Serie N°1 Rapp

Exercice 1 :

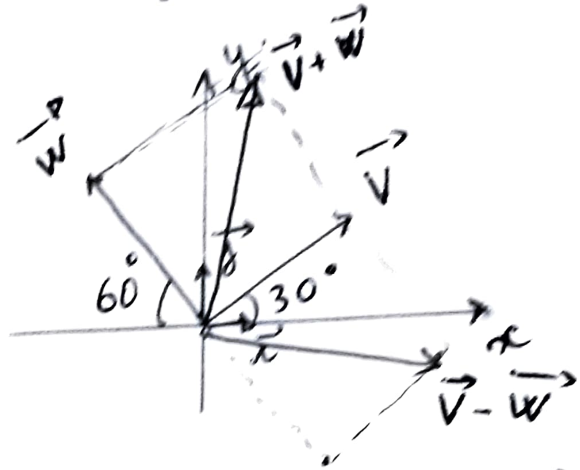
$\|\vec{F}\| = -20\,N$ Faux $\longrightarrow |\vec{F}| > 0$ donc $\|\vec{F}\| = +20\,N$

$\vec{V} = 30\,m/s$ Faux $\longrightarrow \|\vec{V}\| = 30\,m/s$

$\vec{F} = -5\vec{i}$ juste car la force $\vec{F}$ est un vecteur ✓

Si $\vec{F_1} + \vec{F_2} = \vec{0}$ alors $\vec{F_1} = -\vec{F_2}$ et $\|\vec{F_1}\| = \|\vec{F_2}\|$ juste ✓

Exercice 2 :



$\|\vec{V}\| = 3$

$\|\vec{W}\| = 5$

1) Les composante de $\vec{V}$ et $\vec{W}$:

$$\vec{V}\begin{pmatrix} V_x = V\cos 30° = 3\times\frac{\sqrt{3}}{2} \\ V_y = V\sin 30° = \frac{3\times 1}{2} \end{pmatrix} \quad \vec{V}\begin{pmatrix} \frac{3\sqrt{3}}{2} \\ 3/2 \end{pmatrix}$$

, $V = \|\vec{V}\|$ et $W = \|\vec{W}\|$

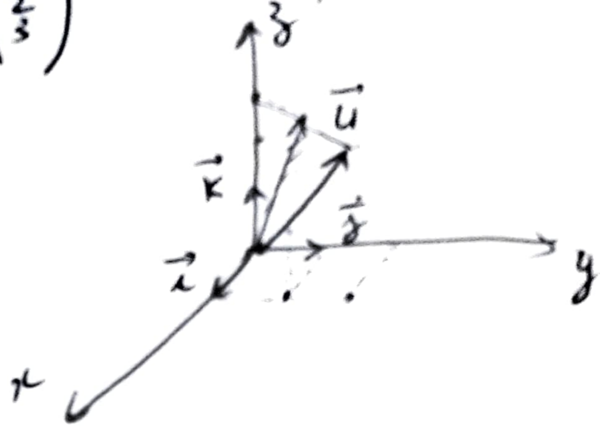
$$\vec{W}\begin{pmatrix} W_x = -W\cos 60° = -\frac{5\times 1}{2} \\ W_y = W\sin 60° = \frac{5\sqrt{3}}{2} \end{pmatrix} \quad \vec{W}\begin{pmatrix} \frac{-5}{2} \\ \frac{5\sqrt{3}}{2} \end{pmatrix}$$

2) • Les composantes de $\vec{V}+\vec{W}$

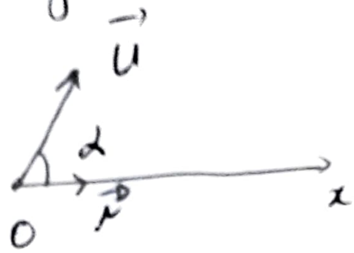
$$\vec{V}+\vec{W}\begin{pmatrix} V_x + W_x = \frac{3\sqrt{3}}{2} - \frac{5}{2} = \frac{3\sqrt{3}-5}{2} \\ W_y + V_y = \frac{3}{2} + \frac{5\sqrt{3}}{2} = \frac{3+5\sqrt{3}}{2} \end{pmatrix}$$

• Les composantes de $\vec{V}-\vec{W}$: $\vec{V}-\vec{W}\Big(V_x - W_x = \frac{3\sqrt{3}+5}{2}$

Exercice 3 : $\vec{u} = \vec{i} + 2\vec{j} + 3\vec{k}$ $\quad \vec{u}\begin{pmatrix}1\\2\\3\end{pmatrix}$



L'angle que fait $\vec{u}$ avec l'axe (Ox)

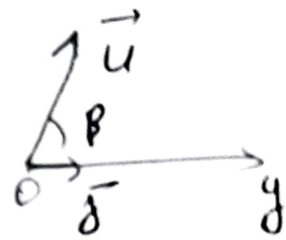


$\vec{i}\begin{pmatrix}1\\0\\0\end{pmatrix}$ $\quad \vec{j}\begin{pmatrix}0\\1\\0\end{pmatrix}$ $\quad \vec{k}\begin{pmatrix}0\\0\\1\end{pmatrix}$

$\vec{u} \cdot \vec{i} = \|\vec{u}\| \times \|\vec{i}\| \cdot \cos\alpha = \|\vec{u}\| \cos\alpha \Rightarrow \cos\alpha = \dfrac{\vec{u} \cdot \vec{i}}{\|\vec{u}\|}$

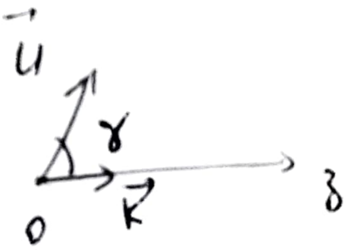
$\|\vec{u}\| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$, $\vec{u} \cdot \vec{i} = 1 \Rightarrow \cos\alpha = \dfrac{1}{\sqrt{14}}$

$\Rightarrow \alpha = 74{,}5^\circ$



. L'angle que fait $\vec{u}$ avec l'axe (Oy) : β

$\cos\beta = \dfrac{\vec{u} \cdot \vec{j}}{\|\vec{u}\|} = \dfrac{2}{\sqrt{14}} \Rightarrow \beta = 57{,}68^\circ$

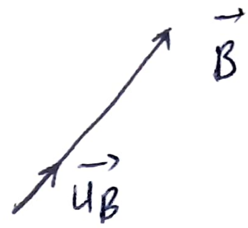


. L'angle que fait $\vec{u}$ avec l'axe (Oz) : γ

$\cos\gamma = \dfrac{\vec{u} \cdot \vec{k}}{\|\vec{u}\|} = \dfrac{3}{\sqrt{14}} \Rightarrow \gamma = 36{,}7^\circ$

Exercice 4 : $\vec{A} = \sqrt{3}\,\vec{i} + \vec{j}$ $\quad \vec{A}\begin{pmatrix}\sqrt{3}\\1\end{pmatrix}$

$\vec{B} = \vec{i} + \sqrt{3}\,\vec{j}$ $\quad \vec{B}\begin{pmatrix}1\\\sqrt{3}\end{pmatrix}$

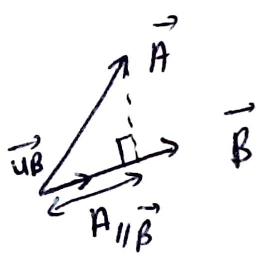


Le vecteur unitaire (شعاع الوحدة) parallèle (يوازي) à $\vec{B}$ est donné par $\vec{u_B}$

$\cancel{\vec{U_B} = \frac{\vec{B}}{\|\vec{B_B}\|}}$ $\quad \vec{U_B} = \frac{\vec{B}}{\|\vec{B}\|}$, $\|\vec{B}\| = \sqrt{1^2 + \sqrt{3}^2} = 2$

$\vec{U_B} = \frac{\vec{B}}{2} = \frac{1}{2}\vec{i} + \frac{\sqrt{3}}{2}\vec{j}$. $\quad \vec{U_B}\begin{pmatrix}\frac{1}{2}\\\frac{\sqrt{3}}{2}\end{pmatrix}$

- La composante de $\vec{A}$ parallèle au vecteur $\vec{B}$: $A_{/\!/\vec{B}}$



$$\boxed{A_{/\!/\vec{B}} = \vec{A} \cdot \vec{u_B} = \|\vec{A}\| \cos(\vec{A}, \vec{u_B})}$$

$$\boxed{A_{/\!/\vec{B}} = \sqrt{3} \times 1 + 1 \times \sqrt{3} = 2\sqrt{3}}$$

$A_{/\!/\vec{B}} \equiv$ la projection (إسقاط) de $\vec{A}$ sur $\vec{B}$.

Exercice 5 : $\vec{A}\begin{pmatrix}-2\\1\\3\end{pmatrix}$ $\quad \vec{B}\begin{pmatrix}2\\-1\\1\end{pmatrix}$ $\quad \vec{C}\begin{pmatrix}x\\1\\z\end{pmatrix}$

a) $\vec{C} /\!/ \vec{A} \Rightarrow \vec{C} \wedge \vec{A} = 0$.

$$\vec{C} \wedge \vec{A} = \begin{array}{c|ccc|} & \vec{i} & \vec{j} & \vec{k} \\ \vec{C} & x & 1 & z \\ \vec{A} & -2 & 1 & 3 \end{array} = \vec{i}(3-z) - \vec{j}(3x+2z) + \vec{k}(x+2)$$

$$\vec{C} \wedge \vec{A} = \vec{0} \Rightarrow \begin{cases} 3 - z = 0 \\ 3x + 2z = 0 \\ x + 2 = 0 \end{cases} \Rightarrow \boxed{z = 3} \quad \boxed{x = -2}$$

b) $\vec{C} \parallel \vec{B} \Rightarrow \vec{C} \wedge \vec{B} = 0 \Rightarrow$

$$\begin{array}{c|ccc|} & \vec{i} & \vec{j} & \vec{k} \\ \vec{C} & x & 1 & z \\ \vec{B} & 2 & -1 & 1 \end{array} = \vec{i}(1+z) - \vec{j}(x-2z) + \vec{k}(-x-2) = 0$$

$$\Rightarrow \begin{cases} 1+z=0 \\ x-2z=0 \\ -x-2=0 \end{cases} \Rightarrow \boxed{x=-2} \quad \boxed{z=-1}$$

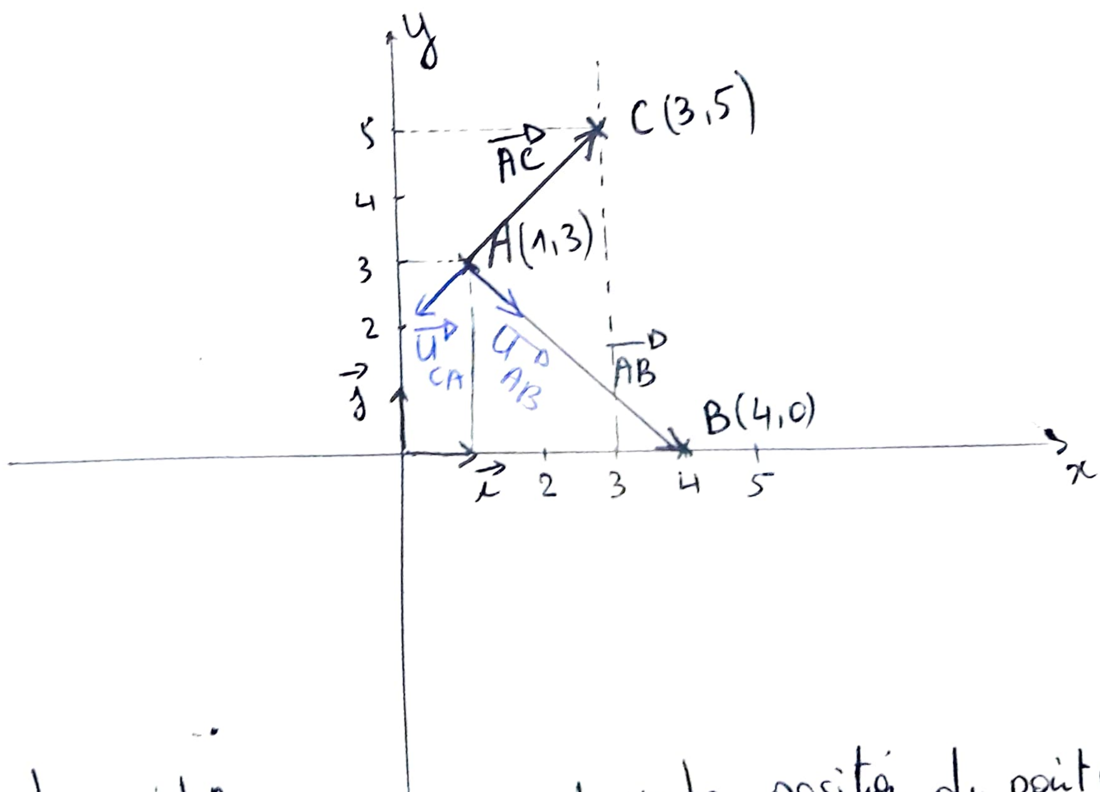
c) $\vec{C} \perp \vec{A}$ et $\vec{C} \perp \vec{B} \Rightarrow \vec{C} \cdot \vec{A} = 0$ et $\vec{C} \cdot \vec{B} = 0$

$$\Rightarrow \begin{cases} 2x - 1 + z = 0 & — ① \\ -2x + 1 + 3z = 0 & — ② \end{cases}$$

① $\Rightarrow z = 1 - 2x$ on remplace dans ② $\Rightarrow -2x + 1 + 3(1-2x) = 0$

$\Rightarrow -2x + 1 + 3 - 6x = 0 \Rightarrow 8x = 4 \Rightarrow \boxed{x = \frac{1}{2}}$, $\boxed{z = 0}$

Exercice 6



- Le vecteur position du point A :

$\vec{OA} = x_A \vec{i} + y_A \vec{j} = \vec{i} + 3\vec{j}$, $\vec{OA} \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

- Le vecteur position du point B : $\vec{OB} \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

$\vec{OB} = 4\vec{i} + 0\vec{j}$

- Le vecteur position du point C :

$\vec{OC} = 3\vec{i} + 5\vec{j}$

$\vec{OC} \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

- Les composantes des vecteurs :

$\vec{AB}\begin{pmatrix} x_B - x_A \\ y_B - y_A \end{pmatrix}$, $\vec{AB}\begin{pmatrix} 4-1 \\ 0-3 \end{pmatrix}$, $\vec{AB}\begin{pmatrix} 3 \\ -3 \end{pmatrix}$

$\vec{BC}\begin{pmatrix} x_C - x_B \\ y_C - y_B \end{pmatrix}$ $\vec{BC}\begin{pmatrix} 3-4 \\ 5-0 \end{pmatrix}$, $\vec{BC}\begin{pmatrix} -1 \\ 5 \end{pmatrix}$

$\vec{AC}\begin{pmatrix} x_C - x_A \\ y_C - y_A \end{pmatrix}$, $\vec{AC}\begin{pmatrix} 3-1 \\ 5-3 \end{pmatrix}$, $\vec{AC}\begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$(\vec{AB} + \vec{CB}) = \vec{AB} - \vec{BC} = (3\vec{i} - 3\vec{j}) - (-\vec{i} + 5\vec{j}) = 4\vec{i} - 7\vec{j}$

$(\vec{AB} + \vec{CB})\begin{pmatrix} 4 \\ -7 \end{pmatrix}$

$\vec{AB} - \vec{AC} = (3\vec{i} - 3\vec{j}) - (2\vec{i} + 2\vec{j}) = \vec{i} - 5\vec{j}$, $(\vec{AB} - \vec{AC})\begin{pmatrix} 1 \\ -5 \end{pmatrix}$

- les composantes des vecteurs unitaires :

- $\vec{U}_{AB} = \dfrac{\vec{AB}}{\|\vec{AB}\|}$

$\|\vec{AB}\| = \sqrt{3^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}$, $\vec{U}_{AB} = \dfrac{(3\vec{i} - 3\vec{j})}{3\sqrt{2}} = \dfrac{1}{\sqrt{2}}\vec{i} - \dfrac{1}{\sqrt{2}}\vec{j}$

$\vec{U}_{AB}\begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$

- $\vec{U}_{CA} = \dfrac{\vec{CA}}{\|\vec{CA}\|}$

$\vec{CA} = -\vec{AC}$, $\vec{CA}\begin{pmatrix} -2 \\ -2 \end{pmatrix}$

$\|\vec{CA}\| = \sqrt{(-2)^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$

$\vec{U}_{CA} = \dfrac{-2\vec{i} - 2\vec{j}}{2\sqrt{2}}$ $\vec{U}_{CA}\begin{pmatrix} \dfrac{-1}{\sqrt{2}} \\ \dfrac{-1}{\sqrt{2}} \end{pmatrix}$

• Les modules :

$\|\vec{OB}\| = \sqrt{4^2+0^2} = 2$, $\|\vec{BC}\| = \sqrt{(-1)^2+5^2} = \sqrt{26}$.

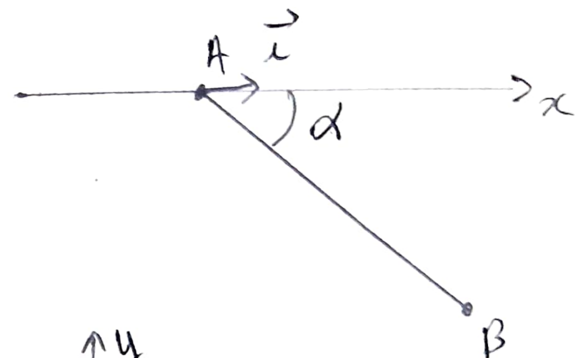
$\|\vec{AB} - \vec{AC}\| = \sqrt{1^2+(-5)^2} = \sqrt{26}$.

• L'angle que fait $\vec{AB}$ avec l'axe des x : $= \alpha$

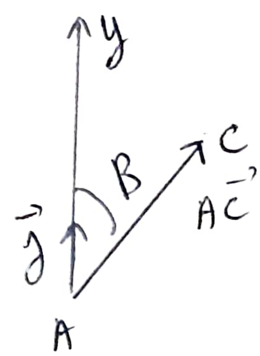
$\vec{AB} \cdot \vec{i} = \|\vec{AB}\| \cos\alpha \Rightarrow \cos\alpha = \dfrac{\vec{AB}\cdot\vec{i}}{\|\vec{AB}\|}$

$\|\vec{AB}\| = \sqrt{3^2+3^2} = 3\sqrt{2}$, $\vec{AB}\cdot\vec{i} = 3$.

$\Rightarrow \cos\alpha = \dfrac{\cancel{3}}{\cancel{3}\sqrt{2}} = \dfrac{\sqrt{2}}{2} \Rightarrow \boxed{\alpha = 45^\circ}$



• L'angle que fait $\vec{AC}$ avec l'axe des $y \equiv \beta$



$\vec{AC}\cdot\vec{j} = \|\vec{AC}\| \cdot \cos\beta$

$\cos\beta = \dfrac{\vec{AC}\cdot\vec{j}}{\|\vec{AC}\|} = \dfrac{2}{\sqrt{2^2+2^2}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$\Rightarrow \boxed{\beta = 45^\circ}$

• $\vec{AB} \perp \vec{AC} \Rightarrow \vec{AB}\cdot\vec{AC} = 0$.

On a $\vec{AB}\cdot\vec{AC} = 3\times 2 + (-3)\times 2 = 0 \Rightarrow \vec{AB} \perp \vec{AC}$

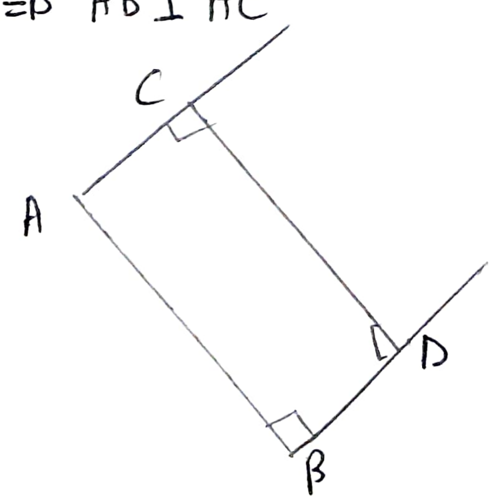
• ABCD forment un rectangle ssi

$\Rightarrow \vec{BD}\cdot\vec{AB} = 0$

et $\|\vec{BD}\| = \|\vec{AC}\|$

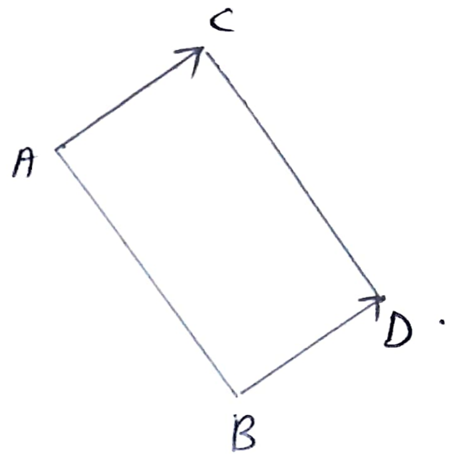
Soit x_D et y_D les coordonnées du point D

~~$D(D_x, D_y)$~~ $D(x_D, y_D)$



- ABCD forment un rectangle $\Rightarrow \overrightarrow{AC} = \overrightarrow{BD}$

مستطيل



$\overrightarrow{AC}\begin{pmatrix}2\\2\end{pmatrix}$, $D(x_D, y_D)$

$\overrightarrow{BD}\begin{pmatrix}x_D - x_B\\ y_D - y_B\end{pmatrix}$, $\overrightarrow{BD}\begin{pmatrix}x_D - 4\\ y_D\end{pmatrix}$

$\overrightarrow{AC} = \overrightarrow{BD} \Rightarrow \begin{cases} x_D - 4 = 2 \Rightarrow \boxed{x_D = 6} \\ \boxed{y_D = 2} \end{cases}$

$\cdot D(6,2)$

<u>Exercice 7</u>

1) $U(x,y,z) = 3x^2yz^2 + 4x^3y^2 \cdot z$

$\overrightarrow{\text{Grad}}\, U = \frac{\partial U}{\partial x}\vec{i} + \frac{\partial U}{\partial y}\vec{j} + \frac{\partial U}{\partial z}\cdot\vec{K}$

$\frac{\partial U}{\partial x} = 6xyz^2 + 12x^2y^2z$, $\frac{\partial U}{\partial y} = 3x^2z^2 + 8x^3yz$

$\frac{\partial U}{\partial z} = 6x^2yz + 4x^3y^2$

$\overrightarrow{\text{Grad}}\, U = (6x + yz^2 + 12x^2y^2z)\vec{i} + (3x^2z^2 + 8x^3yz)\vec{j} + (6x^2yz + 4x^3y^2)\cdot\vec{K}$

2) $\vec{\varphi}(x,y,z) = y\vec{i} + x\vec{j} + \frac{x^2}{\sqrt{x^2+y^2}}\cdot\vec{K}$

Calculons $\overrightarrow{\text{rot}}\,\vec{\varphi}$

$$\vec{\varphi}\begin{pmatrix}\varphi_x\\ \varphi_y\\ \varphi_z\end{pmatrix} \qquad \varphi_x = y \quad , \quad \varphi_y = x \quad , \quad \varphi_z = \frac{x^2}{\sqrt{x^2+y^2}}$$

$$\cdot\ \vec{rot}\,\vec{\varphi} = \vec{\nabla}\wedge\vec{\varphi} \quad \text{tel que} \quad \vec{\nabla}\begin{pmatrix}\partial/\partial x\\ \partial/\partial y\\ \partial/\partial z\end{pmatrix}$$

$$\vec{rot}\,\vec{\varphi} = \begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{\partial}{\partial x} & \partial/\partial y & \partial/\partial z\\ \varphi_x & \varphi_y & \varphi_z\end{vmatrix} = \vec{i}\left(\frac{\partial\varphi_z}{\partial y} - \frac{\partial\varphi_y}{\partial z}\right) - \vec{j}\left(\frac{\partial\varphi_z}{\partial x} - \frac{\partial\varphi_x}{\partial z}\right) + \vec{k}\left(\frac{\partial\varphi_y}{\partial x} - \frac{\partial\varphi_x}{\partial y}\right)$$

$$\frac{\partial\varphi_z}{\partial y} = -\frac{x^2(\cancel{2}y)}{\cancel{2}\sqrt{x^2+y^2}\cdot\left(\sqrt{x^2+y^2}\right)^2}$$

$$= \frac{-x^2y}{\sqrt{x^2+y^2}\,(x^2+y^2)} = \frac{-x^2y}{(x^2+y^2)^{3/2}}$$

$$\frac{\partial\varphi_y}{\partial z} = 0 \quad , \quad \frac{\partial\varphi_z}{\partial x} = \left[2x\sqrt{x^2+y^2} - \frac{x^2\times\cancel{2}x}{\cancel{2}(x^2+y^2)^{1/2}}\right] \Big/ \left(\sqrt{x^2+y^2}\right)^2$$

$$= \frac{2x(x^2+y^2) - x^3}{(x^2+y^2)^{3/2}} = \frac{x^3+2xy^2}{(x^2+y^2)^{3/2}}$$

$$\frac{\partial\varphi_x}{\partial z} = 0 \quad , \quad \frac{\partial\varphi_y}{\partial x} = 1 \quad , \quad \frac{\partial\varphi_x}{\partial y} = 1$$

$$\vec{rot}\,\vec{\varphi} = \left(\frac{-x^2y}{(x^2+y^2)^{3/2}}\right)\vec{i} - \vec{j}\left(\frac{x^3+2xy^2}{(x^2+y^2)^{3/2}}\right) + \vec{k}\,\underbrace{(1-1)}_{0}$$

$$\boxed{\vec{rot}\,\vec{\varphi} = \frac{-x^2y}{(x^2+y^2)^{3/2}}\,\vec{i} - \frac{(x^3+2xy^2)}{(x^2+y^2)^{3/2}}\cdot\vec{j}}$$

Exo 7