit for simplicity, eq. (7.306) generalizes to:

$$m_{h^0}^2 = m_Z^2 \cos^2(2\beta) + \frac{3}{4\pi^2}\sin^2\beta\, y_t^2 \Big[m_t^2 \ln\left(m_{\tilde t_1} m_{\tilde t_2}/m_t^2\right) \\ + c_{\tilde t}^2 s_{\tilde t}^2 (m_{\tilde t_2}^2 - m_{\tilde t_1}^2)\ln(m_{\tilde t_2}^2/m_{\tilde t_1}^2) \\ + c_{\tilde t}^4 s_{\tilde t}^4 \Big\{ (m_{\tilde t_2}^2 - m_{\tilde t_1}^2)^2 - \frac{1}{2}(m_{\tilde t_2}^4 - m_{\tilde t_1}^4)\ln(m_{\tilde t_2}^2/m_{\tilde t_1}^2)\Big\}/m_t^2 \Big]. \quad (7.307)$$

Here $c_{\tilde t}$ and $s_{\tilde t}$ are the cosine and sine of a top squark mixing angle $\theta_{\tilde t}$, defined more specifically below following eq. (7.353). For fixed top-squark masses, the maximum possible h^0 mass occurs for rather large top squark mixing, $c_{\tilde t}^2 s_{\tilde t}^2 = m_t^2/[m_{\tilde t_2}^2 + m_{\tilde t_1}^2 - 2(m_{\tilde t_2}^2 - m_{\tilde t_1}^2)/\ln(m_{\tilde t_2}^2/m_{\tilde t_1}^2)]$ or $1/4$, whichever is less. It follows that the quantity in square brackets in eq. (7.307) is always less than $m_t^2[\ln(m_{\tilde t_2}^2/m_t^2) + 3]$. The LEP constraints on the MSSM Higgs sector make the case of large top-squark mixing noteworthy.

Including these and other important corrections,[185–194] one can obtain only a weaker, but still very interesting, bound

$$m_{h^0} \lesssim 135\,\mathrm{GeV} \quad (7.308)$$

in the MSSM. This assumes that all of the sparticles that can contribute to $m_{h^0}^2$ in loops have masses that do not exceed 1 TeV. By adding extra supermultiplets to the MSSM, this bound can be made even weaker. However, assuming that none of the MSSM sparticles have masses exceeding 1 TeV and that all of the couplings in the theory remain perturbative up to the unification scale, one still has[195]

$$m_{h^0} \lesssim 150\,\mathrm{GeV}. \quad (7.309)$$

This bound is also weakened if, for example, the top squarks are heavier than 1 TeV, but the upper bound rises only logarithmically with the soft masses, as can be seen from eq. (7.306). Thus it is a fairly robust prediction of supersymmetry at the electroweak scale that at least one of the Higgs scalar bosons must be light. (However, if one is willing to extend the MSSM in a completely general way above the electroweak scale, none of these bounds need apply.) For a given set of model parameters, it is always important to take into account the complete set of one-loop corrections and even the dominant two-loop effects in order to get reasonably accurate predictions for the Higgs masses and mixings.[185–194]

In the MSSM, the masses and CKM mixing angles of the quarks and leptons are determined not only by the Yukawa couplings of the superpotential but also the parameter $\tan\beta$. This is because the top, charm and up quark mass matrix is proportional to $v_u = v \sin\beta$ and the bottom, strange, and down quarks and the charge leptons get masses proportional to $v_d = v\cos\beta$. At tree-level,

$$m_t = y_t v \sin\beta, \qquad m_b = y_b v \cos\beta, \qquad m_\tau = y_\tau v \cos\beta. \quad (7.310)$$

These relations hold for the running masses rather than the physical pole masses, which are significantly larger for t, b.[196] Including those corrections, one can relate the Yukawa couplings to $\tan\beta$ and the known fermion masses and CKM mixing angles. It is now clear why we have not neglected y_b and y_τ, even though $m_b, m_\tau \ll m_t$. To a first approximation, $y_b/y_t = (m_b/m_t)\tan\beta$ and $y_\tau/y_t = (m_\tau/m_t)\tan\beta$, so that y_b and y_τ cannot be neglected if $\tan\beta$ is much larger than 1. In fact, there are good theoretical motivations for considering models with large $\tan\beta$. For example, models based on the GUT gauge group $SO(10)$ can unify the running top, bottom and tau Yukawa couplings at the unification scale; this requires $\tan\beta$ to be very roughly of order m_t/m_b.[197,198]

Note that if one tries to make $\sin\beta$ too small, y_t will be nonperturbatively large. Requiring that y_t does not blow up above the electroweak scale, one finds that $\tan\beta \gtrsim 1.2$ or so, depending on the mass of the top quark, the QCD coupling, and other details. In principle, there is also a constraint on $\cos\beta$ if one requires that y_b and y_τ do not become nonperturbatively large. This gives a rough upper bound of $\tan\beta \lesssim 65$. However, this is complicated somewhat by the fact that the bottom quark mass gets significant one-loop non-QCD corrections in the large $\tan\beta$ limit.[198] One can obtain a stronger upper bound on $\tan\beta$ in some models where $m^2_{H_u} = m^2_{H_d}$ at the input scale, by requiring that y_b does not significantly exceed y_t. [Otherwise, X_b would be larger than X_t in eqs. (1.201) and (1.202), so one would expect $m^2_{H_d} < m^2_{H_u}$ at the electroweak scale, and the minimum of the potential would have $\langle H_d^0\rangle > \langle H_u^0\rangle$. This would be a contradiction with the supposition that $\tan\beta$ is large.] The parameter $\tan\beta$ also directly impacts the masses and mixings of the MSSM sparticles, as we will see below.

1.7.2. *Neutralinos and charginos*

The higgsinos and electroweak gauginos mix with each other because of the effects of electroweak symmetry breaking. The neutral higgsinos ($\widetilde{H}_u^0$ and $\widetilde{H}_d^0$) and the neutral gauginos ($\widetilde{B}$, $\widetilde{W}^0$) combine to form four mass eigenstates called *neutralinos*. The charged higgsinos ($\widetilde{H}_u^+$ and $\widetilde{H}_d^-$) and winos ($\widetilde{W}^+$ and $\widetilde{W}^-$) mix to form two mass eigenstates with charge ±1 called *charginos*. We will denote[bb] the neutralino and chargino mass eigenstates by $\widetilde{N}_i$ $(i = 1, 2, 3, 4)$ and $\widetilde{C}_i^{\pm}$ $(i = 1, 2)$. By convention, these are labeled in ascending order, so that $m_{\widetilde{N}_1} < m_{\widetilde{N}_2} < m_{\widetilde{N}_3} < m_{\widetilde{N}_4}$ and $m_{\widetilde{C}_1} < m_{\widetilde{C}_2}$. The lightest neutralino, $\widetilde{N}_1$, is usually assumed to be the LSP, unless there is a lighter gravitino or unless R-parity is not conserved, because it is the only MSSM particle that can make a good dark matter candidate. In this subsection, we will describe the mass spectrum and mixing of the neutralinos and charginos in the MSSM.

In the gauge-eigenstate basis $\psi^0 = (\widetilde{B}, \widetilde{W}^0, \widetilde{H}_d^0, \widetilde{H}_u^0)$, the neutralino mass part of the Lagrangian is

$$\mathcal{L}_{\text{neutralino mass}} = -\frac{1}{2}(\psi^0)^T \mathbf{M}_{\widetilde{N}} \psi^0 + \text{c.c.}, \tag{7.311}$$

where

$$\mathbf{M}_{\widetilde{N}} = \begin{pmatrix} M_1 & 0 & -g' v_d/\sqrt{2} & g' v_u/\sqrt{2} \\ 0 & M_2 & g v_d/\sqrt{2} & -g v_u/\sqrt{2} \\ -g' v_d/\sqrt{2} & g v_d/\sqrt{2} & 0 & -\mu \\ g' v_u/\sqrt{2} & -g v_u/\sqrt{2} & -\mu & 0 \end{pmatrix}. \tag{7.312}$$

The entries M_1 and M_2 in this matrix come directly from the MSSM soft Lagrangian [see eq. (1.152)], while the entries $-\mu$ are the supersymmetric higgsino mass terms [see eq. (1.144)]. The terms proportional to g, g' are the result of Higgs-higgsino-gaugino couplings [see eq. (1.132) and Figure 1.5g,h], with the Higgs scalars replaced by their VEVs [eqs. (7.288), (7.289)]. This can also be written as

$$\mathbf{M}_{\widetilde{N}} = \begin{pmatrix} M_1 & 0 & -c_\beta\, s_W\, m_Z & s_\beta\, s_W\, m_Z \\ 0 & M_2 & c_\beta\, c_W\, m_Z & -s_\beta\, c_W\, m_Z \\ -c_\beta\, s_W\, m_Z & c_\beta\, c_W\, m_Z & 0 & -\mu \\ s_\beta\, s_W\, m_Z & -s_\beta\, c_W\, m_Z & -\mu & 0 \end{pmatrix}. \tag{7.313}$$

Here we have introduced abbreviations $s_\beta = \sin\beta$, $c_\beta = \cos\beta$, $s_W = \sin\theta_W$, and $c_W = \cos\theta_W$. The mass matrix $\mathbf{M}_{\widetilde{N}}$ can be diagonalized by a unitary matrix $\mathbf{N}$ to obtain mass eigenstates:

$$\widetilde{N}_i = \mathbf{N}_{ij}\psi_j^0, \tag{7.314}$$

so that

$$\mathbf{N}^* \mathbf{M}_{\widetilde{N}} \mathbf{N}^{-1} = \begin{pmatrix} m_{\widetilde{N}_1} & 0 & 0 & 0 \\ 0 & m_{\widetilde{N}_2} & 0 & 0 \\ 0 & 0 & m_{\widetilde{N}_3} & 0 \\ 0 & 0 & 0 & m_{\widetilde{N}_4} \end{pmatrix} \tag{7.315}$$

[bb] Other common notations use $\widetilde{\chi}_i^0$ or $\widetilde{Z}_i$ for neutralinos, and $\widetilde{\chi}_i^{\pm}$ or $\widetilde{W}_i^{\pm}$ for charginos.

has real positive entries on the diagonal. These are the magnitudes of the eigenvalues of $\mathbf{M}_{\widetilde{N}}$, or equivalently the square roots of the eigenvalues of $\mathbf{M}^\dagger_{\widetilde{N}}\mathbf{M}_{\widetilde{N}}$. The indices (i, j) on $\mathbf{N}_{ij}$ are (mass, gauge) eigenstate labels. The mass eigenvalues and the mixing matrix $\mathbf{N}_{ij}$ can be given in closed form in terms of the parameters M_1, M_2, μ and $\tan\beta$, by solving quartic equations, but the results are very complicated and not illuminating.

In general, the parameters M_1, M_2, and μ in the equations above can have arbitrary complex phases. A redefinition of the phases of $\widetilde{B}$ and $\widetilde{W}$ always allows us to choose a convention in which M_1 and M_2 are both real and positive. The phase of μ within that convention is then really a physical parameter and cannot be rotated away. [We have already used up the freedom to redefine the phases of the Higgs fields, since we have picked b and $\langle H_u^0\rangle$ and $\langle H_d^0\rangle$ to be real and positive, to guarantee that the off-diagonal entries in eq. (7.313) proportional to m_Z are real.] However, if μ is not real, then there can be potentially disastrous CP-violating effects in low-energy physics, including electric dipole moments for both the electron and the neutron. Therefore, it is usual [although not strictly mandatory, because of the possibility of nontrivial cancellations involving the phases of the (scalar)^3 couplings and the gluino mass] to assume that μ is real in the same set of phase conventions that make M_1, M_2, b, $\langle H_u^0\rangle$ and $\langle H_d^0\rangle$ real and positive. The sign of μ is still undetermined by this constraint.

In models that satisfy eq. (1.189), one has the nice prediction

$$M_1 \approx \frac{5}{3}\tan^2\theta_W\, M_2 \approx 0.5 M_2 \tag{7.316}$$

at the electroweak scale. If so, then the neutralino masses and mixing angles depend on only three unknown parameters. This assumption is sufficiently theoretically compelling that it has been made in most phenomenological studies; nevertheless it should be recognized as an assumption, to be tested someday by experiment.

There is a not-unlikely limit in which electroweak symmetry breaking effects can be viewed as a small perturbation on the neutralino mass matrix. If

$$m_Z \ll |\mu \pm M_1|,\, |\mu \pm M_2|, \tag{7.317}$$

then the neutralino mass eigenstates are very nearly a "bino-like" $\widetilde{N}_1 \approx \widetilde{B}$; a "wino-like" $\widetilde{N}_2 \approx \widetilde{W}^0$; and "higgsino-like" $\widetilde{N}_3, \widetilde{N}_4 \approx (\widetilde{H}_u^0 \pm \widetilde{H}_d^0)/\sqrt{2}$, with mass eigenvalues:

$$m_{\widetilde{N}_1} = M_1 - \frac{m_Z^2 s_W^2 (M_1 + \mu\sin 2\beta)}{\mu^2 - M_1^2} + \dots\,, \tag{7.318}$$

$$m_{\widetilde{N}_2} = M_2 - \frac{m_W^2(M_2 + \mu\sin 2\beta)}{\mu^2 - M_2^2} + \dots, \tag{7.319}$$

$$m_{\widetilde{N}_3}, m_{\widetilde{N}_4} = |\mu| + \frac{m_Z^2(I - \sin 2\beta)(\mu + M_1 c_W^2 + M_2 s_W^2)}{2(\mu + M_1)(\mu + M_2)} + \dots, \tag{7.320}$$

$$|\mu| + \frac{m_Z^2(I + \sin 2\beta)(\mu - M_1 c_W^2 - M_2 s_W^2)}{2(\mu - M_1)(\mu - M_2)} + \dots, \tag{7.321}$$

where we have taken M_1 and M_2 real and positive by convention, and assumed μ is real with sign $I = \pm 1$. The subscript labels of the mass eigenstates may need to be rearranged depending on the numerical values of the parameters; in particular the above labeling of $\widetilde{N}_1$ and $\widetilde{N}_2$ assumes $M_1 < M_2 \ll |\mu|$. This limit, leading to a bino-like neutralino LSP, often emerges from minimal supergravity boundary conditions on the soft parameters, which tend to require it in order to get correct electroweak symmetry breaking.

The chargino spectrum can be analyzed in a similar way. In the gauge-eigenstate basis $\psi^\pm = (\widetilde{W}^+, \widetilde{H}_u^+, \widetilde{W}^-, \widetilde{H}_d^-)$, the chargino mass terms in the Lagrangian are

$$\mathcal{L}_{\text{chargino mass}} = -\frac{1}{2}(\psi^\pm)^T \mathbf{M}_{\widetilde{C}} \psi^\pm + \text{c.c.} \tag{7.322}$$

where, in 2×2 block form,

$$\mathbf{M}_{\widetilde{C}} = \begin{pmatrix} \mathbf{0} & \mathbf{X}^T \\ \mathbf{X} & \mathbf{0} \end{pmatrix}, \tag{7.323}$$

with

$$\mathbf{X} = \begin{pmatrix} M_2 & gv_u \\ gv_d & \mu \end{pmatrix} = \begin{pmatrix} M_2 & \sqrt{2}s_\beta\, m_W \\ \sqrt{2}c_\beta\, m_W & \mu \end{pmatrix}. \tag{7.324}$$

The mass eigenstates are related to the gauge eigenstates by two unitary 2×2 matrices $\mathbf{U}$ and $\mathbf{V}$ according to

$$\begin{pmatrix} \widetilde{C}_1^+ \\ \widetilde{C}_2^+ \end{pmatrix} = \mathbf{V} \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H}_u^+ \end{pmatrix}, \qquad \begin{pmatrix} \widetilde{C}_1^- \\ \widetilde{C}_2^- \end{pmatrix} = \mathbf{U} \begin{pmatrix} \widetilde{W}^- \\ \widetilde{H}_d^- \end{pmatrix}. \tag{7.325}$$

Note that the mixing matrix for the positively charged left-handed fermions is different from that for the negatively charged left-handed fermions. They are chosen so that

$$\mathbf{U}^* \mathbf{X} \mathbf{V}^{-1} = \begin{pmatrix} m_{\widetilde{C}_1} & 0 \\ 0 & m_{\widetilde{C}_2} \end{pmatrix}, \tag{7.326}$$

with positive real entries $m_{\widetilde{C}_i}$. Because these are only 2×2 matrices, it is not hard to solve for the masses explicitly:

$$m^2_{\widetilde{C}_1}, m^2_{\widetilde{C}_2} = \frac{1}{2}\Big[|M_2|^2 + |\mu|^2 + 2m_W^2 \mp \sqrt{(|M_2|^2 + |\mu|^2 + 2m_W^2)^2 - 4|\mu M_2 - m_W^2 \sin 2\beta|^2}\Big]. \quad (7.327)$$

These are the (doubly degenerate) eigenvalues of the 4×4 matrix $\mathbf{M}^\dagger_{\widetilde{C}}\mathbf{M}_{\widetilde{C}}$, or equivalently the eigenvalues of $\mathbf{X}^\dagger\mathbf{X}$, since

$$\mathbf{V}\mathbf{X}^\dagger\mathbf{X}\mathbf{V}^{-1} = \mathbf{U}^*\mathbf{X}\mathbf{X}^\dagger\mathbf{U}^T = \begin{pmatrix} m^2_{\widetilde{C}_1} & 0 \\ 0 & m^2_{\widetilde{C}_2} \end{pmatrix}. \quad (7.328)$$

(But, they are *not* the squares of the eigenvalues of $\mathbf{X}$.) In the limit of eq. (7.317) with real M_2 and μ, the chargino mass eigenstates consist of a wino-like $\widetilde{C}_1^\pm$ and and a higgsino-like $\widetilde{C}_2^\pm$, with masses

$$m_{\widetilde{C}_1} = M_2 - \frac{m_W^2(M_2 + \mu \sin 2\beta)}{\mu^2 - M_2^2} + \ldots \quad (7.329)$$

$$m_{\widetilde{C}_2} = |\mu| + \frac{m_W^2 I(\mu + M_2 \sin 2\beta)}{\mu^2 - M_2^2} + \ldots . \quad (7.330)$$

Here again the labeling assumes $M_2 < |\mu|$, and I is the sign of μ. Amusingly, $\widetilde{C}_1$ is nearly degenerate with the neutralino $\widetilde{N}_2$ in the approximation shown, but that is not an exact result. Their higgsino-like colleagues $\widetilde{N}_3$, $\widetilde{N}_4$ and $\widetilde{C}_2$ have masses of order $|\mu|$. The case of $M_1 \approx 0.5 M_2 \ll |\mu|$ is not uncommonly found in viable models following from the boundary conditions in section 1.6, and it has been elevated to the status of a benchmark framework in many phenomenological studies. However it cannot be overemphasized that such expectations are not mandatory.

The Feynman rules involving neutralinos and charginos may be inferred in terms of $\mathbf{N}$, $\mathbf{U}$ and $\mathbf{V}$ from the MSSM Lagrangian as discussed above; they are collected in Refs. 25, 182. Feynman rules based on two-component spinor notation have also recently been given in Ref. 199. In practice, the masses and mixing angles for the neutralinos and charginos are best computed numerically. Note that the discussion above yields the tree-level masses. Loop corrections to these masses can be significant, and have been found systematically at one-loop order in Ref. 200.

1.7.3. *The gluino*

The gluino is a color octet fermion, so it cannot mix with any other particle in the MSSM, even if R-parity is violated. In this regard, it is unique among all of the MSSM sparticles. In models with minimal supergravity or gauge-mediated boundary conditions, the gluino mass parameter M_3 is related to the bino and wino mass parameters M_1 and M_2 by eq. (1.189), so

$$M_3 \;=\; \frac{\alpha_s}{\alpha}\sin^2\theta_W\, M_2 \;=\; \frac{3}{5}\frac{\alpha_s}{\alpha}\cos^2\theta_W\, M_1 \tag{7.331}$$

at any RG scale, up to small two-loop corrections. This implies a rough prediction

$$M_3 : M_2 : M_1 \approx 6 : 2 : 1 \tag{7.332}$$

near the TeV scale. It is therefore reasonable to suspect that the gluino is considerably heavier than the lighter neutralinos and charginos (even in many models where the gaugino mass unification condition is not imposed).

For more precise estimates, one must take into account the fact that M_3 is really a running mass parameter with an implicit dependence on the RG scale Q. Because the gluino is a strongly interacting particle, M_3 runs rather quickly with Q [see eq. (1.188)]. A more useful quantity physically is the RG scale-independent mass $m_{\tilde{g}}$ at which the renormalized gluino propagator has a pole. Including one-loop corrections to the gluino propagator due to gluon exchange and quark-squark loops, one finds that the pole mass is given in terms of the running mass in the $\overline{\rm DR}$ scheme by[118]

$$m_{\tilde{g}} = M_3(Q)\Big(1 + \frac{\alpha_s}{4\pi}\big[15 + 6\ln(Q/M_3) + \sum A_{\tilde{q}}\big]\Big) \tag{7.333}$$

where

$$A_{\tilde{q}} \;=\; \int_0^1 dx\, x\ln\big[x m_{\tilde{q}}^2/M_3^2 + (1-x)m_q^2/M_3^2 - x(1-x) - i\epsilon\big]. \tag{7.334}$$

The sum in eq. (7.333) is over all 12 squark-quark supermultiplets, and we have neglected small effects due to squark mixing. [As a check, requiring $m_{\tilde{g}}$ to be independent of Q in eq. (7.333) reproduces the one-loop RG equation for $M_3(Q)$ in eq. (1.188).] The correction terms proportional to α_s in eq. (7.333) can be quite significant, because the gluino is strongly interacting, with a large group theory factor [the 15 in eq. (7.333)] due to its color octet nature, and because it couples to all of the squark-quark pairs. The leading two-loop corrections to the gluino pole mass have also been found,[201] and typically increase the prediction by another 1 or 2%.

1.7.4. *The squarks and sleptons*

In principle, any scalars with the same electric charge, R-parity, and color quantum numbers can mix with each other. This means that with completely arbitrary soft terms, the mass eigenstates of the squarks and sleptons of the MSSM should be obtained by diagonalizing three 6×6 squared-mass matrices for up-type squarks ($\widetilde{u}_L$, $\widetilde{c}_L$, $\widetilde{t}_L$, $\widetilde{u}_R$, $\widetilde{c}_R$, $\widetilde{t}_R$), down-type squarks ($\widetilde{d}_L$, $\widetilde{s}_L$, $\widetilde{b}_L$, $\widetilde{d}_R$, $\widetilde{s}_R$, $\widetilde{b}_R$), and charged sleptons ($\widetilde{e}_L$, $\widetilde{\mu}_L$, $\widetilde{\tau}_L$, $\widetilde{e}_R$, $\widetilde{\mu}_R$, $\widetilde{\tau}_R$), and one 3×3 matrix for sneutrinos ($\widetilde{\nu}_e$, $\widetilde{\nu}_\mu$, $\widetilde{\nu}_\tau$). Fortunately, the general hypothesis of flavor-blind soft parameters eqs. (1.158) and (1.159) predicts that most of these mixing angles are very small. The third-family squarks and sleptons can have very different masses compared to their first- and second-family counterparts, because of the effects of large Yukawa (y_t, y_b, y_τ) and soft (a_t, a_b, a_τ) couplings in the RG equations (1.203)-(1.207). Furthermore, they can have substantial mixing in pairs ($\widetilde{t}_L$, $\widetilde{t}_R$), ($\widetilde{b}_L$, $\widetilde{b}_R$) and ($\widetilde{\tau}_L$, $\widetilde{\tau}_R$). In contrast, the first- and second-family squarks and sleptons have negligible Yukawa couplings, so they end up in 7 very nearly degenerate, unmixed pairs ($\widetilde{e}_R, \widetilde{\mu}_R$), ($\widetilde{\nu}_e, \widetilde{\nu}_\mu$), ($\widetilde{e}_L, \widetilde{\mu}_L$), ($\widetilde{u}_R, \widetilde{c}_R$), ($\widetilde{d}_R, \widetilde{s}_R$), ($\widetilde{u}_L, \widetilde{c}_L$), ($\widetilde{d}_L, \widetilde{s}_L$). As we have already discussed in section 1.5.4, this avoids the problem of disastrously large virtual sparticle contributions to flavor-changing processes.

Let us first consider the spectrum of first- and second-family squarks and sleptons. In many models, including both minimal supergravity [eq. (1.246)] and gauge-mediated [eq. (1.262)] boundary conditions, their running squared masses can be conveniently parameterized, to a good approximation, as:

$$m^2_{Q_1} = m^2_{Q_2} = m_0^2 + K_3 + K_2 + \frac{1}{36}K_1, \tag{7.335}$$

$$m^2_{\overline{u}_1} = m^2_{\overline{u}_2} = m_0^2 + K_3 \qquad + \frac{4}{9}K_1, \tag{7.336}$$

$$m^2_{\overline{d}_1} = m^2_{\overline{d}_2} = m_0^2 + K_3 \qquad + \frac{1}{9}K_1, \tag{7.337}$$

$$m^2_{L_1} = m^2_{L_2} = m_0^2 \qquad + K_2 + \frac{1}{4}K_1, \tag{7.338}$$

$$m^2_{\overline{e}_1} = m^2_{\overline{e}_2} = m_0^2 \qquad\qquad + K_1. \tag{7.339}$$

A key point is that the same K_3, K_2 and K_1 appear everywhere in eqs. (7.335)-(7.339), since all of the chiral supermultiplets couple to the same gauginos with the same gauge couplings. The different coefficients in front of K_1 just correspond to the various values of weak hypercharge squared for each scalar.

In minimal supergravity models, m_0^2 is the same common scalar squared mass appearing in eq. (1.246). It can be very small, as in the "no-scale" limit, but it could also be the dominant source of the scalar masses. The contributions K_3, K_2 and K_1 are due to the RG running[cc] proportional to the gaugino masses. Explicitly, they are found at one loop order by solving eq. (1.196):

$$K_a(Q) = \left\{ \begin{matrix} 3/5 \\ 3/4 \\ 4/3 \end{matrix} \right\} \times \frac{1}{2\pi^2} \int_{\ln Q}^{\ln Q_0} dt \; g_a^2(t)\, |M_a(t)|^2 \quad (a = 1,2,3). \quad (7.340)$$

Here Q_0 is the input RG scale at which the minimal supergravity boundary condition eq. (1.246) is applied, and Q should be taken to be evaluated near the squark and slepton mass under consideration, presumably less than about 1 TeV. The running parameters $g_a(Q)$ and $M_a(Q)$ obey eqs. (1.161) and (1.189). If the input scale is approximated by the apparent scale of gauge coupling unification $Q_0 = M_U \approx 2 \times 10^{16}$ GeV, one finds that numerically

$$K_1 \approx 0.15 m_{1/2}^2, \qquad K_2 \approx 0.5 m_{1/2}^2, \qquad K_3 \approx (4.5 \text{ to } 6.5) m_{1/2}^2. \quad (7.341)$$

for Q near the electroweak scale. Here $m_{1/2}$ is the common gaugino mass parameter at the unification scale. Note that $K_3 \gg K_2 \gg K_1$; this is a direct consequence of the relative sizes of the gauge couplings g_3, g_2, and g_1. The large uncertainty in K_3 is due in part to the experimental uncertainty in the QCD coupling constant, and in part to the uncertainty in where to choose Q, since K_3 runs rather quickly below 1 TeV. If the gauge couplings and gaugino masses are unified between M_U and M_P, as would occur in a GUT model, then the effect of RG running for $M_U < Q < M_P$ can be absorbed into a redefinition of m_0^2. Otherwise, it adds a further uncertainty roughly proportional to $\ln(M_P/M_U)$, compared to the larger contributions in eq. (7.340), which go roughly like $\ln(M_U/1 \text{ TeV})$.

In gauge-mediated models, the same parameterization eqs. (7.335)-(7.339) holds, but m_0^2 is always 0. At the input scale Q_0, each MSSM scalar gets contributions to its squared mass that depend only on its gauge interactions, as in eq. (1.262). It is not hard to see that in general these contribute in exactly the same pattern as K_1, K_2, and K_3 in eq. (7.335)-(7.339). The subsequent evolution of the scalar squared masses down to the electroweak scale again just yields more contributions to the K_1, K_2, and K_3

[cc] The quantity S defined in eq. (1.197) vanishes for both minimal supergravity and gauge-mediated boundary conditions, and remains small under RG evolution.

parameters. It is somewhat more difficult to give meaningful numerical estimates for these parameters in gauge-mediated models than in the minimal supergravity models without knowing the messenger mass scale(s) and the multiplicities of the messenger fields. However, in the gauge-mediated case one quite generally expects that the numerical values of the ratios K_3/K_2, K_3/K_1 and K_2/K_1 should be even larger than in eq. (7.341). There are two reasons for this. First, the running squark squared masses start off larger than slepton squared masses already at the input scale in gauge-mediated models, rather than having a common value m_0^2. Furthermore, in the gauge-mediated case, the input scale Q_0 is typically much lower than M_P or M_U, so that the RG evolution gives relatively more weight to RG scales closer to the electroweak scale, where the hierarchies $g_3 > g_2 > g_1$ and $M_3 > M_2 > M_1$ are already in effect.

In general, one therefore expects that the squarks should be considerably heavier than the sleptons, with the effect being more pronounced in gauge-mediated supersymmetry breaking models than in minimal supergravity models. For any specific choice of model, this effect can be easily quantified with a numerical RG computation. The hierarchy $m_{\rm squark} > m_{\rm slepton}$ tends to hold even in models that do not fit neatly into any of the categories outlined in section 1.6, because the RG contributions to squark masses from the gluino are always present and usually quite large, since QCD has a larger gauge coupling than the electroweak interactions.

Regardless of the type of model, there is also a "hyperfine" splitting in the squark and slepton mass spectrum produced by electroweak symmetry breaking. Each squark and slepton ϕ will get a contribution Δ_ϕ to its squared mass, coming from the $SU(2)_L$ and $U(1)_Y$ D-term quartic interactions [see the last term in eq. (1.135)] of the form (squark)^2(Higgs)^2 and (slepton)^2(Higgs)^2, when the neutral Higgs scalars H_u^0 and H_d^0 get VEVs. They are model-independent for a given value of $\tan\beta$:

$$\begin{aligned}\Delta_\phi &= \frac{1}{2}(T_{3\phi}g^2 - Y_\phi g'^2)(v_d^2 - v_u^2) \\ &= (T_{3\phi} - Q_\phi \sin^2\theta_W)\cos(2\beta)\, m_Z^2, \end{aligned} \tag{7.342}$$

where $T_{3\phi}$, Y_ϕ, and Q_ϕ are the third component of weak isospin, the weak hypercharge, and the electric charge of the left-handed chiral supermultiplet to which ϕ belongs. For example, $\Delta_{\tilde{u}_L} = (\frac{1}{2} - \frac{2}{3}\sin^2\theta_W)\cos(2\beta)\, m_Z^2$ and $\Delta_{\tilde{d}_L} = (-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W)\cos(2\beta)\, m_Z^2$ and $\Delta_{\tilde{u}_R} = (\frac{2}{3}\sin^2\theta_W)\cos(2\beta)\, m_Z^2$. These D-term contributions are typically smaller than the m_0^2 and K_1, K_2, K_3 contributions, but should not be neglected. They split apart the components of the $SU(2)_L$-doublet sleptons and squarks. Including them, the first-family squark and slepton masses are now given by:

$$m^2_{\tilde{d}_L} = m_0^2 + K_3 + K_2 + \frac{1}{36}K_1 + \Delta_{\tilde{d}_L}, \tag{7.343}$$

$$m^2_{\tilde{u}_L} = m_0^2 + K_3 + K_2 + \frac{1}{36}K_1 + \Delta_{\tilde{u}_L}, \tag{7.344}$$

$$m^2_{\tilde{u}_R} = m_0^2 + K_3 \qquad + \frac{4}{9}K_1 + \Delta_{\tilde{u}_R}, \tag{7.345}$$

$$m^2_{\tilde{d}_R} = m_0^2 + K_3 \qquad + \frac{1}{9}K_1 + \Delta_{\tilde{d}_R}, \tag{7.346}$$

$$m^2_{\tilde{e}_L} = m_0^2 \qquad + K_2 + \frac{1}{4}K_1 + \Delta_{\tilde{e}_L}, \tag{7.347}$$

$$m^2_{\tilde{\nu}} = m_0^2 \qquad + K_2 + \frac{1}{4}K_1 + \Delta_{\tilde{\nu}}, \tag{7.348}$$

$$m^2_{\tilde{e}_R} = m_0^2 \qquad\qquad + K_1 + \Delta_{\tilde{e}_R}, \tag{7.349}$$

with identical formulas for the second-family squarks and sleptons. The mass splittings for the left-handed squarks and sleptons are governed by model-independent sum rules

$$m^2_{\tilde{e}_L} - m^2_{\tilde{\nu}_e} = m^2_{\tilde{d}_L} - m^2_{\tilde{u}_L} = g^2(v_u^2 - v_d^2)/2 = -\cos(2\beta)\, m_W^2. \tag{7.350}$$

In the allowed range $\tan\beta > 1$, it follows that $m_{\tilde{e}_L} > m_{\tilde{\nu}_e}$ and $m_{\tilde{d}_L} > m_{\tilde{u}_L}$, with the magnitude of the splittings constrained by electroweak symmetry breaking.

Let us next consider the masses of the top squarks, for which there are several non-negligible contributions. First, there are squared-mass terms for $\widetilde{t}_L^*\widetilde{t}_L$ and $\widetilde{t}_R^*\widetilde{t}_R$ that are just equal to $m^2_{Q_3} + \Delta_{\tilde{u}_L}$ and $m^2_{\overline{u}_3} + \Delta_{\tilde{u}_R}$, respectively, just as for the first- and second-family squarks. Second, there are contributions equal to m_t^2 for each of $\widetilde{t}_L^*\widetilde{t}_L$ and $\widetilde{t}_R^*\widetilde{t}_R$. These come from F-terms in the scalar potential of the form $y_t^2 H_u^{0*} H_u^0 \widetilde{t}_L^* \widetilde{t}_L$ and $y_t^2 H_u^{0*} H_u^0 \widetilde{t}_R^* \widetilde{t}_R$ (see Figures 1.8b and 1.8c), with the Higgs fields replaced by their VEVs. (Of course, similar contributions are present for all of the squarks and sleptons, but they are too small to worry about except in the case of the top squarks.) Third, there are contributions to the scalar potential from F-terms of the form $-\mu^* y_t \widetilde{\bar{t}}\widetilde{t} H_d^{0*} + \text{c.c.}$; see eqs. (1.146) and Figure 1.10a. These become $-\mu^* v y_t \cos\beta\, \widetilde{t}_R^* \widetilde{t}_L + \text{c.c.}$ when H_d^0 is replaced by its VEV. Finally, there are contributions to the scalar potential from the soft (scalar)^3 couplings $a_t \widetilde{\bar{t}} \widetilde{Q}_3 H_u^0 + \text{c.c.}$ [see the first term of the second line of eq. (1.152), and eq. (1.190)], which become $a_t v \sin\beta\, \widetilde{t}_L \widetilde{t}_R^* + \text{c.c.}$ when H_u^0 is replaced by its VEV. Putting these all together, we have a squared-mass matrix for the top squarks, which in the gauge-eigenstate basis $(\widetilde{t}_L, \widetilde{t}_R)$ is given by

$$\mathcal{L}_{\text{stop masses}} = -\left(\, \widetilde{t}_L^* \quad \widetilde{t}_R^* \,\right) \mathbf{m}^{\mathbf{2}}_{\widetilde{\mathbf{t}}} \begin{pmatrix} \widetilde{t}_L \\ \widetilde{t}_R \end{pmatrix} \tag{7.351}$$

where

$$\mathbf{m}^{\mathbf{2}}_{\widetilde{\mathbf{t}}} = \begin{pmatrix} m^2_{Q_3} + m_t^2 + \Delta_{\tilde{u}_L} & v(a_t^* \sin\beta - \mu y_t \cos\beta) \\ v(a_t \sin\beta - \mu^* y_t \cos\beta) & m^2_{\overline{u}_3} + m_t^2 + \Delta_{\tilde{u}_R} \end{pmatrix}. \tag{7.352}$$

This hermitian matrix can be diagonalized by a unitary matrix to give mass eigenstates:

$$\begin{pmatrix} \widetilde{t}_1 \\ \widetilde{t}_2 \end{pmatrix} = \begin{pmatrix} c_{\tilde{t}} & -s^*_{\tilde{t}} \\ s_{\tilde{t}} & c_{\tilde{t}} \end{pmatrix} \begin{pmatrix} \widetilde{t}_L \\ \widetilde{t}_R \end{pmatrix}. \tag{7.353}$$

Here $m^2_{\tilde{t}_1} < m^2_{\tilde{t}_2}$ are the eigenvalues of eq. (7.352), and $|c_{\tilde{t}}|^2 + |s_{\tilde{t}}|^2 = 1$. If the off-diagonal elements of eq. (7.352) are real, then $c_{\tilde{t}}$ and $s_{\tilde{t}}$ are the cosine and sine of a stop mixing angle $\theta_{\tilde{t}}$, which can be chosen in the range $0 \leq \theta_{\tilde{t}} < \pi$. Because of the large RG effects proportional to X_t in eq. (1.203) and eq. (1.204), at the electroweak scale one finds that $m^2_{\overline{u}_3} < m^2_{Q_3}$, and both of these quantities are usually significantly smaller than the squark squared masses for the first two families. The diagonal terms m_t^2 in eq. (7.352) tend to mitigate this effect somewhat, but the off-diagonal entries will typically induce a significant mixing, which always reduces the lighter top-squark squared-mass eigenvalue. Therefore, models often predict that $\widetilde{t}_1$ is the lightest squark of all, and that it is predominantly $\widetilde{t}_R$.

A very similar analysis can be performed for the bottom squarks and charged tau sleptons, which in their respective gauge-eigenstate bases $(\widetilde{b}_L, \widetilde{b}_R)$ and $(\widetilde{\tau}_L, \widetilde{\tau}_R)$ have squared-mass matrices:

$$\mathbf{m}^{\mathbf{2}}_{\widetilde{\mathbf{b}}} = \begin{pmatrix} m^2_{Q_3} + \Delta_{\tilde{d}_L} & v(a_b^* \cos\beta - \mu y_b \sin\beta) \\ v(a_b \cos\beta - \mu^* y_b \sin\beta) & m^2_{\overline{d}_3} + \Delta_{\tilde{d}_R} \end{pmatrix}, \tag{7.354}$$

$$\mathbf{m}^{\mathbf{2}}_{\widetilde{\boldsymbol{\tau}}} = \begin{pmatrix} m^2_{L_3} + \Delta_{\tilde{e}_L} & v(a_\tau^* \cos\beta - \mu y_\tau \sin\beta) \\ v(a_\tau \cos\beta - \mu^* y_\tau \sin\beta) & m^2_{\overline{e}_3} + \Delta_{\tilde{e}_R} \end{pmatrix}. \tag{7.355}$$

These can be diagonalized to give mass eigenstates $\widetilde{b}_1, \widetilde{b}_2$ and $\widetilde{\tau}_1, \widetilde{\tau}_2$ in exact analogy with eq. (7.353).

The magnitude and importance of mixing in the sbottom and stau sectors depends on how big $\tan\beta$ is. If $\tan\beta$ is not too large (in practice, this usually means less than about 10 or so, depending on the situation under study), the sbottoms and staus do not get a very large effect from the mixing terms and the RG effects due to X_b and X_τ, because $y_b, y_\tau \ll y_t$

from eq. (7.310). In that case the mass eigenstates are very nearly the same as the gauge eigenstates $\widetilde{b}_L$, $\widetilde{b}_R$, $\widetilde{\tau}_L$ and $\widetilde{\tau}_R$. The latter three, and $\widetilde{\nu}_\tau$, will be nearly degenerate with their first- and second-family counterparts with the same $SU(3)_C \times SU(2)_L \times U(1)_Y$ quantum numbers. However, even in the case of small $\tan\beta$, $\widetilde{b}_L$ will feel the effects of the large top Yukawa coupling because it is part of the doublet containing $\widetilde{t}_L$. In particular, from eq. (1.203) we see that X_t acts to decrease $m^2_{Q_3}$ as it is RG-evolved down from the input scale to the electroweak scale. Therefore the mass of $\widetilde{b}_L$ can be significantly less than the masses of $\widetilde{d}_L$ and $\widetilde{s}_L$.

For larger values of $\tan\beta$, the mixing in eqs. (7.354) and (7.355) can be quite significant, because y_b, y_τ and a_b, a_τ are non-negligible. Just as in the case of the top squarks, the lighter sbottom and stau mass eigenstates (denoted $\widetilde{b}_1$ and $\widetilde{\tau}_1$) can be significantly lighter than their first- and second-family counterparts. Furthermore, $\widetilde{\nu}_\tau$ can be significantly lighter than the nearly degenerate $\widetilde{\nu}_e$, $\widetilde{\nu}_\mu$.

The requirement that the third-family squarks and sleptons should all have positive squared masses implies limits on the magnitudes of $a_t^* \sin\beta - \mu y_t \cos\beta$ and $a_b^* \cos\beta - \mu y_b \sin\beta$ and and $a_\tau^* \cos\beta - \mu y_\tau \sin\beta$. If they are too large, then the smaller eigenvalue of eq. (7.352), (7.354) or (7.355) will be driven negative, implying that a squark or charged slepton gets a VEV, breaking $SU(3)_C$ or electromagnetism. Since this is clearly unacceptable, one can put bounds on the (scalar)^3 couplings, or equivalently on the parameter A_0 in minimal supergravity models. Even if all of the squared-mass eigenvalues are positive, the presence of large (scalar)^3 couplings can yield global minima of the scalar potential, with non-zero squark and/or charged slepton VEVs, which are disconnected from the vacuum that conserves $SU(3)_C$ and electromagnetism.[202] However, it is not always immediately clear whether the mere existence of such disconnected global minima should really disqualify a set of model parameters, because the tunneling rate from our "good" vacuum to the "bad" vacua can easily be longer than the age of the universe.[203]

1.7.5. *Summary: The MSSM sparticle spectrum*

In the MSSM there are 32 distinct masses corresponding to undiscovered particles, not including the gravitino. In this section we have explained how the masses and mixing angles for these particles can be computed, given an underlying model for the soft terms at some input scale. Assuming only that the mixing of first- and second-family squarks and sleptons is negligible, the

Table 1.5. The undiscovered particles in the Minimal Supersymmetric Standard Model (with sfermion mixing for the first two families assumed to be negligible.

Names	Spin	P_R	Gauge Eigenstates	Mass Eigenstates
Higgs bosons	0	+1	H_u^0 H_d^0 H_u^+ H_d^-	h^0 H^0 A^0 $H^\pm$
squarks	0	−1	$\widetilde{u}_L$ $\widetilde{u}_R$ $\widetilde{d}_L$ $\widetilde{d}_R$	(same)
			$\widetilde{s}_L$ $\widetilde{s}_R$ $\widetilde{c}_L$ $\widetilde{c}_R$	(same)
			$\widetilde{t}_L$ $\widetilde{t}_R$ $\widetilde{b}_L$ $\widetilde{b}_R$	$\widetilde{t}_1$ $\widetilde{t}_2$ $\widetilde{b}_1$ $\widetilde{b}_2$
sleptons	0	−1	$\widetilde{e}_L$ $\widetilde{e}_R$ $\widetilde{\nu}_e$	(same)
			$\widetilde{\mu}_L$ $\widetilde{\mu}_R$ $\widetilde{\nu}_\mu$	(same)
			$\widetilde{\tau}_L$ $\widetilde{\tau}_R$ $\widetilde{\nu}_\tau$	$\widetilde{\tau}_1$ $\widetilde{\tau}_2$ $\widetilde{\nu}_\tau$
neutralinos	1/2	−1	$\widetilde{B}^0$ $\widetilde{W}^0$ $\widetilde{H}_u^0$ $\widetilde{H}_d^0$	$\widetilde{N}_1$ $\widetilde{N}_2$ $\widetilde{N}_3$ $\widetilde{N}_4$
charginos	1/2	−1	$\widetilde{W}^\pm$ $\widetilde{H}_u^+$ $\widetilde{H}_d^-$	$\widetilde{C}_1^\pm$ $\widetilde{C}_2^\pm$
gluino	1/2	−1	$\widetilde{g}$	(same)
goldstino (gravitino)	1/2 (3/2)	−1	$\widetilde{G}$	(same)

mass eigenstates of the MSSM are listed in Table 1.5. A complete set of Feynman rules for the interactions of these particles with each other and with the Standard Model quarks, leptons, and gauge bosons can be found in Refs. 25, 182. Feynman rules based on two-component spinor notation have also recently been given in Ref. 199.

Specific models for the soft terms typically predict the masses and the mixing angles angles for the MSSM in terms of far fewer parameters. For example, in the minimal supergravity models, the only free parameters not already measured by experiment are m_0^2, $m_{1/2}$, A_0, μ, and b. In gauge-mediated supersymmetry breaking models, the free parameters include at least the scale Λ, the typical messenger mass scale $M_{\rm mess}$, the integer number N_5 of copies of the minimal messengers, the goldstino decay constant $\langle F\rangle$, and the Higgs mass parameters μ and b. After RG evolving the soft terms down to the electroweak scale, one can demand that the scalar potential gives correct electroweak symmetry breaking. This allows us to trade $|\mu|$ and b (or B_0) for one parameter $\tan\beta$, as in eqs. (7.291)-(7.290). So, to a reasonable approximation, the entire mass spectrum in minimal supergravity models is determined by only five unknown parameters: m_0^2, $m_{1/2}$, A_0, $\tan\beta$, and $\mathrm{Arg}(\mu)$, while in the simplest gauge-mediated supersymmetry breaking models one can pick parameters Λ, $M_{\rm mess}$, N_5, $\langle F\rangle$, $\tan\beta$, and $\mathrm{Arg}(\mu)$. Both frameworks are highly predictive. Of course, it is easy to

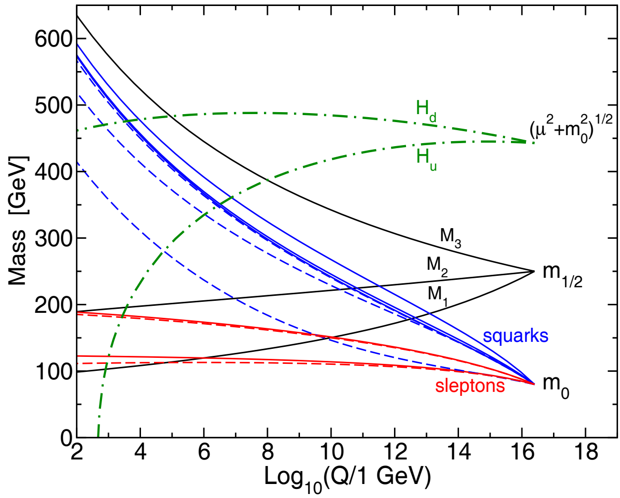

Fig. 1.23. RG evolution of scalar and gaugino mass parameters in the MSSM with typical minimal supergravity-inspired boundary conditions imposed at $Q_0 = 2.5 \times 10^{16}$ GeV. The parameter $\mu^2 + m^2_{H_u}$ runs negative, provoking electroweak symmetry breaking.

imagine that the essential physics of supersymmetry breaking is not captured by either of these two scenarios in their minimal forms. For example, the anomaly mediated contributions could play a role, perhaps in concert with the gauge-mediation or Planck-scale mediation mechanisms.

Figure 1.23 shows the RG running of scalar and gaugino masses in a typical model based on the minimal supergravity boundary conditions imposed at $Q_0 = 2.5 \times 10^{16}$ GeV. [The parameter values used for this illustration were $m_0 = 80$ GeV, $m_{1/2} = 250$ GeV, $A_0 = -500$ GeV, $\tan\beta = 10$, and sign(μ)$= +$.] The running gaugino masses are solid lines labeled by M_1, M_2, and M_3. The dot-dashed lines labeled H_u and H_d are the running values of the quantities $(\mu^2 + m^2_{H_u})^{1/2}$ and $(\mu^2 + m^2_{H_d})^{1/2}$, which appear in the Higgs potential. The other lines are the running squark and slepton masses, with dashed lines for the square roots of the third family parameters $m^2_{\overline{d}_3}$, $m^2_{Q_3}$, $m^2_{\overline{u}_3}$, $m^2_{L_3}$, and $m^2_{\overline{e}_3}$ (from top to bottom), and solid lines for the first and second family sfermions. Note that $\mu^2 + m^2_{H_u}$ runs negative because of the effects of the large top Yukawa coupling as discussed above, providing for electroweak symmetry breaking. At the electroweak scale, the values of the Lagrangian soft parameters can be used to extract the physical masses, cross-sections, and decay widths of the particles, and other observables such as dark matter abundances and rare process rates. There are a variety of publicly available programs that do these tasks, including radiative corrections; see for example Refs. 204–213, 194.

Figure 1.24 shows deliberately qualitative sketches of sample MSSM mass spectrum obtained from three different types of models assumptions. The first is the output from a minimal supergravity-inspired model with relatively low m_0^2 compared to $m_{1/2}^2$ (in fact the same model parameters as used for Figure 1.23). This model features a near-decoupling limit for the Higgs sector, and a bino-like $\widetilde{N}_1$ LSP, nearly degenerate wino-like $\widetilde{N}_2, \widetilde{C}_1$, and higgsino-like $\widetilde{N}_3, \widetilde{N}_4, \widetilde{C}_2$. The gluino is the heaviest superpartner. The squarks are all much heavier than the sleptons, and the lightest sfermion is a stau. Variations in the model parameters have important and predictable effects. For example, taking larger m_0^2 in minimal supergravity models will tend to squeeze together the spectrum of squarks and sleptons and move them all higher compared to the neutralinos, charginos and gluino. Taking larger values of $\tan\beta$ with other model parameters held fixed will usually tend to lower $\widetilde{b}_1$ and $\widetilde{\tau}_1$ masses compared to those of the other sparticles.

The second sample sketch in Figure 1.24 is obtained from a typical minimal GMSB model, with boundary conditions as in eq. (1.269) [with $N_5 = 1$, $\Lambda = 150$ TeV, $\tan\beta = 15$, and sign(μ)= + at a scale $Q_0 = M_{\rm mess} = 300$ TeV for the illustration]. Here we see that the hierarchy between strongly interacting sparticles and weakly interacting ones is quite large. Changing the messenger scale or Λ does not reduce the relative splitting between squark and slepton masses, because there is no analog of the universal m_0^2 contribution here. Increasing the number of messenger fields tends to decrease the squark and slepton masses relative to the gaugino masses, but still keeps the hierarchy between squark and slepton masses intact. In the model shown, the NLSP is a bino-like neutralino, but for larger number of messenger fields it could be either a stau, or else co-NLSPs $\tilde{\tau}_1$, $\tilde{e}_L$, $\tilde{\mu}_L$, depending on the choice of $\tan\beta$.

The third sample sketch in Figure 1.24 is obtained from an AMSB model with an additional universal scalar mass $m_0 = 450$ TeV added at $Q_0 = 2 \times 10^{16}$ GeV to rescue the sleptons, and with $m_{3/2} = 60$ TeV, $\tan\beta = 10$, and sign(μ)= + for the illustration. Here the most striking feature is that the LSP is a wino-like neutralino, with $m_{\tilde{C}_1} - m_{\tilde{N}_1}$ only about 160 MeV.

It would be a mistake to rely too heavily on specific scenarios for the MSSM mass and mixing spectrum, and the above illustrations are only a tiny fraction of the available possibilities. However, it is also useful to keep in mind some general lessons that often recur in various different models.

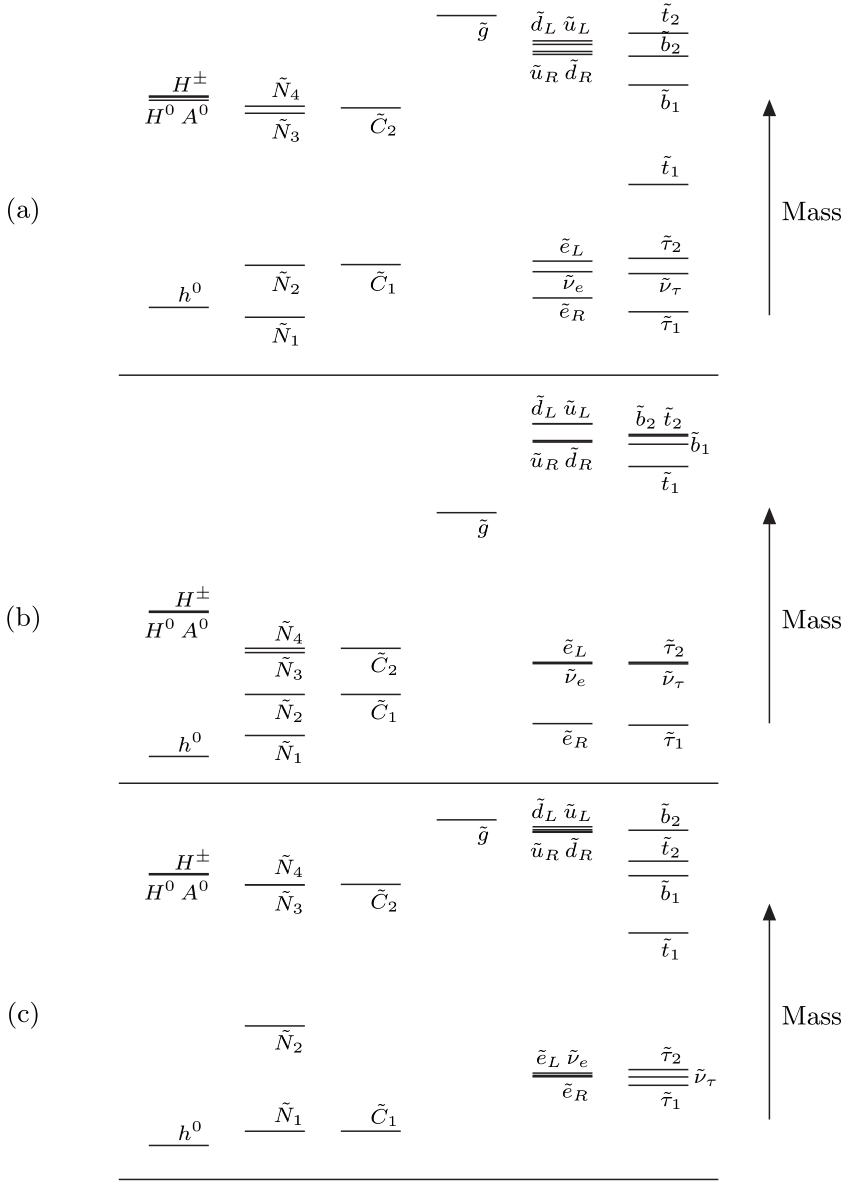

Fig. 1.24. Three sample schematic mass spectra for the undiscovered particles in the MSSM, for (a) minimal supergravity with $m_0^2 \ll m_{1/2}^2$, (b) minimal GMSB with $N_5 = 1$, and (c) AMSB with an extra m_0^2 for scalars. These spectra are presented for entertainment purposes only! No warranty, expressed or implied, guarantees that they look anything like the real world.

Indeed, there has emerged a sort of folklore concerning likely features of the MSSM spectrum, partly based on theoretical bias and partly on the constraints inherent in most known viable softly-broken supersymmetric theories. We remark on these features mainly because they represent the prevailing prejudices among supersymmetry theorists, which is certainly a useful thing to know even if one wisely decides to remain skeptical. For example, it is perhaps not unlikely that:

- The LSP is the lightest neutralino $\widetilde{N}_1$, unless the gravitino is lighter or R-parity is not conserved. If $M_1 < M_2, |\mu|$, then $\widetilde{N}_1$ is likely to be bino-like, with a mass roughly 0.5 times the masses of $\widetilde{N}_2$ and $\widetilde{C}_1$ in many well-motivated models. If, instead, $|\mu| < M_1, M_2$, then the LSP $\widetilde{N}_1$ has a large higgsino content and $\widetilde{N}_2$ and $\widetilde{C}_1$ are not much heavier. And, if $M_2 \ll M_1, |\mu|$, then the LSP will be a wino-like neutralino, with a chargino only very slightly heavier.
- The gluino will be much heavier than the lighter neutralinos and charginos. This is certainly true in the case of the "standard" gaugino mass relation eq. (1.189); more generally, the running gluino mass parameter grows relatively quickly as it is RG-evolved into the infrared because the QCD coupling is larger than the electroweak gauge couplings. So even if there are big corrections to the gaugino mass boundary conditions eqs. (1.245) or (1.260), the gluino mass parameter M_3 is likely to come out larger than M_1 and M_2.
- The squarks of the first and second families are nearly degenerate and much heavier than the sleptons. This is because each squark mass gets the same large positive-definite radiative corrections from loops involving the gluino. The left-handed squarks $\widetilde{u}_L$, $\widetilde{d}_L$, $\widetilde{s}_L$ and $\widetilde{c}_L$ are likely to be heavier than their right-handed counterparts $\widetilde{u}_R$, $\widetilde{d}_R$, $\widetilde{s}_R$ and $\widetilde{c}_R$, because of the effect parameterized by K_2 in eqs. (7.343)-(7.349).
- The squarks of the first two families cannot be lighter than about 0.8 times the mass of the gluino in minimal supergravity models, and about 0.6 times the mass of the gluino in the simplest gauge-mediated models as discussed in section 1.6.7 if the number of messenger squark pairs is $N_5 \leq 4$. In the minimal supergravity case this is because the gluino mass feeds into the squark masses through RG evolution; in the gauge-mediated case it is because the gluino and squark masses are tied together by eqs. (1.265) and (1.266).

- The lighter stop $\widetilde{t}_1$ and the lighter sbottom $\widetilde{b}_1$ are probably the lightest squarks. This is because stop and sbottom mixing effects and the effects of X_t and X_b in eqs. (1.203)-(1.205) both tend to decrease the lighter stop and sbottom masses.
- The lightest charged slepton is probably a stau $\widetilde{\tau}_1$. The mass difference $m_{\tilde{e}_R} - m_{\tilde{\tau}_1}$ is likely to be significant if $\tan\beta$ is large, because of the effects of a large tau Yukawa coupling. For smaller $\tan\beta$, $\widetilde{\tau}_1$ is predominantly $\widetilde{\tau}_R$ and it is not so much lighter than $\widetilde{e}_R$, $\widetilde{\mu}_R$.
- The left-handed charged sleptons $\widetilde{e}_L$ and $\widetilde{\mu}_L$ are likely to be heavier than their right-handed counterparts $\widetilde{e}_R$ and $\widetilde{\mu}_R$. This is because of the effect of K_2 in eq. (7.347). (Note also that $\Delta_{\tilde{e}_L} - \Delta_{\tilde{e}_R}$ is positive but very small because of the numerical accident $\sin^2\theta_W \approx 1/4$.)
- The lightest neutral Higgs boson h^0 should be lighter than about 150 GeV, and may be much lighter than the other Higgs scalar mass eigenstates A^0, $H^\pm$, H^0.

The most important point is that by measuring the masses and mixing angles of the MSSM particles we will be able to gain a great deal of information that can rule out or bolster evidence for competing proposals for the origin and mediation of supersymmetry breaking.

1.8. Sparticle Decays

This section contains a brief qualitative overview of the decay patterns of sparticles in the MSSM, assuming that R-parity is conserved. We will consider in turn the possible decays of neutralinos, charginos, sleptons, squarks, and the gluino. If, as is most often assumed, the lightest neutralino $\widetilde{N}_1$ is the LSP, then all decay chains will end up with it in the final state. Section 1.8.5 discusses the alternative possibility that the gravitino/goldstino $\widetilde{G}$ is the LSP. For the sake of simplicity of notation, we will often not distinguish between particle and antiparticle names and labels in this section, with context and consistency (dictated by charge and color conservation) resolving any ambiguities.

1.8.1. *Decays of neutralinos and charginos*

Let us first consider the possible two-body decays. Each neutralino and chargino contains at least a small admixture of the electroweak gauginos $\widetilde{B}$, $\widetilde{W}^0$ or $\widetilde{W}^\pm$, as we saw in section 1.7.2. So $\widetilde{N}_i$ and $\widetilde{C}_i$ inherit couplings of weak interaction strength to (scalar, fermion) pairs, as shown in

Figure 1.9b,c. If sleptons or squarks are sufficiently light, a neutralino or chargino can therefore decay into lepton+slepton or quark+squark. To the extent that sleptons are probably lighter than squarks, the lepton+slepton final states are favored. A neutralino or chargino may also decay into any lighter neutralino or chargino plus a Higgs scalar or an electroweak gauge boson, because they inherit the gaugino-higgsino-Higgs (see Figure 1.9b,c) and $SU(2)_L$ gaugino-gaugino-vector boson (see Figure 1.5c) couplings of their components. So, the possible two-body decay modes for neutralinos and charginos in the MSSM are:

$$\widetilde{N}_i \to Z\widetilde{N}_j,\ W\widetilde{C}_j,\ h^0\widetilde{N}_j,\ \ell\widetilde{\ell},\ \nu\widetilde{\nu},\ [A^0\widetilde{N}_j,\ H^0\widetilde{N}_j,\ H^\pm\widetilde{C}_j^\mp,\ q\widetilde{q}], \quad (8.356)$$

$$\widetilde{C}_i \to W\widetilde{N}_j,\ Z\widetilde{C}_1,\ h^0\widetilde{C}_1,\ \ell\widetilde{\nu},\ \nu\widetilde{\ell},\ [A^0\widetilde{C}_1,\ H^0\widetilde{C}_1,\ H^\pm\widetilde{N}_j,\ q\widetilde{q}'], \quad (8.357)$$

using a generic notation ν, ℓ, q for neutrinos, charged leptons, and quarks. The final states in brackets are the more kinematically implausible ones. (Since h^0 is required to be light, it is the most likely of the Higgs scalars to appear in these decays. This could even be the best way to discover the Higgs.) For the heavier neutralinos and chargino ($\widetilde{N}_3$, $\widetilde{N}_4$ and $\widetilde{C}_2$), one or more of the two-body decays in eqs. (8.356) and (8.357) is likely to be kinematically allowed. Also, if the decays of neutralinos and charginos with a significant higgsino content into third-family quark-squark pairs are open, they can be greatly enhanced by the top-quark Yukawa coupling, following from the interactions shown in Figure 1.7b,c.

It may be that all of these two-body modes are kinematically forbidden for a given chargino or neutralino, especially for $\widetilde{C}_1$ and $\widetilde{N}_2$ decays. In that case, they have three-body decays

$$\widetilde{N}_i \to ff\widetilde{N}_j, \quad \widetilde{N}_i \to ff'\widetilde{C}_j, \quad \widetilde{C}_i \to ff'\widetilde{N}_j, \quad \text{and} \quad \widetilde{C}_2 \to ff\widetilde{C}_1, \quad (8.358)$$

through the same (but now off-shell) gauge bosons, Higgs scalars, sleptons, and squarks that appeared in the two-body decays eqs. (8.356) and (8.357). Here f is generic notation for a lepton or quark, with f and f' distinct members of the same $SU(2)_L$ multiplet (and of course one of the f or f' in each of these decays must actually be an antifermion). The chargino and neutralino decay widths into the various final states can be found in Refs. 214–216.

The Feynman diagrams for the neutralino and chargino decays with $\widetilde{N}_1$ in the final state that seem most likely to be important are shown in Figure 1.25. In many situations, the decays

$$\widetilde{C}_1^\pm \to \ell^\pm\nu\widetilde{N}_1, \qquad \widetilde{N}_2 \to \ell^+\ell^-\widetilde{N}_1 \quad (8.359)$$

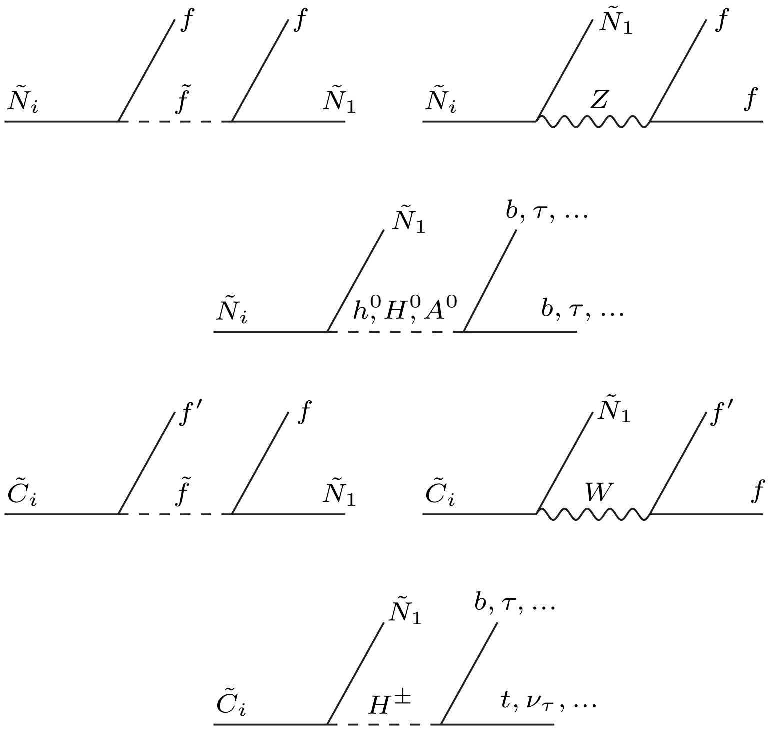

Fig. 1.25. Feynman diagrams for neutralino and chargino decays with $\tilde{N}_1$ in the final state. The intermediate scalar or vector boson in each case can be either on-shell (so that actually there is a sequence of two-body decays) or off-shell, depending on the sparticle mass spectrum.

can be particularly important for phenomenology, because the leptons in the final state often will result in clean signals. These decays are more likely if the intermediate sleptons are relatively light, even if they cannot be on-shell. Unfortunately, the enhanced mixing of staus, common in models, can often result in larger branching fractions for both $\widetilde{N}_2$ and $\tilde{C}_1$ into final states with taus, rather than electrons or muons. This is one reason why tau identification may be a crucial limiting factor in attempts to discover and study supersymmetry.

In other situations, decays without isolated leptons in the final state are more useful, so that one will not need to contend with background events with missing energy coming from leptonic W boson decays in Standard Model processes. Then the decays of interest are the ones with quark partons in the final state, leading to

$$\widetilde{C}_1 \to jj\widetilde{N}_1, \qquad \widetilde{N}_2 \to jj\widetilde{N}_1, \tag{8.360}$$

where j means a jet. If the second of these decays goes through an on-shell (or nearly so) h^0, then these will usually be b-jets.

1.8.2. *Slepton decays*

Sleptons can have two-body decays into a lepton and a chargino or neutralino, because of their gaugino admixture, as may be seen directly from the couplings in Figures 1.9b,c. Therefore, the two-body decays

$$\widetilde{\ell} \to \ell\widetilde{N}_i, \quad \widetilde{\ell} \to \nu\widetilde{C}_i, \quad \widetilde{\nu} \to \nu\widetilde{N}_i, \quad \widetilde{\nu} \to \ell\widetilde{C}_i \tag{8.361}$$

can be of weak interaction strength. In particular, the direct decays

$$\widetilde{\ell} \to \ell\widetilde{N}_1 \quad \text{and} \quad \widetilde{\nu} \to \nu\widetilde{N}_1 \tag{8.362}$$

are (almost[dd]) always kinematically allowed if $\widetilde{N}_1$ is the LSP. However, if the sleptons are sufficiently heavy, then the two-body decays

$$\widetilde{\ell} \to \nu\widetilde{C}_1, \quad \widetilde{\ell} \to \ell\widetilde{N}_2, \quad \widetilde{\nu} \to \nu\widetilde{N}_2, \quad \text{and} \quad \widetilde{\nu} \to \ell\widetilde{C}_1 \tag{8.363}$$

can be important. The right-handed sleptons do not have a coupling to the $SU(2)_L$ gauginos, so they typically prefer the direct decay $\widetilde{\ell}_R \to \ell\widetilde{N}_1$, if $\widetilde{N}_1$ is bino-like. In contrast, the left-handed sleptons may prefer to decay as in eq. (8.363) rather than the direct decays to the LSP as in eq. (8.362), if the former is kinematically open and if $\widetilde{C}_1$ and $\widetilde{N}_2$ are mostly wino. This is because the slepton-lepton-wino interactions in Figure 1.9b are proportional to the $SU(2)_L$ gauge coupling g, whereas the slepton-lepton-bino interactions in Figure 1.9c are proportional to the much smaller $U(1)_Y$ coupling g'. Formulas for these decay widths can be found in Ref. 215.

1.8.3. *Squark decays*

If the decay $\widetilde{q} \to q\widetilde{g}$ is kinematically allowed, it will always dominate, because the quark-squark-gluino vertex in Figure 1.9a has QCD strength. Otherwise, the squarks can decay into a quark plus neutralino or chargino: $\widetilde{q} \to q\widetilde{N}_i$ or $q'\widetilde{C}_i$. The direct decay to the LSP $\widetilde{q} \to q\widetilde{N}_1$ is always kinematically favored, and for right-handed squarks it can dominate because $\widetilde{N}_1$ is mostly bino. However, the left-handed squarks may strongly prefer to decay into heavier charginos or neutralinos instead, for example $\widetilde{q} \to q\widetilde{N}_2$

[dd] An exception occurs if the mass difference $m_{\tilde{\tau}_1} - m_{\tilde{N}_1}$ is less than m_τ.

or $q'\widetilde{C}_1$, because the relevant squark-quark-wino couplings are much bigger than the squark-quark-bino couplings. Squark decays to higgsino-like charginos and neutralinos are less important, except in the cases of stops and sbottoms, which have sizable Yukawa couplings. The gluino, chargino or neutralino resulting from the squark decay will in turn decay, and so on, until a final state containing $\widetilde{N}_1$ is reached. This results in numerous and complicated decay chain possibilities called cascade decays.[217]

It is possible that the decays $\widetilde{t}_1 \to t\widetilde{g}$ and $\widetilde{t}_1 \to t\widetilde{N}_1$ are both kinematically forbidden. If so, then the lighter top squark may decay only into charginos, by $\widetilde{t}_1 \to b\widetilde{C}_1$. If even this decay is kinematically closed, then it has only the flavor-suppressed decay to a charm quark, $\widetilde{t}_1 \to c\widetilde{N}_1$, and the four-body decay $\widetilde{t}_1 \to bff'\widetilde{N}_1$. These decays can be very slow,[218] so that the lightest stop can be quasi-stable on the time scale relevant for collider physics, and can hadronize into bound states.

1.8.4. *Gluino decays*

The decay of the gluino can only proceed through a squark, either on-shell or virtual. If two-body decays $\widetilde{g} \to q\widetilde{q}$ are open, they will dominate, again because the relevant gluino-quark-squark coupling in Figure 1.9a has QCD strength. Since the top and bottom squarks can easily be much lighter than all of the other squarks, it is quite possible that $\widetilde{g} \to t\widetilde{t}_1$ and/or $\widetilde{g} \to b\widetilde{b}_1$ are the only available two-body decay mode(s) for the gluino, in which case they will dominate over all others. If instead all of the squarks are heavier than the gluino, the gluino will decay only through off-shell squarks, so $\widetilde{g} \to qq\widetilde{N}_i$ and $qq'\widetilde{C}_i$. The squarks, neutralinos and charginos in these final states will then decay as discussed above, so there can be many competing gluino decay chains. Some of the possibilities are shown in Figure 1.26. The cascade decays can have final-state branching fractions that are individually small and quite sensitive to the model parameters.

The simplest gluino decays, including the ones shown in Figure 1.26, can have 0, 1, or 2 charged leptons (in addition to two or more hadronic jets) in the final state. An important feature is that when there is exactly one charged lepton, it can have either charge with exactly equal probability. This follows from the fact that the gluino is a Majorana fermion, and does not "know" about electric charge; for each diagram with a given lepton charge, there is always an equal one with every particle replaced by its antiparticle.

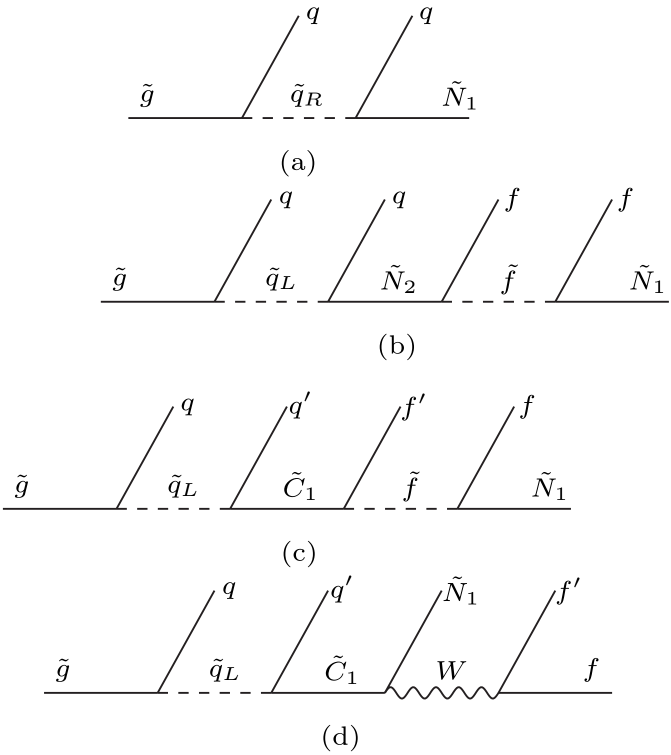

Fig. 1.26. Some of the many possible examples of gluino cascade decays ending with a neutralino LSP in the final state. The squarks appearing in these diagrams may be either on-shell or off-shell, depending on the mass spectrum of the theory.

1.8.5. *Decays to the gravitino/goldstino*

Most phenomenological studies of supersymmetry assume explicitly or implicitly that the lightest neutralino is the LSP. This is typically the case in gravity-mediated models for the soft terms. However, in gauge-mediated models (and in "no-scale" models), the LSP is instead the gravitino. As we saw in section 1.6.5, a very light gravitino may be relevant for collider phenomenology, because it contains as its longitudinal component the goldstino, which has a non-gravitational coupling to all sparticle-particle pairs $(\widetilde{X}, X)$. The decay rate found in eq. (1.239) for $\widetilde{X} \to X\widetilde{G}$ is usually not fast enough to compete with the other decays of sparticles $\widetilde{X}$ as mentioned above, *except* in the case that $\widetilde{X}$ is the next-to-lightest supersymmetric particle (NLSP). Since the NLSP has no competing decays, it should always decay into its superpartner and the LSP gravitino.

In principle, any of the MSSM superpartners could be the NLSP in models with a light goldstino, but most models with gauge mediation of supersymmetry breaking have either a neutralino or a charged lepton playing this role. The argument for this can be seen immediately from eqs. (1.265) and (1.266); since $\alpha_1 < \alpha_2, \alpha_3$, those superpartners with only $U(1)_Y$ interactions will tend to get the smallest masses. The gauge-eigenstate sparticles with this property are the bino and the right-handed sleptons $\widetilde{e}_R$, $\widetilde{\mu}_R$, $\widetilde{\tau}_R$, so the appropriate corresponding mass eigenstates should be plausible candidates for the NLSP.

First suppose that $\widetilde{N}_1$ is the NLSP in light goldstino models. Since $\widetilde{N}_1$ contains an admixture of the photino (the linear combination of bino and neutral wino whose superpartner is the photon), from eq. (1.239) it decays into photon + goldstino/gravitino with a partial width

$$\Gamma(\widetilde{N}_1 \to \gamma\widetilde{G}) \,=\, 2\times 10^{-3}\,\kappa_{1\gamma}\left(\frac{m_{\widetilde{N}_1}}{100\text{ GeV}}\right)^5\left(\frac{\sqrt{\langle F\rangle}}{100\text{ TeV}}\right)^{-4}\text{ eV}. \quad (8.364)$$

Here $\kappa_{1\gamma} \equiv |\mathbf{N}_{11}\cos\theta_W + \mathbf{N}_{12}\sin\theta_W|^2$ is the "photino content" of $\widetilde{N}_1$, in terms of the neutralino mixing matrix $\mathbf{N}_{ij}$ defined by eq. (7.315). We have normalized $m_{\widetilde{N}_1}$ and $\sqrt{\langle F\rangle}$ to (very roughly) minimum expected values in gauge-mediated models. This width is much smaller than for a typical flavor-unsuppressed weak interaction decay, but it is still large enough to allow $\widetilde{N}_1$ to decay before it has left a collider detector, if $\sqrt{\langle F\rangle}$ is less than a few thousand TeV in gauge-mediated models, or equivalently if $m_{3/2}$ is less than a keV or so when eq. (1.238) holds. In fact, from eq. (8.364), the mean decay length of an $\widetilde{N}_1$ with energy E in the lab frame is

$$d = 9.9\times 10^{-3}\,\frac{1}{\kappa_{1\gamma}}\,(E^2/m^2_{\widetilde{N}_1} - 1)^{1/2}\left(\frac{m_{\widetilde{N}_1}}{100\text{ GeV}}\right)^{-5}\left(\frac{\sqrt{\langle F\rangle}}{100\text{ TeV}}\right)^{4}\text{ cm}, \quad (8.365)$$

which could be anything from sub-micron to multi-kilometer, depending on the scale of supersymmetry breaking $\sqrt{\langle F\rangle}$. (In other models that have a gravitino LSP, including certain "no-scale" models,[219] the same formulas apply with $\langle F\rangle \to \sqrt{3}m_{3/2}M_{\text{P}}$.)

Of course, $\widetilde{N}_1$ is not a pure photino, but contains also admixtures of the superpartner of the Z boson and the neutral Higgs scalars. So, one can also have[144] $\widetilde{N}_1 \to Z\widetilde{G}$, $h^0\widetilde{G}$, $A^0\widetilde{G}$, or $H^0\widetilde{G}$, with decay widths given in Ref. 145. Of these decays, the last two are unlikely to be kinematically allowed, and only the $\widetilde{N}_1 \to \gamma\widetilde{G}$ mode is guaranteed to be kinematically allowed for a gravitino LSP. Furthermore, even if they are open, the decays $\widetilde{N}_1 \to Z\widetilde{G}$ and $\widetilde{N}_1 \to h^0\widetilde{G}$ are subject to strong kinematic suppressions proportional to $(1 - m_Z^2/m_{\widetilde{N}_1}^2)^4$ and $(1 - m_{h^0}^2/m_{\widetilde{N}_1}^2)^4$, respectively, in view of eq. (1.239). Still, these decays may play an important role in phenomenology if $\langle F \rangle$ is not too large, $\widetilde{N}_1$ has a sizable zino or higgsino content, and $m_{\widetilde{N}_1}$ is significantly greater than m_Z or m_{h^0}.

A charged slepton makes another likely candidate for the NLSP. Actually, more than one slepton can act effectively as the NLSP, even though one of them is slightly lighter, if they are sufficiently close in mass so that each has no kinematically allowed decays except to the goldstino. In GMSB models, the squared masses obtained by $\widetilde{e}_R$, $\widetilde{\mu}_R$ and $\widetilde{\tau}_R$ are equal because of the flavor-blindness of the gauge couplings. However, this is not the whole story, because one must take into account mixing with $\widetilde{e}_L$, $\widetilde{\mu}_L$, and $\widetilde{\tau}_L$ and renormalization group running. These effects are very small for $\widetilde{e}_R$ and $\widetilde{\mu}_R$ because of the tiny electron and muon Yukawa couplings, so we can quite generally treat them as degenerate, unmixed mass eigenstates. In contrast, $\widetilde{\tau}_R$ usually has a quite significant mixing with $\widetilde{\tau}_L$, proportional to the tau Yukawa coupling. This means that the lighter stau mass eigenstate $\widetilde{\tau}_1$ is pushed lower in mass than $\widetilde{e}_R$ or $\widetilde{\mu}_R$, by an amount that depends most strongly on $\tan\beta$. If $\tan\beta$ is not too large then the stau mixing effect leaves the slepton mass eigenstates $\widetilde{e}_R$, $\widetilde{\mu}_R$, and $\widetilde{\tau}_1$ degenerate to within less than $m_\tau \approx 1.8$ GeV, so they act effectively as co-NLSPs. In particular, this means that even though the stau is slightly lighter, the three-body slepton decays $\widetilde{e}_R \to e\tau^\pm\widetilde{\tau}_1^\mp$ and $\widetilde{\mu}_R \to \mu\tau^\pm\widetilde{\tau}_1^\mp$ are not kinematically allowed; the only allowed decays for the three lightest sleptons are $\widetilde{e}_R \to e\widetilde{G}$ and $\widetilde{\mu}_R \to \mu\widetilde{G}$ and $\widetilde{\tau}_1 \to \tau\widetilde{G}$. This situation is called the "slepton co-NLSP" scenario.

For larger values of $\tan\beta$, the lighter stau eigenstate $\widetilde{\tau}_1$ is more than 1.8 GeV lighter than $\widetilde{e}_R$ and $\widetilde{\mu}_R$ and $\widetilde{N}_1$. This means that the decays $\widetilde{N}_1 \to \tau\widetilde{\tau}_1$ and $\widetilde{e}_R \to e\tau\widetilde{\tau}_1$ and $\widetilde{\mu}_R \to \mu\tau\widetilde{\tau}_1$ are open. Then $\widetilde{\tau}_1$ is the sole NLSP, with all other MSSM supersymmetric particles having kinematically allowed decays into it. This is called the "stau NLSP" scenario.

In any case, a slepton NLSP can decay like $\widetilde{\ell} \to \ell\widetilde{G}$ according to eq. (1.239), with a width and decay length just given by eqs. (8.364) and (8.365) with the replacements $\kappa_{1\gamma} \to 1$ and $m_{\widetilde{N}_1} \to m_{\widetilde{\ell}}$. So, as for the neutralino NLSP case, the decay $\widetilde{\ell} \to \ell\widetilde{G}$ can be either fast or very slow, depending on the scale of supersymmetry breaking.

If $\sqrt{\langle F \rangle}$ is larger than roughly 10^3 TeV (or the gravitino is heavier than a keV or so), then the NLSP is so long-lived that it will usually escape a typical collider detector. If $\widetilde{N}_1$ is the NLSP, then, it might as well be the LSP from the point of view of collider physics. However, the decay of $\widetilde{N}_1$ into the gravitino is still important for cosmology, since an unstable $\widetilde{N}_1$ is clearly not a good dark matter candidate while the gravitino LSP conceivably could be. On the other hand, if the NLSP is a long-lived charged slepton, then one can see its tracks (or possibly decay kinks) inside a collider detector.[144] The presence of a massive charged NLSP can be established by measuring an anomalously long time-of-flight or high ionization rate for a track in the detector.

1.9. Concluding Remarks

In this primer, I have attempted to convey some of the more essential features of supersymmetry as it is understood so far. One of the most amazing qualities of supersymmetry is that so much is known about it already, despite the present lack of direct experimental data. Even the terms and stakes of many of the important outstanding questions, especially the paramount issue "How is supersymmetry broken?", are already rather clear. That this can be so is a testament to the unreasonably predictive quality of the symmetry itself.

We have seen that sensible and economical models for supersymmetry at the TeV scale can be used as convenient templates for experimental searches. Two of the simplest and most popular possibilities are the "minimal supergravity" scenario with new parameters m_0^2, $m_{1/2}$, A_0, $\tan\beta$ and $\mathrm{Arg}(\mu)$, and the "gauge-mediated" scenario with new parameters Λ, $M_{\rm mess}$, N_5, $\langle F\rangle$, $\tan\beta$, and $\mathrm{Arg}(\mu)$. However, one should not lose sight of the fact that the only indispensable idea of supersymmetry is simply that of a symmetry between fermions and bosons. Nature may or may not be kind enough to realize this beautiful idea within one of the specific frameworks that have already been explored well by theorists.

The experimental verification of supersymmetry will not be an end, but rather a revolution in high energy physics. It seems likely to present us with questions and challenges that we can only guess at presently. The measurement of sparticle masses, production cross-sections, and decay modes will rule out some models for supersymmetry breaking and lend credence to others. We will be able to test the principle of R-parity conservation, the idea that supersymmetry has something to do with the dark matter, and possibly make connections to other aspects of cosmology including baryogenesis and inflation. Other fundamental questions, like the origin of the

μ parameter and the rather peculiar hierarchical structure of the Yukawa couplings may be brought into sharper focus with the discovery of the MSSM spectrum. Understanding the precise connection of supersymmetry to the electroweak scale will surely open the window to even deeper levels of fundamental physics.

Appendix: Non-Renormalizable Supersymmetric Lagrangians

In section 1.3, we discussed only renormalizable supersymmetric Lagrangians. However, like all known theories that include general relativity, supergravity is non-renormalizable as a quantum field theory. It is therefore clear that non-renormalizable interactions must be present in any low-energy effective description of the MSSM. Fortunately, these can be neglected for most phenomenological purposes, because non-renormalizable interactions have couplings of negative mass dimension, proportional to powers of $1/M_{\mathrm{P}}$ (or perhaps $1/\Lambda_{\mathrm{UV}}$, where Λ_{UV} is some other cutoff scale associated with new physics). This means that their effects at energy scales E ordinarily accessible to experiment are typically suppressed by powers of E/M_{P} (or by powers of E/Λ_{UV}). For energies $E \lesssim 1$ TeV, the consequences of non-renormalizable interactions are therefore usually far too small to be interesting.

Still, there are several reasons why one might be interested in non-renormalizable contributions to supersymmetric Lagrangians. First, some very rare processes (like proton decay) can only be described using an effective MSSM Lagrangian that includes non-renormalizable terms. Second, one may be interested in understanding physics at very high energy scales where the suppression associated with non-renormalizable terms is not enough to stop them from being important. For example, this could be the case in the study of the very early universe, or in understanding how additional gauge symmetries get broken. Third, the non-renormalizable interactions may play a crucial role in understanding how supersymmetry breaking is transmitted to the MSSM. Finally, it is sometimes useful to treat strongly coupled supersymmetric gauge theories using non-renormalizable effective Lagrangians, in the same way that chiral effective Lagrangians are used to study hadron physics in QCD. Unfortunately, we will not be able to treat these subjects in any sort of systematic way. Instead, we will merely sketch a few of the key elements that go into defining a non-renormalizable supersymmetric Lagrangian. More detailed treatments may be found for example in Refs. 12, 16, 18, 20, 26, 28, 29.

Let us consider a supersymmetric theory containing gauge and chiral supermultiplets whose Lagrangian may contain terms that are non-renormalizable. This includes supergravity as a special case, but applies more generally. It turns out that the part of the Lagrangian containing terms with up to two spacetime derivatives is completely determined by specifying three functions of the complex scalar fields (or more formally, of the chiral superfields). They are:

- The superpotential $W(\phi_i)$, which we have already encountered in the case of renormalizable supersymmetric Lagrangians. It must be an analytic function of the superfields treated as complex variables; in other words it depends only on the ϕ_i and not on the ϕ^{*i}. It must be invariant under the gauge symmetries of the theory, and has dimensions of [mass]^3.
- The *Kähler potential* $K(\phi_i, \phi^{*i})$. Unlike the superpotential, the Kähler potential is a function of both ϕ_i and ϕ^{*i}. It is gauge-invariant, real, and has dimensions of [mass]^2. In the special case of renormalizable theories, we did not have to discuss the Kähler potential explicitly, because at tree-level it is always $K = \phi^{i*}\phi_i$ (with i summed over as usual).
- The *gauge kinetic function* $f_{ab}(\phi_i)$. Like the superpotential, it is an analytic function of the ϕ_i treated as complex variables. It is dimensionless and symmetric under interchange of its two indices a, b, which run over the adjoint representations of the gauge groups of the model. In the special case of renormalizable supersymmetric Lagrangians, it is just a constant (independent of the ϕ_i), and is equal to the identity matrix divided by the gauge coupling squared: $f_{ab} = \delta_{ab}/g_a^2$. More generally, it also determines the non-renormalizable couplings of the gauge supermultiplets.

The whole Lagrangian with up to two derivatives can now be written down in terms of these. This is a non-trivial consequence of supersymmetry, because many different individual couplings in the Lagrangian are determined by the same three functions.

For example, in supergravity models, the part of the scalar potential that does not depend on the gauge kinetic function can be found as follows. First, one may define the real, dimensionless *Kähler function*:

$$G = K/M_{\rm P}^2 + \ln(W/M_{\rm P}^3) + \ln(W^*/M_{\rm P}^3). \tag{A.1}$$

(Just to maximize the confusion, G is also sometimes referred to as the Kähler potential. Also, many references use units with $M_{\rm P} = 1$, which simplifies the expressions but can slightly obscure the correspondence with the global supersymmetry limit of large $M_{\rm P}$.) From G, one can construct its derivatives with respect to the scalar fields and their complex conjugates: $G^i = \delta G/\delta\phi_i$; $G_i = \delta G/\delta\phi^{*i}$; and $G_i^j = \delta^2 G/\delta\phi^{*i}\delta\phi_j$. As in section 1.3.2, raised (lowered) indices i correspond to derivatives with respect to ϕ_i (ϕ^{*i}). Note that $G_i^j = K_i^j/M_{\rm P}^2$, which is sometimes called the Kähler metric, does not depend on the superpotential. The inverse of this matrix is denoted $(G^{-1})_i^j$, or equivalently $M_{\rm P}^2(K^{-1})_i^j$, so that $(G^{-1})_i^k G_k^j = (G^{-1})_k^j G_i^k = \delta_i^j$. In terms of these objects, the generalization of the F-term contribution to the scalar potential in ordinary renormalizable global supersymmetry turns out to be, after a complicated derivation:[146,147]

$$V_F \;=\; M_{\rm P}^4\, e^G \Big[G^i (G^{-1})_i^j G_j - 3 \Big] \tag{A.2}$$

in supergravity. It can be rewritten as

$$V_F \;=\; K_i^j F_j F^{*i} - 3 e^{K/M_{\rm P}^2} W W^*/M_{\rm P}^2, \tag{A.3}$$

where

$$F_i = -M_{\rm P}^2\, e^{G/2}\, (G^{-1})_i^j G_j = -e^{K/2M_{\rm P}^2}\, (K^{-1})_i^j \Big(W_j^* + W^* K_j/M_{\rm P}^2 \Big), \tag{A.4}$$

with $K^i = \delta K/\delta\phi_i$ and $K_j = \delta K/\delta\phi^{*j}$. The F_i are order parameters for supersymmetry breaking in supergravity (generalizing the auxiliary fields in the renormalizable global supersymmetry case). In other words, local supersymmetry will be broken if one or more of the F_i obtain a VEV. The gravitino then absorbs the would-be goldstino and obtains a squared mass

$$m_{3/2}^2 \;=\; \langle K_j^i F_i F^{*j} \rangle / 3M_{\rm P}^2. \tag{A.5}$$

Now, assuming a minimal Kähler potential $K = \phi^{*i}\phi_i$, then $K_i^j = (K^{-1})_i^j = \delta_i^j$, so that expanding eqs. (A.3) and (A.4) to lowest order in $1/M_{\rm P}$ just reproduces the results $F_i = -W_i^*$ and $V = F_i F^{*i} = W^i W_i^*$, which were found in section 1.3.2 for renormalizable global supersymmetric theories [see eqs. (1.108)-(1.111)]. Equation (A.5) also reproduces the expression for the gravitino mass that was quoted in eq. (1.238).

The scalar potential eq. (A.2) does not include the D-term contributions from gauge interactions, which are given by

$$V_D \;=\; \frac{1}{2} {\rm Re}[f_{ab}^{-1}\, \widehat{D}_a \widehat{D}_b], \tag{A.6}$$

where

$$\widehat{D}^a \equiv -G^i (T^a)_i{}^j \phi_j = -\phi^{*j} (T^a)_j{}^i G_i = -K^i (T^a)_i{}^j \phi_j = -\phi^{*j} (T^a)_j{}^i K_i, \tag{A.7}$$

are real order parameters of supersymmetry breaking, with the last three equalities following from the gauge invariance of W and K. The full scalar potential is

$$V = V_F + V_D, \tag{A.8}$$

and it depends on W and K only through the combination G in eq. (A.1). There are many other contributions to the supergravity Lagrangian, which also turn out to depend only on G and f_{ab}, and can be found in Ref. 146, 147. This allows one to consistently redefine W and K so that there are no purely holomorphic or purely anti-holomorphic terms appearing in the latter.

Note that in the tree-level global supersymmetry case $f_{ab} = \delta_{ab}/g_a^2$ and $K^i = \phi^{*i}$, eq. (A.6) reproduces the result of section 1.3.4 for the renormalizable global supersymmetry D-term scalar potential, with $\widehat{D}^a = D^a/g^a$ being the D-term order parameter for supersymmetry breaking.

Unlike in the case of global supersymmetry, the scalar potential in supergravity is *not* necessarily non-negative, because of the -3 term in eq. (A.2). Therefore, in principle, one can have supersymmetry breaking with a positive, negative, or zero vacuum energy. Recent developments in experimental cosmology[220] imply a positive vacuum energy associated with the acceleration of the scale factor of the observable universe,

$$\rho_{\text{vac}}^{\text{observed}} = \frac{\Lambda}{8\pi G_{\text{Newton}}} \approx (2.3 \times 10^{-12}\ \text{GeV})^4, \tag{A.9}$$

but this is also certainly tiny compared to the scales associated with supersymmetry breaking. Therefore, it is tempting to simply assume that the vacuum energy is 0 within the approximations pertinent for working out the supergravity effects on particle physics at high energies. However, it is notoriously unclear *why* the terms in the scalar potential in a supersymmetry-breaking vacuum should conspire to give $\langle V \rangle \approx 0$ at the minimum. A naive estimate, without miraculous cancellations, would give instead $\langle V \rangle$ of order $|\langle F \rangle|^2$, so at least roughly $(10^{10}\ \text{GeV})^4$ for Planck-scale mediated supersymmetry breaking, or $(10^4\ \text{GeV})^4$ for Gauge-mediated supersymmetry breaking. Furthermore, while $\rho_{\text{vac}} = \langle V \rangle$ classically, the former is a very large-distance scale measured quantity, while the latter is associated with effective field theories at length scales comparable to and shorter than those familiar to high energy physics. So, in the absence of a compelling explanation for the tiny value of $\rho_{\rm vac}$, it is not at all clear that $\langle V\rangle \approx 0$ is really the right condition to impose.[221] Nevertheless, with $\langle V\rangle = 0$ imposed as a constraint,[ee] eqs. (A.3)-(A.5) tell us that $\langle K^i_j F_i F^{*j}\rangle = 3M_{\rm P}^4 e^{\langle G\rangle} = 3e^{\langle K\rangle/M_{\rm P}^2}|\langle W\rangle|^2/M_{\rm P}^2$, and an equivalent formula for the gravitino mass is therefore $m_{3/2} = e^{\langle G\rangle/2}M_{\rm P}$.

An instructive special case arises if we assume a minimal Kähler potential and divide the fields ϕ_i into a visible sector including the MSSM fields φ_i, and a hidden sector containing a field X that breaks supersymmetry for us (and other fields that we need not treat explicitly). In other words, suppose that the superpotential and the Kähler potential have the form

$$W = W_{\rm vis}(\varphi_i) + W_{\rm hid}(X), \tag{A.10}$$
$$K = \varphi^{*i}\varphi_i + X^*X. \tag{A.11}$$

Now let us further assume that the dynamics of the hidden sector fields provides non-zero VEVs

$$\langle X\rangle = xM_{\rm P}, \qquad \langle W_{\rm hid}\rangle = wM_{\rm P}^2, \qquad \langle \delta W_{\rm hid}/\delta X\rangle = w' M_{\rm P}, \tag{A.12}$$

which define a dimensionless quantity x, and w, w' with dimensions of [mass]. Requiring $\langle V\rangle = 0$ yields $|w' + x^*w|^2 = 3|w|^2$, and

$$m_{3/2} \;=\; |\langle F_X\rangle|/\sqrt{3}M_{\rm P} \;=\; e^{|x|^2/2}|w|. \tag{A.13}$$

Now we suppose that it is valid to expand the scalar potential in powers of the dimensionless quantities $w/M_{\rm P}$, $w'/M_{\rm P}$, $\varphi_i/M_{\rm P}$, etc., keeping only terms that depend on the visible sector fields φ_i. It is not a difficult exercise to show that in leading order the result is:

$$\begin{aligned} V = {} & (W^*_{\rm vis})_i (W_{\rm vis})^i + m_{3/2}^2 \varphi^{*i}\varphi_i \\ & + e^{|x|^2/2}\left[w^*\varphi_i (W_{\rm vis})^i \;+\; (x^*w'^* + |x|^2 w^* - 3w^*)W_{\rm vis} + {\rm c.c.}\right]. \end{aligned} \tag{A.14}$$

A tricky point here is that we have rescaled the visible sector superpotential $W_{\rm vis} \rightarrow e^{-|x|^2/2}W_{\rm vis}$ everywhere, in order that the first term in eq. (A.14) is the usual, properly normalized, F-term contribution in global supersymmetry. The next term is a universal soft scalar squared mass of the form eq. (1.246) with

$$m_0^2 \;=\; |\langle F_X\rangle|^2/3M_{\rm P}^2 \;=\; m_{3/2}^2. \tag{A.15}$$

[ee]We do this only to follow popular example; as just noted we cannot endorse this imposition.

The second line of eq. (A.14) just gives soft (scalar)^3 and (scalar)^2 analytic couplings of the form eqs. (1.247) and (1.248), with

$$A_0 = -x^* \langle F_X \rangle / M_{\rm P}, \qquad B_0 = \left(\frac{1}{x + w'^*/w^*} - x^* \right) \langle F_X \rangle / M_{\rm P} \quad \text{(A.16)}$$

since $\varphi_i (W_{\rm vis})^i$ is equal to $3W_{\rm vis}$ for the cubic part of $W_{\rm vis}$, and to $2W_{\rm vis}$ for the quadratic part. [If the complex phases of x, w, w' can be rotated away, then eq. (A.16) implies $B_0 = A_0 - m_{3/2}$, but there are many effects that can ruin this prediction.] The Polonyi model mentioned in section 1.6.6 is just the special case of this exercise in which $W_{\rm hid}$ is assumed to be linear in X.

However, there is no particular reason why W and K must have the simple form eq. (A.10) and eq. (A.11). In general, the superpotential can be expanded like

$$W = W_{\rm ren} + \frac{1}{M_{\rm P}} w^{ijkn} \phi_i \phi_j \phi_k \phi_n + \frac{1}{M_{\rm P}^2} w^{ijknm} \phi_i \phi_j \phi_k \phi_n \phi_m + \ldots \quad \text{(A.17)}$$

where $W_{\rm ren}$ is the renormalizable superpotential with terms up to ϕ^3. Similarly, the Kähler potential can be expanded like

$$\begin{aligned} K = \phi_i \phi^{*i} &+ \frac{1}{M_{\rm P}} \left(k^{ij}_k \phi_i \phi_j \phi^{*k} + \text{c.c.} \right) \\ &+ \frac{1}{M_{\rm P}^2} \left(k^{ij}_{kn} \phi_i \phi_j \phi^{*k} \phi^{*n} + k^{ijk}_n \phi_i \phi_j \phi_k \phi^{*n} + \text{c.c.} \right) + \ldots, \quad \text{(A.18)} \end{aligned}$$

where terms in K that are analytic in ϕ (and ϕ^*) are assumed to have been absorbed into W (and W^*), as explained above. The form of the first term is dictated by the requirement of canonical kinetic terms for the chiral supermultiplet fields. If one now plugs eqs. (A.17) and (A.18) with arbitrary hidden sector fields and VEVs into eq. (A.2), one obtains a general form like eq. (1.242) for the soft terms. It is only when special assumptions are made [like eqs. (A.10), (A.11)] that one gets the phenomenologically desirable results in eqs. (1.244)-(1.248). Thus supergravity by itself does not guarantee universality of the soft terms. Furthermore, there is no guarantee that expansions in $1/M_{\rm P}$ of the form given above are valid or appropriate. In superstring models, the dilaton and moduli fields have Kähler potential terms proportional to $M_{\rm P}^2 \ln[(\phi + \phi^*)/M_{\rm P}]$. (The moduli are massless fields that do not appear in the tree-level perturbative superpotential. The dilaton is a special modulus field whose VEV determines the gauge couplings in the theory.)

Gaugino masses arise from non-renormalizable terms through a non-minimal gauge kinetic function f_{ab}. Expanding it in powers of $1/M_{\rm P}$ as

$$f_{ab} \,=\, \delta_{ab}\Bigl[1/g_a^2 \,+\, f_a^i\phi_i/M_{\rm P} \,+\, \ldots\Bigr], \tag{A.19}$$

it is possible to show that the gaugino mass induced by supersymmetry breaking is

$$m_{\lambda^a} \,=\, {\rm Re}[f_a^i]\langle F_i\rangle/2M_{\rm P}. \tag{A.20}$$

The assumption of universal gaugino masses therefore follows if the dimensionless quantities f_a^i are the same for each of the three MSSM gauge groups; this is automatic in certain GUT and superstring-inspired models, but not in general.

Finally, let us mention how gaugino condensates can provide supersymmetry breaking in supergravity models. This again requires that the gauge kinetic function has a non-trivial dependence on the scalar fields, as in eq. (A.19). Then eq. (A.4) is modified to

$$F_i \,=\, -M_{\rm P}^2\, e^{G/2}\,(G^{-1})_i^j G_j - \frac{1}{4}(K^{-1})_i^j\,\frac{\partial f_{ab}^*}{\partial\phi^{*j}}\lambda^a\lambda^b + \ldots. \tag{A.21}$$

Now if there is a gaugino condensate $\langle\lambda^a\lambda^b\rangle \,=\, \delta^{ab}\Lambda^3$ and $\langle(K^{-1})_j^i\partial f_{ab}/\partial\phi_j\rangle \sim 1/M_{\rm P}$, then $|\langle F_i\rangle| \sim \Lambda^3/M_{\rm P}$. Then as above, the non-vanishing F-term gives rise to soft parameters of order $m_{\rm soft} \sim |\langle F_i\rangle|/M_{\rm P} \sim \Lambda^3/M_{\rm P}^2$, as in eq. (1.233).

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