Physics 234A: String Theory

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Homework 4.

1 Problem: bc CFT

Let b and c be anti-commuting fields, with action

$$S = \frac{1}{2\pi} \int d^2 z b \bar{\partial} c$$

1.1

Show this is conformally invariant, for b and c transforming as tensors of weight $(\lambda, 0)$ and $(1 - \lambda, 0)$, respectivelly.

1.2

Show that the OPE is given by

$$b(z)c(w) = -c(w)b(z) = \frac{1}{z - w}.$$

1.3

Verify, by computing the OPE's with the stress tensor

$$T^{(g)} =: (\partial b)c : -\lambda \partial : bc :, \qquad , \tilde{T} = 0$$

that b and c are primaries of above dimensions.

1.4

Show that the central charge of the theory equals

$$c = -3(2\lambda - 1)^2 + 1, \qquad \tilde{c} = 0.$$

For $\lambda = 2$, this is the bc ghost system of the bosonic string.

2 Problem: BRST symmetry and the central charge

Recall that the BRST charge equals

$$Q = \sum_{n\in\mathbb{Z}} \left(c_n L_{-n}^{(m)} + \tilde{c}_n \tilde{L}_{-n}^{(m)} \right) + \frac{1}{2} \sum_{n\in\mathbb{Z}} \left(: c_n L_{-n}^{(g)} : + : \tilde{c}_n \tilde{L}_{-n}^{(g)} : \right) - c_0 - \tilde{c}_0$$

where $L^{(m)}, \tilde{L}^{(m)}$ are left and the right moving modes of the matter stress tensor, and $L^{(g)}, \tilde{L}^{(g)}$ of the bc ghost system.

2.1

Compute $[L_m, b_n]$ and $\{Q, b_n\}$, where $L_m = L_m^{(m)} + L_m^{(g)}$.

2.2

Prove the Jacobi Identity

$$\{[Q, L_m], b_m\} - \{[L_m, b_n], Q\} - [\{b_n, Q\}, L_n] = 0$$

2.3

Use the Jacobi identity to prove that

$$\{[Q, L_m], b_m\}$$

vanishes if the total central charge is zero.

2.4

Show that vanishing of

$$\{[Q, L_m], b_m\}$$

implies that $[Q, L_m]$ itself vanishes. This implies Q is conformally invariant. (Hint: if non-zero, it would have to have ghost number 1. On the other hand, the vanishing of commutator implies it has no ghost, or c_n, modes.)

2.5

Use the Jacobi identity for QQb_n to show that $Q^2 = 0$ if the central charge vanishes. This means that Q is a generator of a global, fermionic symmetry of the theory, on any Riemann surface.

3 Problem: Spectrum from BRST

3.1

B.B.S Problem 3.14

4 Problem: $\beta\gamma$ CFT

Consider now the CFT of two *commuting* fields β and γ

$$S = \frac{1}{2\pi} \int d^2 z \beta \bar{\partial} \gamma$$

4.1

Show this is conformally invariant, for β and γ transforming as tensors of weight $(\lambda, 0)$ and $(1 - \lambda, 0)$, respectively.

4.2

Show that the OPE is given by

$$\beta(z)\gamma(w) = \gamma(w)\beta(z) = -\frac{1}{z-w}.$$

4.3

Verify, by computing the OPE's with the stress tensor

$$T =: (\partial\beta)\gamma : -\lambda\partial : \beta\gamma :, \qquad \tilde{T} = 0$$

that β and γ are primaries of above dimensions.

4.4

Show that the central charge of the theory equals

$$c = 3(2\lambda - 1)^2 - 1, \qquad \tilde{c} = 0$$

For $\lambda = \frac{3}{2}$, this is part of the ghost system of the superstring.