STAT 425 Case Study 3

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1 Introduction and Data Description

In order to maximize production capacity, this study analyzes the *bubble wrap* experiment dataset to establish the ideal operating conditions for the bubble wrap lines. We conducted a thorough analysis using the One way ANOVA, Two way ANOVA, Anova Type III, partial F-tests, and diagnostics tests (α=0.05 is used for all tests in this case study) to evaluate a model with an ideal combination of *line speed* and *percent loading of additives* that will result in the highest production rate.

2 Data Pre-Processing

Before starting model selection, the experiment mentions that the bubble wrap quality is unacceptable in Replication 2, Runs 2, 3, and 5, so we removed the data with those conditions. After that, we modified the two categorical variables (*line speed* and *percent loading of additives*) as factors.

3 Model Selection Process

3.1 Testing the Significance of Interaction term

To start the model selection process, we constructed an interaction plot to examine the presence of interactions (shown in Figure 1)) and found that there seems to be an interaction between *loading* and *line speed*. However, further investigation by running Anova (type III) on the full model gives us a p-value of 0.6426, indicating that the interaction term is insignificant.

3.2 Testing the Significance of Predictors for Additive Model

We tested a reduced model with only *loading* against an additive model with both *line speed* and *loading* by a partial F-test. The p-value given by the partial F-test is 0.1002, meaning that the reduced model is better and thus *line speed* can be removed. We then ran a second partial F-test to compare the model with *loading* against our null model (Intercept only). The partial F-test has a p-value of 0.0061 which means the reduced model with *loading* is better, implying that *loading* is significant, and we will keep it in our model.

3.3 Diagnostics for the Model with *loading* as Predictor

As the first step in our diagnosis, we looked at the QQ-Plot (shown in Figure 2) and observed a roughly straight line, while the Shapiro-Wilk test yields a p-value of 0.0638, supporting the normality assumption. Unfortunately, the Residual Plot appears to be fan-shaped (shown in Figure 3), which, when combined with Levene's test, gave a p-value of 0.0069, indicating that constant variance was not satisfied. In order to address the non-constant variance issue, we decided to try some remedial measures.

4 Remedial Measures

4.1 WLS Model with *loading* as Predictor

We then carried out Weighted Least Squares on the model, hoping to fix the non-constant variance issue. The weights were calculated by taking the reciprocal of the variances within all *rate* values grouped by different levels of *loading*.

4.2 Diagnostics for the WLS Model with *loading* as Predictor

The Shapiro-Wilk test with a p-value of 0.0638, which is greater than 0.05, means that the model satisfies the normality assumption. And using Levene's test, we found a p-value of 0.0069, which is less than 0.05. Levene's test indicates that the constant variance assumption is still unsatisfied even though the normality assumption is upheld.

4.3 WLS Model with *loading*, *line speed* as Predictors

Even if *line speed* is not statistically significant, we tried introducing *line speed* in our model, hoping to stabilize the variance. We then tried Weighted Least Squares on both variables, *loading* and *line speed*, to see if it would satisfy both model assumptions. The weights were calculated by taking the reciprocal of the variances within all *rate* values grouped by the same level of *loading* and *line speed*.

4.4 Diagnostics for the WLS Model with *loading*, *line speed* as Predictors

The Shapiro-Wilk test yields a p-value of 0.0007, which indicates that the normality assumption is not satisfied while using Levene's test, we got a p-value of 0.0476, indicating that the constant variance assumption is still not met. This trial failed as both assumptions were not satisfied, and we decided to do a Box-Cox transformation to fix the normality issue.

4.5 Box-Cox Transformation for WLS Model with *loading*, *line speed* as Predictors

We ran Box-Cox on the model with both *loading*, and *line speed* predictors included. The plot (shown in Figure 4) shows that $\lambda = 1.5$ may be an optimal transformation.

4.6 Diagnostics for the Box-Cox Transformation for WLS Model with *loading*, *line speed* as Predictors

We conducted another diagnosis after running the WLS model with our response set to the power of 1.5 to check if the model assumptions were supported. With a p-value of 0.0015, the Shapiro-Wilk test shows that the normality assumption was still not valid. We used Levene's test, and the result was a p-value of 0.0473, indicating that the model once more failed to adhere to the constant-variance assumption.

5 Selection of the Optimal Combination of Factor Levels

We decided that the optimal model is the one-way ANOVA because every remedial measures we did didn't help fix the non-constant variance issue. So we chose the simplest model and kept the diagnostics in our mind. To find the optimal combination of the factor levels, we adopted the Tukey test (shown in Figure 5). According to the results, *percent loading of additives* with level 4 significantly yields higher production rates than that of level 2. However, level 0 is not statistically different from levels 2 and 4. Thus, we do not completely conclude that loading level 4 is optimal. Still, we suggest the manufacturer employ *percent loading of additives* at level 4 because it is statistically better than level 2. We also suggest the manufacturer employ more experiments to know more about level 0.

6 Conclusion

We only advise the company to use the *percent loading of additives* at level 4 to achieve the highest output rate as *line speed* will not have a significant impact on it. Additionally, we advise the engineers at *BubbleBobble* to employ more experiments on level 0 for a more comprehensive understanding.

7 Appendix

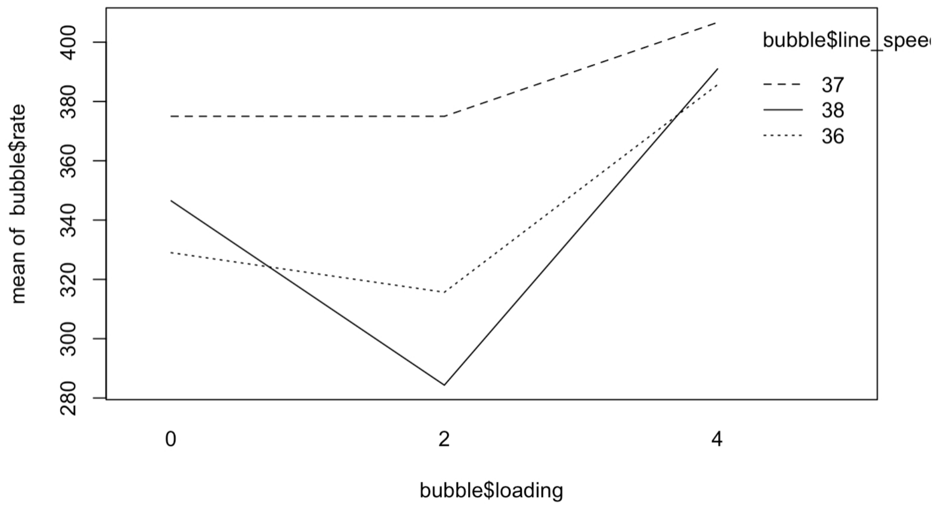

Figure 1: Interaction Plot

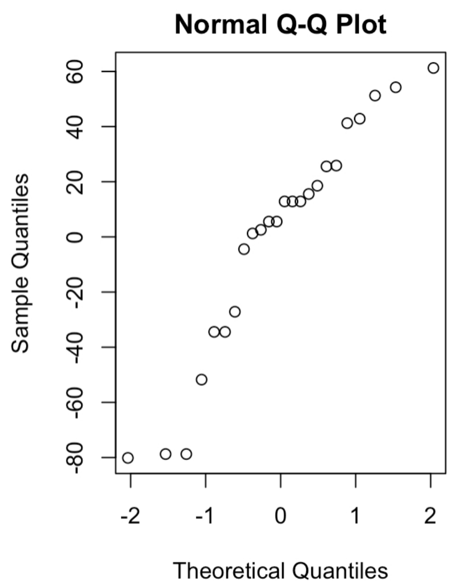

Figure 2: Q-Q Plot for the model with only *loading* as the predictor

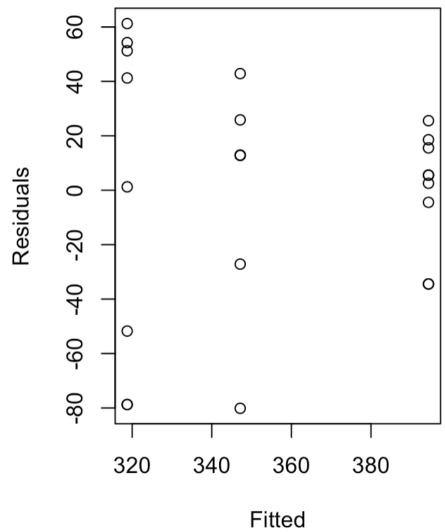

Figure 3: Residual Plot for the model with only *loading* as the predictor

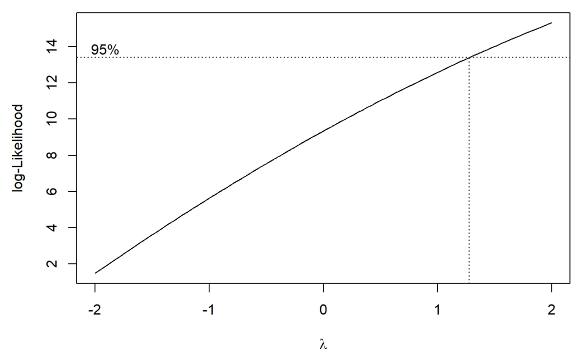

Figure 4: Box-Cox Result

```
##   Tukey multiple comparisons of means
##     95% family-wise confidence level
##
## Fit: aov(formula = rate ~ loading, data = bubble)
##
## $loading
##          diff        lwr       upr     p adj
## 2-0 -28.39286 -85.302946  28.51723 0.4339225
## 4-0  47.30159  -8.113416 102.71659 0.1035384
## 4-2  75.69444  22.263130 129.12576 0.0049169
```

Figure 5: Tukey Test for the WLS model with only *loading* as the predictor