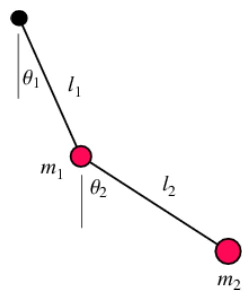

DOUBLE PENDULUM

Exemplifies chaos theory: a small change in the initial conditions of the system creates a large change in the future

Assumptions: rigid, massless rods connecting the masses; masses are point masses (1D), no energy losses due to resistive forces

Numerical solver: ODE45, RK78

Polar to cartesian coordinates of position and component velocity

$x_1 = L_1 * \sin(\theta_1)$

$x_2 = L_1 * \sin(\theta_1) + L_2 * \sin(\theta_2)$

$y_1 = -L_1 * \cos(\theta_1)$

$y_2 = -L_1 * \cos(\theta_1) - L_2 * \cos(\theta_2)$

$\dot{x_1} = L_1 * \cos(\theta_1) * \dot{\theta}_1$

$\dot{x_2} = L_2 * \cos(\theta_1) * \dot{\theta}_1 + L_2 * \cos(\theta_2) * \dot{\theta}_2$

$\dot{y_1} = -L_1 * \sin(\theta_1) * \dot{\theta}_1$

$\dot{y_2} = -L_1 * \sin(\theta_1) * \dot{\theta}_1 - L_2 * \sin(\theta_2) * \dot{\theta}_2$

Potential energy of system

$P = m * g * h$

$P = m_1 * g * y_1 + m_2 * g * y_2$

$P = m_1 * g * [-L_1 * \cos(\theta_1)] + m_2 * g * [-L_1 * \cos(\theta_1) - L_2 * \cos(\theta_2)]$

$P = -m_1 * g * L_1 * \cos(\theta_1) - m_2 * g * [L_1 * \cos(\theta_1) + L_2 * \cos(\theta_2)]$

$P = -m_1 * g * L_1 * \cos(\theta_1) - m_2 * g * L_1 * \cos(\theta_1) - m_2 * g * L_2 * \cos(\theta_2)$

Final potential energy expression

$P = -(m_1 + m_2) * g * L_1 * \cos(\theta_1) - m_2 * g * L_2 * \cos(\theta_2)$

Kinetic energy of system

$K = \frac{1}{2} * m * v^2 = \frac{1}{2} * m * (\dot{x}^2 + \dot{y}^2)$

$K = \frac{1}{2} * m_1 * \left(\dot{x}_1^2 + \dot{y}_1^2\right) + \frac{1}{2} * m_2 * \left(\dot{x}_2^2 + \dot{y}_2^2\right)$

$K = \frac{1}{2} * m_1 * \left[\left(L_1 * \cos(\theta_1) * \dot{\theta}_1\right)^2 + \left(-L_1 * \sin(\theta_1) * \dot{\theta}_1\right)^2\right] + \frac{1}{2} * m_2 * \left[\left(L_1 * \cos(\theta_1) * \dot{\theta}_1 + L_2 * \cos(\theta_2) * \dot{\theta}_2\right)^2 + \left(-L_1 * \sin(\theta_1) * \dot{\theta}_1 - L_2 * \sin(\theta_2) * \dot{\theta}_2\right)^2\right]$

Expansion of squared terms in kinetic energy

FOR $\left(\dot{x}_1^2 + \dot{y}_1^2\right) = \left[\left(L_1 * \cos(\theta_1) * \dot{\theta}_1\right)^2 + \left(-L_1 * \sin(\theta_1) * \dot{\theta}_1\ \right)^2\right]$

$\left[\dot{x}_1^2, \dot{y}_1^2\right]^2 = [L_1^2 * \cos^2(\theta_1) * \dot{\theta}_1^2\ \ ,\ \ L_1^2 * \sin^2(\theta_1) * \dot{\theta}_1^2]$

FOR $\dot{x}_2^2 = \left(L_1 * \cos(\theta_1) * \dot{\theta}_1 + L_2 * \cos(\theta_2) * \dot{\theta}_2\right)^2$

$= L_1^2 * \cos^2(\theta_1) * \dot{\theta}_1^2 + L_2^2 * \cos^2(\theta_2) * \dot{\theta}_2^2 + 2 * L_1 * L_2 * \dot{\theta}_1 * \dot{\theta}_2 * \cos(\theta_1) * \cos(\theta_2)$

FOR $\dot{y}_2^2 = \left(-L_1 * \sin(\theta_1) * \dot{\theta}_1 - L_2 * \sin(\theta_2) * \dot{\theta}_2\right)^2$

$= L_1^2 * \sin^2(\theta_1) * \dot{\theta}_1^2 + L_2^2 * \sin^2(\theta_2) * \dot{\theta}_2^2 + 2 * L_1 * L_2 * \dot{\theta}_1 * \dot{\theta}_2 * \sin(\theta_1) * \sin(\theta_2)$

$K = \frac{1}{2} * m_1 * \left[L_1^2 * \cos^2(\theta_1) * \dot{\theta}_1^2 + L_1^2 * \sin^2(\theta_1) * \dot{\theta}_1^2\right] +$

$\frac{1}{2} * m_2 * \left[L_1^2 * \cos^2(\theta_1) * \dot{\theta}_1^2 + L_2^2 * \cos^2(\theta_2) * \dot{\theta}_2^2 + 2 * L_1 * L_2 * \dot{\theta}_1 * \dot{\theta}_2 * \cos(\theta_1) * \cos(\theta_2)\right] +$

$\frac{1}{2} * m_2 * \left[L_1^2 * \sin^2(\theta_1) * \dot{\theta}_1^2 + L_2^2 * \sin^2(\theta_2) * \dot{\theta}_2^2 + 2 * L_1 * L_2 * \dot{\theta}_1 * \dot{\theta}_2 * \sin(\theta_1) * \sin(\theta_2)\right]$

$K = \frac{1}{2} * m_1 * \left[L_1^2 * \dot{\theta}_1^2 * [\sin^2(\theta_1) + \cos^2(\theta_1)\right] +$

$\frac{1}{2} * m_2 * \left[L_1^2 * \dot{\theta}_1^2 * [\sin^2(\theta_1) + \cos^2(\theta_1)] * + L_2^2 * \dot{\theta}_2^2 * [\sin^2(\theta_2) + \cos^2(\theta_2)]\right]$

$\frac{1}{2} * m_2 * \left[2 * L_1 * L_2 * \dot{\theta}_1 * \dot{\theta}_2 * [\sin(\theta_1) * \sin(\theta_2) + \cos(\theta_1) * \cos(\theta_2)]\right]$

$K = \frac{1}{2} * m_1 * L_1^2 * \dot{\theta}_1^2 + \frac{1}{2} * m_2 * L_1^2 * \dot{\theta}_1^2 + \frac{1}{2} * m_2 * L_2^2 * \dot{\theta}_2^2 + \frac{1}{2} * m_2 * 2 * L_1 * L_2 * \dot{\theta}_1 * \dot{\theta}_2 * \cos(\theta_1 - \theta_2)$

Final kinetic energy expression

$K = \frac{1}{2} * m_1 * L_1^2 * \dot{\theta}_1^2 + \frac{1}{2} * m_2 * \left[L_1^2 * \dot{\theta}_1^2 + L_2^2 * \dot{\theta}_2^2 + 2 * L_1 * L_2 * \dot{\theta}_1 * \dot{\theta}_2 * \cos(\theta_1 - \theta_2)\ \right]$

FOR THETA 1

Definition of Lagrangian /// Lagrange's general equation of motion

$L = K - P \quad /// \quad \frac{d}{dt}\left(\frac{dL}{d\dot{\theta}}\right) - \frac{dL}{d\theta} = 0$

$K = \frac{1}{2} * m_1 * L_1^2 * \dot{\theta}_1^2 + \frac{1}{2} * m_2 * \left[L_1^2 * \dot{\theta}_1^2 + L_2^2 * \dot{\theta}_2^2 + 2 * L_1 * L_2 * \dot{\theta}_1 * \dot{\theta}_2 * \cos(\theta_1 - \theta_2)\right]$

$P = -(m_1 + m_2) * g * L_1 * \cos(\theta_1) - m_2 * g * L_2 * \cos(\theta_2)$

Lagrangian

$L = \frac{1}{2} * m_1 * L_1^2 * \dot{\theta}_1^2 + \frac{1}{2} * m_2 * \left[L_1^2 * \dot{\theta}_1^2 + L_2^2 * \dot{\theta}_2^2 + 2 * L_1 * L_2 * \dot{\theta}_1 * \dot{\theta}_2 * \cos(\theta_1 - \theta_2)\right] + (m_1 + m_2) * g * L_1 * \cos(\theta_1) + m_2 * g * L_2 * \cos(\theta_2)$

$\frac{dL}{d\theta_1} = -(m_1 + m_2) * g * L_1 * \sin(\theta_1) - m_2 * L_1 * L_2 * \dot{\theta}_1 * \dot{\theta}_2 * \sin(\theta_1 - \theta_2)$

$\frac{dL}{d\dot{\theta}_1} = m_1 * L_1^2 * \dot{\theta}_1 + m_2 * L_1^2 * \dot{\theta}_1 + m_2 * L_1 * L_2 * \dot{\theta}_2 * \cos(\theta_1 - \theta_2)$

$\frac{dL}{d\dot{\theta}_1} = (m_1 + m_2) * L_1^2 * \dot{\theta}_1 + m_2 * L_1 * L_2 * \dot{\theta}_2 * \cos(\theta_1 - \theta_2)$

$\frac{d}{dt}\left(\frac{dL}{d\dot{\theta}_1}\right) = (m_1 + m_2) * L_1^2 * \ddot{\theta}_1 + m_2 * L_1 * L_2 * \left[-\dot{\theta}_2 * \sin(\theta_1 - \theta_2) * \left(\dot{\theta}_1 - \dot{\theta}_2\right) + \ddot{\theta}_2 * \cos(\theta_1 - \theta_2)\right]$

$\frac{d}{dt}\left(\frac{dL}{d\dot{\theta}_1}\right) = (m_1 + m_2) * L_1^2 * \ddot{\theta}_1 + m_2 * L_1 * L_2 * \ddot{\theta}_2 * \cos(\theta_1 - \theta_2) - m_2 * L_1 * L_2 * \dot{\theta}_2 * \sin(\theta_1 - \theta_2) * \left(\dot{\theta}_1 - \dot{\theta}_2\right)$

$\frac{d}{dt}\left(\frac{dL}{d\dot{\theta}_1}\right) - \frac{dL}{d\theta_1} = 0$

Preliminary (non-simplified) expression for angular acceleration of theta 1

$$(m_1 + m_2) * L_1^2 * \ddot{\theta}_1 + m_2 * L_1 * L_2 * \ddot{\theta}_2 * \cos(\theta_1 - \theta_2) - m_2 * L_1 * L_2 * \dot{\theta}_2 * \sin(\theta_1 - \theta_2) * \left(\dot{\theta}_1 - \dot{\theta}_2\right) - \left[-(m_1 + m_2) * g * L_1 * \sin(\theta_1) - m_2 * L_1 * L_2 * \dot{\theta}_1 * \dot{\theta}_2 * \sin(\theta_1 - \theta_2)\right] = 0$$

$$(m_1 + m_2) * L_1^2 * \ddot{\theta}_1 + m_2 * L_1 * L_2 * \ddot{\theta}_2 * \cos(\theta_1 - \theta_2) + (m_1 + m_2) * g * L_1 * \sin(\theta_1) + \left\{\left[-m_2 * L_1 * L_2 * \dot{\theta}_2 * \sin(\theta_1 - \theta_2) * \left(\dot{\theta}_1 - \dot{\theta}_2\right) + m_2 * L_1 * L_2 * \dot{\theta}_1 * \dot{\theta}_2 * \sin(\theta_1 - \theta_2)\right]\right\} = 0$$

$\left\{\left[m_2 * L_1 * L_2 * \dot{\theta}_2 * \sin(\theta_1 - \theta_2) * \left[-\left(\dot{\theta}_1 - \dot{\theta}_2\right) + \dot{\theta}_1\right]\right]\right\} = \left\{m_2 * L_1 * L_2 * \dot{\theta}_2 * \sin(\theta_1 - \theta_2) * \dot{\theta}_2\right\}$

$= \left\{m_2 * L_1 * L_2 * \dot{\theta}_2^2 * \sin(\theta_1 - \theta_2)\right\}$

$$(m_1 + m_2) * L_1^2 * \ddot{\theta}_1 + m_2 * L_1 * L_2 * \ddot{\theta}_2 * \cos(\theta_1 - \theta_2) + (m_1 + m_2) * g * L_1 * \sin(\theta_1) + m_2 * L_1 * L_2 * \dot{\theta}_2^2 * \sin(\theta_1 - \theta_2) = 0$$

$$\ddot{\theta}_1 = \frac{-m_2 * L_1 * L_2 * \ddot{\theta}_2 * \cos(\theta_1 - \theta_2) - (m_1 + m_2) * g * L_1 * \sin(\theta_1) - m_2 * L_1 * L_2 * \dot{\theta}_2^2 * \sin(\theta_1 - \theta_2)}{(m_1 + m_2) * L_1^2}$$

Simplified, and solved for $\ddot{\theta}_1$

$$\ddot{\theta}_1 = \frac{-m_2 * L_2 * \ddot{\theta}_2 * \cos(\theta_1 - \theta_2) - (m_1 + m_2) * g * \sin(\theta_1) - m_2 * L_2 * \dot{\theta}_2^2 * \sin(\theta_1 - \theta_2)}{(m_1 + m_2) * L_1}$$

FOR MASS 2

$L = K - P \quad /// \quad \frac{d}{dt}\left(\frac{dL}{d\dot{\theta}}\right) - \frac{dL}{d\theta} = 0$

$K = \frac{1}{2} * m_1 * L_1^2 * \dot{\theta}_1^2 + \frac{1}{2} * m_2 * \left[L_1^2 * \dot{\theta}_1^2 + L_2^2 * \dot{\theta}_2^2 + 2 * L_1 * L_2 * \dot{\theta}_1 * \dot{\theta}_2 * \cos(\theta_1 - \theta_2)\right]$

$P = -(m_1 + m_2) * g * L_1 * \cos(\theta_1) - m_2 * g * L_2 * \cos(\theta_2)$

Same Lagrangian as above

$L = \frac{1}{2} * m_1 * L_1^2 * \dot{\theta}_1^2 + \frac{1}{2} * m_2 * \left[L_1^2 * \dot{\theta}_1^2 + L_2^2 * \dot{\theta}_2^2 + 2 * L_1 * L_2 * \dot{\theta}_1 * \dot{\theta}_2 * \cos(\theta_1 - \theta_2)\right] + (m_1 + m_2) * g * L_1 * \cos(\theta_1) + m_2 * g * L_2 * \cos(\theta_2)$

$\frac{dL}{d\theta_2} = -m_2 * L_1 * L_2 * \dot{\theta}_1 * \dot{\theta}_2 * \sin(\theta_1 - \theta_2) - m_2 * g * L_2 * \sin(\theta_2)$

$\frac{dL}{d\dot{\theta}_2} = m_2 * L_2^2 * \dot{\theta}_2 + m_2 * L_1 * L_2 * \dot{\theta}_1 * \cos(\theta_1 - \theta_2)$

$\frac{d}{dt}\left(\frac{dL}{d\dot{\theta}_2}\right) = m_2 * L_2^2 * \ddot{\theta}_2 + m_2 * L_1 * L_2 * \left[-\dot{\theta}_1 * \sin(\theta_1 - \theta_2) * \left(\dot{\theta}_1 - \dot{\theta}_2\right) + \cos(\theta_1 - \theta_2) * \ddot{\theta}_1\right]$

$\frac{d}{dt}\left(\frac{dL}{d\dot{\theta}_2}\right) = m_2 * L_2^2 * \ddot{\theta}_2 - m_2 * L_1 * L_2 * \dot{\theta}_1 * \sin(\theta_1 - \theta_2) * \left(\dot{\theta}_1 - \dot{\theta}_2\right) + m_2 * L_1 * L_2 * \ddot{\theta}_1 * \cos(\theta_1 - \theta_2)$

$\frac{d}{dt}\left(\frac{dL}{d\dot{\theta}_2}\right) - \frac{dL}{d\theta_2} = 0$

Preliminary (non-simplified) expression for angular acceleration of theta 2

$$m_2 * L_2^2 * \ddot{\theta}_2 - m_2 * L_1 * L_2 * \dot{\theta}_1 * \sin(\theta_1 - \theta_2) * \left(\dot{\theta}_1 - \dot{\theta}_2\right) + m_2 * L_1 * L_2 * \ddot{\theta}_1 * \cos(\theta_1 - \theta_2) - \left[-m_2 * L_1 * L_2 * \dot{\theta}_1 * \dot{\theta}_2 * \sin(\theta_1 - \theta_2) - m_2 * g * L_2 * \sin(\theta_2)\right] = 0$$

$$m_2 * L_2^2 * \ddot{\theta}_2 + m_2 * L_1 * L_2 * \ddot{\theta}_1 * \cos(\theta_1 - \theta_2) + m_2 * g * L_2 * \sin(\theta_2) + \left\{\left[-m_2 * L_1 * L_2 * \dot{\theta}_1 * \sin(\theta_1 - \theta_2) * \left(\dot{\theta}_1 - \dot{\theta}_2\right) + m_2 * L_1 * L_2 * \dot{\theta}_1 * \dot{\theta}_2 * \sin(\theta_1 - \theta_2)\right]\right\} = 0$$

$\left\{\left[m_2 * L_1 * L_2 * \dot{\theta}_1 * \sin(\theta_1 - \theta_2) * \left[-\left(\dot{\theta}_1 - \dot{\theta}_2\right) + \dot{\theta}_2\right]\right]\right\} = \left\{-m_2 * L_1 * L_2 * \dot{\theta}_1 * \sin(\theta_1 - \theta_2) * \dot{\theta}_1\right\}$

$= \left\{-m_2 * L_1 * L_2 * \dot{\theta}_1^2 * \sin(\theta_1 - \theta_2)\right\}$

$$m_2 * L_2^2 * \ddot{\theta}_2 + m_2 * L_1 * L_2 * \ddot{\theta}_1 * \cos(\theta_1 - \theta_2) + m_2 * g * L_2 * \sin(\theta_2) - m_2 * L_1 * L_2 * \dot{\theta}_1^2 * \sin(\theta_1 - \theta_2) = 0$$

$$\ddot{\theta}_2 = \frac{m_2 * L_1 * L_2 * \dot{\theta}_1^2 * \sin(\theta_1 - \theta_2) - m_2 * L_1 * L_2 * \cos(\theta_1 - \theta_2) * \ddot{\theta}_1 - m_2 * g * L_2 * \sin(\theta_2)}{m_2 * L_2^2}$$

Simplified, and solved for $\ddot{\theta}_2$

$$\ddot{\theta}_2 = \frac{L_1 * \dot{\theta}_1^2 * \sin(\theta_1 - \theta_2) - L_1 * \cos(\theta_1 - \theta_2) * \ddot{\theta}_1 - g * \sin(\theta_2)}{L_2}$$

Both equations of angular acceleration for angles 1 and 2 with respect to $\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2, \ddot{\theta}_1, \ddot{\theta}_2$

$$\ddot{\theta}_1 = \frac{-m_2 * L_2 * \ddot{\theta}_2 * \cos(\theta_1 - \theta_2) - (m_1 + m_2) * g * \sin(\theta_1) - m_2 * L_2 * \dot{\theta}_2^2 * \sin(\theta_1 - \theta_2)}{(m_1 + m_2) * L_1} \dots (a)$$

$$\ddot{\theta}_2 = \frac{L_1 * \dot{\theta}_1^2 * \sin(\theta_1 - \theta_2) - L_1 * \ddot{\theta}_1 * \cos(\theta_1 - \theta_2) - g * \sin(\theta_2)}{L_2} \dots (b)$$

Sub equation (b) into equation (a)

$$\ddot{\theta}_1 = \frac{-m_2 * L_2 * \left[\frac{L_1 * \dot{\theta}_1^2 * \sin(\theta_1 - \theta_2) - L_1 * \ddot{\theta}_1 * \cos(\theta_1 - \theta_2) - g * \sin(\theta_2)}{L_2}\right] * \cos(\theta_1 - \theta_2) - (m_1 + m_2) * g * \sin(\theta_1) - m_2 * L_2 * \dot{\theta}_2^2 * \sin(\theta_1 - \theta_2)}{(m_1 + m_2) * L_1}$$

$$\ddot{\theta}_1 = \frac{-m_2 * \left[L_1 * \dot{\theta}_1^2 * \sin(\theta_1 - \theta_2) - L_1 * \ddot{\theta}_1 * \cos(\theta_1 - \theta_2) - g * \sin(\theta_2)\right] * \cos(\theta_1 - \theta_2) - (m_1 + m_2) * g * \sin(\theta_1) - m_2 * L_2 * \dot{\theta}_2^2 * \sin(\theta_1 - \theta_2)}{(m_1 + m_2) * L_1}$$

$$\ddot{\theta}_1 = \frac{-m_2 * \cos(\theta_1 - \theta_2) * \left[L_1 * \dot{\theta}_1^2 * \sin(\theta_1 - \theta_2)\right] - m_2 * \cos(\theta_1 - \theta_2) * \left[-L_1 * \ddot{\theta}_1 * \cos(\theta_1 - \theta_2)\right] - m_2 * \cos(\theta_1 - \theta_2) * [-g * \sin(\theta_2)] - (m_1 + m_2) * g * \sin(\theta_1) - m_2 * L_2 * \dot{\theta}_2^2 * \sin(\theta_1 - \theta_2)}{(m_1 + m_2) * L_1}$$

$$\ddot{\theta}_1 = \frac{-m_2 * \cos(\theta_1 - \theta_2) * L_1 * \dot{\theta}_1^2 * \sin(\theta_1 - \theta_2) + m_2 * \cos(\theta_1 - \theta_2) * L_1 * \ddot{\theta}_1 * \cos(\theta_1 - \theta_2) + m_2 * \cos(\theta_1 - \theta_2) * g * \sin(\theta_2) - (m_1 + m_2) * g * \sin(\theta_1) - m_2 * L_2 * \dot{\theta}_2^2 * \sin(\theta_1 - \theta_2)}{(m_1 + m_2) * L_1}$$

$$\ddot{\theta}_1 = \frac{-m_2 * L_1 * \dot{\theta}_1^2 * \cos(\theta_1 - \theta_2) * \sin(\theta_1 - \theta_2) + m_2 * L_1 * \ddot{\theta}_1 * \cos^2(\theta_1 - \theta_2) + m_2 * g * \cos(\theta_1 - \theta_2) * \sin(\theta_2) - (m_1 + m_2) * g * \sin(\theta_1) - m_2 * L_2 * \dot{\theta}_2^2 * \sin(\theta_1 - \theta_2)}{(m_1 + m_2) * L_1}$$

$$\ddot{\theta}_1 = \frac{-m_2 * L_1 * \dot{\theta}_1^2 * \cos(\theta_1 - \theta_2) * \sin(\theta_1 - \theta_2) + m_2 * g * \cos(\theta_1 - \theta_2) * \sin(\theta_2) - (m_1 + m_2) * g * \sin(\theta_1) - m_2 * L_2 * \dot{\theta}_2^2 * \sin(\theta_1 - \theta_2)}{(m_1 + m_2) * L_1} + \frac{m_2 * L_1 * \ddot{\theta}_1 * \cos^2(\theta_1 - \theta_2)}{(m_1 + m_2) * L_1}$$

$$\ddot{\theta}_1 - \frac{m_2 * L_1 * \ddot{\theta}_1 * \cos^2(\theta_1 - \theta_2)}{(m_1 + m_2) * L_1} = \frac{-m_2 * L_1 * \dot{\theta}_1^2 * \cos(\theta_1 - \theta_2) * \sin(\theta_1 - \theta_2) + m_2 * g * \cos(\theta_1 - \theta_2) * \sin(\theta_2) - (m_1 + m_2) * g * \sin(\theta_1) - m_2 * L_2 * \dot{\theta}_2^2 * \sin(\theta_1 - \theta_2)}{(m_1 + m_2) * L_1}$$

$$\ddot{\theta}_1 * \left(1 - \frac{m_2 * L_1 * \cos^2(\theta_1 - \theta_2)}{(m_1 + m_2) * L_1}\right) = \frac{-m_2 * L_1 * \dot{\theta}_1^2 * \cos(\theta_1 - \theta_2) * \sin(\theta_1 - \theta_2) + m_2 * g * \cos(\theta_1 - \theta_2) * \sin(\theta_2) - (m_1 + m_2) * g * \sin(\theta_1) - m_2 * L_2 * \dot{\theta}_2^2 * \sin(\theta_1 - \theta_2)}{(m_1 + m_2) * L_1}$$

$$\ddot{\theta}_1 * \left(\frac{\left((m_1 + m_2) * L_1\right) - (m_2 * L_1 * \cos^2(\theta_1 - \theta_2))}{(m_1 + m_2) * L_1}\right) = \frac{-m_2 * L_1 * \dot{\theta}_1^2 * \cos(\theta_1 - \theta_2) * \sin(\theta_1 - \theta_2) + m_2 * g * \cos(\theta_1 - \theta_2) * \sin(\theta_2) - (m_1 + m_2) * g * \sin(\theta_1) - m_2 * L_2 * \dot{\theta}_2^2 * \sin(\theta_1 - \theta_2)}{(m_1 + m_2) * L_1}$$

$$\ddot{\theta}_1 = \frac{-m_2 * L_1 * \dot{\theta}_1^2 * \cos(\theta_1 - \theta_2) * \sin(\theta_1 - \theta_2) + m_2 * g * \cos(\theta_1 - \theta_2) * \sin(\theta_2) - (m_1 + m_2) * g * \sin(\theta_1) - m_2 * L_2 * \dot{\theta}_2^2 * \sin(\theta_1 - \theta_2)}{(m_1 + m_2) * L_1} * \left(\frac{(m_1 + m_2) * L_1}{\left((m_1 + m_2) * L_1\right) - (m_2 * L_1 * \cos^2(\theta_1 - \theta_2))}\right)$$

$$\ddot{\theta}_1 = \frac{-m_2 * L_1 * \dot{\theta}_1^2 * \cos(\theta_1 - \theta_2) * \sin(\theta_1 - \theta_2) + m_2 * g * \cos(\theta_1 - \theta_2) * \sin(\theta_2) - (m_1 + m_2) * g * \sin(\theta_1) - m_2 * L_2 * \dot{\theta}_2^2 * \sin(\theta_1 - \theta_2)}{(m_1 + m_2) * L_1 - m_2 * L_1 * \cos^2(\theta_1 - \theta_2)}$$

Sub equation (a) into equation (b)

$$\ddot{\theta_2} = \frac{L_1 * \dot{\theta_1^2} * \sin(\theta_1 - \theta_2) - L_1 * \left[\frac{-m_2 * L_2 * \ddot{\theta_2} * \cos(\theta_1 - \theta_2) - (m_1 + m_2) * g * \sin(\theta_1) - m_2 * L_2 * \dot{\theta_2^2} * \sin(\theta_1 - \theta_2)}{(m_1 + m_2) * L_1}\right] * \cos(\theta_1 - \theta_2) - g * \sin(\theta_2)}{L_2}$$

$$\ddot{\theta_2} = \frac{(m_1 + m_2) * L_1 * \dot{\theta_1^2} * \sin(\theta_1 - \theta_2) - \left[-m_2 * L_2 * \ddot{\theta_2} * \cos(\theta_1 - \theta_2) - (m_1 + m_2) * g * \sin(\theta_1) - m_2 * L_2 * \dot{\theta_2^2} * \sin(\theta_1 - \theta_2)\right] * \cos(\theta_1 - \theta_2) - (m_1 + m_2) * g * \sin(\theta_2)}{(m_1 + m_2) * L_2}$$

$$\ddot{\theta_2} = \frac{(m_1 + m_2) * L_1 * \dot{\theta_1^2} * \sin(\theta_1 - \theta_2) - \cos(\theta_1 - \theta_2) * \left[-m_2 * L_2 * \ddot{\theta_2} * \cos(\theta_1 - \theta_2)\right] - \cos(\theta_1 - \theta_2) * [-(m_1 + m_2) * g * \sin(\theta_1)] - \cos(\theta_1 - \theta_2) * \left[-m_2 * L_2 * \dot{\theta_2^2} * \sin(\theta_1 - \theta_2)\right] - (m_1 + m_2) * g * \sin(\theta_2)}{(m_1 + m_2) * L_2}$$

$$\ddot{\theta_2} = \frac{(m_1 + m_2) * L_1 * \dot{\theta_1^2} * \sin(\theta_1 - \theta_2) + \cos(\theta_1 - \theta_2) * m_2 * L_2 * \ddot{\theta_2} * \cos(\theta_1 - \theta_2) + \cos(\theta_1 - \theta_2) * (m_1 + m_2) * g * \sin(\theta_1) + \cos(\theta_1 - \theta_2) * m_2 * L_2 * \dot{\theta_2^2} * \sin(\theta_1 - \theta_2) - (m_1 + m_2) * g * \sin(\theta_2)}{(m_1 + m_2) * L_2}$$

$$\ddot{\theta_2} = \frac{(m_1 + m_2) * L_1 * \dot{\theta_1^2} * \sin(\theta_1 - \theta_2) + m_2 * L_2 * \ddot{\theta_2} * \cos^2(\theta_1 - \theta_2) + (m_1 + m_2) * g * \cos(\theta_1 - \theta_2) * \sin(\theta_1) + m_2 * L_2 * \dot{\theta_2^2} * \cos(\theta_1 - \theta_2) * \sin(\theta_1 - \theta_2) - (m_1 + m_2) * g * \sin(\theta_2)}{(m_1 + m_2) * L_2}$$

$$\ddot{\theta_2} = \frac{(m_1 + m_2) * L_1 * \dot{\theta_1^2} * \sin(\theta_1 - \theta_2) + m_2 * L_2 * \ddot{\theta_2} * \cos^2(\theta_1 - \theta_2) + (m_1 + m_2) * g * \cos(\theta_1 - \theta_2) * \sin(\theta_1) + m_2 * L_2 * \dot{\theta_2^2} * \cos(\theta_1 - \theta_2) * \sin(\theta_1 - \theta_2) - (m_1 + m_2) * g * \sin(\theta_2)}{(m_1 + m_2) * L_2} + \frac{m_2 * L_2 * \ddot{\theta_2} * \cos^2(\theta_1 - \theta_2)}{(m_1 + m_2) * L_2}$$

$$\ddot{\theta_2} - \frac{m_2 * L_2 * \ddot{\theta_2} * \cos^2(\theta_1 - \theta_2)}{(m_1 + m_2) * L_2} = \frac{(m_1 + m_2) * L_1 * \dot{\theta_1^2} * \sin(\theta_1 - \theta_2) + (m_1 + m_2) * g * \cos(\theta_1 - \theta_2) * \sin(\theta_1) + m_2 * L_2 * \dot{\theta_2^2} * \cos(\theta_1 - \theta_2) * \sin(\theta_1 - \theta_2) - (m_1 + m_2) * g * \sin(\theta_2)}{(m_1 + m_2) * L_2}$$

$$\ddot{\theta_2} * \left(1 - \frac{m_2 * L_2 * \cos^2(\theta_1 - \theta_2)}{(m_1 + m_2) * L_2}\right) = \frac{(m_1 + m_2) * L_1 * \dot{\theta_1^2} * \sin(\theta_1 - \theta_2) + (m_1 + m_2) * g * \cos(\theta_1 - \theta_2) * \sin(\theta_1) + m_2 * L_2 * \dot{\theta_2^2} * \cos(\theta_1 - \theta_2) * \sin(\theta_1 - \theta_2) - (m_1 + m_2) * g * \sin(\theta_2)}{(m_1 + m_2) * L_2}$$

$$\ddot{\theta_2} * \left(\frac{\left((m_1 + m_2) * L_2\right) - (m_2 * L_2 * \cos^2(\theta_1 - \theta_2))}{(m_1 + m_2) * L_2}\right) = \frac{(m_1 + m_2) * L_1 * \dot{\theta_1^2} * \sin(\theta_1 - \theta_2) + (m_1 + m_2) * g * \cos(\theta_1 - \theta_2) * \sin(\theta_1) + m_2 * L_2 * \dot{\theta_2^2} * \cos(\theta_1 - \theta_2) * \sin(\theta_1 - \theta_2) - (m_1 + m_2) * g * \sin(\theta_2)}{(m_1 + m_2) * L_2}$$

$$\ddot{\theta_2} = \frac{(m_1 + m_2) * L_1 * \dot{\theta_1^2} * \sin(\theta_1 - \theta_2) + (m_1 + m_2) * g * \cos(\theta_1 - \theta_2) * \sin(\theta_1) + m_2 * L_2 * \dot{\theta_2^2} * \cos(\theta_1 - \theta_2) * \sin(\theta_1 - \theta_2) - (m_1 + m_2) * g * \sin(\theta_2)}{(m_1 + m_2) * L_2} * \left(\frac{(m_1 + m_2) * L_2}{\left((m_1 + m_2) * L_2\right) - (m_2 * L_2 * \cos^2(\theta_1 - \theta_2))}\right)$$

$$\ddot{\theta_2} = \frac{(m_1 + m_2) * L_1 * \dot{\theta_1^2} * \sin(\theta_1 - \theta_2) + (m_1 + m_2) * g * \cos(\theta_1 - \theta_2) * \sin(\theta_1) + m_2 * L_2 * \dot{\theta_2^2} * \cos(\theta_1 - \theta_2) * \sin(\theta_1 - \theta_2) - (m_1 + m_2) * g * \sin(\theta_2)}{(m_1 + m_2) * L_2 - m_2 * L_2 * \cos^2(\theta_1 - \theta_2)}$$

Both equations of angular acceleration for angles 1 and 2 with respect to $\theta_1, \theta_2, \dot{\theta_1}, \dot{\theta_2}$

$$\ddot{\theta_1} = \frac{-m_2 * L_1 * \dot{\theta_1^2} * \cos(\theta_1 - \theta_2) * \sin(\theta_1 - \theta_2) + m_2 * g * \cos(\theta_1 - \theta_2) * \sin(\theta_2) - (m_1 + m_2) * g * \sin(\theta_1) - m_2 * L_2 * \dot{\theta_2^2} * \sin(\theta_1 - \theta_2)}{(m_1 + m_2) * L_1 - m_2 * L_1 * \cos^2(\theta_1 - \theta_2)}$$

$$\ddot{\theta_2} = \frac{(m_1 + m_2) * L_1 * \dot{\theta_1^2} * \sin(\theta_1 - \theta_2) + (m_1 + m_2) * g * \cos(\theta_1 - \theta_2) * \sin(\theta_1) + m_2 * L_2 * \dot{\theta_2^2} * \cos(\theta_1 - \theta_2) * \sin(\theta_1 - \theta_2) - (m_1 + m_2) * g * \sin(\theta_2)}{(m_1 + m_2) * L_2 - m_2 * L_2 * \cos^2(\theta_1 - \theta_2)}$$

System of equations represented by x

ODE45 will solve the array of equations represented by $\dot{x}$

$$x_1 = \theta_1 \qquad \dot{x_1} = \dot{\theta_1}$$

$$x_2 = \theta_2 \qquad \dot{x_2} = \dot{\theta_2}$$

$$x_3 = \dot{\theta_1} \qquad \dot{x_3} = \ddot{\theta_1}$$

$$x_4 = \dot{\theta_2} \qquad \dot{x_4} = \ddot{\theta_2}$$

Rewritten expressions for ODE45 to understand, where $\dot{x_1}, \dot{x_2}, \dot{x_3}, \dot{x_4}$ are array elements

$$\dot{x_1} = x_3$$

$$\dot{x_2} = x_4$$

$$\dot{x_3} = \frac{-m_2 * L_1 * x_3^2 * \cos(x_1 - x_2) * \sin(x_1 - x_2) + m_2 * g * \cos(x_1 - x_2) * \sin(x_2) - (m_1 + m_2) * g * \sin(x_1) - m_2 * L_2 * x_4^2 * \sin(x_1 - x_2)}{(m_1 + m_2) * L_1 - m_2 * L_1 * \cos^2(x_1 - x_2)}$$

$$\dot{x_4} = \frac{(m_1 + m_2) * L_1 * x_3^2 * \sin(\theta_1 - \theta_2) + (m_1 + m_2) * g * \cos(x_1 - x_2) * \sin(x_1) + m_2 * L_2 * x_4^2 * \cos(x_1 - x_2) * \sin(x_1 - x_2) - (m_1 + m_2) * g * \sin(x_2)}{(m_1 + m_2) * L_2 - m_2 * L_2 * \cos^2(x_1 - x_2)}$$

To reduce the complexity of the final expression, common pieces are substituted with symbolic values

$$A = \cos(x_1 - x_2)$$

$$B = \sin(x_1 - x_2)$$

$$C = A * B = \cos(x_1 - x_2) * \sin(x_1 - x_2)$$

$$D = A * A = \cos^2(x_1 - x_2)$$

$$E = m_1 + m_2$$

$$F = E - m_2 * D = (m_1 + m_2) - m_2 * \cos^2(x_1 - x_2)$$

$$G = -m_2 * L_1 * x_3^2 * C + m_2 * g * \mathrm{A} * \sin(x_2) - E * g * \sin(x_1) - m_2 * L_2 * x_4^2 * \mathrm{B}$$

$$H = E * L_1 * x_3^2 * \mathrm{B} + E * g * \mathrm{A} * \sin(x_1) + m_2 * L_2 * x_4^2 * \mathrm{C} - E * g * \sin(x_2)$$

$$\dot{x_3} = \frac{G}{L_1 * F}$$

$$\dot{x_4} = \frac{H}{L_2 * F}$$