Nathan Haouzi

Phys 234A

Problem Set 2

(you can use as solution if you want)

2/2

Problem 1

1) $H = \frac{\dot{X}^2}{2} + \frac{X^2}{2} = \frac{p^2}{2} + \frac{X^2}{2}$

Define $a = \frac{1}{\sqrt{2}}(p - iX)$ and $a^+ = \frac{1}{\sqrt{2}}(p + iX)$

Then $\boxed{H = a^+ a + \frac{1}{2}}$

Define the ground state $|0\rangle$ as $a|0\rangle = 0$ with energy $E_0 = \frac{1}{2}$

Then the Hilbert space is spanned by $\boxed{|n\rangle = (a^+)^n |0\rangle}$

with energy $\boxed{E_n = n + \frac{1}{2}}$. (This follows from the fact that if $|\varphi\rangle$ is an energy eigenstate, $H|\varphi\rangle = E|\varphi\rangle$, then we have $H a^+|\varphi\rangle = (E+1)a^+|\varphi\rangle$)

2) $$Z(\beta) = \mathrm{Tr}\left(e^{-\beta H}\right) = \sum_{n=0}^{+\infty} e^{-\beta\left(n+\frac{1}{2}\right)}$$

$$= e^{-\beta/2} \frac{1}{1-e^{-\beta}}$$

$$= \frac{1}{e^{\beta/2} - e^{-\beta/2}}$$

So $\boxed{Z(\beta) = \frac{1}{2\sinh(\beta/2)}}$

3) We have $Z(\beta) = \int\limits_{x(\tau)=x(\tau+\beta)} \mathcal{D}x(\tau)\, e^{-\oint d\tau \left(\frac{1}{2}\dot{x}^2 + \frac{1}{2}x^2\right)}$

$$= \int \mathcal{D}x(\tau)\, e^{-\oint d\tau \left(\frac{1}{2} x \left(-\frac{d^2}{d\tau^2}+1\right) x\right)}$$

Let $\psi_n(\tau)$ be the (orthonormal) eigenfunctions of $\left(-\frac{d^2}{d\tau^2}+1\right)$:

$$\left(-\frac{d^2}{d\tau^2}+1\right)\psi_n(\tau) = \lambda_n \psi_n(\tau)$$

Now expand $x(\tau)$ in this eigenbasis: $x(\tau) = \sum_n a_n \psi_n(\tau)$

and $\mathcal{D}x(\tau) = \prod_n \frac{da_n}{\sqrt{2\pi}}$ where $\frac{1}{\sqrt{2\pi}}$ is for normalization.

Then $Z(\beta) = \int \prod_n \frac{da_n}{\sqrt{2\pi}}\, e^{-\frac{1}{2}\sum_{n,m}\int d\tau\, a_n^* \psi_n^*(\tau)\, \lambda_m a_m \psi_m(\tau)}$

$$= \int \prod_n \frac{da_n}{\sqrt{2\pi}}\, e^{-\frac{1}{2}\sum_n |a_n|^2 \lambda_n}$$

$$= \prod_n \frac{1}{\sqrt{\lambda_n}}$$

Now, $\psi_n(\tau) = e^{i2\pi n\tau/\beta}$ are the eigenstates of $\left(-\frac{d^2}{d\tau^2}+1\right)$, with eigenvalues $\left(\frac{2\pi n}{\beta}\right)^2 + 1$.

So $Z(\beta) = \prod_{n=-\infty}^{+\infty}\left(\left(\frac{2\pi n}{\beta}\right)^2+1\right)^{-1/2}$

So $\boxed{Z(\beta) = \prod_{n=1}^{\infty}\left(\left(\frac{2\pi n}{\beta}\right)^2+1\right)^{-1}}$, which needs regularization

4) We use $\prod_{n=1}^{+\infty}\left(1+\frac{x^2}{n^2}\right)=\frac{\sinh(\pi x)}{\pi x}$

So $Z(\beta)=\prod_{n=1}^{+\infty}\left(\left(\frac{2\pi n}{\beta}\right)^2+1\right)^{-1}=\left[\prod_{n=1}^{+\infty}\left(1+\frac{(\beta/2\pi)^2}{n^2}\right)\right]^{-1}\left[\prod_{n=1}^{+\infty}\left(\frac{2\pi n}{\beta}\right)^{-2}\right]$

$$=\left(\frac{\sinh(\pi\beta/2\pi)}{\pi\beta/2\pi}\right)^{-1}\prod_{n=1}^{+\infty}\left(\frac{2\pi n}{\beta}\right)^{-2}$$

$$=\frac{\beta}{2\sinh(\beta/2)}\prod_{n=1}^{+\infty}\left(\frac{2\pi n}{\beta}\right)^{-2}$$

But by ζ-function regularization, we have:

$$\prod_{n=1}^{+\infty}\left(\frac{2\pi}{\beta}\right)^{-2}=\left(\frac{2\pi}{\beta}\right)^{-2\zeta(0)}=\left(\frac{2\pi}{\beta}\right)^{-2\cdot(-1/2)}=\left(\frac{2\pi}{\beta}\right)$$

And $\prod_{n=1}^{+\infty} n^{-2}=e^{-(-2)\zeta'(0)}=e^{+2\cdot(-1/2)\ln(2\pi)}=\frac{1}{2\pi}$

So $Z(\beta)=\frac{\beta}{2\sinh(\beta/2)}\left(\frac{2\pi}{\beta}\right)\left(\frac{1}{2\pi}\right)=\boxed{\frac{1}{2\sinh(\beta/2)}}$

This is the same as before!

2.1 Eqn of moth : $\partial_\tau \frac{\partial \mathcal{L}}{\partial(\partial_\tau X)} + \partial_\theta \frac{\partial \mathcal{L}}{\partial(\partial_\theta X)} = 0$

$$\therefore \quad \partial_\tau^2 X - \partial_\theta^2 X = 0 \quad \cdots (*)$$

Using "light cone" coord $\xi^\pm = \tau \pm \theta$.

$(*)$: $\partial_+ \partial_- X = 0 \Rightarrow \boxed{X = X_L(\tau+\theta) + X_R(\tau-\theta)}$

left Right mover

In order to quantize the theory, it is better to think in Fourier expansion form. $\boxed{X(\tau,\theta) = X_0(\tau) + \sum_{n\neq 0} X_n(\tau) e^{in\theta}}$: mode expansion

(Since $X(\tau,\theta)$ is a real valued, $(X_n(\tau))^* = X_{-n}(\tau)$)

$$\Rightarrow S = \frac{1}{4\pi}\int d\tau d\theta \left((\partial_\tau X)^2 - (\partial_\theta X)^2 \right)$$

$$= \frac{1}{4\pi}\int d\tau d\theta \left(\left(\dot{X}_0 + \sum_{n\neq 0} \dot{X}_n e^{in\theta}\right)^2 - \left(\sum_{n\neq 0} in X_n(\tau) e^{in\theta}\right)^2 \right)$$

$$= \int d\tau \left\{ \frac{1}{2}(\dot{X}_0)^2 + \sum_{n=1}^{\infty} \left(|\dot{X}_n|^2 - n^2 |X_n|^2 \right) \right\}$$

$\Rightarrow \{X_n\}_{n=0}^{\infty}$ are harmonic oscillators w/ frequency n

(Note that X_0 is real but $X_{n=1,2,\cdots}$ are complex)

(So conjugate momenta are

$$P_0 = \frac{\partial \mathcal{L}}{\partial \dot{X}_0} = \dot{X}_0 , \quad P_{n\neq 0} = \frac{\partial \mathcal{L}}{\partial \dot{X}_n} = \dot{X}_n^* \quad)$$

$\Rightarrow$ Commutator relation $[X_n, P_m] = i\delta_{mn}$

$\underline{n=0}$: $\boxed{[X_0, P_0] = i}$

real / imaginary part

let $X_n \equiv \frac{1}{\sqrt{2}}(X_{1n} + i X_{2n})$ for $n \neq 0$.

and define $a_{in} \equiv \frac{1}{\sqrt{2}}\left(\frac{P_{in}}{\sqrt{n}} - i\sqrt{n}X_{in}\right)$, $a_{in}^\dagger = \frac{1}{\sqrt{2}}\left(\frac{P_{in}}{\sqrt{n}} + i\sqrt{n}X_{in}\right)$ $\left(\begin{matrix} i=1,2 \\ n \neq 0\end{matrix}\right)$

$\Rightarrow [a_{in}, a_{jn}^\dagger] = \delta_{ij}$, $[a_{in}, a_{jn}] = 0$, $[a_{in}^\dagger, a_{jn}^\dagger] = 0$

and then define, $\alpha_n \equiv \sqrt{\frac{n}{2}}(a_{1n} + i a_{2n})$, $\alpha_{-n} \equiv \alpha_n^\dagger \equiv \sqrt{\frac{n}{2}}(a_{1n}^\dagger - i a_{2n}^\dagger)$

$\tilde{\alpha}_n \equiv \sqrt{\frac{n}{2}}(a_{1n} - i a_{2n})$, $\tilde{\alpha}_{-n} \equiv \tilde{\alpha}_n^\dagger = \sqrt{\frac{n}{2}}(a_{1n}^\dagger + i a_{2n}^\dagger)$

$\Rightarrow [\alpha_n, \alpha_{-n}] = [\tilde{\alpha}_n, \tilde{\alpha}_{-n}] = n$, $n > 0$

(usual expansion in string theory)

$[\alpha_n, \tilde{\alpha}_m] = 0$, $n, m \in \mathbb{Z}\setminus\{0\}$

together with $[X_0, P_0] = i$ gives us the full commutation relation

2.2 Theory is invariant under translation

$\Rightarrow$ action is invariant under $\delta_\alpha X = \alpha^\mu \partial_\mu X$: infinitesimal transl.

$\Rightarrow$ conserved current: $T^\mu{}_\nu =$

$\Rightarrow T^\tau{}_\tau = \frac{1}{2}\{(\partial_\tau X)^2 + (\partial_\theta X)^2\}$,

$T^\tau{}_\theta = \partial_\tau X \partial_\theta X$

$\Rightarrow$ Hamiltonian $H = \frac{1}{2\pi}\int_0^{2\pi} d\theta\, T^\tau{}_\tau = \frac{1}{4\pi}\int_0^{2\pi}\{(\partial_\tau X)^2 + (\partial_\theta X)^2\}\, d\theta$

Momentum: $P = \frac{1}{2\pi}\int_0^{2\pi} d\theta\, T^\tau{}_\theta = \frac{1}{2\pi}\int_0^{2\pi} \partial_\tau X \partial_\theta X\, d\theta$

So far, this is a classical Hamiltonian / momentum.

Using our quantized theory,

4) We use $\prod_{n=1}^{+\infty}\left(1+\frac{x^2}{n^2}\right)=\frac{\sinh(\pi x)}{\pi x}$

$$\text{So } Z(\beta)=\prod_{n=1}^{+\infty}\left(\left(\frac{2\pi n}{\beta}\right)^2+1\right)^{-1}=\left[\prod_{n=1}^{+\infty}\left(1+\frac{(\beta/2\pi)^2}{n^2}\right)\right]^{-1}\left[\prod_{n=1}^{+\infty}\left(\frac{2\pi n}{\beta}\right)^{-2}\right]$$

$$=\left(\frac{\sinh(\pi\beta/2\pi)}{\pi\beta/2\pi}\right)^{-1}\prod_{n=1}^{+\infty}\left(\frac{2\pi n}{\beta}\right)^{-2}$$

$$=\frac{\beta}{2\sinh(\beta/2)}\prod_{n=1}^{+\infty}\left(\frac{2\pi n}{\beta}\right)^{-2}$$

But by ζ-function regularization, we have:

$$\prod_{n=1}^{+\infty}\left(\frac{2\pi}{\beta}\right)^{-2}=\left(\frac{2\pi}{\beta}\right)^{-2\zeta(0)}=\left(\frac{2\pi}{\beta}\right)^{-2\cdot(-1/2)}=\left(\frac{2\pi}{\beta}\right)$$

$$\text{And } \prod_{n=1}^{+\infty}n^{-2}=e^{-(-2)\zeta'(0)}=e^{+2\cdot(-1/2)\ln(2\pi)}=\frac{1}{2\pi}$$

$$\text{So } Z(\beta)=\frac{\beta}{2\sinh(\beta/2)}\left(\frac{2\pi}{\beta}\right)\left(\frac{1}{2\pi}\right)=\boxed{\frac{1}{2\sinh(\beta/2)}}$$

This is the same as before!

$$\underline{\hat{H}} = \frac{1}{4\pi}\int d\theta\, (\partial_\tau \hat{X})^2 + (\partial_\theta \hat{X})^2 = \int d\theta \left\{ \left(\dot{X}_0 + \sum_{n\neq 0} \dot{X}_n e^{in\theta} \right)^2 + \left(\sum_{n\neq 0} in X_n e^{in\theta} \right)^2 \right\} \frac{1}{4\pi}$$

$$= \frac{1}{2}\hat{p}_0^2 + \sum_{n\geq 1}^{\infty} \left(|\dot{X}_n|^2 + n^2 X_n^2 \right)$$

change variables

$X_n \to \alpha_n, \alpha_{-n}$

by $X_n = \frac{1}{\sqrt{2}in}(\hat{\tilde{\alpha}}_{-n} - \hat{\alpha}_n)$

$\dot{X}_n = \frac{1}{\sqrt{2}in}(\tilde{\alpha}_{-n} + \hat{\alpha}_n)$

$$\stackrel{\partial}{=} \frac{1}{2}\hat{p}_0^2 + \sum_{n=1}^{\infty} \left(\alpha_{-n}\alpha_n + \tilde{\alpha}_{-n}\tilde{\alpha}_n + n \right)$$

$$= \frac{1}{2}\hat{p}_0^2 + \sum_{n=1}^{\infty} \hat{\alpha}_{-n}\alpha_n + \sum_{n=1}^{\infty} \tilde{\alpha}_{-n}\tilde{\alpha}_n + \underbrace{\sum_{n=1}^{\infty} n}_{=\zeta(-1)}$$

zeta function regularization

$$\stackrel{\partial}{=} \frac{1}{2}\hat{p}_0^2 + \sum_{n=1}^{\infty} \hat{\alpha}_{-n}\alpha_n + \sum_{n=1}^{\infty} \hat{\tilde{\alpha}}_{-n}\hat{\tilde{\alpha}}_n - \frac{1}{12}$$

$$\underline{\hat{P}} = \frac{1}{2\pi}\int_0^{2\pi} d\theta\, \partial_\tau X \partial_\theta X = \frac{1}{2\pi}\int_0^{2\pi} d\theta \left(\dot{X}_0 + \sum_{n\neq 0} \dot{X}_n e^{in\theta} \right)\left(\sum_{m\neq 0} in X_m e^{im\theta} \right)$$

$$= \sum_{n\neq 0} in \dot{X}_{-n} X_n$$

$X_n = \frac{1}{\sqrt{2}in}(\hat{\tilde{\alpha}}_{-n} - \hat{\alpha}_n)$

$\dot{X}_n = \frac{1}{\sqrt{2}in}(\hat{\tilde{\alpha}}_{-n} + \hat{\alpha}_n)$

$$\stackrel{\partial}{=} -\sum_{n=1}^{\infty} \hat{\alpha}_{-n}\hat{\alpha}_n + \sum_{n=1}^{\infty} \hat{\tilde{\alpha}}_{-n}\tilde{\alpha}_n$$

$\therefore$ Hamiltonian operator: $\hat{H} = \frac{1}{2}\hat{p}_0^2 + \sum_{n=1}^{\infty} \hat{\alpha}_{-n}\hat{\alpha}_n + \sum_{n=1}^{\infty} \hat{\tilde{\alpha}}_{-n}\hat{\tilde{\alpha}}_n - \frac{1}{12}$

Momentum operator: $\hat{P} = -\sum_{n=1}^{\infty} \hat{\alpha}_{-n}\hat{\alpha}_n + \sum_{n=1}^{\infty} \hat{\tilde{\alpha}}_{-n}\hat{\tilde{\alpha}}_n$

2.3. Our Hamiltonian compose of free part : $\frac{1}{2}\hat{p}_0^2$ plus H.O. parts

$\Rightarrow$ let $|k\rangle \equiv |k\rangle_0 \otimes \bigotimes_{n=1}^{\infty} |0\rangle_n$

$\Rightarrow$ Hilbert space $\mathcal{H} = \left\{ \prod_{n=1}^{\infty} (\alpha_{-n})^{m_n} (\tilde{\alpha}_{-n})^{\tilde{m}_n} |k\rangle \;\middle|\; \begin{array}{c} m_n, \tilde{m}_n \geq 0 \\ k \in \mathbb{R} \end{array} \right\}$

Where $\prod_{n=1}^{\infty} (\alpha_{-n})^{m_n} (\tilde{\alpha}_n)^{\tilde{m}_n} |k\rangle$ has energy $E = \frac{k^2}{2} + \sum_{n=1}^{\infty} n(m_n + \tilde{m}_n) - \frac{1}{12}$

and momentum $P = \sum_{n=1}^{\infty} n(-m_n + \tilde{m}_n)$

2.4 $H_R = \frac{1}{2}(H-P) = \frac{1}{4}p_0^2 + \sum_{n=1}^{\infty} \alpha_{-n}\alpha_n - \frac{1}{24}$: involves only right movers

$H_L = \frac{1}{2}(H+P) = \frac{1}{4}p_0^2 + \sum_{n=1}^{\infty} \tilde{\alpha}_{-n}\tilde{\alpha}_n - \frac{1}{24}$: " " Left "

2.5. $Z(\tau_1,\tau_2) = \mathrm{tr}_{\mathcal{H}}\, e^{-2\pi\tau_2 H} e^{-2\pi\tau_1 P}$ $\Rightarrow$ geometric meaning of partition function is that it is a path integral over torus

letting $\tau = \tau_1 + i\tau_2 \Rightarrow Z = Z(\tau,\bar\tau) = \mathrm{tr}_{\mathcal{H}}\, q^{H_L} \bar{q}^{H_R}$

$$= (q\bar q)^{-\frac{1}{24}}\, \mathrm{tr}_{\mathcal{H}_0} (q\bar q)^{p_0^2/2} \prod_{n=1}^{\infty} \mathrm{tr}_{\mathcal{H}_n^R}\, q^{\alpha_{-n}\alpha_n} \prod_{n=1}^{\infty} \mathrm{tr}_{\mathcal{H}_n^L}\, \bar q^{\tilde\alpha_{-n}\tilde\alpha_n}$$

$$\Rightarrow \mathrm{tr}_{\mathcal{H}_n^R}\, q^{\alpha_{-n}\alpha_n} = \sum_{k=0}^{\infty} q^{nk} = \frac{1}{1-q^n}$$

$$\mathrm{tr}_{\mathcal{H}_n^L}\, \bar q^{\tilde\alpha_{-n}\tilde\alpha_n} = \sum_{k=0}^{\infty} \bar q^{nk} = \frac{1}{1-\bar q^n}$$

$$\mathrm{tr}_{\mathcal{H}_0} (q\bar q)^{p_0^2/4} = \mathrm{tr}_{\mathcal{H}_0}\, e^{-2\pi\tau_2 H_0} = V\int_{-\infty}^{\infty} \frac{dp_0}{2\pi}\, e^{-2\pi\tau_2\left(\frac{1}{2}p^2\right)} = \frac{V}{2\pi\sqrt{\tau_2}}$$

$$\Rightarrow Z(\tau,\bar\tau) = (q\bar q)^{-1/24}\, \frac{V}{2\pi}\frac{1}{\sqrt{\tau_2}} \prod_{n=1}^{\infty} \left|\frac{1}{1-q^n}\right|^2$$

$$= \frac{V}{2\pi}\frac{1}{\sqrt{\tau_2}}\, |\eta(\tau)|^2$$

, when $\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty}(1-q^n)$ is a Dedekind eta function

$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$

2.6 $SL(2,\mathbb{Z})$ action on T^2 : $\tau \longmapsto \dfrac{a\tau+b}{c\tau+d}$ is gen'd

by $\tau \mapsto \tau+1$, $\tau \mapsto -\frac{1}{\tau}$

Since $\eta(\tau+1) = e^{\pi i/12}\eta(\tau)$ & $\eta\left(-\frac{1}{\tau}\right) = (-i\tau)^{1/2}\eta(\tau)$

$\Rightarrow$ $Z(\tau+1,\bar{\tau}+1) = Z(\tau,\bar{\tau})$ and $Z\left(-\frac{1}{\tau}, -\frac{1}{\bar{\tau}}\right) = Z(\tau,\bar{\tau})$

$\Rightarrow$ Theory is invariant under $SL(2,\mathbb{Z})$ action

As for $\tau \mapsto -\tau^{-1}$, we first compute the q dependence of the integral appearing in the expression for Z. First of all, note that

$$|q| = \mathrm{e}^{2\pi\mathrm{i}\,\tau}\mathrm{e}^{-2\pi\mathrm{i}\,\tau^*} = \mathrm{e}^{2\pi\mathrm{i}\,(\tau-\tau^*)} = e^{\pi\tau_2},$$

so

$$\int \mathrm{d}p\, |q|^{-\frac{1}{2}p^2} = \int \mathrm{d}p\, e^{-\frac{1}{2}\pi\tau_2 p^2} \propto \left(\frac{2\pi}{\pi\tau_2}\right)^{D/2}. \tag{8}$$

In particular, Z will be invariant iff

$$|q|^{1/12}(\tau_2)^{-1/2}|\eta(q)|^{-2}$$

is invariant. However,

$$\mathrm{Im}[-\tau^{-1}] = \mathrm{Im}[\tau]/|\tau|^2. \tag{9}$$

Furthermore, it is well-known that

$$\eta(q(-\tau^{-1})) = \sqrt{-\mathrm{i}\tau}\,\eta(q(\tau)). \tag{10}$$

Unfortunately, however, something has gone wrong; as it stands, Z will not be invariant under this transformation. However, we recall that the quantum hamiltonian and total momentum were defined only up to a constant. We see now that we may take this constant to be such that

$$\boxed{Z(q) = \left(\int \mathrm{d}p\, |q|^{-\frac{1}{2}p^2}\right) |\eta(q)|^{-2D}}.$$

However, given (8) and the transformation laws (9) and (10), this is invariant under $\tau \mapsto -\tau^{-1}$.

Problem 3. In light-cone gauge quantization, let α^i_m, $1 \le i \le 24$, be the physical excitation modes of the open string, and similarly for the closed string with $\tilde{\alpha}^i_m$ denoting the corresponding left-moving modes. In the open case, at level $N - 3$, we have the possible states generated from the vacuum:

$$\alpha^i_{-3}|0\rangle; \alpha^i_{-1}\alpha^j_{-2}|0\rangle; \alpha^i_{-1}\alpha^i_{-2}|0\rangle; \alpha^i_{-1}\alpha^j_{-1}\alpha^k_{-1}|0\rangle; \alpha^i_{-1}\alpha^i_{-1}\alpha^j_{-1}|0\rangle; \alpha^i_{-1}\alpha^i_{-1}\alpha^i_{-1}|0\rangle$$

(i, j, and k are always taken to be distinct). For the closed string, we obtain a state for any choice of two of the above states (one for the right-moving and one for the left-moving). Counting using the above list, we see that there are

$$24 + 24 \cdot 23 + 24 + \frac{24 \cdot 23 \cdot 22}{3!} + 24 \cdot 23 + 24 = 3200$$

states for the open string at this level. It follows that there are $3200^2 = 10240000$ states for the closed string at this level.

All these states a priori transform in representations of $SO(24)$, but as they are massive, they should transform in representations of $SO(25)$. If we wish to put these states into representations of $SO(25)$, we should first decompose the representations that appear in the states above into irreps of $SO(24)$. In this list of irreps, we should obtain the trivial representation, the fundamental representation v^i, the symmetric trace-less representation s^{ij}, the anti-symmetric trace-less representation a^{ij}, the symmetric trace-less representation s^{ijk}, ~~and the anti-symmetric representation a^{ijk}~~. The trivial representation of $SO(24)$ also obviously transforms as an irrep of $SO(25)$, namely the trivial representation again. We can put the remaining components into representations of $SO(25)$ by defining

$$S^{000} := 0, S^{i00} := v^i, S^{ij0} := s^{ij}, S^{ijk} := s^{ijk} \text{ and } A^{i00} := v^i, A^{ij0} := a^{ij}, \cancel{A^{ijk} := a^{ijk}},$$

and symmetrizing and anti-symmetrizing respectively (so that, for example, we have $S^{0i0} = v^i$ as well, etc.). As the indices on the S^{ijk} and A^{ijk} actually run from 0 to 24, we see that that furnish representations of $SO(25)$. To get irreps of $SO(25)$, we need only decompose these representations; however, if we did our job right, S^{ijk} and A^{ijk} (along with the trivial representation of course) should already be irreducible.