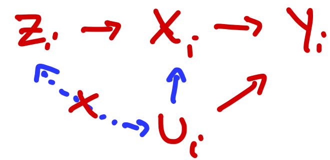
Lecture Note 10: IV

$$Y_i = \beta_0 + \underset{\substack{\uparrow \\ \text{homogeneous} \\ \text{effect}}}{\beta_1} X_i + U_i$$



Assumptions: 1. Relevance: $cov(Z_i, X_i) \neq 0$

2. Exogeneity: $cov(Z_i, U_i) = 0$

a) Z_i has no direct effect on Y_i

b) Z_i "as good as random"

$$cov(Z_i, Y_i) = cov(Z_i, \beta_0 + \beta_1 X_i + U_i) = \underbrace{\cancel{cov(Z_i, \beta_0)}}_{0} + \beta_1 cov(Z_i, X_i) + \underbrace{\cancel{cov(Z_i, U_i)}}_{0}$$

$$cov(Z_i, Y_i) = \beta_1 cov(Z_i, X_i)$$

$$\beta_1 = \frac{cov(Z_i, Y_i) / V[Z_i]}{cov(Z_i, X_i) / V[Z_i]}$$

① Regress Y_i on Z_i (reduced form)

② Regress X_i on Z_i (first stage)

③ Ratio: β_{RF} / β_{FS}

Two-stage least squares

1. Regress X_i on Z_i: $X_i = \tilde{\pi}_0 + \tilde{\pi}_1 Z_i + V_i$

   Form predicted values: $\hat{X}_i = \hat{\tilde{\pi}}_0 + \hat{\tilde{\pi}}_1 Z_i$

2. Regress Y_i on $\hat{X}_i$: $Y = \beta_0 + \beta_1 \hat{X}_i + \varepsilon_i$

$\hat{\beta}_1^{TSLS}$ same as RF/1st and consistent estimator for β_1

In R using fixest, regress Y on X, W_1, W_2, instrument for X using Z

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feols(y ~ w1+w2 | x ~ z, data=df)
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Wald estimator: Z_i binary

- $\bar{Y}_1$, $\bar{Y}_0$, $\bar{X}_1$, $\bar{X}_0$
- $\hat{\beta}_1^{wald} = \dfrac{\bar{Y}_1 - \bar{Y}_0}{\bar{X}_1 - \bar{X}_0}$

$\bar{X}_0 = 0$ in eligibility experiment

Building toward the heterogeneous effects setup
w/ binary Z_i, binary X_i

	$Z_i = 1$: $X_i = 0$	$Z_i = 1$: $X_i = 1$
$Z_i = 0$: $X_i = 0$	never takers	compliers
$Z_i = 0$: $X_i = 1$	~~defiers~~	always takers

monotonicity

TOT → never, compliers

LATE → never, compliers, always

Heterogeneous effects

- single binary Z_i, single binary X_i
- potential outcome: $Y_i(x,z)$ for treatment level x
  instrument level z
- potential treatment status: $X_i(z)$
- to interpret IV/2SLS under heterogeneity, 3 assumptions:

① Independence: $\{Y_i(x,z), X_i(z)\} \perp Z_i$ ← Z_i is "as good as random"
← allows us to estimate RF and 1st

② Exclusion: $Y_i(x,z) = Y_i(x)$ ← $Z_i \to X_i \to Y_i$
$Y_i(x,1) = Y_i(x,0)$

③ Monotonicity: either $X_i(1) \geq X_i(0)$ for all i
or $X_i(1) \leq X_i(0)$ for all i

Under ①-③:

$\hat{\beta}_1^{TSLS} \xrightarrow{p}$ LATE = avg effect of X on Y among compliers

local

Generalizing to non-binary Z_i and X_i:

$X_i = \pi_{0i} + \pi_{1i} Z_i + V_i$ 1st

$Y_i = \beta_{0i} + \beta_{1i} X_i + U_i$ 2nd

Then:

$$\hat{\beta}_1^{TSLS} \xrightarrow{p} E\left[\frac{\pi_{1i}}{E[\pi_{1i}]}\beta_{1i}\right]$$

ind. causal effect of X_i on Y_i