Lecture Note 1: Parameters and Estimators

Population parameters: properties of random variables

X, μ_X, σ_X^2 β θ

μ_X: central tendency; σ_X^2: dispersion

Statistics: properties of samples

Sample: N observations of X: $X_1, X_2, \cdots, X_N$

"iid" → independent and identically distributed

Estimators are statistics that approximate parameters

Greek letters for parameters: μ, σ^2, β, θ

Hats for estimators: $\hat{\mu}$, $\hat{\sigma}^2$, $\hat{\beta}$, $\hat{\theta}$

Desirable properties for $\hat{\theta}$:

① Unbiasedness: $E[\hat{\theta}] = \theta$

② Consistency: as $N \to \infty$, $\Pr[|\hat{\theta} - \theta| > \varepsilon] \to 0$ for any $\varepsilon > 0$

$\hat{\theta} \not\to \theta$

③ Efficiency: $\hat{\theta}$ has smallest possible $V[\hat{\theta}]$ ← how "noisy"

$\hat{\theta}$ has a distribution!

"Variance"

$V[X] = \sigma_X^2$ variance of X → $\sqrt{V[X]}$ std. dev. of X

$V[\hat{\theta}] = \sigma_{\hat{\theta}}^2$ variance of $\hat{\theta}$ → $\sqrt{V[\hat{\theta}]}$ std. error of $\hat{\theta}$