

One Loop Tests of Higher Spin AdS/CFT

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Higher spins in AdS

- Vasiliev wrote down a set of consistent higher spin gauge invariant equations of motion. They admit a vacuum solution which is AdS space.
- In the simplest bosonic 4d theory, the linearized spectrum around the AdS vacuum consists of an infinite tower of higher spin fields plus a scalar

$$\begin{array}{lll} \text{Spectrum :} & s = 1, 2, 3, \dots, \infty & \text{gauge fields} \\ & s = 0, \quad m^2 = -2/\ell_{AdS}^2 & \text{scalar} \end{array}$$

Vasiliev equations

- The Vasiliev equations in 4d

$$d\mathcal{A} + \mathcal{A} * \mathcal{A} = e^{i\theta_0} B * \kappa dz^\alpha dz_\alpha + e^{-i\theta_0} B * \bar{\kappa} d\bar{z}^{\dot{\alpha}} d\bar{z}_{\dot{\alpha}}$$

$$dB + \mathcal{A} * B - B * \pi(\mathcal{A}) = 0$$

Vasiliev's frame-like formalism

twistors Y, Z

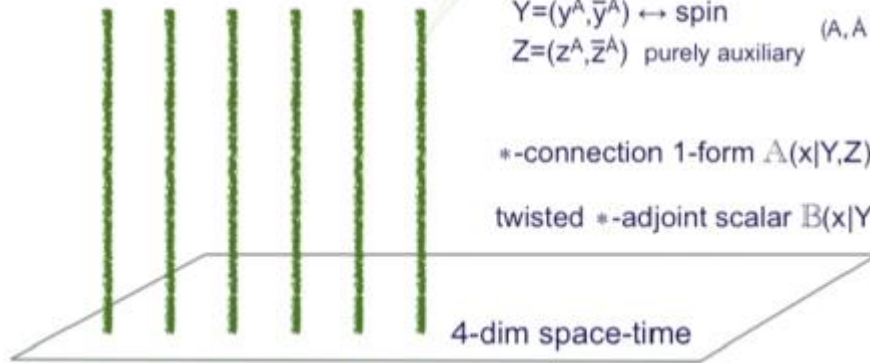
noncommutative $*$ product

$Y = (y^A, \bar{y}^{\dot{A}}) \leftrightarrow \text{spin}$
 $Z = (z^A, \bar{z}^{\dot{A}})$ purely auxiliary ($A, \dot{A} = 1, 2$)

$*$ -connection 1-form $A(x|Y, Z)$

twisted $*$ -adjoint scalar $B(x|Y, Z)$

4-dim space-time



Vasiliev equations

- The Vasiliev equations in 4d

$$\begin{aligned} d\mathcal{A} + \mathcal{A} * \mathcal{A} &= e^{i\theta_0} B * \kappa dz^\alpha dz_\alpha + e^{-i\theta_0} B * \bar{\kappa} d\bar{z}^{\dot{\alpha}} d\bar{z}_{\dot{\alpha}} \\ dB + \mathcal{A} * B - B * \pi(\mathcal{A}) &= 0 \end{aligned}$$

- “Type A”: $\theta_0 = 0$. “Type B”: $\theta_0 = \pi/2$. General θ_0 : parity breaking HS theory \longleftrightarrow Chern-Simons vector models.
- Essentially, the master field \mathcal{A} contains the metric and all other higher spin fields (more precisely, \mathcal{A} contains “vielbein” and “spin connections”), and B contains the scalar field and the curvatures (Weyl tensors) of the HS fields.

Vasiliev equations

$$\begin{aligned} d\mathcal{A} + \mathcal{A} * \mathcal{A} &= e^{i\theta_0} B * \kappa dz^\alpha dz_\alpha + e^{-i\theta_0} B * \bar{\kappa} d\bar{z}^{\dot{\alpha}} d\bar{z}_{\dot{\alpha}} \\ dB + \mathcal{A} * B - B * \pi(\mathcal{A}) &= 0 \end{aligned}$$

- AdS vacuum solution:

$$\mathcal{A} = (e_0)_{\alpha\dot{\beta}} y^\alpha \bar{y}^{\dot{\beta}} + (\omega_0)_{\alpha\beta} y^\alpha y^\beta + (\omega_0)_{\dot{\alpha}\dot{\beta}} \bar{y}^{\dot{\alpha}} \bar{y}^{\dot{\beta}}, \quad B = 0$$

Flatness of \mathcal{A} implies (translating to 4-vector notation)

$$d_x e_a + \omega_{ab} \wedge e_b = 0$$

$$d_x \omega_{ab} + \omega_{ac} \wedge \omega_{cb} + 6e_a \wedge e_b = 0$$

whose solution corresponds indeed to AdS_4

Vasiliev equations

- Linearizing the equations around the AdS vacuum, one finds that they describe the propagation of massless HS gauge fields with all integer spins, plus a scalar.
- The linearized equations are equivalent to Fronsdal's equations for totally symmetric fields. After gauge fixing, they take the form

$$\begin{aligned} (\nabla^2 - \kappa^2) \varphi_{\mu_1 \mu_2 \dots \mu_s} &= 0, & \kappa^2 &= (s-2)(s+d-3) - 2 \\ \nabla^\mu \varphi_{\mu \mu_2 \dots \mu_s} &= 0, & \varphi_{\mu \mu_3 \dots \mu_s}^\mu &= 0 \end{aligned}$$

- Studying the behavior of solutions to this wave equation, one can deduce the conformal dimension of dual operators

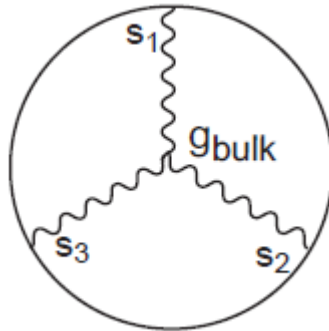
$$\Delta(\Delta - d) - s = \kappa^2 \quad \rightarrow \quad \Delta = s + d - 2 \quad (\text{or } \Delta = 2 - s)$$

Vasiliev equations

- Going to higher order in perturbation theory, one can read off the cubic, quartic, etc. couplings. In principle, one could reconstruct a Lagrangian in terms of the physical HS fields this way (in practice very hard!).
- At quadratic order around AdS, however, we can assume that the Lagrangian is just a sum of free Fronsdal's Lagrangians for massless higher spin fields. Kinetic operators are fixed.

Tests of the duality

- The HS/vector model dualities have been explicitly tested at the level of 3-point correlation functions (SG-Yin, Maldacena-Zhiboedov, Didenko-Skorvstov...)



$$\langle J_{s_1} J_{s_2} J_{s_3} \rangle = \cos^2 \theta_0 \langle J_{s_1} J_{s_2} J_{s_3} \rangle_{\text{sc}} + \sin^2 \theta_0 \langle J_{s_1} J_{s_2} J_{s_3} \rangle_{\text{fer}} + \cos \theta_0 \sin \theta_0 \langle J_{s_1} J_{s_2} J_{s_3} \rangle_{\text{odd}}$$

Free energy on S^3

- We would like to make a new type of test based on a different observable of the CFT: the partition function of the free energy on a round sphere S^3 , $F = -\log Z$.
- It is an interesting quantity that for any RG flow satisfies $F_{UV} > F_{IR}$. (“F-theorem”).
- For a CFT, it is also related to the entanglement entropy across a circle (this relation was used by Casini-Huerta to prove the 3d F-theorem)

Free energy on S^3

- In the CFT, it is simply defined as the log of the partition function of the theory on S^3 (generalization to S^d is straightforward)

$$F = -\log Z \qquad Z = \int D\phi e^{-S}$$
$$S = \int d^3x \sqrt{g} \left(\partial_\mu \phi^i \partial^\mu \phi^i + \frac{R}{8} \phi^i \phi^i \right)$$

- This is straightforward to compute in the free theory: need to evaluate the determinant of the kinetic operator

$$F = \frac{1}{2} \log \det (-\nabla^2 + 3/4)$$

Free energy on S^3

- The explicit computation gives (Klebanov, Pufu, Safdi)

$$F = \frac{N}{2} \sum_{n=0}^{\infty} (n+1)^2 \log[(n+1/2)(n+3/2)] = N \left(\frac{\log 2}{8} - \frac{3\zeta_3}{16\pi^2} \right)$$

for N real scalars, and twice this result for N complex scalars. Trivial N dependence (free theory!)

- One can also perform the calculation in the critical theory (in the large N expansion), with the result

$$F^{\text{critical}} = F^{\text{free}} - \frac{\zeta_3}{8\pi^2} + O(1/N)$$

Free energy on S^3 from the bulk

- The challenge is: can we reproduce these results from the bulk? In particular, can we see the vanishing of the subleading corrections in the large N expansion of the free energy from the HS dual to the free theory?
- How do we compute F from the bulk?

$$Z_{\text{CFT}} = Z_{\text{bulk}}$$

- We “simply” have to compute the partition function of the Vasiliev’s theory on the Euclidean AdS_4 vacuum (hyperbolic 4-space)

$$ds^2 = d\rho^2 + \sinh^2 \rho d\Omega_3$$

Free energy on S^3 from the bulk

- In practice, we should compute the path integral of the bulk theory, where we expand the metric around AdS_4 and integrate over all quantum fluctuations

$$\begin{aligned} Z_{\text{bulk}} &= \int D\varphi_{(0)} Dg_{\mu\nu} D\varphi_{(s)} e^{-S[g=g_0+h, \varphi_{(0)}, \varphi_{(s)}]} \\ &= e^{-\frac{1}{G_N} F^{(0)} - F^{(1)} - G_N F^{(2)} + \dots} = e^{-F_{\text{bulk}}} \end{aligned}$$

- Here G_N is Newton's constant, which scales as $1/G_N \sim N$.

Free energy on S^3 from the bulk

- The explicit bulk action is not well understood (proposals by *Douroud, Smolin; Boulanger, Sundell*), but in terms of physical fields is expected to take a form

$$S \sim \frac{1}{G_N} \int d^4x \sqrt{g} \left(R + \Lambda + R^3 + R^4 + \dots \right. \\ \left. + \varphi_{(s)} \Delta_s \varphi_{(s)} + \sum C_{s_1 s_2 s_3} \partial^{k_1} \varphi_{s_1} \partial^{k_2} \varphi_{s_2} \partial^{k_3} \varphi_{s_3} + \dots \right)$$

- The leading term $\frac{1}{G_N} F^{(0)}$ in the bulk free energy corresponds to evaluating this action on the AdS_4 background metric, with all other fields set to zero.
- This is already very non-trivial, as it requires to know the form of all the higher derivative corrections in the metric sector (we know they are non-trivial from knowledge of correlation functions).

Free energy on S^3 from the bulk

- One would like to show that

$$S_{\text{classical}}[g_{\mu\nu} = AdS_4, \varphi_{(s)} = 0] = \frac{1}{G_N} F^{(0)} = N \left(\frac{\log 2}{4} - \frac{3\zeta_3}{8\pi^2} \right)$$

- Alternatively, one would need a generalization of Ryu-Takayanagi prescription to compute holographic entanglement entropy in HS theory.
- This is one of the outstanding open problems in testing HS/vector model dualities.
- While we cannot show this (yet), we can start by something simpler, namely assume that this tree-level piece works, and compute the one-loop contribution $F^{(1)}$ to the bulk free energy.

The one-loop piece

- Let us now concentrate on the calculation of the one-loop contribution $F^{(1)}$ to the bulk free energy

$$e^{-\frac{1}{G_N}F^{(0)} - F^{(1)} - G_N F^{(2)} + \dots} = e^{-F_{\text{bulk}}}$$

- Even if we don't know the full action, we know that the linearized equations correspond to standard kinetic terms for all the higher spin fields, so we assume a canonical quadratic action

$$S_{(2)} = \int d^4x \sqrt{g} \left(\varphi_{(0)}(-\nabla^2 - 2)\varphi_{(0)} + \sum_{s=1,2,\dots} \varphi_{(s)} \Delta_s \varphi_{(s)} \right)$$

The one-loop piece

- The one-loop free energy is then obtained by computing the log of the determinants for the corresponding operators and summing over all spins
- The HS fields have a linearized gauge invariance $\delta\varphi_{(s)} = D\epsilon_{(s-1)}$ that must be gauge fixed.
- The gauge fixing for spin 1 and 2 is well known. For higher spins, it has been worked out in detail by various authors (Gaberdiel, Grumiller, Saha, Gupta, Lal...)

The one-loop piece

- One can introduce spin $s-1$ ghosts, then after decomposing physical and ghost fields into their irreducible parts, the contribution to the one-loop free energy of each HS field is the ratio of determinants on symmetric traceless transverse (STT) fields

$$\frac{\left[\det_{s-1}^{STT} (-\nabla^2 + s^2 - 1)\right]^{\frac{1}{2}}}{\left[\det_s^{STT} (-\nabla^2 + s(s-2) - 2)\right]^{\frac{1}{2}}}$$

- Note that degrees of freedom work $(2s+1-(2(s-1)+1)=2)$. And the “mass terms” correspond to dual CFT dimensions $\Delta=s+1$ for the physical part, and $\Delta=s+2$ for the ghost part (these may be seen as the dimensions of the spin s current and its divergence).

One-loop free energy

- We have to compute

$$F_{1\text{-loop}} = \frac{1}{2} \log \det (-\nabla^2 - 2) + \frac{1}{2} \sum_{s=1}^{\infty} [\log \det_s (-\nabla^2 - 2 + s(s-2)) - \log \det_{s-1} (-\nabla^2 + s^2 - 1)]$$

- Luckily, a large part of the calculation was already done in a series of papers by Camporesi and Higuchi in the '90's.
- They computed the spectral zeta function (Mellin transform of the heat kernel) for $-\nabla^2 + \kappa^2$ operators acting on STT fields of arbitrary spin in hyperbolic space.

Spectral zeta function

- More precisely, they computed the *spectral zeta function*. This is related to the heat kernel by a Mellin transform.
- For operators with discrete eigenvalues, the spectral zeta function is defined as

$$\zeta(z) = \sum_n d_n \lambda_n^{-z}$$

- In non-compact spaces such as AdS, this becomes

$$\zeta(z) = \int du \mu(u) \lambda_u^{-z}$$

where $\mu(u)$ is a “spectral density”.

AdS Spectral zeta function

- The explicit spectral zeta function in AdS is

$$\zeta_{(\Delta,s)}(z) = \left(\frac{\int \text{vol}_{AdS_{d+1}}}{\int \text{vol}_{S^d}} \right) \frac{2^{d-1}}{\pi} g(s) \int_0^\infty du \frac{\mu_s(u)}{\left[u^2 + \left(\Delta - \frac{d}{2} \right)^2 \right]^z}$$

with $\Delta(\Delta - d) - s = \kappa^2$

- In the present case of $d=3$

$$\text{vol}_{AdS_4} = \frac{4}{3}\pi^2, \quad \text{vol}_{S^3} = 2\pi^2$$
$$\mu_s(u) = \frac{\pi u}{16} \left[u^2 + \left(s + \frac{1}{2} \right)^2 \right] \tanh \pi u, \quad g(s) = 2s + 1$$

AdS Spectral zeta function

- In terms of the spectral zeta function, the contribution to the one-loop free energy is then obtained as

$$F_{(\Delta,s)}^{(1)} = -\frac{1}{2}\zeta'_{(\Delta,s)}(0) - \frac{1}{2}\zeta_{(\Delta,s)}(0) \log(\ell^2 \Lambda^2)$$

- Importantly, in every even dimensional bulk spacetime, there is a logarithmic divergence proportional to the value of the spectral zeta function at $z=0$. (It is related to the bulk conformal anomaly).

UV finiteness

- For the duality to be exact and Vasiliev theory to be “UV complete”, this divergence should not be present in the full HS theory: the bulk theory should be finite.
- While each spin contributes a log divergence, can the divergence cancel in the sum over the infinite tower of fields?

$$\begin{aligned} F^{(1)} \Big|_{\log-\text{div}} &= -\frac{1}{2} \left(\zeta_{(1,0)}(0) + \sum_{s=1}^{\infty} (\zeta_{(s+1,s)}(0) - \zeta_{(s+2,s-1)}(0)) \right) \log(\ell^2 \Lambda^2) \\ &= \left(\frac{1}{360} + \sum_{s=1}^{\infty} \left(\frac{1}{180} - \frac{s^2}{24} + \frac{5s^4}{24} \right) \right) \log(\ell^2 \Lambda^2) \end{aligned}$$

UV finiteness

- It appears natural to regulate this sum with the usual Riemann zeta-function regularization. Recall that $\zeta(0)=-1/2$, and $\zeta(-2)=\zeta(-4)=0$ (equivalently, use exponential regulator and discard divergences). Then:

$$\frac{1}{360} + \sum_{s=1}^{\infty} \left(\frac{1}{180} - \frac{s^2}{24} + \frac{5s^4}{24} \right) = 0$$

- So Vasiliev's theory is one-loop finite (at least accepting this regulator of the sum over spins).
- Regularization can be understood as natural analytic continuation of spectral zeta function in the spectral parameter z .
- The same result holds for the theory with even spins only, and regardless of boundary conditions on the scalar.
- This is similar to the cancellation of UV divergences in $N > 4$ SUGRA in AdS_4 (*Allen-Davis '83*), but here we have a purely bosonic theory (with an *infinite* number of fields).

The finite part

- Having shown that the log divergence cancels, we can move on to the computation of the finite contribution. This is considerably more involved. Computing the derivative of the spectral zeta-function, the result is expressed as

$$F^{(1)} = -\frac{1}{2}\mathcal{I}(-1/2, 0) - \frac{1}{2} \sum_{s=1}^{\infty} [\mathcal{I}(s - 1/2, s) - \mathcal{I}(s + 1/2, s - 1)]$$

with:

$$\mathcal{I}(\nu, s) = \frac{1}{3}(2s + 1) \int_0^{\nu} dx \left[\left(s + \frac{1}{2} \right)^2 x - x^3 \right] \psi(x + \frac{1}{2})$$

where $\nu = \Delta - d/2$.

The finite part

- The $\Delta=1$ scalar contribution is

$$\mathcal{I}\left(-\frac{1}{2}, 0\right) = -\frac{1}{3} \int_{-1/2}^0 dx \left(\frac{x}{4} - x^3\right) \psi\left(x + \frac{1}{2}\right) = \frac{11}{1152} - \frac{11 \log 2}{2880} - \frac{\log A}{8} - \frac{\zeta(3)}{8\pi^2} + \frac{5\zeta'(-3)}{8}$$

- Getting the contribution of the HS tower is more involved. One way to do it is to use the integral representation of the digamma function

$$\psi(y) = \int_0^\infty dt \left(\frac{e^{-t}}{t} - \frac{e^{-yt}}{1 - e^{-t}} \right)$$

- Then one can do the x-integral and sum over spins (with zeta-regulator) at fixed t.

The finite part

- This procedure gives

$$\begin{aligned} & \sum_{s=1}^{\infty} \left[\mathcal{I} \left(s - \frac{1}{2}, s \right) - \mathcal{I} \left(s + \frac{1}{2}, s - 1 \right) \right] \\ &= \int_0^{\infty} dt \left[\frac{191e^{-t} + 1349e^{-2t} + 1334e^{-3t} + 202e^{-4t} - 5e^{-5t} + e^{-6t}}{192(1 - e^{-t})^5 t} \right. \\ & \quad \left. + \frac{e^{-\frac{t}{2}} + 18e^{-t} - e^{-\frac{3}{2}t} - 2e^{-2t}}{12(1 - e^{-t})^2 t^2} - \frac{3e^{-t} + 6e^{-2t} - e^{-3t}}{(1 - e^{-t})^3 t^3} - 2 \frac{e^{-\frac{t}{2}} + 3e^{-t} - e^{-\frac{3}{2}t} + e^{-2t}}{(1 - e^{-t})^2 t^4} \right] \end{aligned}$$

- Now one needs to do this t-integral (importantly, it does not have 1/t logarithmic divergences).

The finite part

- The integral can be done analytically with the help of the Hurwitz-Lerch function

$$\Phi(z, s, v) = \frac{1}{\Gamma(s)} \int_0^\infty dt \frac{t^{s-1} e^{-vt}}{1 - ze^{-t}} = \sum_{n=0}^{\infty} (n+v)^{-s} z^n$$

- Need its derivatives at $z=+/-1$, and the relation to the more familiar Riemann-Hurwitz zeta function at these values.

The finite part

- After a somewhat lengthy calculation (or numerically) we find

$$\sum_{s=1}^{\infty} \left[\mathcal{I} \left(s - \frac{1}{2}, s \right) - \mathcal{I} \left(s + \frac{1}{2}, s - 1 \right) \right] \\ = -\frac{11}{1152} + \frac{11 \log 2}{2880} + \frac{\log A}{8} - \frac{5\zeta'(-3)}{8} - \frac{\zeta'(-2)}{2}$$

- The contribution of the scalar was

$$\mathcal{I} \left(-\frac{1}{2}, 0 \right) = -\frac{1}{3} \int_{-1/2}^0 dx \left(\frac{x}{4} - x^3 \right) \psi(x + \frac{1}{2}) = \frac{11}{1152} - \frac{11 \log 2}{2880} - \frac{\log A}{8} - \frac{\zeta(3)}{8\pi^2} + \frac{5\zeta'(-3)}{8}$$

- Using the fact that $\zeta'(-2) = -\frac{\zeta(3)}{4\pi^2}$, the higher spin tower precisely cancels the scalar!

The finite part

- So we conclude that the one-loop bulk free energy in Vasiliev's theory with $\Delta=1$ boundary condition for the scalar is exactly zero

$$F^{(1)} = 0$$

- This is precisely consistent with the fact that in the dual free CFT the large N expansion should be trivial.

$\Delta=2$ and the critical vector model

- We can also easily do the calculation with $\Delta=2$ boundary condition on the scalar. Only the scalar contribution is affected, and one finds

$$-\frac{1}{2}\mathcal{I}(\Delta = 2, 0) = -\frac{1}{2}\mathcal{I}(\Delta = 1, 0) - \frac{\zeta_3}{8\pi^2}$$

- So the final result is

$$F^{(1)} = -\frac{\zeta_3}{8\pi^2}$$

exactly consistent with the non-trivial large N expansion in the critical scalar theory.

The minimal HS theory

- We can repeat the same calculation in the minimal theory, with even spins only, which should be dual to the $O(N)$ vector model.
- Here we find a surprise. The total one loop free energy is *not* zero, but it is equal to

$$F_{\min}^{(1)} = \frac{\log 2}{8} - \frac{3\zeta(3)}{16\pi^2}$$

- This is precisely equal to the value of the S^3 free energy of a single real conformal scalar field...Why?

The minimal HS theory

- So far we have always assumed that Newton's constant is given by $G_N^{-1} = cN$. But there can in principle be subleading corrections in the map between G_N and N .
- Because the one-loop piece is precisely proportional to the expected classical piece, this suggests that the result can be consistent with the duality if we assume a shift $N \rightarrow N-1$, i.e. $G_N^{-1} = c(N-1)$, so that the classical piece is

$$\frac{1}{G_N} F_{\min}^{(0)} = (N-1) \left(\frac{\log 2}{8} - \frac{3\zeta(3)}{16\pi^2} \right)$$

combined with the one-loop piece would give the expected result for F .

One-loop shift

- This effect may perhaps be thought as a finite “one-loop renormalization” of the bare coupling constant in Vasiliev’s theory, somewhat similar to the one-loop shift of the level in Chern-Simons gauge theory
- The fact that the shift is simply an integer is consistent with the idea that the coupling constant in Vasiliev’s theory should be quantized (an argument for this was given by Maldacena-Zhiboedov based on higher spin symmetry).
- This shift will also affect correlation functions. For instance, it implies that the graviton 2-point function should receive non-trivial one-loop corrections (proportional to the classical result) in the minimal theory, but not in the “non-minimal” one. Would be interesting (but quite non-trivial) to check.

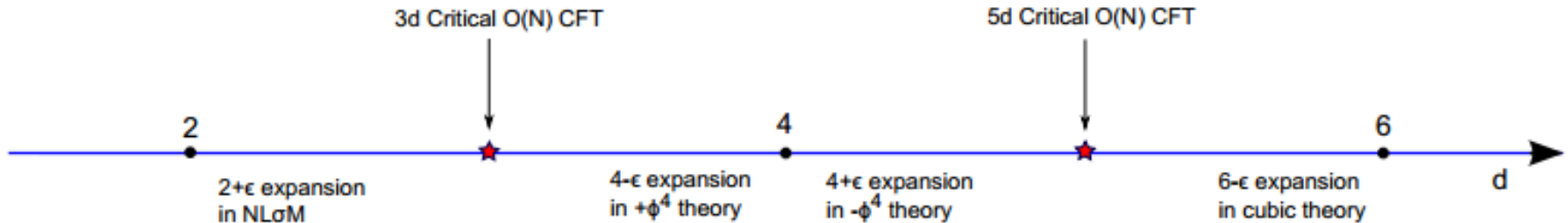
General dimensions

- There is a formulation of Vasiliev's theory in arbitrary dimensions. The equations of motion have a AdS_{d+1} vacuum solution, and the linearized spectrum around this background is

$$\begin{aligned} \text{Spectrum :} \quad & s = 1, 2, 3, \dots, \infty \quad \text{gauge fields} \\ & s = 0, \quad m^2 = -2(d-2) \quad \text{scalar} \end{aligned}$$

- This spectrum is in one-to-one correspondence with the single trace primaries of a free scalar vector model in dimension d (the scalar bilinear has dimension $\Delta=d-2$). Above $d=3$, there are no interacting IR fixed points dual to alternate boundary conditions. But there is a UV fixed point in $d=5$ of the scalar theory with ϕ^4 interaction: dual to Vasiliev theory in AdS_6 with alternate b.c. on scalar.

The critical $O(N)$ theory in $d=5$



- The 5d fixed point may be understood as either the UV fixed point of the ϕ^4 theory, or (*Fei, SG, Klebanov*) as the *IR fixed point* of the cubic theory

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^i)^2 + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{g_1}{2}\sigma \phi^i \phi^i + \frac{g_2}{6}\sigma^3$$

One loop in general dimensions

- The spectral zeta functions of the totally symmetric HS fields in general dimension are known (*Camporesi-Higuchi*). It is then natural to repeat the one loop calculations in general dimensions (*SG, Klebanov, Safdi*).
- In all odd d (even dim. AdS), there are UV logarithmic divergences spin by spin. Summing over all spins, the UV divergence always vanishes. Vasiliev theory is one-loop UV finite in *any* dimension.
- Finite part of $F^{(1)}$ is consistent with AdS/CFT in all dimensions (for minimal theories, this requires the shift $N \rightarrow N-1$ as found earlier). Warning: regularization of the sum over spins has to be done with care!

Conclusion and summary

- Consistent interacting theories of massless higher spins can be constructed if the cosmological constant is non-zero. They involve infinite towers of fields of all spins.
- The Vasiliev theory in AdS was conjectured to be exactly dual to simple vector model CFT's.
- Vasiliev theories provide exact AdS dual not only to free theories, but also to interesting interacting theories such as the critical (Wilson-Fisher) $O(N)$ model, the Gross-Neveu model, CP^N model, theories involving Chern-Simons gauge fields...Also suggested the existence of new interacting CFT's with $O(N)$ symmetry in 5 dimensions (*Fei, SG, Klebanov*), dual to Vasiliev theory in AdS_6 .

Conclusion and summary

- We have recently obtained new simple tests of higher spin/vector model dualities, by comparing partition functions on both sides of the duality.
- The classical bulk contribution is still out of reach (lacking understanding of the Lagrangian), but the one-loop calculation is well defined and can be done explicitly in general dimensions.
- In all dimensions, we find that one-loop UV divergences in the Vasiliev theory vanish due to the contribution of the infinite tower of spins. Is higher spin gravity a “UV complete” model of quantum gravity? Connection to string theory?

Conclusion and summary

- More to be done: loop corrections to correlation functions, action for Vasiliev equations, study other non-trivial solutions of the theory (e.g. mass deformations of CFT? Black holes?)...