# Lecture Note 11: Regression Discontinuity Designs

## Sharp RD designs

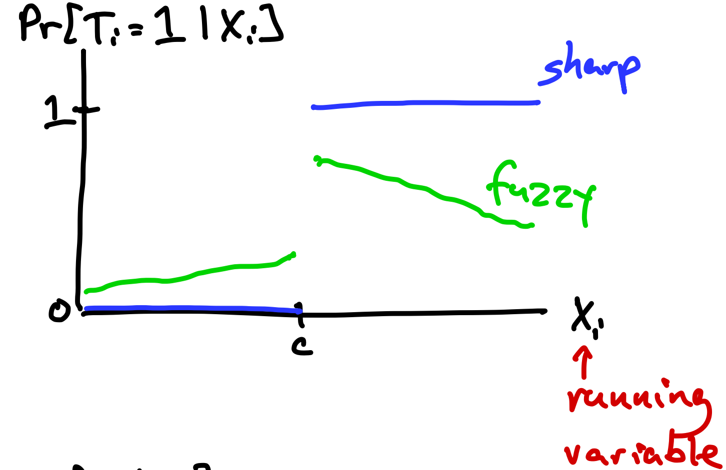
$$Y_i = Y_i(T_i) = T_i Y_i(1) + (1-T_i) Y_i(0)$$

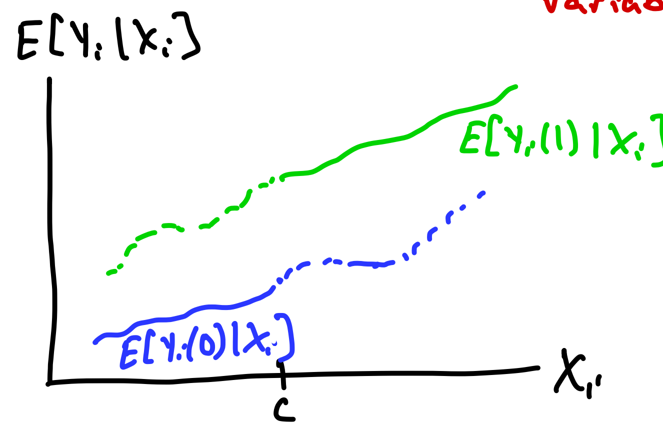
$$= \begin{cases} Y_i(0) & \text{if } T_i = 0 \\ Y_i(1) & \text{if } T_i = 1 \end{cases}$$

Assignment rule:

$$T_i = \left.\begin{cases} 1 & \text{if } X_i \geq c \\ 0 & \text{if } X_i < c \end{cases}\right] \text{sharp}$$

Assumption: $E[Y_i(0) \mid X = x]$ and $E[Y_i(1) \mid X = x]$ are continuous



Under Assumption...

$$\alpha_{SRD} = \lim_{x \downarrow c} E[Y_i \mid X = x] - \lim_{x \uparrow c} E[Y_i \mid X_i = x]$$

$$= \lim_{x \downarrow c} E[Y_i(1) \mid X_i = x] - \lim_{x \uparrow c} E[Y_i(0) \mid X_i = x] \quad \text{continuity}$$

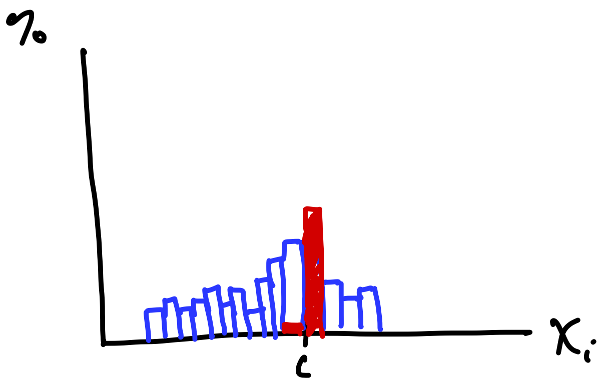
$$= E[Y_i(1) \mid X_i = c] - E[Y_i(0) \mid X_i = c]$$

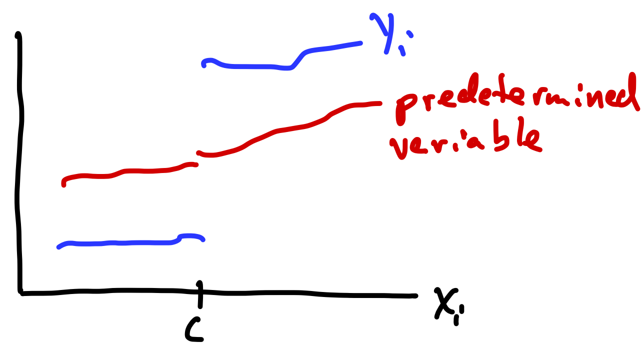
$$= \underbrace{E[\underbrace{Y_i(1) - Y_i(0)}_{\alpha_i} \mid X_i = c]}_{\text{avg effect of } T_i \text{ among } i \text{ with } X_i = c}$$

Alternative assumption: local random assignment
imperfect control

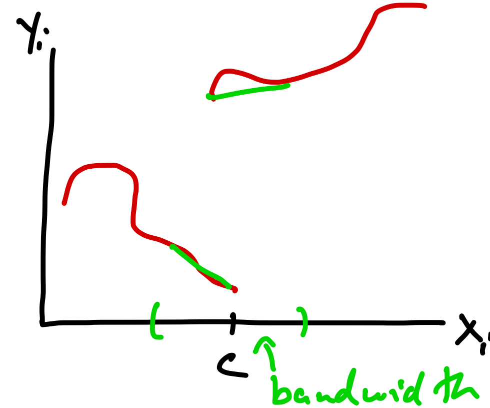
Checks on RD designs

1. Balance check: predetermined variables similar above/below $c$
2. Density continuity: histogram of $X_i$



# Two methods for sharp RD designs




$$Y_i = \alpha T_i + f(X_i) + U_i$$

$1[X_i \geq c]$

① Global polynomial

$$Y_i = \alpha T_i + \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \beta_3 X_i^3 + \beta_4 X_i T_i + \beta_5 X_i^2 T_i + \beta_6 X_i^3 T_i + U_i$$

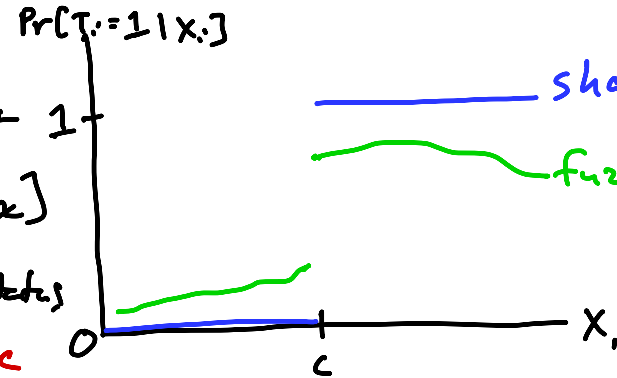
② Local linear

$$Y_i = \alpha T_i + \beta_0 + \beta_1 X_i + \beta_1 X_i T_i + U_i$$

for $X_i \in [c-h, c+h]$

bandwidth

# Fuzzy RD designs




→ discontinuity in probability of treatment

$$\lim_{x\downarrow c} Pr[T_i=1|X_i=x] > \lim_{x\uparrow c} Pr[T_i=1|X_i=x]$$

→ monotonicity: for potential treatment status

$T_i(x)$ is non-decreasing in $x$ at $x=c$

→ compliers: $\lim_{x\downarrow c} T_i(x)=1$ and $\lim_{x\uparrow c} T_i(x)=0$

→ Use IV to estimate LATE at $x=c$

→ RF/1$^{st}$: $\alpha_{FRD} = \dfrac{\lim_{x\downarrow c} E[Y_i|X_i=x] - \lim_{x\uparrow c} E[Y_i|X_i=x]}{\lim_{x\downarrow c} Pr[T_i=1|X_i=x] - \lim_{x\uparrow c} Pr[T_i=1|X_i=x]}$ ← RF ← 1$^{st}$ stage

→ TSLS: 1$^{st}$: $T_i = \pi_1 1[X_i \geq c] + g(X_i) + v_i$

2$^{nd}$: $Y_i = \alpha \hat{T}_i + f(X_i) + v_i$

$g(X_i)$ and $f(X_i)$ are flexible functions of $X_i$. Use same method!