Simulation of Traffic Flow

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Abstract
We have built a simplistic Monte Carlo simulation of vehicles travelling a closed loop of road, in order to understand how various parameters impact the flow of traffic. We find that traffic can be described by two states, one of free flow and one in which jams spontaneously occur and propagate.

1. Introduction

We wrote a simple model to simulate the flow of vehicle traffic, using the Monte Carlo method of simulating the randomised motion of a large number of particles. This is done by setting the behaviour of the motion of individual vehicles and then observing their interactions for a period of time and noting any recurrent phenomena. Despite its simplicity, our model displays characteristics observed in reality. As such, it allows us to explore how the traffic flow (i.e. the number of cars passing a point per unit time) is impacted by the variation of several parameters, namely the car density, the maximum allowed speed, and the probability of random deceleration. Thus we can make suggestions on how to improve real-world traffic management.
Section 2 discusses the implementation of the model and section 3 expands on error reduction. The results are presented and discussed in section 4.

2. Method

The basic premise of our model is a one-dimensional cellular automation; the road is divided into cells that each has a state dependent on whether or not they are occupied by a vehicle, and the velocity of said vehicle. Time is also discretised into steps. In each time step, the model is updating according to specified rules. For simplicity our boundary conditions are such that the model simulates a closed loop of road, meaning that the density of cars will remain constant. Traffic flow can also be modelled using fluid dynamics[1], but this requires far greater computational time.

To begin the simulation we must first populate the model with a random spread of cars, with random velocities. The program selects a random cell, and if that cell is unoccupied its state is updated with a randomly chosen velocity v between zero and the maximum allowed velocity V_{max} (velocity is also discretised into integer steps). This is repeated until n cells are occupied, with n being the specified number of cars. The rules are then applied at each time step. They are as follows:

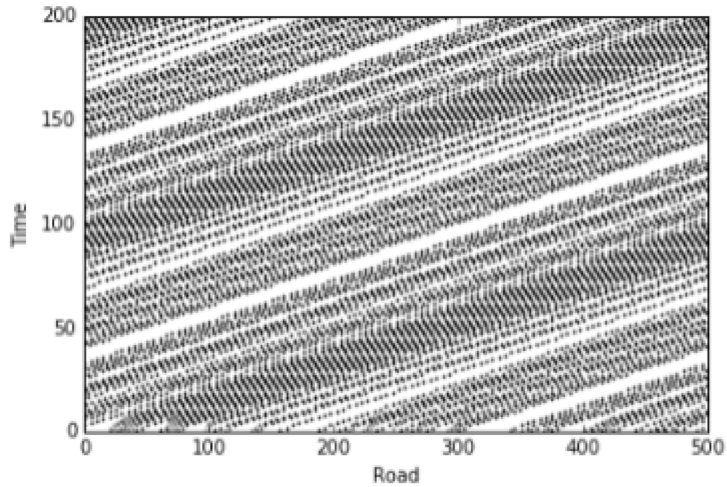

FIG. 1: A space-time diagram for $p = 0$, $T = 200$, $\rho = 0.25$. The vehicles (represented by black dots) are travelling to the right.

1.) For each car, if $v < V_{max}$, v is increased to $v + 1$.

This imitates driver behaviour of accelerating to the maximum allowed velocity.

2.) If a car is in cell i and the car in front is in cell $i + d$ then if $v \geq d$, v is set equal to $d - 1$. This overrules the tendency to accelerate.

This imitates drivers slowing down to a the maximum allowed velocity that doesn't result in them colliding with the car in front.

3.) Unexpected events cause slowing down. This is a random component that is intended to model external influences, like what is happening on the other carriageway, or just irrational behaviour by drivers. If $v > 0$ then, with a probability p, v is reduced to $v - 1$. This happens after v has been updated by the first two rules.

This is a significant rule. If there was no randomisation, the model would be completely deterministic. If $p = 0$, we find that the model quickly reaches the same steady configuration every time (figure 1), very dissimilar to real situations.

4.) Each vehicle is moved forward v places, using the new value of v.

After the other rules have been applied, this updates the model with the new car positions.

Employing just these four rules may seem too simplistic to yield results with any connection to real traffic, but this model does provide some powerful results.

3. Model Accuracy

In order to measure the flow of traffic, 'checkpoints' were used that count the number of cars passing a certain point in order to reduce the number of times the simulation has to be run (to get reliable results), and thus significantly reduce computational time. Four checkpoints were used on the road, set equal distances apart.

To obtain an accurate measurement of flow the simulation must be run over a long time (many iterations), to account for times of both free-flowing traffic and heavy jams. A graph of global flow against the number of iterations was created (figure 2) to better judge the optimum T value. Large errors in flow can be seen for $T < 5000$. It was judged that $T = 8000$ would be an appropriate value to use, with respect to the balance between accuracy and computational time- higher T values would have provided little gain in accuracy, but a large increase in computational time (generating a flow vs density graph with $T = 8000$ took 28 minutes, compared to 35 minutes with $T = 10000$).

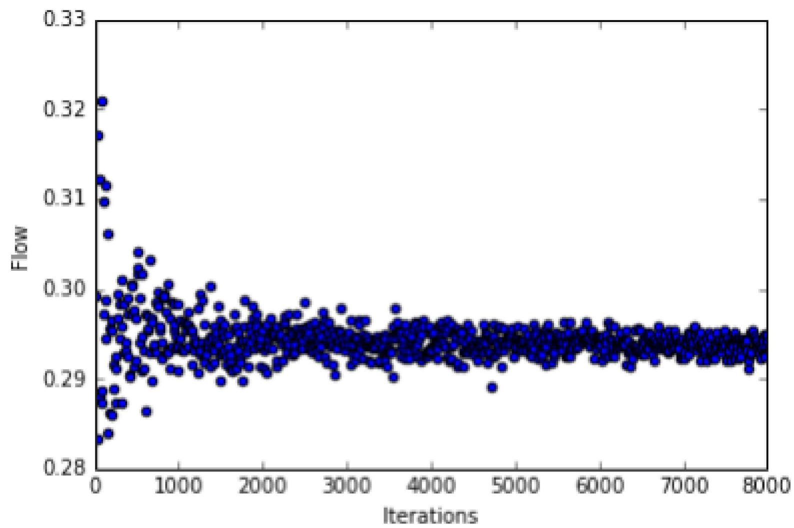

FIG. 2: Global flow vs. the number of iterations, with $p = 0.5$, $V_{max} = 5$, $n = 100$, $f = 0.3$ and $\delta n = 3$.

Measurements were only taken after a certain number of iterations, once a statistically steady state is achieved. This is done to avoid the initial configuration of the traffic influencing the results (as can be seen in the curving at the very beginning of figure 1). To find the fraction of T only after which recording data should begin, several simulations for different values were performed (as shown in figure 7 of appendix B). It was determined that a fraction of $f = 0.3$ would be appropriate.

Measurements were also averaged over five runs for each density to further reduce the effect of this bias.

Several simulations (figure 8 of appendix B) were also run to determine the appropriate step in density that should be employed. $\delta n = 3$ ($\delta \rho = 0.006$) was deemed sufficient.

When deciding on these optimum parameters to take in order to minimise errors, judgement by eye using the

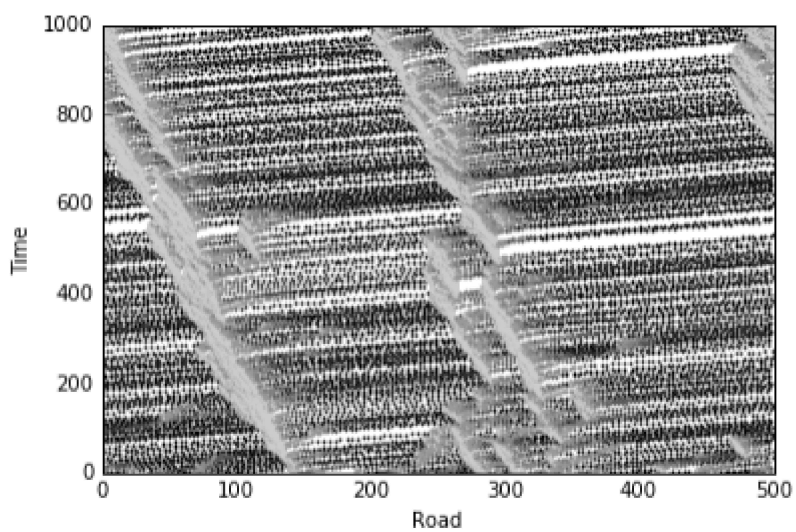

FIG 3. A space-time diagram with $p = 0.75$, $T = 1000$, $\rho = 0.25$.

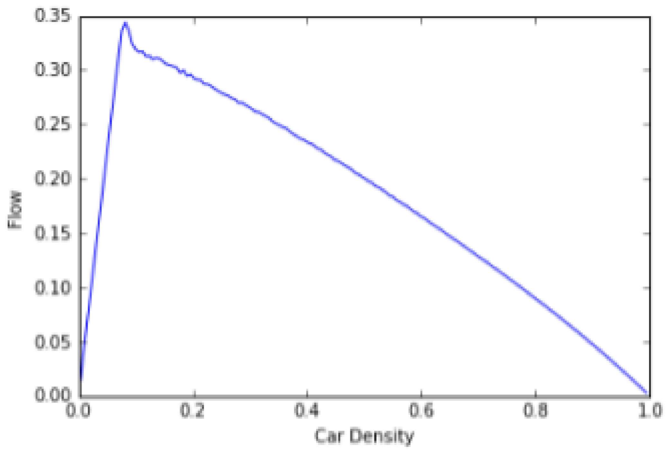

FIG. 4: The flow vs. density graph for $p = 0.5$, $T = 8000$, $V_{max} = 5$, $f = 0.3$, $\delta n = 3$.

generated graphs was sufficient, given that we are interested in finding general trends rather than accurate values, and were somewhat limited by computational time. Still, our results were consistent with far more accurate models[2].

4. Results & Discussion

Figure 3 is a space-time diagram where each black dot represents one vehicle (with faster vehicles a darker shade, slower vehicles lighter, and white spaces an empty cell), thus showing how the density distribution evolves with time. The lighter 'ridges' that are visible represent the densest, slowest regions; the spontaneously occurring traffic jams when $p > 0$. The jams move opposite to vehicle direction, a consequence of the second rule as a vehicle's velocity is affected only by the position of the vehicle immediately ahead. Thus causality travels backwards, a phenomenon seen in real traffic.

The flow vs. density graph is displayed in figure 4. It initially displays a steep, linear relationship before peaking, after which there is a more gentle decline until the traffic stops completely at full capacity.

Observed is the existence of two distinct phases. The initial climb shows a state of free-flowing traffic in which no vehicles velocity is impeded by its proximity to another, result in a linear relation between the density (i.e. the number of cars on the road) and the flow (cars passing a certain point). At the peak a critical density is reached and the flow transitions into start-stop traffic where the average velocity is lower than V_{max}, after which increasing the density adds to the jam, reducing the global flow.

Figure 5 displays several flow-density diagrams for various maximum velocity values. A special case is observed when $V_{max} = 1$ in which the plot is symmetric about $\rho = 0.5$. This can be considered analogous to particle-hole symmetry in which occupied cells (vehicles i.e. particles) and empty cells (holes) are equal and opposite, giving rise to the symmetric plot as the empty cells behave as physical vehicles. For $V_{max} > 1$, this symmetry is broken.

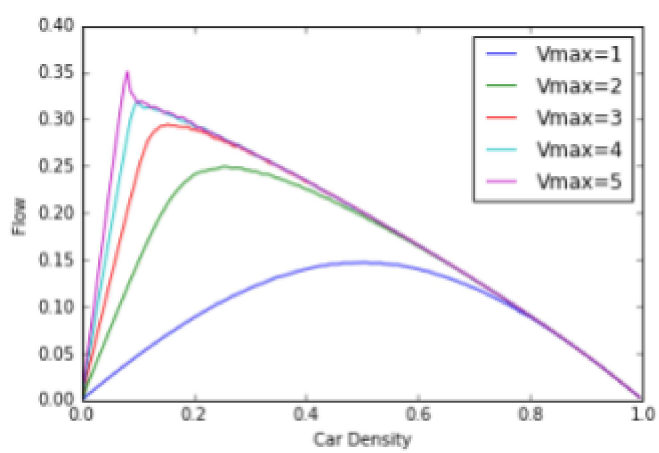

FIG. 5: Flow vs. density plots for a range of V_{max} values when $p = 0.5$.

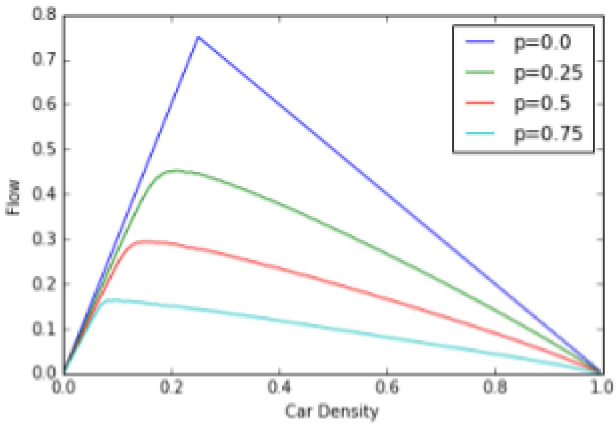

FIG. 6: Flow vs. density plots for a range of p values when $V_{max} = 3$.

As the minimum distance between vehicles before which they begin to break scales with V_{max}, we observe that the maximum flow is reached at lower number densities as V_{max} increases. As would be expected the maximum flow increases with V_{max}, as this is still determined by the free flow state (where vehicles are travelling at V_{max}). It is also clear that the peak tends to a sharp point with increasing V_{max}, due to the smaller range in velocities.

Flow-density diagram variation was also explored for differing values of p (figure 6). We observe that increasing p causes the peak flow and density to decrease. The lower peak flow is logically expected given that average velocity will be lower, and the decrease in peak density is a consequence of the larger fluctuations in velocities and distances between vehicles, causing the free-flowing state to collapse at lower densities. Interestingly, the initial positive gradient is unaffected by the variation of p, implying p is not a parameter of the free flowing state unlike the jammed state.
As V_{max} is increased the improvement in flow for any given density becomes smaller. Improvements in p give much higher gains. To give an example, for a density of ~0.4 (corresponding to a vehicle roughly every 19 metres in reality, assuming each cell is 7.5 metres long) $V_{max} = 5$ (with $p = 0.5$) results in a flow of ~0.25 (corresponding to a journey time of ~800 seconds), whilst $p = 0.25$ (with $V_{max} = 3$) gives a flow of ~0.4 (journey time of ~500 seconds). For any given density in the jammed state, reducing p will help alleviate congestion, but this is not necessarily true for increasing V_{max}. This suggests that reducing diversions that cause breaking is more effective at improving the traffic flow than increasing the speed limit.

5. Conclusions

We have shown that our simple model of single lane traffic predicts the free-flowing state and jammed state of traffic, including the critical density at which traffic flow is maximum and transitions between states.
When no range in velocities is permitted ($V_{max} = 1$) the model displays particle-hole symmetry. It was also observed that increasing the maximum velocity resulted in a sharpening of the peak flow and a decrease in the critical density. Increasing the deceleration probability resulted in a decrease of the critical density. These results suggest that traffic management should aim to achieve this critical density to minimise the formation of jams and increase efficiency. Our results also suggest the removal of factors that encourage random breaking should be prioritised, rather than increasing speed limits to help traffic flow.

It would be of interest to extend this model to multiple-lane systems or to include more complex features such as junctions and on/off-ramps in order to explore subsequent phenomena that emerge for models more closely resembling reality.

References

1. M. Treiber, A. Hennecke, and D. Helbing, Phys. Rev. E 59, 239 ~1999
2. K Nagel and M Schreckenberg, Journal de Physique I, vol 2, pages 2221-2229, Dec 1992.

Appendix A

Computer Specifications:
CPU: Intel Core i5-4460 @3.20GHz
GPU: AMD Radeon R9 380
System Memory: 16 GB (GDDR5)
Operating System: Windows 10 (64 bit)

Appendix B

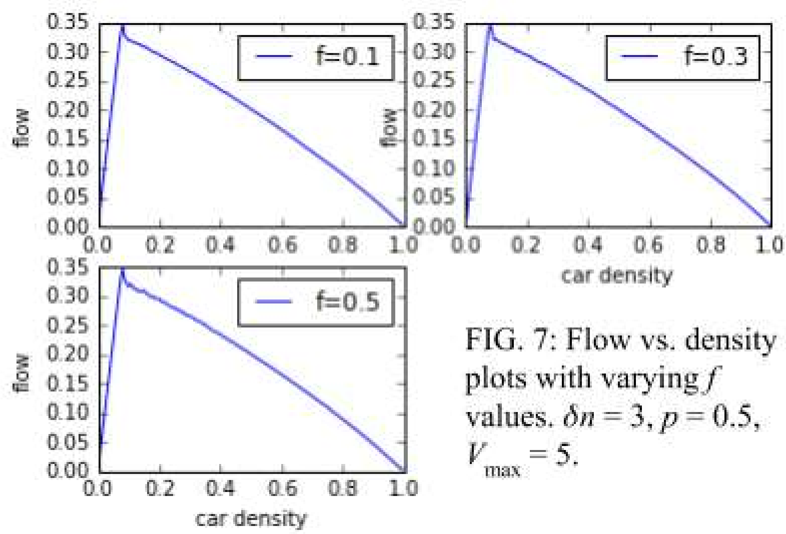

FIG. 7: Flow vs. density plots with varying f values. $\delta n = 3$, $p = 0.5$, $V_{max} = 5$.

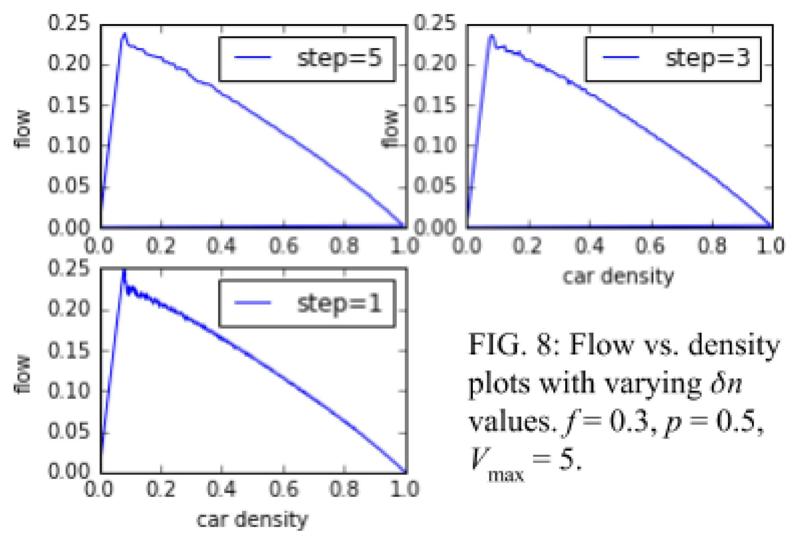

FIG. 8: Flow vs. density plots with varying δn values. $f = 0.3$, $p = 0.5$, $V_{max} = 5$.