Home assignment 2

Numerical Optimization and its Applications - Spring 2019

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1 The efficiency of different iterative methods for solving a linear system

(a) Following are the implementation for the four methods:
Jacobi:

```
from numpy import diag, matmul, array
from numpy.linalg import inv, norm


def weighted_jacobi(A, b, x_0, maxIter, epsilon, w):
    D = diag(diag(A))
    x = x_0
    res = [norm(matmul(A, x) - b)]
    for i in range(maxIter):
        x = x + w * matmul(inv(D), b - matmul(A, x))
        res.append(norm(matmul(A, x) - b))
        if res[-1] / norm(b) < epsilon:
            break
    return x, array(res)
```

Gauss-Seidel:

```python
from numpy import matmul, tril, array
from numpy.linalg import inv, norm


def weighted_gauss_seidel(A, b, x_0, maxIter, epsilon, w):
    L_D = tril(A, k=0)
    x = x_0
    res = [norm(matmul(A, x) - b)]
    for i in range(maxIter):
        x = x + w * matmul(inv(L_D), b - matmul(A, x))
        res.append(norm(matmul(A, x) - b))
        if res[-1] / norm(b) < epsilon:
            break
    return x, array(res)
```

Steepest Descent:

```python
from numpy import matmul, array, dot, copy
from numpy.linalg import norm
from py_files.utils import is_pos_def


def steepest_decent(A, b, x_0, max_iter, epsilon):
    if not is_pos_def(A):
        print("matrix is not SPD, can't solve using steepest decent...")
        return None, None
    x = copy(x_0)
    r = b - matmul(A, x)
    all_r = [norm(r)]
    for k in range(max_iter):
        alph = dot(r, r) / dot(r, matmul(A, r))
        x = x + alph * r
        # res.append(norm(matmul(A, x) - b))
        r = b - matmul(A, x)
        all_r.append(norm(r))
        if norm(r) / norm(b) < epsilon:
            break
    return x, array(all_r)
```

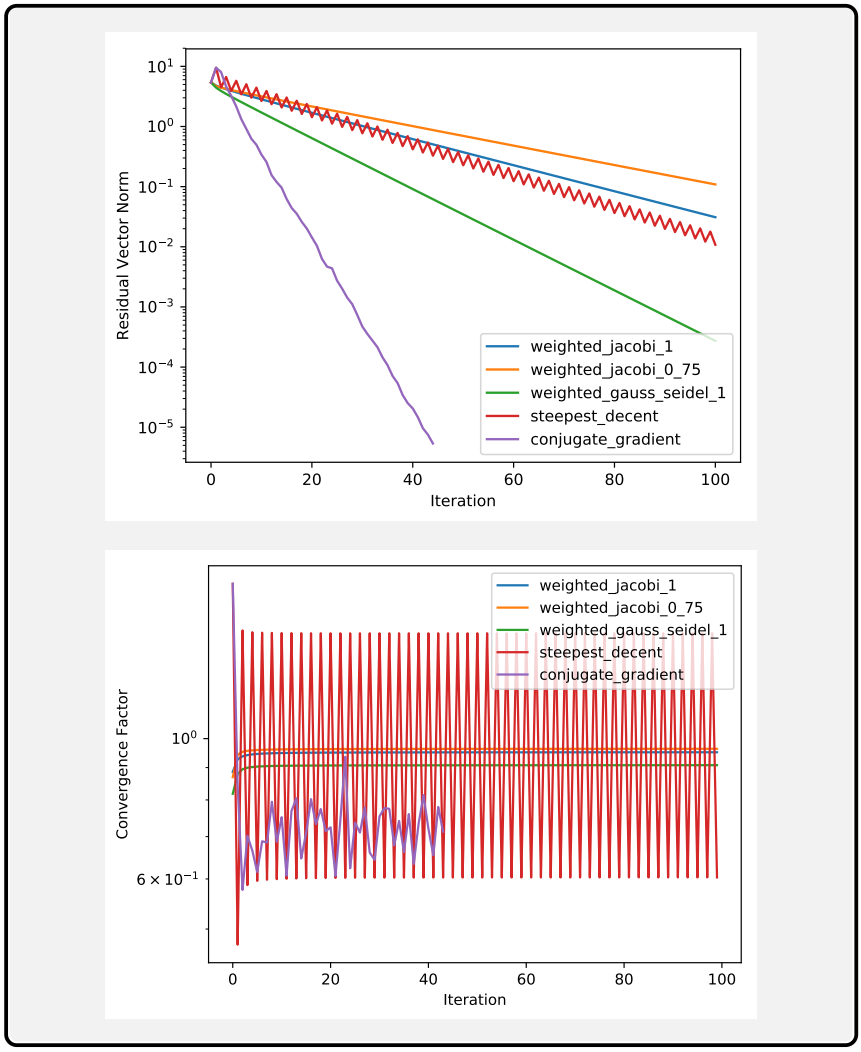
Conjugate Gradient

```python
from numpy import matmul, array, dot, copy
from numpy.linalg import norm
from py_files.utils import is_pos_def


def conjugate_gradient(A, b, x_0, max_iter, epsilon):
    if not is_pos_def(A):
        print("matrix is not SPD, can't solve using steepest decent...")
        return None, None
    x = copy(x_0)
    r = b - matmul(A, x)
    p = r
    all_r = [norm(r)]
    for k in range(max_iter):
        alph = dot(r, p) / dot(p, matmul(A, p))
        x = x + alph * p
        # res.append(norm(matmul(A, x) - b))
        r_prev = copy(r)
        r = b - matmul(A, x)
        all_r.append(norm(r))
        if norm(r) / norm(b) < epsilon:
            break
        beta = dot(r, r) / dot(r_prev, r_prev)
        p = r + beta * p
    return x, array(all_r)
```

(b) Following are the system and parameters definition, methods calls, residual vector norm and convergence factor plotting:

```python
import numpy as np
from scipy.sparse import spdiags
import matplotlib.pyplot as plt
from py_files.part_1_gauss_seidel import weighted_gauss_seidel
from py_files.part_1_jacobi import weighted_jacobi
from py_files.part_1_sd import steepest_decent
from py_files.part_1_cg import conjugate_gradient

n = 100
# TODO: not sure if .toarray() is the right approach
A = spdiags(np.array([-np.ones(n), 2.1 * np.ones(n), -np.ones(n)]),
            np.array([-1, 0, 1]), n, n).toarray()
x_0 = np.zeros(n)
b = np.random.rand(n)
```

```
maxIter = 100
epsilon = 1e-6

res = dict()
x, res['weighted_jacobi_1'] = weighted_jacobi(A, b, x_0, maxIter, epsilon, 1)
#print('weighted_jacobi_1 result:', x)
x, res['weighted_jacobi_0_75'] = weighted_jacobi(A, b, x_0, maxIter, epsilon, 0.75)
#print('weighted_jacobi_0_75 result:', x)
x, res['weighted_gauss_seidel_1'] = weighted_gauss_seidel(A, b, x_0, maxIter, epsilo
#print('weighted_gauss_seidel_1 result:', x)
x, res['steepest_decent'] = steepest_decent(A, b, x_0, maxIter, epsilon)
#print('steepest_decent result:', x)
x, res['conjugate_gradient'] = conjugate_gradient(A, b, x_0, maxIter, epsilon)
#print('conjugate_gradient result:', x)

convergence_factor = dict()
for alg_res in res:
    convergence_factor[alg_res] = res[alg_res][1:] / res[alg_res][:-1]

plt.figure()
for alg_res in res:
    plt.semilogy(res[alg_res], label=alg_res)
plt.legend()
plt.xlabel("Iteration")
plt.ylabel("Residual Vector Norm")

# Save the plot as .pdf and include it in the .tex document
plt.savefig("myplot1.pdf", bbox_inches="tight")
print(r"\saveandshowplot{myplot1.pdf}")

plt.figure()
for alg_con in convergence_factor:
    plt.semilogy(convergence_factor[alg_con], label=alg_con)
plt.legend()
plt.xlabel("Iteration")
plt.ylabel("Convergence Factor")

# Save the plot as .pdf and include it in the .tex document
plt.savefig("myplot2.pdf", bbox_inches="tight")
print(r"\saveandshowplot{myplot2.pdf}")
```



2 Convergence properties

(a) t

(b) t

(c) t

3 GMRES(1) method

(a) t

(b) t

(c) t

(d) t

(e) t

4 Convexity

(a) t

(b) t

(c) t

5 Non Linear Optimization

(a) t

(b) t

(c) t