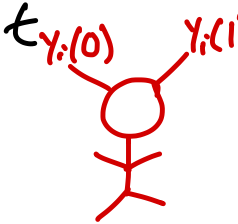
Lecture Note 9: Causality

- Potential outcomes framework (Rubin causal model)
- $Y_i(t)$ - potential outcome at treatment level t

  → we are interested in $t = 0, 1$



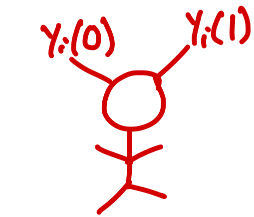
- only observe Y_i, T_i

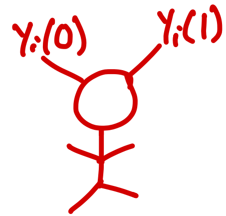
  Y_i: realized outcome

  T_i: treatment status

- so: $Y_i = Y_i(T_i) = T_i Y_i(1) + (1 - T_i) Y_i(0)$

- heterogeneous treatment effect: $\alpha_i = Y_i(1) - Y_i(0)$


$Y_i(0)$
$Y_i(1)$


$Y_i(0)$
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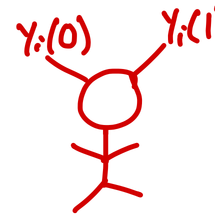
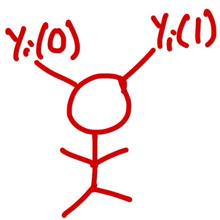
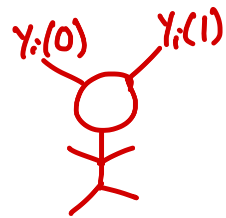
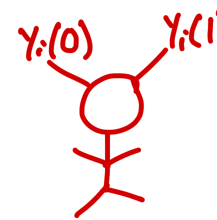

$Y_i(0)$
$Y_i(1)$


$Y_i(0)$
$Y_i(1)$


$Y_i(0)$
$Y_i(1)$


$Y_i(0)$
$Y_i(1)$

Estimands of interest

1. Average treatment effect: $ATE = E[Y_i(1) - Y_i(0)] = E[\alpha_i]$

2. Treatment on the treated: $TOT = ATT = E[Y_i(1) - Y_i(0) \mid T_i = 1]$

$$= E[\alpha_i \mid T_i = 1]$$

compare with

$E[\alpha_i \mid T_i = 0]$

not bias!

heterogeneity.

Selection bias

$$E[Y_i \mid T_i=1] - E[Y_i \mid T_i=0]$$

$$= E[Y_i(1) \mid T_i=1] - E[Y_i(0) \mid T_i=0] + E[Y_i(0) \mid T_i=1] - E[Y_i(0) \mid T_i=1]$$

$$= \underbrace{E[Y_i(1) \mid T_i=1] - E[Y_i(0) \mid T_i=1]} + E[Y_i(0) \mid T_i=1] - E[Y_i(0) \mid T_i=0]$$

$$= \underbrace{E[\underbrace{Y_i(1)-Y_i(0)}_{\alpha_i} \mid T_i=1]}_{\text{TOT / ATT}} + \underbrace{E[Y_i(0) \mid T_i=1] - E[Y_i(0) \mid T_i=0]}_{\text{selection bias}}$$

if T_i randomly assigned, then...

1. no selection bias
2. TOT = ATE

- For causal interpretation, need $E[Y_i(0) \mid T_i = 1] = E[Y_i(0) \mid T_i = 0]$

- Unconfoundedness:

$$\{Y_i(1), Y_i(0)\} \perp T_i$$

balance check

- Selection on observables (conditional independence)

$$\{Y_i(1), Y_i(0)\} \perp T_i \mid X_i$$

Randomized experiments

① Direct randomization: researcher directly randomizes T_i

② Eligibility randomization (non-compliance): researcher randomizes eligibility Z_i, not treatment T_i

$$ITT = E[Y_i \mid Z_i = 1] - E[Y_i \mid Z_i = 0]$$

intent to treat

can we still learn about effect of T_i?

To learn about effect of T_i, need 3 assumptions:

① Z_i randomly assigned (independence)

② Z_i only affects Y_i through T_i (exclusion restriction)

$Z_i \to T_i \to Y_i$

↳ potential outcomes: $Y_i(z,t) = Y_i(t)$

③ No treatment for ineligibles: $Pr[T_i = 1 \mid Z_i = 0] = 0$

$Z_i = 1$

	$T_i = 0$	$T_i = 1$
$Z_i = 0 \to T_i = 0$	never takers	compliers

compliance rate $= Pr[T_i = 1 \mid Z_i = 1]$

↳ by ass. 1, can estimate compliance rate with the share of eligibles who take up T_i

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ITT = (avg effect of Z_i among compliers)(compliance rate)

~~+ (avg effect of Z_i among never takers)(1 − compliance rate)~~ ass. 2

ITT = TOT × compliance rate

↳ $$TOT = \frac{ITT}{\text{compliance rate}}$$ ← share $T_i = 1$ among $Z_i = 1$

offered job training → 50% participated → 40% found jobs

not → 0% participated → 20% found jobs

ITT = 20 % points

compliance = 50%

$$TOT = \frac{ITT}{\text{compliance}} = \frac{20\%\text{ pts}}{.5}$$

$$= 40\%\text{ pts.}$$