Lecture Note 5: Multivariate Models

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_K X_K + U$$

$$= \beta_0 + \sum_{k=1}^{K} \beta_k X_k + U$$

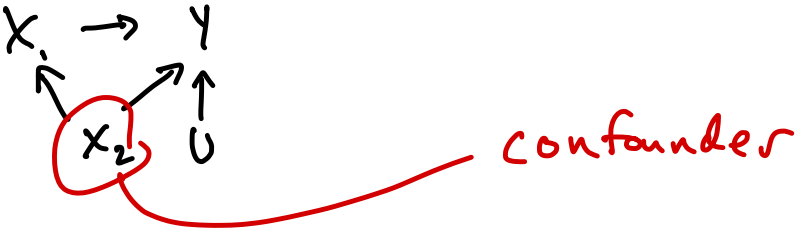
$$= X'\beta + U$$

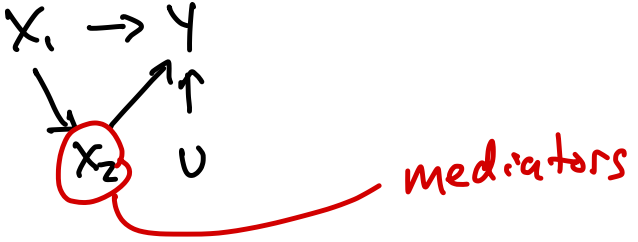
$$X = \begin{bmatrix} 1 \\ X_1 \\ X_2 \\ \vdots \\ X_K \end{bmatrix} \qquad \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_K \end{bmatrix}$$

$$Y = \alpha_0 + \alpha_1 X_1 + U$$

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + U$$

see OVB formula in Lecture Note 5





Linear combinations of coefs

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + U$$

$$a\hat{\beta}_1 + b\hat{\beta}_2 \rightarrow a\beta_1 + b\beta_2$$

$$V\left[a\hat{\beta}_1 + b\hat{\beta}_2\right] = a^2 V[\hat{\beta}_1] + b^2 V[\hat{\beta}_2] + 2ab\,\mathrm{cov}(\hat{\beta}_1, \hat{\beta}_2)$$

$$= a^2 \sigma^2_{\hat{\beta}_1} + b^2 \sigma^2_{\hat{\beta}_2} + 2ab\,\sigma_{\hat{\beta}_1,\hat{\beta}_2}$$

$\sigma^2_{\hat{\beta}_1} \uparrow SE[\hat{\beta}_1]^2$ $\qquad$ $\sigma^2_{\hat{\beta}_2} \uparrow SE[\hat{\beta}_2]$

Example: null $\beta_1 = \beta_2 \Rightarrow \beta_1 - \beta_2 = 0 \Rightarrow a = 1,\ b = -1$

Nonlinear functions of coefs

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + U$$

$$g(\beta_1, \beta_2) = \frac{\beta_1}{\beta_2}$$

Delta method: 1st-order Taylor approximation

$$V[a\hat{\beta}_1 + b\hat{\beta}_2] = a^2 V[\hat{\beta}_1] + b^2 V[\hat{\beta}_2] + 2ab\,\mathrm{cov}(\hat{\beta}_1, \hat{\beta}_2)$$

$\partial g/\partial \beta_1$ $\partial g/\partial \beta_2$

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