Physics 234A: String Theory

Prof. Mina Aganagic
Fall 2013

Homework 3.

1 Problem: Virasoro Algebra and Central Extension

1.1

B.B.S Problem 2.13

1.2

B.B.S. Problem 2.14

2 Problem: Free Boson CFT

Consider a theory of a single free boson $X(z, \bar{z})$ on a plane,

$$S = \frac{1}{2\pi\alpha'} \int d^2 z \partial_z X \bar{\partial} X$$

where $\partial = \partial_z$, $\bar{\partial} = \partial_{\bar{z}}$.

2.1

Work out the relation between

$$X(z_1, \bar{z}_1) X(z_2, \bar{z}_2)$$

and the normal ordered product

$$: X(z_1, \bar{z}_1) X(z_2, \bar{z}_2) :$$

where :: denotes oscillator normal ordering. Compute from this

$$\langle X(z_1, \bar{z}_1) X(z_2, \bar{z}_2) \rangle$$

The expectation values are implicitly taken in vacuum, $\langle X(z_1, \bar{z}_1) X(z_2, \bar{z}_2) \rangle = \langle 0 | X(z_1, \bar{z}_1) X(z_2, \bar{z}_2) | 0 \rangle$. The former is the notation of Polchinski's book, another standard string theory textbook; the later is used by B.B. S. Recall also that, in writing the product of operators, time ordering is implicit: we write operators at earlier times to the right of those at later times. In going from the cylinder to the plane, the slices of constant time are slices of constant $|z|$.

2.2

Compute

$$\langle : \exp(ikX(z_1, \bar{z}_1)) : : \exp(ik'X(z_2, \bar{z}_2)) : \rangle$$

what constraint do k and k' have to satisfy for this to be non vanishing?

2.3

Show that operator-state correspondence relates $: \exp(ikX(z, \bar{z})) :$ to the eigenstate $|k\rangle$ of space-time momentum p.

3 Problem: OPE of Stress energy Tensor

In the free CFT of the previous problem, show that

$$T(z) = -\frac{1}{\alpha'} : \partial_z X \partial_z X :$$

equals the T_{zz} component of the stress tensor that follows from Noethers theorem, up to an additive constant. The normal ordering :: defines this constant.

3.1

B.B.S. Problem 3.4

3.2

BBS Problem 3.5

4 Problem: Correlation functions of primary operators in a CFT

4.1

B.B. S. Problem 3.7

4.2

B.B.S. Problem 3.8

4.3

B.B.S. Problem 3.9.

4.4

B.B.S. Problem 3.10