```
<< LocalQFTToolKit`
Put[SaveFile = NBname["stub"] <> ".out"]

DefineTensorShortcuts[{{η, t, δ, ε, λ, D, F, U, H, g, τ, v, H, m, μ, n, Q, V, u, e, T}, 1},
{{U, x, y, γ, h, T, e}, 2},
{{σ, λ, T, g}, 3},
{{σ, R, K}, 4}
]

PR1["4.9.1 The Lagrange density for electromagnetism in curved space is ",
e4156 = ℒ->(*√(-g)*)(-Fuu[μ, ν]Fdd[μ, ν]/4 + Ad[μ]Ju[μ]),
" where ", Ju[μ], " is the conserved current.",
NL,
"(a) Derive the energy-momentum tensor by functional differentiation with respect to the metric. You can ass
Ad[μ]Ju[μ], " term does not contribute to the energy-meomentum tensor.",
NL, "(b) Consider adding a new term to the Lagrangian, ",
tmpL1 = ℒ1->βRuu[μ, ν]guu[ρ, σ]Fdd[μ, ρ]Fdd[ν, σ],
" How is Maxwell's equations altered in the presence of this term? Einstein's equation? Is the current still co
]
```

4.9.1 The Lagrange density for electromagnetism in curved space is $\mathcal{L} \to -\frac{1}{4}F^{\mu\nu}_{\mu\nu}F^{\mu\nu}_{\mu\nu}+$
$A^{\mu}_{\mu}J^{\mu}_{\mu}$ where J^{μ}_{μ} is the conserved current.\n(a) Derive the energy-momentum tensor by functional differentiati
$\beta F^{\mu\rho}_{\mu\rho}F^{\nu\sigma}_{\nu\sigma}g^{\rho\sigma}_{\rho\sigma}R^{\mu\nu}_{\mu\nu}$ How is Maxwell's equations altered in the presence of this term? Einstein's equation? Is the

```
PR1["Compute energy-momentum tensor for (a) and (b) and set β->0 for (a). We follow general theory from
tmpS = S->IntegralOp[{x}, ℒ + ℒ1],
NL, "The functional variation wrt ", gdd[μ, ν],
Yield, tmp = Map[δ[#]&, tmpS],
yield, tmp = tmp/.e4156/.tmpL1,
yield, tmp = tmp/.δ[IntegralOp[a_, b_]]->IntegralOp[a, δ[b]],
"POFF",
```

Yield, tmp = tmp/.δExpand[δ, {}],

NL, “(4.69) ”, subg = {$\delta[g]$-> $-g$gdd[$\mu9, \nu9$]δ[guu[$\mu9, \nu9$]], $\delta\left[\sqrt{-g}\right]$-> $-\sqrt{-g}$gdd[$\mu9, \nu9$]δ[guu[$\mu9, \nu9$]]/2} ,

Yield, tmp = tmp/.subg,

NL, “with: ”,

sub = Fuu[μ, ν]->guu[$\mu, \mu1$]guu[$\nu, \nu1$]Fdd[$\mu1, \nu1$],

Yield, tmp = tmp/.sub,

Yield, tmp = tmp//ExpandAll,

Yield, tmp = tmp//.δExpand[δ, {Fdd[_, _], Ju[_], Ad[_], β}],

Yield, tmp = tmp//.subg[[1]]//ExpandAll,

NL, “Change indices, get δ[guu[]] indices to be the same: ”,

Yield, tmp = tmp/.{δ[guu[a_, b_]]A_:>(δ[guu[a, b]]A/.{a->$\mu9$, b->$\nu9$})/;a=!=$\mu9$||b=!=$\mu9$

},

Yield, tmp = tmp/.{δ[Ruu[a_, b_]]->δd[δ[guu[$\mu9, \nu9$]]][Ruu[a, b]]δ[guu[$\mu9, \nu9$]]

},

Yield, tmp = tmp/.IntegralOp[a_, δ[b_]A_]->$\delta[b]$IntegralOp[a, A],

Yield, tmp = Map $\left[-2\# / (\sqrt{-g}\delta[\text{guu}[\mu9, \nu9]])\ \&, \text{tmp}\right]$,

“PONdd”,

NL, “Here the idea of density is confusing. If (4.75) is a density equation: ”,

Yield, tmp = Tdd[$\mu9, \nu9$]->tmp/.IntegralOp[a_, b_]->b//Simplify,

Yield, tmp = tmp//.Tensor[F, a_, b_]:>OrderAntiSymmetricTensor[Tensor[F, a, b]]//Expand;

Yield, tmp = tmp//MetricContractGexp[g],

NL, “Change term ”, pos = tmp//ExtractPositionPatterns[{Fdu[a_, $\nu1$]}];

yield, pos = Drop[pos[[1, 1]], -1];

yield, tmp1 = Extract[tmp, pos], “ so that it merges with other terms: ”,

yield, tmp = MapAt[#/.$\nu1$->$\mu1$&, tmp, pos];

Yield, Framed[tmpT = tmp//.Tensor[F, a_, b_]:>OrderAntiSymmetricTensor[Tensor[F, a, b]]],

NL, “which is the EM energy-momentum tensor with η->g plus other terms from L1 ”

];

Compute energy-momentum tensor for (a) and (b) and set β->0 for (a). We follow general theory from the a

$\int_x\ [\mathcal{L}+\mathcal{L}1]$\nThe functional variation wrt $g^{\mu\nu}_{\mu\nu}$\n$\to$ $\delta[S] \to \delta\left[\int_x\ [\mathcal{L}+\mathcal{L}1]\right] \longrightarrow \delta[S] \to$

$\delta\left[\int_x\ \left[-\frac{1}{4}F^{\mu\nu}_{\mu\nu}F^{\mu\nu}_{\mu\nu}+A^{\mu}_{\mu}J^{\mu}_{\mu}+\beta F^{\mu\rho}_{\mu\rho}F^{\nu\sigma}_{\nu\sigma}g^{\rho\sigma}_{\rho\sigma}R^{\mu\nu}_{\mu\nu}\right]\right] \longrightarrow \delta[S] \to \int_x\ \left[\delta\left[-\frac{1}{4}F^{\mu\nu}_{\mu\nu}F^{\mu\nu}_{\mu\nu}+A^{\mu}_{\mu}J^{\mu}_{\mu}+\beta F^{\mu\rho}_{\mu\rho}F^{\nu\sigma}_{\nu\sigma}g^{\rho\sigma}_{\rho\sigma}R^{\mu\nu}_{\mu\nu}\right.\right.$

$-\frac{2\delta[S]}{\sqrt{-g}\delta\left[g^{\mu9\nu9}_{\mu9\nu9}\right]} \to \frac{F^{\mu9\nu}_{\mu9\nu}F^{\nu9\nu1}_{\nu9\nu1}g^{\nu\nu1}_{\nu\nu1}+F^{\mu\mu9}_{\mu\mu9}\left(F^{\mu1\nu9}_{\mu1\nu9}g^{\mu\mu1}_{\mu\mu1}-4\beta F^{\nu\nu9}_{\nu\nu9}R^{\mu\nu}_{\mu\nu}\right)-4\beta F^{\mu\rho}_{\mu\rho}F^{\nu\sigma}_{\nu\sigma}g^{\rho\sigma}_{\rho\sigma}\delta^{\delta\left[g^{\mu9\nu9}_{\mu9\nu9}\right]}_{\delta\left[g^{\mu9\nu9}_{\mu9\nu9}\right]}\left[R^{\mu\nu}_{\mu\nu}\right]}{2\sqrt{-g}}$\n$\to$ \n$\to$ $T^{\mu9\nu9}_{\mu9\nu9} \to$

$\frac{2\sqrt{-g}\delta[S]}{g\delta\left[g^{\mu9\nu9}_{\mu9\nu9}\right]} \to \frac{\sqrt{-g}F^{\nu1\nu9}_{\nu1\nu9}F^{\mu9\nu1}_{\mu9\nu1}}{2g}-\frac{\sqrt{-g}F^{\mu1\nu9}_{\mu1\nu9}F^{\mu1\mu9}_{\mu1\mu9}}{2g}+\frac{2\sqrt{-g}\beta F^{\mu\mu9}_{\mu\mu9}F^{\nu\nu9}_{\nu\nu9}R^{\mu\nu}_{\mu\nu}}{g}+\frac{2\sqrt{-g}\beta F^{\nu\sigma}_{\nu\sigma}F^{\mu\sigma}_{\mu\sigma}\delta^{\delta\left[g^{\mu9\nu9}_{\mu9\nu9}\right]}_{\delta\left[g^{\mu9\nu9}_{\mu9\nu9}\right]}\left[R^{\mu\nu}_{\mu\nu}\right]}{g}$\nChange term $\longrightarrow$

PR1["Einstein equation (4.44): ",

e444 = Rdd[μ,ν] − Rgdd[μ,ν]/2->8πGTdd[μ,ν],

NL, "The energy-momentum tensor ", Tdd[μ,ν], " is give by ", tmpT,

NL, " changes the classical Einstein equation."

]

Einstein equation (4.44): $-\frac{1}{2}Rg^{\mu\nu}_{\mu\nu}+R^{\mu\nu}_{\mu\nu} \to 8G\pi T^{\mu\nu}_{\mu\nu}$\nThe energy-momentum tensor $T^{\mu\nu}_{\mu\nu}$ is give by $T^{\mu9\nu9}_{\mu9\nu9} \to$

$\frac{2\sqrt{-g}\delta[S]}{g\delta\left[g^{\mu9\nu9}_{\mu9\nu9}\right]} \to -\frac{\sqrt{-g}F^{\mu1\nu9}_{\mu1\nu9}F^{\mu1\mu9}_{\mu1\mu9}}{g}+\frac{2\sqrt{-g}\beta F^{\mu\mu9}_{\mu\mu9}F^{\nu\nu9}_{\nu\nu9}R^{\mu\nu}_{\mu\nu}}{g}+\frac{2\sqrt{-g}\beta F^{\nu\sigma}_{\nu\sigma}F^{\mu\sigma}_{\mu\sigma}\delta^{\delta\left[g^{\mu9\nu9}_{\mu9\nu9}\right]}_{\delta\left[g^{\mu9\nu9}_{\mu9\nu9}\right]}\left[R^{\mu\nu}_{\mu\nu}\right]}{g}$\n changes the classical Einstein e

PR1["We compute Maxwell's equations (equation of motion) from Lagrangian: ",

Yield, tmp = $\mathcal{L}+\mathcal{L}$1,

Yield, tmp = tmp/.e4156/.tmpL1,

NL, "Take the definition of ",

subF = {Fdd[μ_, ν_]->2AntiSymmetrize2[{μ,ν}][xCovariantD[Ad[μ], ν]]},

NL, "and lowering F indices by the metric: ",

subFl = {Fuu[μ_, ν_]->Fdd[μ9, ν9]guu[μ9, μ]guu[ν9, ν]},

Yield, subLt = $\mathcal{L}$t->tmp/.subFl/.subF,

NL, "The use (4.49) as the Euler-Lagrange equation: ",

e449 = xPartialD[$\mathcal{L}$t, Ad[α1]] − xCovariantD[xPartialD[$\mathcal{L}$t, xCovariantD[Ad[α1], α]], α],

NL, "Calculate the term ",

tmp = e449[[2]],

Yield, tmp = tmp->(tmp/.subLt),

Yield, tmp = tmp//DerivativeExpand[{Ju[_], Tensor[g, _, _], β, Ruu[_, _]}],

NL, “Use the relationships ”,

subdL = {xPartialD[xPartialD[Ad[a_], b_], xPartialD[Ad[c_], d_]] → δdd[a, c]δdd[b, d],

xPartialD[xCovariantD[Ad[a_], b_], xPartialD[Ad[c_], d_]] → δdd[a, c]δdd[b, d],

xPartialD[xCovariantD[Ad[a_], b_], xCovariantD[Ad[c_], d_]] → δdd[a, c]δdd[b, d],

xPartialD[Ad[a_], xCovariantD[Ad[c_], d_]] → 0,

xPartialD[Ad[a_], Ad[c_]] → δdd[a, c],

xPartialD[xCovariantD[Ad[a_], b_], Ad[c_]] → 0

},

Yield, Framed[tmp0 = tmp/.subdL//ContractNot[δ, {α1}]], (****)

NL, “For the term ”, tmp = e449[[1]],

Yield, tmp = tmp->(tmp/.subLt), check,

“POFF”,

Yield, tmp = tmp//DerivativeExpand[constants = {Ju[_], Tensor[g, _, _], Tensor[δ, _, _], β, Ruu[_, _]}],

Yield, tmp = tmp/.subdL, check,

Yield, tmp = tmp//DerivativeExpand[constants]//Expand, check,

Yield, tmp = tmp//ContractNot[δ, {α, α1}],

Yield, tmp = tmp//ContractUpDnNot[g, {α, α1}],

“PONdd”,

Yield, Framed[tmp1 = tmp//Simplify],

NL, “Combining to get Euler equations: ”, tmp3 = tmp0[[2]] + tmp1[[2]]

]

We compute Maxwell’s equations (equation of motion) from Lagrangian: \n→ $\mathcal{L}+$
$\mathcal{L}1$\n→ $-\frac{1}{4}F^{\mu\nu}_{\mu\nu}F^{\mu\nu}_{\mu\nu}+A^{\mu}_{\mu}J^{\mu}_{\mu}+\beta F^{\mu\rho}_{\mu\rho}F^{\nu\sigma}_{\nu\sigma}g^{\rho\sigma}_{\rho\sigma}R^{\mu\nu}_{\mu\nu}$\nTake the definition of $\left\{F^{\mu\_\nu\_}_{\mu\_\nu\_}\to\underline{\mathfrak{D}}_{\nu}\left[A^{\mu}_{\mu}\right]-\underline{\mathfrak{D}}_{\mu}\left[A^{\nu}_{\nu}\right]\right\}$ \nand lo
$A^{\mu}_{\mu}J^{\mu}_{\mu}-\frac{1}{4}g^{\mu9\mu}_{\mu9\mu}g^{\nu9\nu}_{\nu9\nu}\left(\underline{\mathfrak{D}}_{\nu}\left[A^{\mu}_{\mu}\right]-\underline{\mathfrak{D}}_{\mu}\left[A^{\nu}_{\nu}\right]\right)\left(\underline{\mathfrak{D}}_{\nu9}\left[A^{\mu9}_{\mu9}\right]-\underline{\mathfrak{D}}_{\mu9}\left[A^{\nu9}_{\nu9}\right]\right)+\beta g^{\rho\sigma}_{\rho\sigma}R^{\mu\nu}_{\mu\nu}\left(\underline{\mathfrak{D}}_{\rho}\left[A^{\mu}_{\mu}\right]-\underline{\mathfrak{D}}_{\mu}\left[A^{\rho}_{\rho}\right]\right)\left(\underline{\mathfrak{D}}_{\sigma}\left[A^{\nu}_{\nu}\right]-\right.$
$\underline{\mathfrak{D}}_{\alpha}\left[\underline{\partial}_{\underline{\mathfrak{D}}_{\alpha}\left[A^{\alpha1}_{\alpha1}\right]}[\mathcal{L}\mathrm{t}]\right]+\underline{\partial}_{A^{\alpha1}_{\alpha1}}[\mathcal{L}\mathrm{t}]$\nCalculate the term $\underline{\partial}_{A^{\alpha1}_{\alpha1}}[\mathcal{L}\mathrm{t}]$\n→ $\underline{\partial}_{A^{\alpha1}_{\alpha1}}[\mathcal{L}\mathrm{t}]\to\underline{\partial}_{A^{\alpha1}_{\alpha1}}\left[A^{\mu}_{\mu}J^{\mu}_{\mu}-\frac{1}{4}g^{\mu9\mu}_{\mu9\mu}g^{\nu9\nu}_{\nu9\nu}\left(\underline{\mathfrak{D}}_{\nu}\left[A^{\mu}_{\mu}\right.\right.\right.$

$J^{\mu}_{\mu}\underline{\partial}_{A^{\alpha1}_{\alpha1}}\left[A^{\mu}_{\mu}\right]-\frac{1}{4}g^{\mu9\mu}_{\mu9\mu}g^{\nu9\nu}_{\nu9\nu}\left(\left(\underline{\mathfrak{D}}_{\nu9}\left[A^{\mu9}_{\mu9}\right]-\underline{\mathfrak{D}}_{\mu9}\left[A^{\nu9}_{\nu9}\right]\right)\left(\underline{\partial}_{A^{\alpha1}_{\alpha1}}\left[\underline{\mathfrak{D}}_{\nu}\left[A^{\mu}_{\mu}\right]\right]-\underline{\partial}_{A^{\alpha1}_{\alpha1}}\left[\underline{\mathfrak{D}}_{\mu}\left[A^{\nu}_{\nu}\right]\right]\right)+\left(\underline{\mathfrak{D}}_{\nu}\left[A^{\mu}_{\mu}\right]-\underline{\mathfrak{D}}_{\mu}\left[A^{\nu}_{\nu}\right.\right.\right.$
$\beta g^{\rho\sigma}_{\rho\sigma}R^{\mu\nu}_{\mu\nu}\left(\left(\underline{\mathfrak{D}}_{\sigma}\left[A^{\nu}_{\nu}\right]-\underline{\mathfrak{D}}_{\nu}\left[A^{\sigma}_{\sigma}\right]\right)\left(\underline{\partial}_{A^{\alpha1}_{\alpha1}}\left[\underline{\mathfrak{D}}_{\rho}\left[A^{\mu}_{\mu}\right]\right]-\underline{\partial}_{A^{\alpha1}_{\alpha1}}\left[\underline{\mathfrak{D}}_{\mu}\left[A^{\rho}_{\rho}\right]\right]\right)+\left(\underline{\mathfrak{D}}_{\rho}\left[A^{\mu}_{\mu}\right]-\underline{\mathfrak{D}}_{\mu}\left[A^{\rho}_{\rho}\right]\right)\left(\underline{\partial}_{A^{\alpha1}_{\alpha1}}\left[\underline{\mathfrak{D}}_{\sigma}\left[A^{\nu}_{\nu}\right]\right]-\right.\right.$

$\underline{\mathfrak{D}}_{\alpha}\left[\underline{\partial}_{\underline{\mathfrak{D}}_{\alpha}\left[A^{\alpha1}_{\alpha1}\right]}[\mathcal{L}\mathrm{t}]\right]$ \n$\rightarrow -\underline{\mathfrak{D}}_{\alpha}\left[\underline{\partial}_{\underline{\mathfrak{D}}_{\alpha}\left[A^{\alpha1}_{\alpha1}\right]}[\mathcal{L}\mathrm{t}]\right]\rightarrow -\underline{\mathfrak{D}}_{\alpha}\left[\underline{\partial}_{\underline{\mathfrak{D}}_{\alpha}\left[A^{\alpha1}_{\alpha1}\right]}\left[A^{\mu}_{\mu}J^{\mu}_{\mu}-\frac{1}{4}g^{\mu9\mu}_{\mu9\mu}g^{\nu9\nu}_{\nu9\nu}\left(\underline{\mathfrak{D}}_{\nu}\left[A^{\mu}_{\mu}\right]-\underline{\mathfrak{D}}_{\mu}\left[A^{\nu}_{\nu}\right]\right)\left(\underline{\mathfrak{D}}_{\nu9}\right.\right.\right.$
$\beta R^{\mu\alpha}_{\mu\alpha}\underline{\mathfrak{D}}_{\alpha}\left[\underline{\mathfrak{D}}_{\mu}\left[A^{\alpha1}_{\alpha1}\right]\right]-\beta R^{\alpha\nu}_{\alpha\nu}\underline{\mathfrak{D}}_{\alpha}\left[\underline{\mathfrak{D}}_{\nu}\left[A^{\alpha1}_{\alpha1}\right]\right]+\beta R^{\mu\alpha1}_{\mu\alpha1}\left(\underline{\mathfrak{D}}_{\alpha}\left[\underline{\mathfrak{D}}_{\mu}\left[A^{\alpha}_{\alpha}\right]\right]-\underline{\mathfrak{D}}_{\alpha}\left[\underline{\mathfrak{D}}^{\alpha}\left[A^{\mu}_{\mu}\right]\right]\right)+$
$\beta R^{\mu\alpha}_{\mu\alpha}\underline{\mathfrak{D}}_{\alpha}\left[\underline{\mathfrak{D}}^{\alpha1}\left[A^{\mu}_{\mu}\right]\right]+\beta R^{\alpha1\nu}_{\alpha1\nu}\left(\underline{\mathfrak{D}}_{\alpha}\left[\underline{\mathfrak{D}}_{\nu}\left[A^{\alpha}_{\alpha}\right]\right]-\underline{\mathfrak{D}}_{\alpha}\left[\underline{\mathfrak{D}}^{\alpha}\left[A^{\nu}_{\nu}\right]\right]\right)+\beta R^{\alpha\nu}_{\alpha\nu}\underline{\mathfrak{D}}_{\alpha}\left[\underline{\mathfrak{D}}^{\alpha1}\left[A^{\nu}_{\nu}\right]\right]-$
$\underline{\mathfrak{D}}_{\alpha}\left[\underline{\mathfrak{D}}^{\alpha1}\left[A^{\alpha}_{\alpha}\right]\right]+\underline{\mathfrak{D}}_{\alpha}\left[\underline{\mathfrak{D}}^{\alpha}\left[A^{\alpha1}_{\alpha1}\right]\right]$

PR1["Checking for standard Maxwell's equations, β->0: ",

tmp = tmp3/.β->0/.CombineDeriv,

NL, "With ", sub = subF[[1]],

yield, sub = sub//RemovePatterns//RaiseIndexTU1[{μ,ν}, {μ,ν}]//Reverse,

yield, sub = Map[#&, sub]//RuleVarPattern[{μ,ν}],

Yield, tmp0 = tmp/.sub//DerivativeExpand[{}], CR[" Off by a sign (1.169)"],

Yield, tmp = (tmp0->0)//LowerIndexTU[α1, α1],

yield, sub4j = RuleX1[tmp, xCovariantD[a_, b_], {α, α1}] [[1]]

];

Checking for standard Maxwell's equations, β->0: $J^{\alpha1}_{\alpha1}+\underline{\mathfrak{D}}_{\alpha}\left[-\underline{\mathfrak{D}}^{\alpha1}\left[A^{\alpha}_{\alpha}\right]+\underline{\mathfrak{D}}^{\alpha}\left[A^{\alpha1}_{\alpha1}\right]\right]$ \nWith $F^{\mu\_\nu\_}_{\mu\_\nu\_}\rightarrow$
$\underline{\mathfrak{D}}_{\nu}\left[A^{\mu}_{\mu}\right]-\underline{\mathfrak{D}}_{\mu}\left[A^{\nu}_{\nu}\right]\longrightarrow\underline{\mathfrak{D}}^{\nu}\left[A^{\mu}_{\mu}\right]-\underline{\mathfrak{D}}^{\mu}\left[A^{\nu}_{\nu}\right]\rightarrow F^{\mu\nu}_{\mu\nu}\longrightarrow\underline{\mathfrak{D}}^{\nu\_}\left[A^{\mu\_}_{\mu\_}\right]-\underline{\mathfrak{D}}^{\mu\_}\left[A^{\nu\_}_{\nu\_}\right]\rightarrow$
$F^{\mu\nu}_{\mu\nu}$\n$\rightarrow J^{\alpha1}_{\alpha1}+\underline{\mathfrak{D}}_{\alpha}\left[F^{\alpha1\alpha}_{\alpha1\alpha}\right]$ Off by a sign (1.169)\n$\rightarrow J^{\alpha1}_{\alpha1}+\underline{\mathfrak{D}}_{\alpha}\left[F^{\alpha1\alpha}_{\alpha1\alpha}\right]\rightarrow 0\longrightarrow\underline{\mathfrak{D}}_{\alpha_}\left[F^{\alpha1_\alpha_}_{\alpha1_\alpha_}\right]\rightarrow$
$-J^{\alpha1}_{\alpha1}$

PR1["What is the β term: ",

tmp = tmp3,

Yield, tmp = CoefficientList[tmp, β][[2]],

Yield, tmp = tmp/.ν->μ//Expand,

Yield, tmp = tmp/.Ruu[a_, b_]:>OrderSymmetricTensor[Ruu[a,b]]//Simplify,

Yield, tmp = tmp/.CombineDeriv,

NL, "Using ",

sub = subF[[1]],

yield, sub = sub//RemovePatterns//RaiseIndexTU1[{ν}, {ν}]//Reverse,

yield, sub = Map[#&, sub]//RuleVarPattern[{μ,ν}],

Yield, tmp = tmp/.sub/.(Map[−#&, sub])//DerivativeExpand[{}],

NL, "Adding the β->0 term for the complete Maxwell's equation",

Yield, Framed[tmpj = βtmp + tmp0->0]

];

What is the β term: $J^{\alpha 1}_{\alpha 1} - \beta R^{\mu\alpha}_{\mu\alpha}\underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}_\mu\left[A^{\alpha 1}_{\alpha 1}\right]\right] - \beta R^{\alpha\nu}_{\alpha\nu}\underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}_\nu\left[A^{\alpha 1}_{\alpha 1}\right]\right] + \beta R^{\mu\alpha 1}_{\mu\alpha 1}\left(\underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}_\mu\left[A^{\alpha}_{\alpha}\right]\right] - \underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}^\alpha\left[A^{\mu}_{\mu}\right]\right]\right)$ $\beta R^{\mu\alpha}_{\mu\alpha}\underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}^{\alpha 1}\left[A^{\mu}_{\mu}\right]\right] + \beta R^{\alpha 1\nu}_{\alpha 1\nu}\left(\underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}_\nu\left[A^{\alpha}_{\alpha}\right]\right] - \underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}^\alpha\left[A^{\nu}_{\nu}\right]\right]\right) + \beta R^{\alpha\nu}_{\alpha\nu}\underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}^{\alpha 1}\left[A^{\nu}_{\nu}\right]\right] -$ $\underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}^{\alpha 1}\left[A^{\alpha}_{\alpha}\right]\right] + \underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}^{\alpha}\left[A^{\alpha 1}_{\alpha 1}\right]\right]$ \n$\rightarrow -R^{\mu\alpha}_{\mu\alpha}\underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}_\mu\left[A^{\alpha 1}_{\alpha 1}\right]\right] - R^{\alpha\nu}_{\alpha\nu}\underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}_\nu\left[A^{\alpha 1}_{\alpha 1}\right]\right] +$ $R^{\mu\alpha 1}_{\mu\alpha 1}\left(\underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}_\mu\left[A^{\alpha}_{\alpha}\right]\right] - \underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}^\alpha\left[A^{\mu}_{\mu}\right]\right]\right) + R^{\mu\alpha}_{\mu\alpha}\underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}^{\alpha 1}\left[A^{\mu}_{\mu}\right]\right] + R^{\alpha 1\nu}_{\alpha 1\nu}\left(\underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}_\nu\left[A^{\alpha}_{\alpha}\right]\right] - \underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}^\alpha\left[A^{\nu}_{\nu}\right]\right]\right) +$ $R^{\alpha\nu}_{\alpha\nu}\underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}^{\alpha 1}\left[A^{\nu}_{\nu}\right]\right]$ \n$\rightarrow R^{\alpha 1\mu}_{\alpha 1\mu}\underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}_\mu\left[A^{\alpha}_{\alpha}\right]\right] + R^{\mu\alpha 1}_{\mu\alpha 1}\underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}_\mu\left[A^{\alpha}_{\alpha}\right]\right] - R^{\alpha\mu}_{\alpha\mu}\underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}_\mu\left[A^{\alpha 1}_{\alpha 1}\right]\right] -$ $R^{\mu\alpha}_{\mu\alpha}\underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}_\mu\left[A^{\alpha 1}_{\alpha 1}\right]\right] - R^{\alpha 1\mu}_{\alpha 1\mu}\underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}^\alpha\left[A^{\mu}_{\mu}\right]\right] - R^{\mu\alpha 1}_{\mu\alpha 1}\underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}^\alpha\left[A^{\mu}_{\mu}\right]\right] + R^{\alpha\mu}_{\alpha\mu}\underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}^{\alpha 1}\left[A^{\mu}_{\mu}\right]\right] +$ $R^{\mu\alpha}_{\mu\alpha}\underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}^{\alpha 1}\left[A^{\mu}_{\mu}\right]\right]$ \n$\rightarrow 2\left(R^{\alpha 1\mu}_{\alpha 1\mu}\left(\underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}_\mu\left[A^{\alpha}_{\alpha}\right]\right] - \underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}^\alpha\left[A^{\mu}_{\mu}\right]\right]\right) + R^{\alpha\mu}_{\alpha\mu}\left(-\underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}_\mu\left[A^{\alpha 1}_{\alpha 1}\right]\right] + \underline{\mathfrak{D}}_\alpha\left[\underline{\mathfrak{D}}^{\alpha 1}\left[A^{\mu}_{\mu}\right]\right]\right)\right)$ $\underline{\mathfrak{D}}_\nu\left[A^{\mu}_{\mu}\right] - \underline{\mathfrak{D}}_\mu\left[A^{\nu}_{\nu}\right] \longrightarrow -\underline{\mathfrak{D}}_\mu\left[A^{\nu}_{\nu}\right] + \underline{\mathfrak{D}}^\nu\left[A^{\mu}_{\mu}\right] \rightarrow F^{\mu\nu}_{\mu\nu} \longrightarrow -\underline{\mathfrak{D}}_{\mu\_}\left[A^{\nu\_}_{\nu\_}\right] + \underline{\mathfrak{D}}^{\nu\_}\left[A^{\mu\_}_{\mu\_}\right] \rightarrow$ $F^{\mu\nu}_{\mu\nu}$\n$\rightarrow 2\left(-R^{\alpha 1\mu}_{\alpha 1\mu}\underline{\mathfrak{D}}_\alpha\left[F^{\mu\alpha}_{\mu\alpha}\right] + R^{\alpha\mu}_{\alpha\mu}\underline{\mathfrak{D}}_\alpha\left[F^{\mu\alpha 1}_{\mu\alpha 1}\right]\right)$ \nAdding the β->0 term for the complete Maxwell's equatio

PR1["Is current conserved? From ", tmp = tmpj,

Yield, xtmp = tmp = Map[xCovariantD[#, α1]&, tmp]//DerivativeExpand[{β}],

NL, "Using ",

sub = {xCovariantD[xCovariantD[Fuu[a_, b_], b_], a_]->0},

Yield, tmp = tmp/.sub,

NL, "Replace ",

sub1 = ExtractPositionPattern[tmp, Ruu[α, μ]xCovariantD[xCovariantD[Fdu[a_, b_], α], α1]],

yield, sub1 = sub1//Swap[{α, α1}],

Yield,

tmp = ReplacePart[tmp, sub1]/.xCovariantD[xCovariantD[A_, a_], b_]:>xCovariantD[xCovariantD[A, b], a]/;!Or

NL, "Simplify further: ",

sub = ExtractPositionPattern[tmp, xCovariantD[Fdu[μ, α], α]A_],

yield, sub = SwitchLabel[sub, {α, α1}],

Yield, Framed[tmp = ReplacePart[tmp, sub]],

NL, "Current does not appear to be conserved."

];

Is current conserved? From $J^{\alpha1}_{\alpha1}+2\beta\left(-R^{\alpha1\mu}_{\alpha1\mu}\underline{\mathfrak{D}}_{\alpha}\left[F^{\mu\alpha}_{\mu\alpha}\right]+R^{\alpha\mu}_{\alpha\mu}\underline{\mathfrak{D}}_{\alpha}\left[F^{\mu\alpha1}_{\mu\alpha1}\right]\right)+\underline{\mathfrak{D}}_{\alpha}\left[F^{\alpha1\alpha}_{\alpha1\alpha}\right]\to$
0\n$\to\underline{\mathfrak{D}}_{\alpha1}\left[J^{\alpha1}_{\alpha1}\right]+2\beta\left(\underline{\mathfrak{D}}_{\alpha}\left[F^{\mu\alpha1}_{\mu\alpha1}\right]\underline{\mathfrak{D}}_{\alpha1}\left[R^{\alpha\mu}_{\alpha\mu}\right]-\underline{\mathfrak{D}}_{\alpha}\left[F^{\mu\alpha}_{\mu\alpha}\right]\underline{\mathfrak{D}}_{\alpha1}\left[R^{\alpha1\mu}_{\alpha1\mu}\right]-R^{\alpha1\mu}_{\alpha1\mu}\underline{\mathfrak{D}}_{\alpha1}\left[\underline{\mathfrak{D}}_{\alpha}\left[F^{\mu\alpha}_{\mu\alpha}\right]\right]+R^{\alpha\mu}_{\alpha\mu}\underline{\mathfrak{D}}_{\alpha1}\left[\underline{\mathfrak{D}}_{\alpha}\right.\right.$
$\underline{\mathfrak{D}}_{\alpha1}\left[\underline{\mathfrak{D}}_{\alpha}\left[F^{\alpha1\alpha}_{\alpha1\alpha}\right]\right]\to$ 0\nUsing $\left\{\underline{\mathfrak{D}}_{\text{a\_}}\left[\underline{\mathfrak{D}}_{\text{b\_}}\left[F^{\text{a\_b\_}}_{\text{a\_b\_}}\right]\right]\to 0\right\}$ \n$\to\underline{\mathfrak{D}}_{\alpha1}\left[J^{\alpha1}_{\alpha1}\right]+2\beta\left(\underline{\mathfrak{D}}_{\alpha}\left[F^{\mu\alpha1}_{\mu\alpha1}\right]\underline{\mathfrak{D}}_{\alpha1}\left[R^{\alpha\mu}_{\alpha\mu}\right]-\underline{\mathfrak{D}}_{\alpha}\left[F^{\mu\alpha}_{\mu\alpha}\right.\right.$
0\nReplace $\left\{\{1,2,3,4\}\to R^{\alpha\mu}_{\alpha\mu}\underline{\mathfrak{D}}_{\alpha1}\left[\underline{\mathfrak{D}}_{\alpha}\left[F^{\mu\alpha1}_{\mu\alpha1}\right]\right]\right\}\longrightarrow\left\{\{1,2,3,4\}\to R^{\alpha1\mu}_{\alpha1\mu}\underline{\mathfrak{D}}_{\alpha}\left[\underline{\mathfrak{D}}_{\alpha1}\left[F^{\mu\alpha}_{\mu\alpha}\right]\right]\right\}$ \n$\to\underline{\mathfrak{D}}_{\alpha1}\left[J^{\alpha1}_{\alpha1}\right.$
$2\beta\left(\underline{\mathfrak{D}}_{\alpha}\left[F^{\mu\alpha1}_{\mu\alpha1}\right]\underline{\mathfrak{D}}_{\alpha1}\left[R^{\alpha\mu}_{\alpha\mu}\right]-\underline{\mathfrak{D}}_{\alpha}\left[F^{\mu\alpha}_{\mu\alpha}\right]\underline{\mathfrak{D}}_{\alpha1}\left[R^{\alpha1\mu}_{\alpha1\mu}\right]\right)\to$ 0\nSimplify further: $\left\{\{1,2,3,2\}\to-\underline{\mathfrak{D}}_{\alpha}\left[F^{\mu\alpha}_{\mu\alpha}\right]\underline{\mathfrak{D}}_{\alpha1}\left[\right.\right.$

PR1[

"We showed how to derive Einsteins's equation by varying the Hilbert action with respect to the metric.

They can also be derived by treating the metric and connection a independent degrees of freedom and

varying separately with respect to them; this is known as the Palatini formalism. That is, we consider the act

tmpS = S**->IntegralOp** $[\{x\},\sqrt{-g}$**guu**$[\mu,\nu]$**Rdd**$[\mu,\nu][\Gamma]]$ **,**

" where the Ricci tensor is thought of as constructed purely fromm the connection, not using the metric.

Variation with respect to the metric gives the usual Einstein's equations, but for a Ricci tensor

constructed from a connection that has no a priori relationship to the metric. Imaginig from the start

that the connection is symmetric (torsion free), show that the variation of this action with respect to

the connection coefficients leads to the requirement that the connection be metric compatible, that is,

the Cristoffel connection. Remember that Stokes's theorem, relating the integral of the covariant

divergence of a vector to an integral of the vector over the boundary, does not work for a general

covariant derivative. The best strategy is to write the connection coefficients as a sum of the Christoffel symb

$T\left[\tilde{\Gamma},\text{“udd”}\right][\lambda,\mu,\nu]$**, “ and tensor ”,** $T[C,\text{“udd”}][\lambda,\mu,\nu]$**,**

Yield,

subG = $T[\Gamma,\text{“udd”}][\lambda,\mu,\nu]$**->**$T\left[\tilde{\Gamma},\text{“udd”}\right][\lambda,\mu,\nu]+T[C,\text{“udd”}][\lambda,\mu,\nu]$**, “ and then show that ”,** $T[C,\text{“udd”}]$

“ must vanish.”

]

We showed how to derive Einsteins's equation by varying the Hilbert action with respect to the metric. They
$\int_x\left[\sqrt{-g}g^{\mu\nu}_{\mu\nu}R^{\mu\nu}_{\mu\nu}[\Gamma]\right]$ where the Ricci tensor is thought of as constructed purely fromm the connection, not u
$C^{\lambda\mu\nu}_{\lambda\mu\nu}+\tilde{\Gamma}^{\lambda\mu\nu}_{\lambda\mu\nu}$ and then show that $C^{\lambda\mu\nu}_{\lambda\mu\nu}$ must vanish.

PR1[“Let's try the variation over Γ **of ”, tmp = tmpS,**

Yield, tmp = Map[δ**[#]&, tmp],**

yield, tmp = tmp/.δ[IntegralOp[a_, b_]]->IntegralOp[a, δ[b]],

Yield, tmp = tmp//.δExpand[δ, {}],

NL, "(4.69) ", subg = {δ[g]-> − $g$gdd[μ9, ν9]δ[guu[μ9, ν9]], δ [$\sqrt{-g}$] -> − $\sqrt{-g}$gdd[μ9, ν9]δ[guu[μ9, ν9]]/2} ,

Yield, tmp = tmp/.subg//ExpandAll,

Yield, tmp = tmp//.IntegralOp[a_, b_ + c_]->IntegralOp[a, b] + IntegralOp[a, c],

NL, "The last 2 terms generate Einstein's equations for the vacuum. Examine first term which must equal 0:

tmp0 = tmp[[2, 1]]

];

Let's try the variation over Γ of $S \to \int_x \left[\sqrt{-g} g^{\mu\nu}_{\mu\nu} R^{\mu\nu}_{\mu\nu}[\Gamma]\right]$ \n$\to \delta[S] \to \delta\left[\int_x \left[\sqrt{-g} g^{\mu\nu}_{\mu\nu} R^{\mu\nu}_{\mu\nu}[\Gamma]\right]\right] \longrightarrow \delta[S]$ -
$\int_x \left[\delta\left[\sqrt{-g} g^{\mu\nu}_{\mu\nu} R^{\mu\nu}_{\mu\nu}[\Gamma]\right]\right]$ \n$\to \delta[S] \to \int_x \left[-\frac{g^{\mu\nu}_{\mu\nu}\delta[g] R^{\mu\nu}_{\mu\nu}[\Gamma]}{2\sqrt{-g}} + \sqrt{-g}\left(g^{\mu\nu}_{\mu\nu}\delta\left[R^{\mu\nu}_{\mu\nu}[\Gamma]\right] + \delta\left[g^{\mu\nu}_{\mu\nu}\right] R^{\mu\nu}_{\mu\nu}[\Gamma]\right)\right]$ \n(4.69)
$\int_x \left[\sqrt{-g} g^{\mu\nu}_{\mu\nu}\delta\left[R^{\mu\nu}_{\mu\nu}[\Gamma]\right] + \sqrt{-g}\delta\left[g^{\mu\nu}_{\mu\nu}\right] R^{\mu\nu}_{\mu\nu}[\Gamma] - \frac{1}{2}\sqrt{-g} g^{\mu9\nu9}_{\mu9\nu9} g^{\mu\nu}_{\mu\nu}\delta\left[g^{\mu9\nu9}_{\mu9\nu9}\right] R^{\mu\nu}_{\mu\nu}[\Gamma]\right]$ \n$\to \delta[S] \to$
$\int_x \left[\sqrt{-g} g^{\mu\nu}_{\mu\nu}\delta\left[R^{\mu\nu}_{\mu\nu}[\Gamma]\right]\right] + \int_x \left[\sqrt{-g}\delta\left[g^{\mu\nu}_{\mu\nu}\right] R^{\mu\nu}_{\mu\nu}[\Gamma]\right] + \int_x \left[-\frac{1}{2}\sqrt{-g} g^{\mu9\nu9}_{\mu9\nu9} g^{\mu\nu}_{\mu\nu}\delta\left[g^{\mu9\nu9}_{\mu9\nu9}\right] R^{\mu\nu}_{\mu\nu}[\Gamma]\right]$ \nThe last 2 te

PR1["Check (4.62) from (3.4): ",

tmp =

e34 = Ruddd[ρ, μ, σ, ν]->xPartialD[Γudd[ρ, ν, μ], σ] − xPartialD[Γudd[ρ, σ, μ], ν] + Γudd[ρ, σ, λ1]Γudd[λ1, ν, μ]−

Γudd[ρ, ν, λ1]Γudd[λ1, σ, μ],

Yield, tmp = Map[δ[#]&, tmp]//.δExpand[δ, {}],

NL, "Using (4.61) the Covariant Derivative of : ", δ[Γudd[ρ, ν, μ]],

yield,

e461 = xCovariantD[δ[Γudd[ρ, ν, μ]], λ] $\to$ xPartialD[δ[Γudd[ρ, ν, μ]], λ] + Γudd[ρ, λ, σ1]δ[Γudd[σ1, ν, μ]]−

Γudd[σ1, λ, ν]δ[Γudd[ρ, σ1, μ]] − Γudd[σ1, λ, μ]δ[Γudd[ρ, ν, σ1]],

yield, sub = RuleX1[e461, xPartialD[δ[Γudd[ρ, ν, μ]], λ], {ρ, ν, μ, λ}];

yield, sub = sub//SwitchDeriv[{δ}, {xPartialD, xPartialDu}],

NL, "Eliminating Partial Derivative: ",

Imply, tmp = tmp/.sub,

Yield, tmp = tmp//Symmetrize[{{Γ, 3, {2, 3}}}],

Yield, Framed[e462 = tmp/.A_B_:>(AB/.λ1->σ1)/;FreeQ[AB, σ1]//Symmetrize[{{Γ, 3, {2, 3}}}]], "(4.62)",

NL, "For the Ricci tensor: ",

subdRicci = e462/.{σ->ρ1, ρ->ρ1}/.Ruddd[a_, b_, c_, d_]->Rdd[b, d]

];

Check (4.62) from (3.4): $R^{\rho\mu\sigma\nu}_{\rho\mu\sigma\nu} \;\rightarrow\; -\Gamma^{\lambda1\sigma\mu}_{\lambda1\sigma\mu}\Gamma^{\rho\nu\lambda1}_{\rho\nu\lambda1} + \Gamma^{\lambda1\nu\mu}_{\lambda1\nu\mu}\Gamma^{\rho\sigma\lambda1}_{\rho\sigma\lambda1} + \underline{\partial}_{\sigma}\left[\Gamma^{\rho\nu\mu}_{\rho\nu\mu}\right] -$
$\underline{\partial}_{\nu}\left[\Gamma^{\rho\sigma\mu}_{\rho\sigma\mu}\right]$ \n$\rightarrow \delta\left[R^{\rho\mu\sigma\nu}_{\rho\mu\sigma\nu}\right] \rightarrow \Gamma^{\rho\sigma\lambda1}_{\rho\sigma\lambda1}\delta\left[\Gamma^{\lambda1\nu\mu}_{\lambda1\nu\mu}\right] - \Gamma^{\rho\nu\lambda1}_{\rho\nu\lambda1}\delta\left[\Gamma^{\lambda1\sigma\mu}_{\lambda1\sigma\mu}\right] - \Gamma^{\lambda1\sigma\mu}_{\lambda1\sigma\mu}\delta\left[\Gamma^{\rho\nu\lambda1}_{\rho\nu\lambda1}\right] +$
$\Gamma^{\lambda1\nu\mu}_{\lambda1\nu\mu}\delta\left[\Gamma^{\rho\sigma\lambda1}_{\rho\sigma\lambda1}\right] + \delta\left[\underline{\partial}_{\sigma}\left[\Gamma^{\rho\nu\mu}_{\rho\nu\mu}\right]\right] - \delta\left[\underline{\partial}_{\nu}\left[\Gamma^{\rho\sigma\mu}_{\rho\sigma\mu}\right]\right]$ \nUsing (4.61) the Covariant Derivative of : $\delta\left[\Gamma^{\rho\nu\mu}_{\rho\nu\mu}\right] \longrightarrow \underline{\mathfrak{D}}_{\lambda}\left[\delta\right.$
$\underline{\partial}_{\lambda}\left[\delta\left[\Gamma^{\rho\nu\mu}_{\rho\nu\mu}\right]\right] - \Gamma^{\sigma1\lambda\mu}_{\sigma1\lambda\mu}\delta\left[\Gamma^{\rho\nu\sigma1}_{\rho\nu\sigma1}\right] - \Gamma^{\sigma1\lambda\nu}_{\sigma1\lambda\nu}\delta\left[\Gamma^{\rho\sigma1\mu}_{\rho\sigma1\mu}\right] + \Gamma^{\rho\lambda\sigma1}_{\rho\lambda\sigma1}\delta\left[\Gamma^{\sigma1\nu\mu}_{\sigma1\nu\mu}\right] \;\longrightarrow\; \longrightarrow\; \left\{\delta\left[\underline{\partial}_{\lambda\_}\left[\Gamma^{\rho\_\nu\_\mu\_}_{\rho\_\nu\_\mu\_}\right]\right] \rightarrow \underline{\mathfrak{D}}_{\lambda}\left[\delta\left[\Gamma^{\rho\nu\mu}_{\rho\nu\mu}\right]\right] +\right.$
$\underline{\mathfrak{D}}_{\sigma}\left[\delta\left[\Gamma^{\rho\nu\mu}_{\rho\nu\mu}\right]\right] - \underline{\mathfrak{D}}_{\nu}\left[\delta\left[\Gamma^{\rho\sigma\mu}_{\rho\sigma\mu}\right]\right] + \Gamma^{\rho\sigma\lambda1}_{\rho\sigma\lambda1}\delta\left[\Gamma^{\lambda1\nu\mu}_{\lambda1\nu\mu}\right] - \Gamma^{\rho\nu\lambda1}_{\rho\nu\lambda1}\delta\left[\Gamma^{\lambda1\sigma\mu}_{\lambda1\sigma\mu}\right] - \Gamma^{\lambda1\sigma\mu}_{\lambda1\sigma\mu}\delta\left[\Gamma^{\rho\nu\lambda1}_{\rho\nu\lambda1}\right] +$
$\Gamma^{\sigma1\sigma\mu}_{\sigma1\sigma\mu}\delta\left[\Gamma^{\rho\nu\sigma1}_{\rho\nu\sigma1}\right] + \Gamma^{\lambda1\nu\mu}_{\lambda1\nu\mu}\delta\left[\Gamma^{\rho\sigma\lambda1}_{\rho\sigma\lambda1}\right] - \Gamma^{\sigma1\nu\mu}_{\sigma1\nu\mu}\delta\left[\Gamma^{\rho\sigma\sigma1}_{\rho\sigma\sigma1}\right] - \Gamma^{\sigma1\nu\sigma}_{\sigma1\nu\sigma}\delta\left[\Gamma^{\rho\sigma1\mu}_{\rho\sigma1\mu}\right] + \Gamma^{\sigma1\sigma\nu}_{\sigma1\sigma\nu}\delta\left[\Gamma^{\rho\sigma1\mu}_{\rho\sigma1\mu}\right] -$
$\Gamma^{\rho\sigma\sigma1}_{\rho\sigma\sigma1}\delta\left[\Gamma^{\sigma1\nu\mu}_{\sigma1\nu\mu}\right] + \Gamma^{\rho\nu\sigma1}_{\rho\nu\sigma1}\delta\left[\Gamma^{\sigma1\sigma\mu}_{\sigma1\sigma\mu}\right]$ \n$\rightarrow \delta\left[R^{\rho\mu\sigma\nu}_{\rho\mu\sigma\nu}\right] \rightarrow \underline{\mathfrak{D}}_{\sigma}\left[\delta\left[\Gamma^{\rho\mu\nu}_{\rho\mu\nu}\right]\right] - \underline{\mathfrak{D}}_{\nu}\left[\delta\left[\Gamma^{\rho\mu\sigma}_{\rho\mu\sigma}\right]\right] +$
$\Gamma^{\rho\lambda1\sigma}_{\rho\lambda1\sigma}\delta\left[\Gamma^{\lambda1\mu\nu}_{\lambda1\mu\nu}\right] - \Gamma^{\rho\lambda1\nu}_{\rho\lambda1\nu}\delta\left[\Gamma^{\lambda1\mu\sigma}_{\lambda1\mu\sigma}\right] - \Gamma^{\lambda1\mu\sigma}_{\lambda1\mu\sigma}\delta\left[\Gamma^{\rho\lambda1\nu}_{\rho\lambda1\nu}\right] + \Gamma^{\lambda1\mu\nu}_{\lambda1\mu\nu}\delta\left[\Gamma^{\rho\lambda1\sigma}_{\rho\lambda1\sigma}\right] + \Gamma^{\sigma1\mu\sigma}_{\sigma1\mu\sigma}\delta\left[\Gamma^{\rho\nu\sigma1}_{\rho\nu\sigma1}\right] -$
$\Gamma^{\sigma1\mu\nu}_{\sigma1\mu\nu}\delta\left[\Gamma^{\rho\sigma\sigma1}_{\rho\sigma\sigma1}\right] - \Gamma^{\rho\sigma\sigma1}_{\rho\sigma\sigma1}\delta\left[\Gamma^{\sigma1\mu\nu}_{\sigma1\mu\nu}\right] + \Gamma^{\rho\nu\sigma1}_{\rho\nu\sigma1}\delta\left[\Gamma^{\sigma1\mu\sigma}_{\sigma1\mu\sigma}\right]$ \n$\rightarrow \boxed{\delta\left[R^{\rho\mu\sigma\nu}_{\rho\mu\sigma\nu}\right] \rightarrow \underline{\mathfrak{D}}_{\sigma}\left[\delta\left[\Gamma^{\rho\mu\nu}_{\rho\mu\nu}\right]\right] - \underline{\mathfrak{D}}_{\nu}\left[\delta\left[\Gamma^{\rho\mu\sigma}_{\rho\mu\sigma}\right]\right]}$ (4.62)\nFor
$\underline{\mathfrak{D}}_{\rho1}\left[\delta\left[\Gamma^{\rho1\mu\nu}_{\rho1\mu\nu}\right]\right] - \underline{\mathfrak{D}}_{\nu}\left[\delta\left[\Gamma^{\rho1\mu\rho1}_{\rho1\mu\rho1}\right]\right]$

PR1["Continuing manipulation of: ",

tmp = tmp0,

yield, tmp = tmp/.Rdd[a_, b_][c_]->Rdd[a, b]/.subdRicci,

NL, "As suggested if we assume that ", Γudd[a, b, c], " is a sum of Christoffer connection and a tenor ",

sub = Γudd[a, b, c]->T[Γc, "udd"][a, b, c] + T[C, "udd"][a, b, c],

" where Γc is the Christoffel connection and ", T[C, "udd"][a, b, c], " is a Tensor and in case we may equate it

δ[Γudd[a, b, c]], ". Then we can write: ", sub = RuleX1[sub, T[C, "udd"][a, b, c]],

yield, sub = δ[Γudd[a, b, c]]->sub[[1, 1]]; sub = RuleX2PatternVar[sub, {a, b, c}],

NL, "The integrand: ",

tmp = tmp/.IntegralOp[a_, $\sqrt{-g}$b_]->b,

yield, tmp = tmp/.sub//Expand,

Yield, tmp = tmp/.guu[a_, b_]xCovariantD[x_, y_]->xCovariantD[$x$guu[a, b], y],

yield, tmp = tmp//ContractUpDn[g],

yield, tmp = MapAt[#/.{ν->ρ2, ρ1->ν}&, tmp, {2}]/.ρ1->ρ2,

Yield, tmp = tmp/. − xCovariantD[a_, b_]->xCovariantD[−a, b]/.xCovariantD[a_, b_] + xCovariantD[c_, b_]->xC

NL, “Since the components of ”, T[C, “udd”][a, b, c], “ are arbitary they must be zero.”

];

Continuing manipulation of: $\int_x \left[\sqrt{-g} g^{\mu\nu}_{\mu\nu} \delta\left[R^{\mu\nu}_{\mu\nu}[\Gamma]\right]\right] \longrightarrow \int_x \left[\sqrt{-g} g^{\mu\nu}_{\mu\nu} \left(\underline{\mathfrak{D}}_{\rho 1}\left[\delta\left[\Gamma^{\rho 1 \mu\nu}_{\rho 1 \mu\nu}\right]\right] - \underline{\mathfrak{D}}_{\nu}\left[\delta\left[\Gamma^{\rho 1 \mu \rho 1}_{\rho 1 \mu \rho 1}\right.\right.\right.\right.$
C^{abc}_{abc}+Γc^{abc}_{abc} where Γc is the Christoffel connection and C^{abc}_{abc} is a Tensor and in case we may equate it to $\delta\left[\Gamma^{a}_{a}\right.$
C^{abc}_{abc}\nThe integrand: $g^{\mu\nu}_{\mu\nu}\left(\underline{\mathfrak{D}}_{\rho 1}\left[\delta\left[\Gamma^{\rho 1\mu\nu}_{\rho 1\mu\nu}\right]\right] - \underline{\mathfrak{D}}_{\nu}\left[\delta\left[\Gamma^{\rho 1\mu\rho 1}_{\rho 1\mu\rho 1}\right]\right]\right) \longrightarrow g^{\mu\nu}_{\mu\nu}\underline{\mathfrak{D}}_{\rho 1}\left[C^{\rho 1\mu\nu}_{\rho 1\mu\nu}\right] -$
$g^{\mu\nu}_{\mu\nu}\underline{\mathfrak{D}}_{\nu}\left[C^{\rho 1\mu\rho 1}_{\rho 1\mu\rho 1}\right]$ \n$\rightarrow$ $\underline{\mathfrak{D}}_{\rho 1}\left[C^{\rho 1\mu\nu}_{\rho 1\mu\nu} g^{\mu\nu}_{\mu\nu}\right] - \underline{\mathfrak{D}}_{\nu}\left[C^{\rho 1\mu\rho 1}_{\rho 1\mu\rho 1} g^{\mu\nu}_{\mu\nu}\right] \longrightarrow \underline{\mathfrak{D}}_{\rho 1}\left[C^{\rho 1\nu\nu}_{\rho 1\nu\nu}\right] - \underline{\mathfrak{D}}_{\nu}\left[C^{\rho 1\nu\rho 1}_{\rho 1\nu\rho 1}\right] \longrightarrow -$
$\underline{\mathfrak{D}}_{\rho 2}\left[C^{\nu\rho 2\nu}_{\nu\rho 2\nu}\right] + \underline{\mathfrak{D}}_{\rho 2}\left[C^{\rho 2\nu\nu}_{\rho 2\nu\nu}\right]$ \n$\rightarrow$ $\underline{\mathfrak{D}}_{\rho 2}\left[-C^{\nu\rho 2\nu}_{\nu\rho 2\nu} + C^{\rho 2\nu\nu}_{\rho 2\nu\nu}\right]$ \nSince the components of C^{abc}_{abc} are arbitary they mus

PR1[

"4. Show that the energy-momentum tesnors for electromagnetism and for scalar field theory satisfy the

dominate energy condition, and thus also the weak, null, and null dominant conditions. Show that they also s

w>= − 1

]

4. Show that the energy-momentum tesnors for electromagnetism and for scalar field theory satisfy the domin
−1