Dispersion Relation of GKP Strings

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1 Dispersion Relation of GKP Strings (arXiv:1311.5800)

1.1 Lambert W-Function

We want to check the following formula that gives the dispersion relation of what is known as the $\mathbb{R}\times\mathrm{S}^2$ Gubser-Klebanov-Polyakov (GKP) string:

$$\mathcal{E}-\mathcal{J}=1-\frac{1}{4\mathcal{J}}\left(2W+W^2\right)-\frac{1}{16\mathcal{J}^2}\left(W^2+W^3\right)-\frac{1}{256\mathcal{J}^3}\frac{W^3\left(11\,W^2+26\,W+16\right)}{1+W}+\ldots, \tag{1.1}$$

- leading terms: $-\dfrac{1}{4\mathcal{J}}\left(2W+W^2\right)=\displaystyle\sum_{n=1}^{\infty}\mathfrak{a}_n\,\mathcal{J}^{n-1}\,\left(e^{-2\mathcal{J}-2}\right)^n.$
- subleading terms: $-\dfrac{1}{16\mathcal{J}^2}\left(W^2+W^3\right)=\displaystyle\sum_{n=2}^{\infty}\mathfrak{b}_n\,\mathcal{J}^{n-2}\,\left(e^{-2\mathcal{J}-2}\right)^n.$
- next-to-subleading terms: $-\dfrac{1}{256\mathcal{J}^3}\dfrac{W^3\left(11\,W^2+26\,W+16\right)}{1+W}=\displaystyle\sum_{n=3}^{\infty}\mathfrak{c}_n\,\mathcal{J}^{n-3}\,\left(e^{-2\mathcal{J}-2}\right)^n.$

where $\mathcal{E}\equiv\pi\,E/2\sqrt{\lambda}$ and $\mathcal{J}\equiv\pi\,J/2\sqrt{\lambda}$ are the (scaled) energy and angular momentum of the GKP string and the argument of the Lambert W-function is $W\left(8\mathcal{J}e^{-2\mathcal{J}-2}\right)$.

Upon expansion of Lambert's W-function, the second, third and fourth term on the r.h.s. of (1.1) provide three infinite series of coefficients which completely determine the L, NL and NNL contributions to the large-J finite-size corrections to the dispersion relation of a closed folded single-spin string rotating in $\mathbb{R}\times\mathrm{S}^2$.

1.2 Series Inversion

To check (1.1), we write down the expressions for the conserved string charges:

$$E\left(\omega\right)=\frac{2\sqrt{\lambda}}{\pi\,\omega}\cdot\mathbb{K}\left(\frac{1}{\omega^2}\right)\Rightarrow\mathcal{E}\equiv\frac{\pi\,E}{2\,\sqrt{\lambda}}=\sqrt{1-x}\cdot\mathbb{K}\left(1-x\right), \qquad g=\frac{\sqrt{\lambda}}{4\pi}, \quad \lambda=g_{\mathrm{YM}}^2\,N=R^4/\alpha'^2 \tag{1.2}$$

$$J\left(\omega\right)=\frac{2\sqrt{\lambda}}{\pi}\cdot\left[\mathbb{K}\left(\frac{1}{\omega^2}\right)-\mathbb{E}\left(\frac{1}{\omega^2}\right)\right]\Rightarrow\mathcal{J}\equiv\frac{\pi\,J}{2\,\sqrt{\lambda}}=\mathbb{K}\left(1-x\right)-\mathbb{E}\left(1-x\right), \tag{1.3}$$

where ω is the angular velocity of the GKP string and $x\equiv1-1/\omega^2$ the complementary parameter of $1/\omega^2$. For long folded strings on S^2 ($\omega\to1^+$), we expand the string energy and spin in terms of $x\to0^+$:

$$\mathcal{E}\equiv\frac{\pi\,E}{2\sqrt{\lambda}}=\sqrt{1-x}\cdot\sum_{n=0}^{\infty}x^n\left(d_n\ln x+h_n\right)=-\sum_{n=0}^{\infty}x^n\cdot\sum_{k=0}^{n}\frac{(2k-3)!!}{(2k)!!}\left(d_{n-k}\ln x+h_{n-k}\right) \tag{1.4}$$

$$\mathcal{J}\equiv\frac{\pi\,J}{2\sqrt{\lambda}}=\sum_{n=0}^{\infty}x^n\left(c_n\ln x+b_n\right). \tag{1.5}$$

The coefficients that appear in series (1.4) and (1.5) are given by:

$$d_n=-\frac{1}{2}\left(\frac{(2n-1)!!}{(2n)!!}\right)^2, \qquad h_n=-4\,d_n\cdot\left(\ln2+H_n-H_{2n}\right)$$

$$c_n=-\frac{d_n}{2n-1}, \qquad b_n=-4\,c_n\cdot\left[\ln2+H_n-H_{2n}+\frac{1}{2\left(2n-1\right)}\right], \qquad n=0\,,1\,,2\,,\ldots \tag{1.6}$$

We will use Mathematica to invert series (1.5) in terms of $x=x\left(\mathcal{J}\right)$, then plug the result into (1.4) and obtain the first few terms of the finite-size corrections to the dispersion relation of the GKP string, $\mathcal{E}=\mathcal{E}\left(\mathcal{J}\right)$.

Our result will be compared with the one for $\mathfrak{a}_n$, $\mathfrak{b}_n$, $\mathfrak{c}_n$ that was found above.