Exercice 1

1)

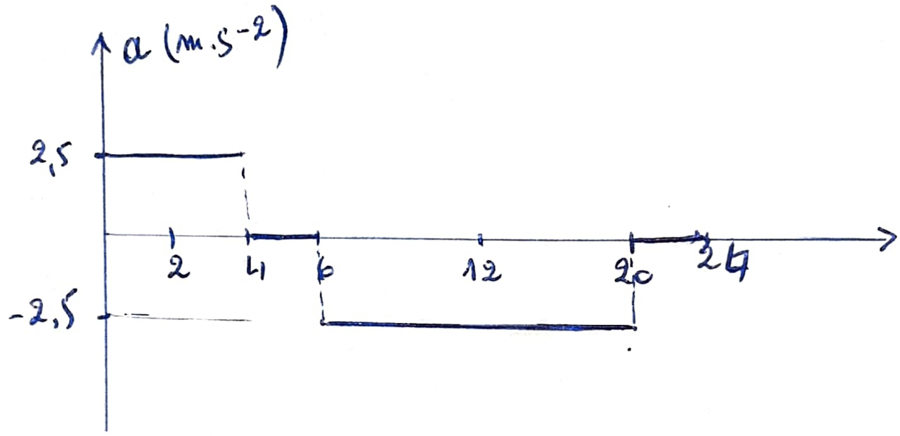
$t \in [0,4]$, $a = \frac{\Delta V}{\Delta t} = \frac{15-5}{4-0} = \frac{5}{2} = 2{,}5\ m.s^{-2}$

$t \in [4,6]$. $a = 0\ m.s^{-2}$

$t \in [6,12]$ $a = \frac{\Delta V}{\Delta t} = \frac{0-15}{12-6} = \frac{-15}{6} = \frac{-5}{2} = -2{,}5\ m.s^{-2}$

$t \in [12,20]$ $a = -2{,}5\ m.s^{-2}$

$t \in [20,24]$ $a = \frac{\Delta V}{\Delta t} = 0\ m.s^{-2}$



2) Les différentes phases du mouvement :

La nature du mouvement dépend du produit : $\vec{a}.\vec{V}$

phase 1 : $t \in [0,4]$. $\vec{V} > 0$, $\vec{a} > 0 \Rightarrow \vec{a}\vec{V} > 0$, $\|\vec{a}\| =$ cste

$\Rightarrow$ Mouvement réctiligne ~~ac~~ uniformément accéléré

phase 2 : $t \in [4,6]$, $\vec{V} =$ cste $\Rightarrow$ Mouvement réctiligne uniforme.

phase 3 : $t \in [6,12]$, $\vec{V} > 0$, $\vec{a} < 0 \Rightarrow \vec{a}.\vec{V} < 0$ et $\|\vec{a}\| =$ cste

$\Rightarrow$ Mouvement réctiligne uniformément retardé.

phase 4 : $t \in [12,20]$, $\vec{a} < 0$, $\vec{V} < 0 \Rightarrow \vec{a}\vec{V} > 0$ et $\|\vec{a}\| =$ cste

$\Rightarrow$ Mouvement réctiligne uniformément accéléré

phase 5 ; $t \in [20,24]$ $\vec{V} =$ cste $\Rightarrow$ Mouvement réctiligne uniforme

3)

-) Les équations de la vitesse en fct du temps : $V(t)$

$$dV = a\,dt \Rightarrow \int dV = \int a\,dt$$

phase 1 : $t \in [0,4]$.

$$\int_{V(0)}^{V(t)} dV = \int_0^t a\,dt \Rightarrow V(t) - V(0) = \int_0^t 2{,}5\,dt = \left[2{,}5 \cdot t\right]_0^t = 2{,}5t - 0.$$

$V(0) = 5$ m/s (d'après le graphe $V(t)$)

$$\Rightarrow V(t) - 5 = 2{,}5t \Rightarrow \boxed{V_1(t) = 2{,}5t + 5}$$

phase 2 : $t \in [4,6]$.

$V_2(t) = $ cste $= 15$ m/s.

phase 3 : $t \in [6,12]$

$$dV = -2{,}5\,dt \Rightarrow \int_{V(6)}^{V(t)} dV = \int_6^t -2{,}5 \cdot dt \quad , \quad V(6) = 15 \text{ m/s}$$

$$\Rightarrow V(t) - V(6) = -2{,}5t\Big]_6^t = -2{,}5t - (-2{,}5 \times 6)$$

$$V(t) - 15 = -2{,}5t + 15 \Rightarrow \boxed{V_3(t) = -2{,}5t + 30}$$

$V(12) = -2{,}5 \times 12 + 30 = 0$ m/s.

phase 4 : $t \in [12,20]$.

$$\int_{V(12)}^{V(t)} dV = \int_{12}^t -2{,}5\,.dt \Rightarrow V(t) - \underbrace{V(12)}_{0} = -2{,}5\,t\Big]_{12}^t = -2{,}5t - (-2{,}5 \times 12)$$

$$\Rightarrow \boxed{V_4(t) = -2{,}5 \cdot t + 30}.$$

phase 5 :

$$\int_{20}^{V(t)} dV = 0 \Rightarrow \boxed{V_5 = \text{cste} = -20 \text{ m/s}}.$$

Exo 1

•) Les équations horaires $x(t)$

$x(0) = x_0 = 10\,m$. , $dx = V.dt \Rightarrow \int dx = \int v.dt$

phase 1 : $t \in [0,4]$

$$\int_{x(0)=x_0}^{x(t)} dx = \int_0^t (2{,}5t+5).dt \Rightarrow x(t) - x_0 = \left[2{,}5\frac{t^2}{2} + 5t\right]_0^t$$

$$\Rightarrow x(t) - 10 = 2{,}5\frac{t^2}{2} + 5t \Rightarrow \boxed{x_1(t) = \frac{2{,}5}{2}t^2 + 5t + 10}$$

$$x(4) = \frac{2{,}5}{2}4^2 + 5\times4 + 10 = 50\,m$$

• phase 2 : $t \in [4,6[$.

$$\int_{x(4)=50}^{x(t)} dx = \int_4^t 15.dt = 15t\Big]_4^t \Rightarrow x(t) - 50 = 15t - (15\times4)$$

$$\Rightarrow x(t) = 15t - 60 + 50$$

$$\Rightarrow \boxed{x_2(t) = 15t - 10}$$

$$x(6) = 15\times6 - 10 = 80\,m$$

• phase 3 + phase 4 : $t \in [6,20[$

$$\int_{x(6)=80}^{x(t)} dx = \int_6^t (-2{,}5t+30).dt \Rightarrow x(t) - 80 = -2{,}5\frac{t^2}{2} + 30t\Big]_6^t$$

$$\Rightarrow x(t) - 80 = -2{,}5\frac{t^2}{2} + 30t - \left(-2{,}5\times\frac{6^2}{2} + 30\times6\right) = -2{,}5\frac{t^2}{2} + 30t - 135$$

$$\Rightarrow \boxed{x_3(t) = -2{,}5\frac{t^2}{2} + 30t - 55} = x_4(t)$$

$$x(20) = -2{,}5\times\frac{(20)^2}{2} + 30\times20 - 55 = 45\,m$$

phase 5 : $t \in [20,24]$.

$$\int_{x(20)}^{x(t)} dx = \int_{20}^t -20.dt \Rightarrow x(t) - x(20) = -20t\Big]_{20}^t$$

$$\Rightarrow x(t) - 45 = -20t - (-20\times20) = -20t + 400$$

$$\Rightarrow \boxed{x_5(t) = -20t + 445}$$

Exo 1

4) La position du mobile : $x(t)$

• Analytiquement : En utilisant l'expression de $x(t)$

$x(6) = x_2(6)$. $6\,s \in$ phase 2

$= 15 \times 6 - 10 = 80\ m$

$x(10) = x_3(10)$ $t = 10\,s \in$ phase 3.

$= -2,5 \times \frac{10^2}{2} + 30 \times 10 - 55 = \cancel{130}\ m = 120\ m$.

$x(20) = x_4(20) = -2,5 \times \frac{20^2}{2} + 30 \times 20 - 55 = \cancel{45}\ m = 45\ m$

• Graphiquement (بيانيا) en utilisant le graphe $V(t)$

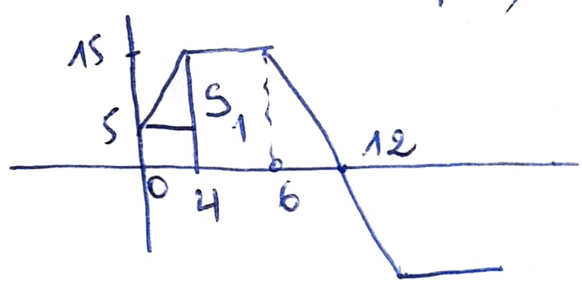
$x(6) = x(0) +$ l'air sous la courbe $V(t)$ entre $0\,s$ et $6\,s$.

مساحة تحت المنحنى V(t)

Remarque : En prend les surfaces en valeurs algébriques (قيم جبرية)

$x(6) = x_0 + S_1 = 10 + 15 \times 2 + 5 \times 4 + \frac{10 \times 4}{2}$.

$x(6) = 10 + 30 + 20 + 20 = 80\ m$

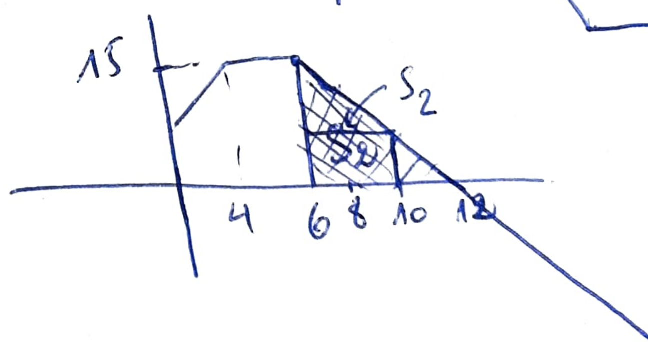


$x(10) = x(6) + S_2$

$= \cancel{80} + \frac{6 \times \cancel{15}}{2} =$

$= 80 + 4 \times 5 + \frac{4 \times 10}{2}$

$x(10) = 120\ m$

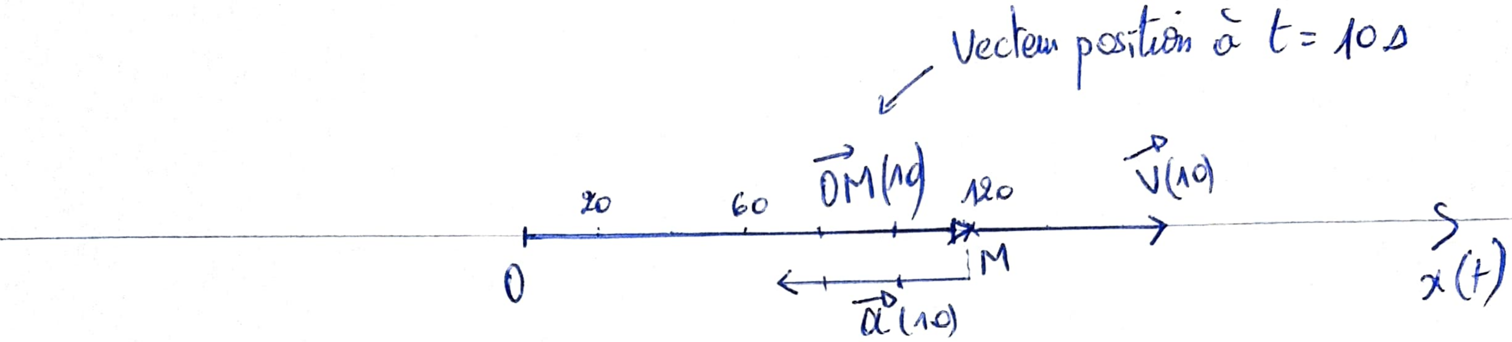


6) Mvt réctiligne ⇒ la trajectoire est une droite.

. Vecteur position à $t = 10s$: $\vec{OM}(10) = x(10) \cdot \vec{i} = 120\,\vec{i}$

. " vitesse " " : $\vec{V}(10) = 5\,\vec{i}$

. " accélération " " : $\vec{a}(10) = -2{,}5\,\vec{i}$



Exo 1

Exercice 2

$V(0) = 0\,m/s$

1)

phase 1 : $t \in [0,10]$, $\bar{a} = 2\,m/s^2$

$dV = a\,dt \Rightarrow \int_{V(0)=0}^{V(t)} dU = \int_0^t 2 \cdot dt = 2t\Big]_0^t = 2t$

$\boxed{V_1(t) = 2t}$

phase 2 : $t \in [10, 60]$. $\bar{a}(t) = 0\,m/s^2$

$dV = 0 \Rightarrow V_2(t) = cste = V_1(10) = 2 \times 10 = 20\,m/s$, $\boxed{V_2(t) = 20\,m/s}$

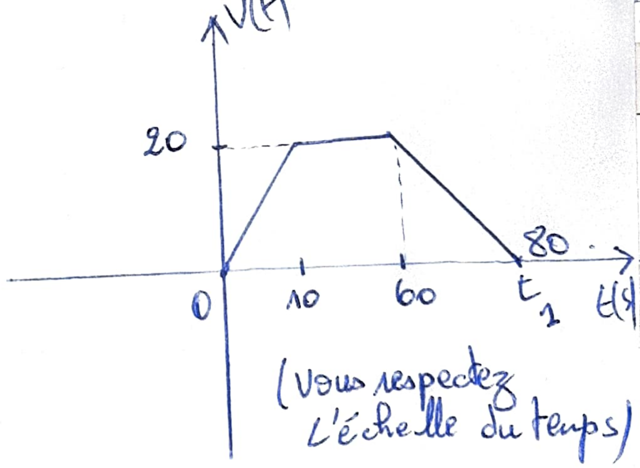
phase 3 : $t \in [60, t_1]$, $\bar{a} = -1\,m/s^2$

$dV = (-1).dt \Rightarrow \int_{V(60)}^{V(t)} dV = \int_{60}^{t} (-1).dt \Rightarrow V_2(t) - V(60) = -t\Big]_{60}^{t}$

$V(60) = 20\,m/s \Rightarrow V_3(t) - 20 = -t - (-60) = -t + 60$.

$\Rightarrow \boxed{V_3(t) = -t + 80}$

2) Tracer des graphe $V(t)$



(Vous respectez L'échelle du temps)

3) $t_1 = 80\,s$, $V_3(t_1) = 0$

5) Les équations horaires $x(t)$

$x(0) = 0\,m$

• phase 1 : $t \in [0,10]$

$dx = V_1 dt \Rightarrow dx = 2t.dt \Rightarrow \int_{x(0)=0}^{x(t)} dx = \int_0^t 2t.dt \Rightarrow x_1(t) - 0 = 2\frac{t^2}{2}\Big]_0^t$

$\Rightarrow \boxed{x_1(t) = t^2}$ $x_1(10) = 100$

phase 2 $t \in [10, 60]$

$$dx = V_2(t).dt \Rightarrow \int_{x(10)}^{x(t)} dx = \int_{10}^{t} 20.dt$$

$$\Rightarrow x_2(t) - x(10) = 20t\Big]_{10}^{t} = 20t - (20\times 10) = 20t - 200.$$

$$x_2(10) = x_1(10) = 10^2 = 100\ m$$

$$\Rightarrow x_2(t) - 100 = 20t - 200 \Rightarrow \boxed{x_2(t) = 20t - 100}$$

phase 3: $t \in [60, 80]$.

$$dx = V_3(t).dt \Rightarrow \int_{x_3(60)}^{x_3(t)} dx = \int_{60}^{t} (-t+80).dt = \frac{-t^2}{2} + 80t\Big]_{60}^{t}$$

$$x_3(60) = x_2(60) = 20\times 60 - 100 = 1100\ m$$

$$\Rightarrow x_3(t) - 1100 = \frac{-t^2}{2} + 80t - \left(-\frac{60^2}{2} + 80\times 60\right) = \frac{-t^2}{2} + 80t + 3000.$$

$$\Rightarrow \boxed{x_3(t) = -\frac{t^2}{2} + 80t + 4100}$$

EXO 2

Exercice 3

$x(t) = 2t^3 + 4t + 2$, $y(t) = t^2 - 2t + 1$, $z(t) = 2t$

1) $\vec{V} = V_x \vec{i} + V_y \vec{j} + V_z \vec{k}$, $\vec{V} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$.

$$V_x = \frac{dx}{dt} = \frac{d}{dt}(2t^3 + 4t + 2) = 6t^2 + 4$$

$$V_y = \frac{dy}{dt} = 2t - 2 \quad , \quad V_z = \frac{dz}{dt} = 2 .$$

$$\|\vec{V}\| = \sqrt{V_x^2 + V_y^2 + V_z^2} = \sqrt{(6t^2+4)^2 + (2t-2)^2 + 2^2} = \sqrt{36t^4 + 16 + 48t^2 + 4t^2 + 4 - 8t + 4}$$

$$\|\vec{V}\| = \sqrt{36t^4 + 52t^2 - 8t + 20} .$$

$\vec{a} \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$

$\cancel{a_x}$

$$a_x = \frac{dV_x}{dt} = 6t , \quad a_y = \frac{dV_y}{dt} = 2 , \quad a_z = \frac{dV_z}{dt} = 0 .$$

$$\|\vec{a}\| = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{(6t)^2 + (2)^2 + 0^2} = \sqrt{36t^2 + 4} = 2\sqrt{9t^2 + 1}$$

- Le point matériel se trouve dans le plan XOZ quand $y(t) = 0$.

$$y(t) = 0 \Rightarrow t^2 - 2t + 1 = 0 \Rightarrow (t-1)^2 = 0 \Rightarrow \boxed{t_0 = 1\,s}$$

$x(t_0) = x(1) = 8\,m$, $z(t_0) = z(1) = 2$

$M_0(x(t_0), y(t_0), z(t_0))$, $M_0(8, 0, 2)$

2) Sa vitesse à $t = t_0$:

$V_x = 6 \times 1^2 + 4 = 10$, $V_y = 2 \times 1 - 2 = 0$, $V_z(t_0) = V_z(1) = 2$

$$\|\vec{V}\| = \sqrt{10^2 + 0^2 + 2^2} = \sqrt{104} \simeq 10{,}20\ m/s .$$

Exo 3

Exercice 4

$x(t) = at^2 + b$, $y(t) = ct^2$; $z(t) = dt + e$

1)

- L'unité de a : $[x(t)] = [a][t^2] + b$.

$L = [a] \cdot T^2 + L$

$\Rightarrow [a] \cdot T^2 = L \Rightarrow [a] = \frac{L}{T^2}$

$\Rightarrow$ l'unité de a : $\frac{m}{s^2} = m \cdot s^{-2}$ (accélération)

- L'unité de b : m

- L'unité de c : $[y] = [c] \cdot [t^2] \Rightarrow L = [c] \cdot T^2$

$\Rightarrow [c] = \frac{L}{T^2}$

$\Rightarrow$ l'unité de c : $m \cdot s^{-2}$

- L'unité de e : $[z(t)] = [d] \cdot [t] + [e]$

$\Rightarrow [e] = [z] = L \Rightarrow$ l'unité de e : m

- L'unité de d : $L = [d] \cdot T \Rightarrow [d] = \frac{L}{T}$

$\Rightarrow$ l'unité de d : $\frac{m}{s} = m \cdot s^{-1}$

2) $a = 2, b = 1, c = 2, d = 4, e = -3$.

$x(t) = 2t^2 + 1$, $y(t) = 2t^2$, $z(t) = 4t - 3$

- Les composantes du vecteur vitesse : V_x, V_y et V_z

$V_x = \frac{dx}{dt} = 4t$, $V_y = \frac{dy}{dt} = 4t$, $V_z = \frac{dz}{dt} = 4$

Le module de $\vec{V}$: $\|\vec{V}\| = \sqrt{V_x^2 + V_y^2 + V_z^2}$

$= \sqrt{(4t)^2 + (4t)^2 + 4^2} = \sqrt{32t^2 + 16} = 4\sqrt{(2t^2+1)}$

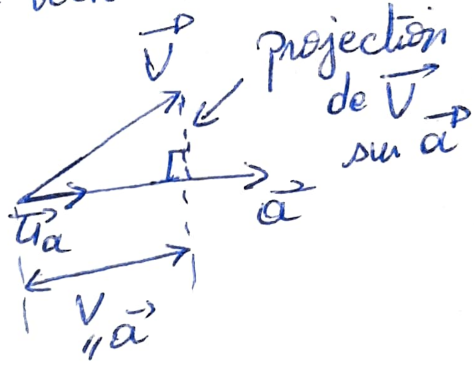
- Les composantes du vecteur accélération : a_x, a_y et a_z

$$a_x = \frac{dV_x}{dt} = \frac{d(4t)}{dt} = 4 \quad , \quad a_y = \frac{dV_y}{dt} = 4 \quad , \quad a_z = \frac{dV_z}{dt} = 0$$

- Le module de $\vec{a}$: $\|\vec{a}\| = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{4^2 + 4^2 + 0^2} = 4\sqrt{2}$

3) La composante du vecteur vitesse sur la direction du vecteur accélération

$= V_{/\!/\vec{a}}$



$$V_{/\!/\vec{a}} = \vec{V} \cdot \vec{u}_a$$

$\vec{u}_a$: vecteur unitaire $/\!/$ à $\vec{a}$

$$\vec{u}_a = \frac{\vec{a}}{\|\vec{a}\|}$$

$$V_{/\!/\vec{a}} = \frac{\vec{V} \cdot \vec{a}}{\|\vec{a}\|} = \frac{a_x V_x + V_y a_y + V_z a_z}{\sqrt{a_x^2 + a_y^2 + a_z^2}} = \frac{4(4t) + 4(4t) + 0}{4\sqrt{2}}$$

$$\boxed{V_{/\!/\vec{a}} = \frac{32t}{4\sqrt{2}} = \frac{16}{\sqrt{2}}t}$$

4. Les composantes tangentielles et normales du vect accélération.

a_t : comp tangentielle : $a_t = \frac{d\|\vec{V}\|}{dt} = \frac{d\left(4\sqrt{2t^2+1}\right)}{dt} = \frac{4 \times 4t}{2\sqrt{2t^2+1}}$

$$a_t = \frac{8t}{\sqrt{2t^2+1}}$$

a_N : comp normale : $a_N = \sqrt{a^2 - a_t^2} = \sqrt{32 - \frac{64t^2}{2t^2+1}} = \sqrt{\frac{32}{2t^2+1}} = 4\sqrt{\frac{2}{2t^2+1}}$

5) Le rayon de courbure : $a_N = \frac{V^2}{\rho} \Rightarrow \rho = \frac{V^2}{a_N}$.

$$\rho = \frac{16(2t^2+1)}{4\sqrt{\frac{2}{2t^2+1}}} = \frac{4(2t^2+1)^{3/2}}{\sqrt{2}}$$

Exo 4

Exercice 5

$x(t) = 5t$, $y(t) = 20t - 2{,}5t^2$.

1) L'équation de la trajectoire :

$t = \frac{x}{5} \Rightarrow y(t) = 20 \times \frac{x}{5} - 2{,}5\left(\frac{x}{5}\right)^2 = \frac{20x}{5} - 2{,}5\frac{x^2}{25}$

$\Rightarrow \boxed{y(t) = 4x - 0{,}1x^2}$

2) • Les composantes de $\vec{V}$: $V_x = \frac{dx}{dt} = 5$, $V_y = \frac{dy}{dt} = 20 - 5t$

$\|\vec{V}\| = \sqrt{V_x^2 + V_y^2} = \sqrt{5^2 + (-5t)^2} = \sqrt{25 + 25t^2} = 5\sqrt{t^2 + 1}$

• Les composantes de $\vec{a}$: $\cancel{\|\vec{a}\|}$

$a_x = \frac{dV_x}{dt} = 0$, $a_y = \frac{dV_y}{dt} = -5$, $\|\vec{a}\| = \sqrt{a_x^2 + a_y^2} = \sqrt{0^2 + (-5)^2}$

$\|\vec{a}\| = \sqrt{25} = 5 \ m.s^{-2}$

3) Les composantes intrinsèques de $\vec{a}$:

$a_t = \frac{d\|\vec{V}\|}{dt} = \frac{d(5\sqrt{t^2+1})}{dt} = \frac{5 \times 2t}{2\sqrt{t^2+1}} = \frac{5t}{\sqrt{t^2+1}}$

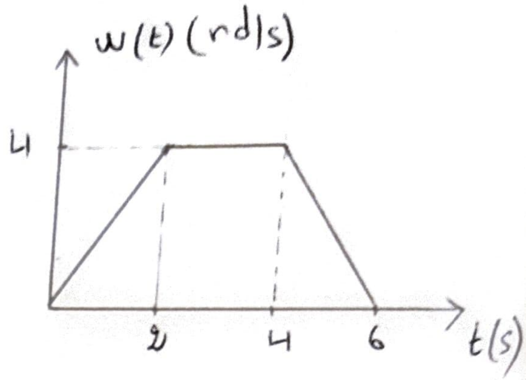
$a_N = \sqrt{a^2 - a_t^2} = \sqrt{5^2 - \left(\frac{5t}{\sqrt{t^2+1}}\right)^2}$

$a_N = \sqrt{25 - \frac{25t^2}{t^2+1}} = \sqrt{\frac{25}{t^2+1}} = \frac{5}{\sqrt{t^2+1}}$

Exo 5

Serie N° 2

Exercice 6 :



1) $\theta(t)$ de chaque phase

$w = \frac{d\theta}{dt} \Rightarrow d\theta = w\,dt$

• phase 1 : $2 \leq t \leq 4$

$w(t) = at \quad , a = \frac{\Delta w}{\Delta t} = \frac{w(2) - w(0)}{2-0} = \frac{4-0}{2-0} = 2$

$\boxed{w(t) = 2t} \quad , \quad d\theta = 2t.dt \Rightarrow \int_{\theta(0)=0}^{\theta(t)} d\theta = \int_0^t 2t.dt \Rightarrow \theta(t) - 0 = t^2\Big|_0^t = t^2$

$\boxed{\theta(t) = t^2}$

• phase 2 : $2 \leq t \leq 4$

$\boxed{w(t) = cste = 4\,rd/s} \Rightarrow d\theta = 4.dt \Rightarrow \int_{\theta(2)}^{\theta(t)} d\theta = \int_2^t 4.dt$

$\theta(2) = 2^2 = 4$

$\Rightarrow \theta(t) - \theta(2) = 4t\Big|_2^t = 4(t-2)$

$\Rightarrow \theta(t) - 4 = 4t - 8$

$\Rightarrow \boxed{\theta(t) = 4t - 4}$

• phase 3 : $t \in [4,6]$.

$w(t) = at + b \quad , w(4) = 4$ et $w(6) = 0 \Rightarrow \begin{cases} 4a + b = 4 & — (1) \\ 6a + b = 0 & — (2) \end{cases}$

$(2) - (1) \Rightarrow 2a = -4 \Rightarrow a = -2$

$(2) \Rightarrow b = -6a = 12 \Rightarrow \boxed{w(t) = -2t + 12}$

$d\theta = (-2t + 12)\,dt \Rightarrow \int_{\theta(4)=12}^{\theta(t)} d\theta = \int_4^t (-2t + 12)\,dt$

$\theta(4) = 4 \times 4 - 4 = 12$

$\theta(t) - 12 = -t^2 + 12t\Big|_4^t = -t^2 + 12t - (-16 + 48)$

$\theta(t) - 12 = -t^2 + 12t - 32$

$\Rightarrow \boxed{\theta(t) = -t^2 + 12t - 20}$

2) L'accélération angulaire : $\alpha(t) = \frac{dw}{dt}$ (la pente)

- phase 1 : $w(t) = 2t \Rightarrow \alpha(t) = \frac{d}{dt}(2t) = 2 \ rd/s^2$
- phase 2 : $\alpha(t) = \frac{d}{dt}(4) = 0$
- phase 3 : $\alpha(t) = \frac{d}{dt}(-2t+12) = -2 \ rd/s^2$

3) pour la 2ème phase :

a/ • Les composantes cartésiennes du vecteur position : x et y

M^{vt} circulaire (s'effectue sur un plan $\Rightarrow$ repère à 2 dimensions)

$\vec{OM} = x\vec{i} + y\vec{j}$, $x = r\cos\theta = R\cos t^2 = \boxed{2\cos t^2 = x}$

$r = $ cste $= R = 2m$ (M^{vt} circulaire) $\theta(t) = t^2$

$\cancel{y = R\sin\theta = 2t}$

$y = R\sin\theta = \boxed{2\sin t^2 = y}$

b/ Les composantes cartésiennes de $\vec{a}$; a_x et a_y

$a_x = \frac{dV_x}{dt}$, $a_y = \frac{dV_y}{dt}$, $V_x = \frac{dx}{dt} = \frac{d}{dt}(2\cos t^2) = -2t \times 2\sin t$

$= -4t\sin t^2$

$V_y = \frac{dy}{dt} = \frac{d}{dt}(2\sin t^2) = 4t\cos t^2$

$a_x = \frac{d}{dt}(-4t\sin t^2)$

$= -8t^2\cos t^2 - 4\sin t^2$

$a_y = \frac{d}{dt}(4t\cos t^2) = -8t^2\sin t^2 + 4\cos t^2$

c/ Les composantes intrinsèques du vecteur $\vec{a}$: $\vec{a} = a_t \vec{u_t} + a_N . \vec{u_N}$

$a_t = \frac{d\|\vec{V}\|}{dt}$, $\|\vec{V}\| = \sqrt{V_x^2 + V_y^2} = \sqrt{V_r^2 + V_\theta^2}$, $V_r = \dot{r} = 0$ ($r = $ cste $= R$)

$V_\theta = r\dot{\theta} = R \times \frac{d\theta}{dt} = R.\frac{dt^2}{dt}$

$= 2t.R$

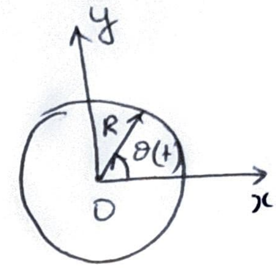
$= 4t$

$\|\vec{V}\| = \sqrt{0^2 + (4t)^2} = 4t$

$a_t = \frac{d}{dt}(4t) = 4$

Exo 6

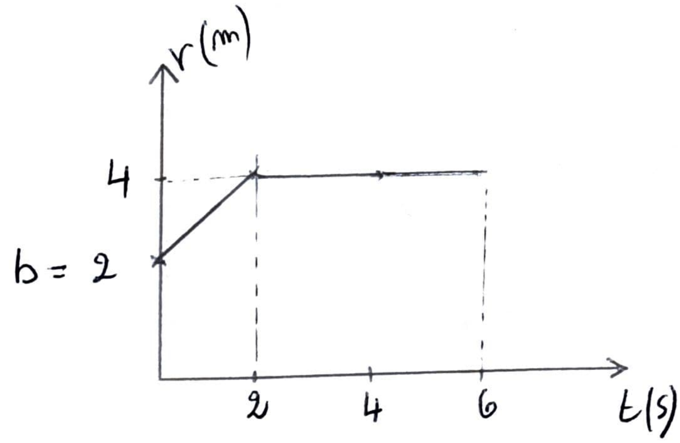
$a_N = \frac{V^2}{\rho}$, Le mvt est circulaire $\Rightarrow$ $\rho(t) = R = 2$
dont le centre est "O"



$$\boxed{a_N = \frac{(4t)^2}{R} = \frac{4^2 \times t^2}{2} = 8t^2}$$

Exercice 7 :

1) Les équations $r(t)$ et $\theta(t)$



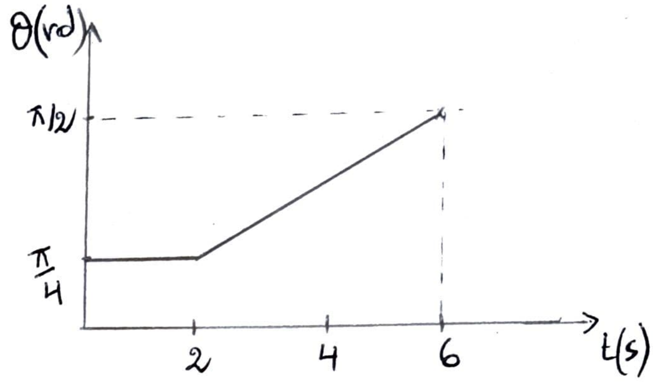
• phase 1 : $0 \leq t \leq 2$

$r(t) = at + b$, $b = 2$ (Voir graphe)

$t = 2 \Rightarrow r = 4$

$\Rightarrow 2a + 2 = 4 \Rightarrow a = 1$

$\Rightarrow \boxed{r(t) = t + 2}$



$\boxed{\theta(t) = \frac{\pi}{4} \text{ (rd)}}$

• phase 2 : $t \in [2, 6]$

$\boxed{r(t) = 4 \text{ m}}$

$\theta(t) = at + b$. $\quad t = 2 \Rightarrow \theta = \frac{\pi}{4}$ $\quad t = 6 \Rightarrow \theta = \frac{\pi}{2}$ $\Rightarrow \begin{cases} 2a + b = \frac{\pi}{4} & — (1) \\ 6a + b = \frac{\pi}{2} & — (2) \end{cases}$

$(2) - (1) \Rightarrow 4a = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4} \Rightarrow a = \frac{\pi}{16}$, $b = \frac{\pi}{4} - 2 \times \frac{\pi}{16} = \frac{\pi}{8}$

$\boxed{\theta(t) = \frac{\pi}{16} t + \frac{\pi}{8}}$

2) Dans le repère des coordonnées polaires :

• Le vecteur position : $\vec{OM} = r \cdot \vec{U_r}$, $\vec{V} = V_r \vec{U_r} + V_\theta \vec{U_\theta}$, $V_r = \dot{r}$, $V_\theta = r\dot{\theta}$.

$\vec{a} = a_r \cdot \vec{U_r} + a_\theta \cdot \vec{U_\theta}$, $\quad a_r = \ddot{r} - r\dot{\theta}^2$

$a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta}$

• phase 1 : $r = t + 2$, $\dot{r} = \frac{d}{dt}(t+2) = 1$, $\theta = \frac{\pi}{4}$, $\dot{\theta} = \frac{d}{dt}\left(\frac{\pi}{4}\right) = 0$, $V_\theta = 0$

$\ddot{r} = 0$, $\ddot{\theta} = 0$. $\quad a_r = 0$, $a_\theta = 0$

$\boxed{\vec{OM} = (t+2) \cdot \vec{U_r}}$, $\boxed{\vec{V} = \vec{U_r}}$, $\boxed{\vec{a} = \vec{0}}$

• Phase 2 : $r=4$, $\dot{r}=0$, $\ddot{r}=0$. , $V_r=0$, $V_\theta = 4\times\frac{\pi}{16} = \frac{\pi}{4}$

$\theta = \frac{\pi}{16}t + \frac{\pi}{8}$, $\dot{\theta} = \frac{\pi}{16}$, $\ddot{\theta}=0$, $a_r = -4\frac{\pi^2}{16^2} = -\frac{\pi^2}{64}$

$\boxed{\vec{OM} = 4.\vec{U_r}}$, $\boxed{\vec{V} = \frac{\pi}{4}.\vec{U_\theta}}$ $a_\theta = 0$

$\boxed{\vec{a} = -\frac{\pi^2}{64}\vec{a_r}}$

phase 1 : $\boxed{\|\vec{V}\| = \sqrt{V_r^2+V_\theta^2} = \sqrt{1^2+0^2} = 1}$

$\boxed{\|\vec{a}\| = \sqrt{a_r^2+a_\theta^2} = \sqrt{0^2+0^2} = 0}$.

phase 2 : $\boxed{\|\vec{V}\| = \sqrt{\left(\frac{\pi}{4}\right)^2+0^2} = \frac{\pi}{4}}$

$\boxed{\|\vec{a}\| = \sqrt{(-64\pi^2)^2+0^2} = +64\pi^2}$

3) Dans le repère cartésien :

$\vec{OM} = x\vec{i}+y\vec{j}$, $x = r\cos\theta$, $y = r\sin\theta$, $V_x = \frac{dx}{dt}$, $V_y = dy/dt$, $a_x = dV_x/dt$, $a_y = dV_y/dt$

phase 1 : $x = (t+2)\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}(t+2)$

$y = (t+2)\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}(t+2)$

$V_x = \frac{\sqrt{2}}{2}$, $V_y = \frac{\sqrt{2}}{2}$, $a_x = 0$, $a_y = 0$.

$\|\vec{V}\| = \sqrt{V_x^2+V_y^2} = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2+\left(\frac{\sqrt{2}}{2}\right)^2} = 1$, $\|\vec{a}\| = 0$

phase 2 : $x = 4\cos\left(\frac{\pi}{16}t+\frac{\pi}{8}\right)$, $y = 4\sin\left(\frac{\pi}{16}t+\frac{\pi}{8}\right)$

$V_x = 4\left(\frac{\pi}{16}\right)\left(-\sin\left(\frac{\pi}{16}t+\frac{\pi}{8}\right)\right) = -\frac{\pi}{4}\sin\left(\frac{\pi}{16}t+\frac{\pi}{8}\right)$

$V_y = 4.\frac{\pi}{16}\cos\left(\frac{\pi}{16}t+\frac{\pi}{8}\right) = \frac{\pi}{4}\cos\left(\frac{\pi}{16}t+\frac{\pi}{8}\right)$

$\|\vec{V}\| = \sqrt{\left(\frac{\pi}{4}\cos\left(\frac{\pi}{16}t+\frac{\pi}{8}\right)\right)^2+\left(-\frac{\pi}{4}\sin\left(\frac{\pi}{16}t+\frac{\pi}{8}\right)\right)^2}$

$= \sqrt{\frac{\pi^2}{4^2}\left[\cos^2(\,)+\sin^2(\,)\right]} = \frac{\pi}{4}$

$\|\vec{V}\| = \pi/4$

EXO 7

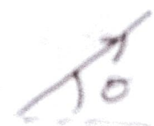
Conclusion : $\|\vec{V}\|$ et $\|\vec{a}\|$ ne dépendent pas de repère choisi

4) Les composantes intrinsèque de $\vec{a}$:

phase 1 : $a_t = \frac{d\|\vec{V}\|}{dt} = 0$, $a_N = \sqrt{a^2 - a_t^2} = 0$

phase 2 : $a_t = 0$, $a_N = \sqrt{a^2 - \underbrace{a_t^2}_{0}} = a = \frac{\pi^2}{64}$

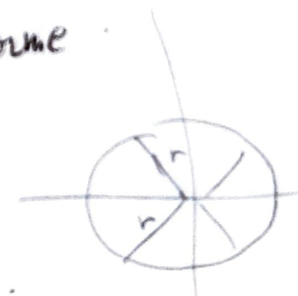
5) Nature du M^{vt}



phase 1 : $\theta = cste$

$\Rightarrow M^{vt}$ réctiligne

$\|\vec{v}\| = 1 = cste \Rightarrow M^{vt}$ réctiligne uniforme



phase 2 : $r = cste \Rightarrow M^{vt}$ circulaire

$\|\vec{V}\| = cste \Rightarrow M^{vt}$ circulaire uniforme

Série N° 2

Exercice 08 :

$x(t) = R\cos\theta$, $y = R\sin\theta$, $z = h\theta$, h, R sont des constantes

1) Les composantes cartésiennes de $\vec{V}$ et de $\vec{a}$

$\vec{V}\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$ $V_x = \frac{dx}{dt} = \frac{d}{dt}(R\cos\theta) = R\dot{\theta}(-\sin\theta) = -R\dot{\theta}\sin\theta$, $\dot{\theta} = \frac{d\theta}{dt}$

$V_y = \frac{dy}{dt} = \frac{d}{dt}(R\sin\theta) = R\dot{\theta}\cos\theta$

$V_z = \frac{dz}{dt} = \dot{z} = h\dot{\theta}$

$\vec{a}\begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$ $a_x = \frac{dV_x}{dt} = \frac{d}{dt}(-R\dot{\theta}\sin\theta) = -R(\ddot{\theta}\sin\theta + (\dot{\theta})^2\cos\theta)$

$a_y = \frac{dV_y}{dt} = \frac{d}{dt}(R\dot{\theta}\cos\theta) = R(\ddot{\theta}\cos\theta + (\dot{\theta})^2(-\sin\theta))$

$= R(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta)$

$a_z = \frac{dV_z}{dt} = \frac{d}{dt}(h\dot{\theta}) = h\ddot{\theta}$

$(f \times g)' = f'g + g'f$

Les composantes de $\vec{V}$ et de $\vec{a}$ en coordonnées polaires cylindriques :

$\vec{r} = \vec{OM}$, $r = \sqrt{x^2+y^2} = \sqrt{(R\cos\theta)^2 + (R\sin\theta)^2} = \sqrt{R^2(\underbrace{\cos^2\theta + \sin^2\theta}_{=1})} = R$

$r = R = cste$

$\dot{r} = 0$, $\ddot{r} = 0$, $\vec{V}\begin{pmatrix} V_r \\ V_\theta \\ V_z \end{pmatrix}$ $V_r = \dot{r} = 0$, $V_\theta = r\dot{\theta} = R\dot{\theta}$

$V_z = \dot{z} = h\dot{\theta}$

$\vec{V} = R\dot{\theta}\vec{U_\theta} + h\dot{\theta}\vec{K} + 0.\vec{U_r} = R\dot{\theta}\vec{U_\theta} + h\dot{\theta}.\vec{K}$

$\|\vec{V}\| = \sqrt{V_r^2 + V_\theta^2 + V_z^2} = \sqrt{0^2 + (R\dot{\theta})^2 + (h\dot{\theta})^2} = \sqrt{\dot{\theta}^2(R^2+h^2)}$

$\|\vec{V}\| = |\dot{\theta}|\sqrt{R^2+h^2}$

$$\vec{a}\begin{pmatrix} a_r \\ a_\theta \\ a_z \end{pmatrix} \quad , \quad \begin{aligned} a_r &= \ddot{r} - r\dot{\theta}^2 \\ a_\theta &= 2\dot{r}\dot{\theta} + r\ddot{\theta} \\ a_z &= \ddot{z} \end{aligned}$$

$r = cste = R \Rightarrow \dot{r} = \ddot{r} = 0.$

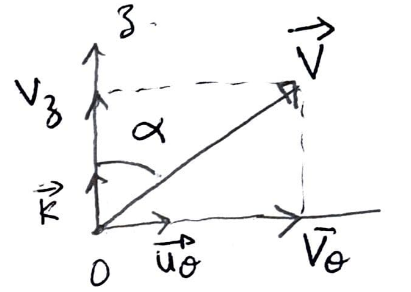
$a_r = -R\dot{\theta}^2 \quad , \quad a_\theta = 2\times 0 + R\ddot{\theta} = R\ddot{\theta} \quad , \quad a_z = \frac{d}{dt}(\dot{z}) = h\ddot{\theta}$

$\|\vec{a}\| = \sqrt{a_r^2 + a_\theta^2 + a_z^2} = \sqrt{(-R\dot{\theta}^2)^2 + (R\ddot{\theta})^2 + (h\ddot{\theta})^2} = \sqrt{R^2\dot{\theta}^4 + \ddot{\theta}^2(R^2 + h^2)}$

2) $\frac{d\theta}{dt} = \dot{\theta} = \omega = cste \Rightarrow \ddot{\theta} = 0$.

$V_r = 0 \quad , \quad V_\theta = R\dot{\theta} = \omega R, \quad V_z = h\omega$

$\|\vec{V}\| = |\omega|\sqrt{R^2 + h^2}$



$\vec{u}_\theta, \vec{u}_r$ et $\vec{k}$ sont $\perp$

$\vec{V} = V_\theta \vec{u}_\theta + V_z \vec{k}$

$\vec{u}_\theta \perp \vec{k}$

. L'angle entre $\vec{V}$ et (Oz) est : α

$tg\,\alpha = \frac{V_\theta}{V_z} = \frac{R\cancel{\omega}}{h\cancel{\omega}} = \frac{R}{h} = cste.$

3) On a : $\omega = cste \rightarrow \dot{\theta} = cste \Rightarrow \ddot{\theta} = 0 \Rightarrow \begin{cases} a_r = -R\omega^2 \\ a_\theta = 0 \\ a_z = 0 \end{cases}$

. $a_N = \frac{V^2}{\rho}$, $a_t = \frac{d\|\vec{V}\|}{dt}$

$a_t = \frac{d}{dt}\sqrt{h^2 + R^2}\,\omega = 0$, $a^2 = a_N^2 + \cancel{a_t^2} \Rightarrow a_N = a = \sqrt{a_r^2 + a_\theta^2 + a_z^2} = R\omega^2$

$\rho = \frac{V^2}{a_N} = \frac{(\omega\sqrt{R^2 + h^2})^2}{R\omega^2} \Rightarrow \boxed{\rho = \frac{R^2 + h^2}{R}}$ $\quad \rho = cste$.

. pour une trajectoire spirale : $\boxed{\rho = cste}$

Exo 8

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