

An Introduction to the Interval Template Library

Lecture held at the Boost Library Conference 2009

2009-05-08

Updated version 3.2.0 2009-12-02

Lecture Outline

- Background and Motivation
- Design
- Examples
- Semantics
- Implementation
- Future Works
- Availability

Background and Motivation

- Interval containers simplified the implementation of date and time related tasks
 - Decomposing "histories" of attributed events into segments with constant attributes.
 - Working with time grids, e.g. a grid of months.
 - Aggregations of values associated to date or time intervals.
- ... that occurred frequently in programs like
 - Billing modules
 - Therapy scheduling programs
 - Hospital and controlling statistics

Design

- Background is the date time problem domain ...
- ... but the scope of the ItI as a generic library is more general:

an interval_set is a set that is implemented as a set of intervals

an interval_map is a map that is implemented as a map of interval value pairs

Aspects

- There are two aspects in the design of interval containers
- Fundamental aspect

```
interval_set<int> mySet;
mySet.insert(42);
bool has_answer = mySet.contains(42);
```

- On the fundamental aspect an interval_set can be used just as a set of elements
- Set theoretic operations are supported
- Abstracts from sequential and segmental information
- Segmental aspect
 - Allows to access and iterate over the segments of interval containers

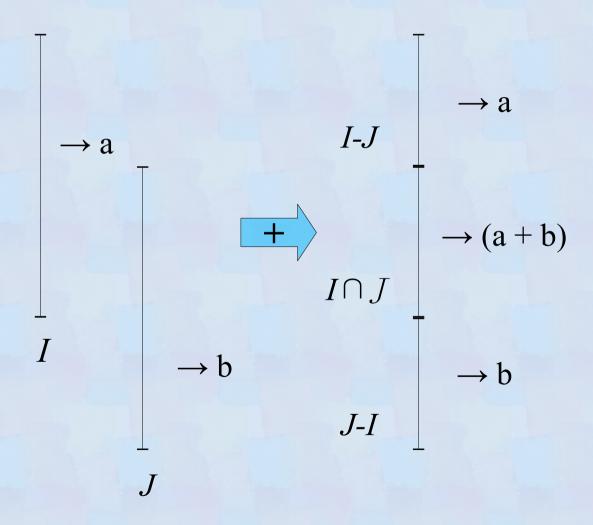
Design

- Addability and Subtractability
 - All of itl's (interval) containers are Addable and Subtractable
 - They implement operators +=, +, -= and -

	+=	-=
sets	set union	set difference
maps	?	?

- A possible implementation for maps
 - Propagate addition/subtraction to the associated values
 - ... or aggregate on overlap
 - ... or aggregate on collision

Aggregate on overlap



- Decompositional effect on Intervals
- Accumulative effect on associated values

I, J: intervals, a,b: associated values

Aggregate on overlap, a minimal example

```
typedef itl::set<string> guests;
interval_map<time, guests> party;

party += make_pair(
   interval<time>::rightopen(20:00, 22:00), guests("Mary"));

party += make_pair(
   interval<time>::rightopen(21:00, 23:00), guests("Harry"));

// party now contains
[20:00, 21:00)->{"Mary"}
[21:00, 22:00)->{"Harry","Mary"} //guest sets aggregated
[22:00, 23:00)->{"Harry"}
```

Design

The Itl's class templates

Granu- larity	Style	Sets	Maps
interval		interval	
	joining	interval_set	interval_map
	separating	separate_interval_set	
	splitting	split_interval_set	split_interval_map
element		set	map

- Interval Combining Styles: Joining
 - Intervals are joined on overlap or on touch
 - . . . for maps, if associated values are equal
 - Keeps interval_maps and sets in a minimal form

- Interval Combining Styles: Splitting
 - Intervals are split on overlap and kept separate on touch
 - All interval borders are preserved (insertion memory)

- Interval Combining Styles: Separating
 - Intervals are joined on overlap but kept separate on touch
 - Preserves borders that are never crossed (preserves a hidden grid).

Examples

A few instances of intervals (interval.cpp)

A way to iterate over months and weeks

```
(month and week grid.cpp)
```

```
#include <boost/itl/gregorian.hpp> //boost::gregorian plus adapter code
#include <boost/itl/split interval set.hpp>
// A split interval set of gregorian dates as date grid.
typedef split interval set < boost:: gregorian::date > date grid;
// Compute a date grid of months using boost::gregorian.
date grid month grid (const interval < date > & scope)
    date grid month grid;
    // Compute a date grid of months using boost::gregorian.
    return month grid;
// Compute a date grid of weeks using boost::gregorian.
date grid week grid(const interval < date > & scope)
    date grid week grid;
    // Compute a date grid of weeks using boost::gregorian.
    return week grid;
```

A way to iterate over months and weeks

```
void month and time grid()
    date someday = day clock::local day();
    date thenday = someday + months(2);
    interval < date > scope = interval < date > :: rightopen (someday, thenday);
    // An intersection of the month and week grids ...
    date grid month and week grid
        = month grid(scope) \overline{\&} week grid(scope);
    // ... allows to iterate months and weeks. Whenever a month
    // or a week changes there is a new interval.
    for(date grid::iterator it = month and week grid.begin();
        it != month and week grid.end(); it++)
    // We can also intersect the grid into an interval map to make
    // shure that all intervals are within months and week bounds.
    interval map<boost::gregorian::date, some type> accrual;
    compute some result (accrual, scope);
    accrual &= month and week grid;
```

- Aggregating with interval_maps
 - Computing averages via implementing operator +=
 (partys guest average.cpp)

```
class counted sum
public:
   counted sum(): sum(0), count(0){}
   counted sum(int sum): sum(sum), count(1){}
   int sum()const {return sum;}
   int count()const{return count;}
   double average()const
    { return count==0 ? 0.0 : sum/static cast<double>( count); }
   counted sum& operator += (const counted sum& right)
   { _sum += right.sum(); _count += right.count(); return *this; }
private:
   int sum;
   int count;
};
bool operator == (const counted sum& left, const counted sum& right)
{ return left.sum() == right.sum() && left.count() == right.count(); }
```

- Aggregating with interval_maps
 - Computing averages via implementing operator +=

```
void partys height average()
    interval map<ptime, counted sum> height sums;
    height sums += (
      make pair (
        interval interval ptime>::rightopen(
          time from string("2008-05-20 19:30"),
          time from string("2008-05-20 23:00")),
          counted sum(165)) // Mary is 1,65 m tall.
    );
    // Add height of more pary quests . . .
    interval map<ptime, counted sum>::iterator height sum =
        height sums.begin();
    while(height sum != height sums.end())
        interval<ptime> when = height sum ->first;
        double height average = (*height sum ++).second.average();
        cout << "[" << when.first() << " - " << when.upper() << ")"
             << ": " << height average << " cm" << endl;
```

- Interval containers allow to express a variety of date and time operations in an easy way.
 - Example man_power.cpp ...
 - Subtract weekends and holidays from an interval_set worktime -= weekends(scope) worktime -= german reunification day
 - Intersect an interval_map with an interval_set
 claudias_working_hours &= worktime
 - Subtract and interval_set from an interval map claudias_working_hours -= claudias_absense_times
 - Adding interval_maps
 interval_map<date,int> manpower;
 manpower += claudias_working_hours;
 manpower += bodos working hours;

Interval_maps can also be intersected Example user_groups.cpp

```
typedef boost::itl::set<string> MemberSetT;
typedef interval map<date, MemberSetT> MembershipT;
void user groups()
    MembershipT med users;
    // Compute membership of medical staff
   med users += make pair(member interval 1, MemberSetT("Dr.Jekyll"));
    med users += . . .
    MembershipT admin users;
    // Compute membership of administation staff
    med users += make pair(member interval 2, MemberSetT("Mr.Hyde"));
    MembershipT all users = med users + admin users;
    MembershipT super users = med users ← admin users;
```

The semantics of itl sets is based on a concept itl::Set

itl::set, interval_set, split_interval_set
and separate_interval_set are models of concept
itl::Set

```
// Abstract part
empty set:
             Set::Set()
subset relation: bool Set::contained in(const Set& s2)const
           bool is element equal(const Set& s1, const Set& s2)
equality:
set union:
                 Set& operator += (Set& s1, const Set& s2)
                 Set operator + (const Set& s1, const Set& s2)
set difference: Set& operator -= (Set& s1, const Set& s2)
                 Set operator - (const Set& s1, const Set& s2)
set intersection: Set& operator &= (Set& s1, const Set& s2)
                 Set operator & (const Set& s1, const Set& s2)
// Part related to sequential ordering
sorting order: bool operator < (const Set& s1, const Set& s2)
lexicographical equality:
                 bool operator == (const Set& s1, const Set& s2)
```

The semantics of itl maps is based on a concept itl:: Map

```
itl::map, interval_map and split_interval_map
are models of concept
itl::Map
```

```
// Abstract part
empty map:
                 Map::Map()
submap relation: bool Map::contained in(const Map& m2)const
                 bool is element equal (const Map& m1, const Map& m2)
equality:
map union:
                 Map& operator += (Map& m1, const Map& m2)
                 Map operator + (const Map& m1, const Map& m2)
map difference:
                Map& operator -= (Map& m1, const Map& m2)
                 Map operator - (const Map& m1, const Map& m2)
map intersection: Map& operator &= (Map& m1, const Map& m2)
                 Map operator & (const Map& m1, const Map& m2)
// Part related to sequential ordering
sorting order: bool operator < (const Map& m1, const Map& m2)
lexicographical equality:
                 bool operator == (const Map& m1, const Map& m2)
```

- Defining semantics of itl concepts via sets of laws
 - aka c++0x axioms
- Checking law sets via automatic testing:
 - A Law Based Test Automaton LaBatea



Generate law instance



apply law to instance



collect violations

Commutativity<T a, U b, +>: a + b = b + a;

- Lexicographical Ordering and Equality
 - For all itl containers operator < implements a strict weak ordering.</p>
 - The induced equivalence of this ordering is lexicographical equality which is implemented as operator ==
 - This is in line with the semantics of SortedAssociativeContainers

- Subset Ordering and Element Equality
 - For all itl containers function contained_in implements a partial ordering.
 - The induced equivalence of this ordering is equality of elements which is implemented as function is element equal.

- itl::Sets
- All itl sets implement a Set Algebra, which is to say satisfy a "classical" set of laws . . .
 - ... using is_element_equal as equality
 - Associativity, Neutrality, Commutativity (for + and &)
 - Distributivity, DeMorgan, Symmetric Difference
- Most of the itl sets satisfy the classical set of laws even if . . .
 - Iexicographical equality: operator == is used
 - The differences reflect proper inequalities in sequence that occur for separate_interval_set and split_interval_set.

- Concept Induction / Concept Transition
 - The semantics of itl::Maps appears to be determined by the codomain type of the map

```
is model of if example

Map<D,Monoid> Monoid interval_map<int,string>
Map<D,Set> Set C1 interval_map<int,set<int>>
Map<D,CommutMonoid> CommutMonoid interval_map<int,unsigned>
Map<D,AbelianGroup> AbelianGroup C2 interval_map<int,int,total>
```

- Conditions C1 and C2 restrict the Concept Induction to specific map traits
 - C1: Value pairs that carry a neutral element as associated value are always deleted (Trait: absorbs_neutrons).
 - C2: The map is total: Non existing keys are implicitly mapped to neutral elements (Trait: is total).

Implementation

- Itl containers are implemented based on std::set and std::map
 - Basic operations like adding and subtracting intervals or interval value pairs perform with a time complexity between* amortized O(log n) and O(n), where n is the number of intervals of a container.
 - Operations like addition and subtraction of whole containers are having a worst case complexity of O(m log(n+m)), where n and m are the numbers of intervals of the containers to combine.
 - * : Consult the library documentation for more detailed information.

Future Works

- Implementing interval maps of sets more efficiently
- Revision of features of the extended itl (itl plus.zip)
 - Decomposition of histories: k histories h, with attribute types $A_1, ..., A_k$ are "decomposed" to a product history of tuples of attribute sets:

$$(h_1 < T, A_1 >, ..., h < T, A_k >) \rightarrow h < T, (set < A_1 >, ..., set < A_k >) >$$

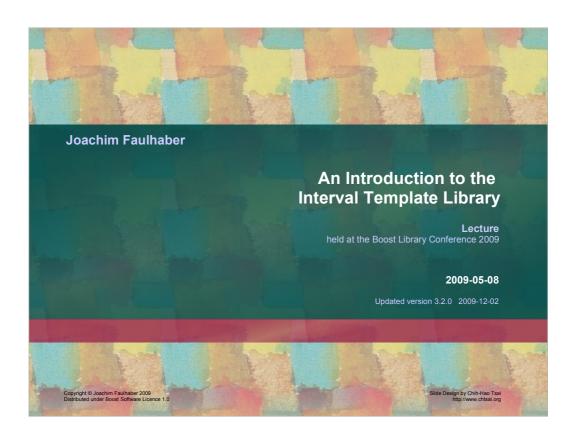
Cubes (generalized crosstables): Applying aggregate on collision to maps of tuple value pairs in order to organize hierachical data and their aggregates.

Availability

- Itl project on sourceforge (version 2.0.1) http://sourceforge.net/projects/itl
- Latest version on boost vault/Containers (3.1.0) http://www.boostpro.com/vault/ → containers
 - itl_3_1_0.zip : Core itl in preparation for boost
 - itl_plus_3_1_0.zip : Extended itl including histories, cubes and automatic validation (LaBatea).
- Online documentation at http://www.herold-faulhaber.de/
 - Doxygen generated docs for (version 2.0.1) http://www.herold-faulhaber.de/itl/
 - Latest boost style documentation (version 3.1.0) http://www.herold-faulhaber.de/boost_itl/doc/libs/itl/doc/html/

Availability

- Boost sandbox https://svn.boost.org/svn/boost/sandbox/itl/
 - Core itl: Interval containers in prepartion for boost https://svn.boost.org/svn/boost/sandbox/itl/boost/itl/ https://svn.boost.org/svn/boost/sandbox/itl/libs/itl/
 - Extended itl_xt: "histories" and cubes https://svn.boost.org/svn/boost/sandbox/itl/boost/itl_xt/ https://svn.boost.org/svn/boost/sandbox/itl/libs/itl_xt/
 - Validater LaBatea: Compiles with msvc-8.0 or newer, gcc-4.3.2 or newer https://svn.boost.org/svn/boost/sandbox/itl/boost/validate/ https://svn.boost.org/svn/boost/sandbox/itl/libs/validate/



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Background and Motivation

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 - Decomposing "histories" of attributed events into segments with constant attributes.
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 - Billing modules
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Design

- Background is the date time problem domain ...
- ... but the scope of the ItI as a generic library is more general:

an interval_set is a set

that is implemented as a set of intervals

an interval_map is a map

that is implemented as a map of interval value pairs

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Aspects

- There are two aspects in the design of interval containers
- Fundamental aspect

```
interval_set<int> mySet;
mySet.insert(42);
bool has_answer = mySet.contains(42);
```

- On the fundamental aspect an interval_set can be used just as a set of elements
- Set theoretic operations are supported
- Abstracts from sequential and segmental information
- Segmental aspect
 - Allows to access and iterate over the segments of interval containers

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Design

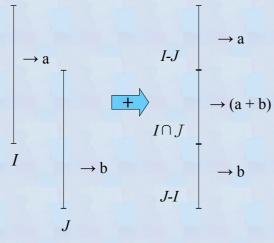
- Addability and Subtractability
 - All of itl's (interval) containers are Addable and Subtractable
 - They implement operators +=, +, -= and -

	+=	-=
sets	set union	set difference
maps	?	?

- A possible implementation for maps
 - Propagate addition/subtraction to the associated values
 - . . . or aggregate on overlap
 - . . . or aggregate on collision

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Aggregate on overlap



- Decompositional effect on Intervals
- Accumulative effect on associated values

I, J: intervals, a,b: associated values

Aggregate on overlap, a minimal example

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```
typedef itl::set<string> guests;
interval_map<time, guests> party;

party += make_pair(
   interval<time>::rightopen(20:00, 22:00), guests("Mary"));

party += make_pair(
   interval<time>::rightopen(21:00, 23:00), guests("Harry"));

// party now contains
[20:00, 21:00) ->{"Mary"}
[21:00, 22:00) ->{"Harry", "Mary"} //guest sets aggregated
[22:00, 23:00) ->{"Harry"}
```

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The Iti's class templates

Granu- larity	Style	Sets	Maps
interval		interval	
	joining	interval_set	interval_map
	separating	separate_interval_set	
	splitting	split_interval_set	split_interval_map
element		set	map

- Interval Combining Styles: Joining
 - Intervals are joined on overlap or on touch
 - . . . for maps, if associated values are equal
 - Keeps interval_maps and sets in a minimal form

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```
interval_map

{ [1      3) ->1      }
+      [2      4) ->1
+      [4     5) ->1

={ [1     2) [2     3) [3     4)      }
      ->1      ->2      ->1

={ [1     2) [2     3) [3      5)     }
      ->1      ->2      ->1
```

- Interval Combining Styles: Splitting
 - Intervals are split on overlap and kept separate on touch
 - All interval borders are preserved (insertion memory)

```
split_interval_set

{[1     3)     }
+     [2     4)
+     [4 5)

= {[1 2)[2 3)[3 4)     }

= {[1 2)[2 3)[3 4)[4 5)}
```

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```
split_interval_map

{ [1      3) ->1      }
+      [2      4) ->1
+      [4     5) ->1

={ [1     2) [2     3) [3     4)      }
->1     ->2     ->1

={ [1     2) [2     3) [3     4) [4     5)     }
->1     ->2     ->1     ->1
```

- Interval Combining Styles: Separating
 - Intervals are joined on overlap but kept separate on touch
 - Preserves borders that are never crossed (preserves a hidden grid).

A few instances of intervals (interval.cpp)

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A way to iterate over months and weeks (month and week grid.cpp)

A way to iterate over months and weeks

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Aggregating with interval_maps

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Computing averages via implementing operator +=
 (partys guest average.cpp)

```
class counted_sum {
    public:
        counted_sum():_sum(0),_count(0) {}
        counted_sum(int sum):_sum(sum),_count(1) {}

    int sum()const {return _sum;}
    int count()const{return _count;}
    double average()const
        { return _count==0 ? 0.0 : _sum/static_cast<double>(_count); }

        counted_sum& operator += (const counted_sum& right)
        { _sum += right.sum(); _count += right.count(); return *this; }

private:
    int _sum;
    int _count;
};

bool operator == (const counted_sum& left, const counted_sum& right)
{ return left.sum()==right.sum() && left.count()==right.count(); }
```

Aggregating with interval_maps

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Computing averages via implementing operator +=

- Interval containers allow to express a variety of date and time operations in an easy way.
 - Example man power.cpp ...
 - Subtract weekends and holidays from an interval_set worktime -= weekends (scope) worktime -= german reunification day
 - Intersect an interval_map with an interval_set claudias_working_hours &= worktime
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 - Adding interval_maps
 interval_map<date,int> manpower;
 manpower += claudias_working_hours;
 manpower += bodos_working_hours;

Interval_maps can also be intersected Example user_groups.cpp

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```
typedef boost::itl::set<string> MemberSetT;
typedef interval_map<date, MemberSetT> MembershipT;

void user_groups()
{
     . . .
     MembershipT med_users;
     // Compute membership of medical staff
     med_users += make_pair(member_interval_1, MemberSetT("Dr.Jekyll"));
     med_users += . .

     MembershipT admin_users;
     // Compute membership of administation staff
     med_users += make_pair(member_interval_2, MemberSetT("Mr.Hyde"));
     . . .

     MembershipT all_users = med_users + admin_users;
     MembershipT super_users = med_users & admin_users;
     . . .
}
```

- The semantics of itl sets is based on a concept itl::Set
 - itl::set, interval_set, split_interval_set
 and separate_interval_set are models of concept
 itl::Set

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- The semantics of itl maps is based on a concept itl:: Map
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- Defining semantics of itl concepts via sets of laws
 - aka c++0x axioms
- Checking law sets via automatic testing:
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Generate law instance

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apply law to instance



collect violations

Commutativity<T a, U b, +>: a + b = b + a;

- Lexicographical Ordering and Equality
 - For all itl containers operator < implements a strict weak ordering.
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 - This is in line with the semantics of SortedAssociativeContainers

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- Subset Ordering and Element Equality
 - For all itl containers function contained_in implements a partial ordering.
 - The *induced equivalence* of this ordering is *equality of elements* which is implemented as function is_element_equal.

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- itl::Sets
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 - ...using is_element_equal as equality
 - Associativity, Neutrality, Commutativity (for + and &)
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Concept Induction / Concept Transition

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■ The semantics of itl::Maps appears to be determined by the codomain type of the map

```
is model of if example

Map<D,Monoid> Monoid interval_map<int,string>

Map<D,Set> Set C1 interval_map<int,set<int>>

Map<D,CommutMonoid> CommutMonoid interval_map<int,unsigned>

Map<D,AbelianGroup> AbelianGroup C2 interval_map<int,int,total>
```

- Conditions C1 and C2 restrict the Concept Induction to specific map traits
 - C1: Value pairs that carry a neutral element as associated value are always deleted (Trait: absorbs_neutrons).
 - c2: The map is total: Non existing keys are implicitly mapped to neutral elements (Trait: is_total).

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Future Works

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- Revision of features of the extended itl (itl_plus.zip)
 - Decomposition of histories: k histories h_k with attribute types $A_1, ..., A_k$ are "decomposed" to a product history of tuples of attribute sets:

 $(h_1 < T, A_1 >, ..., h < T, A_k >) \rightarrow h < T, (set < A_1 >, ..., set < A_k >) >$

Cubes (generalized crosstables): Applying aggregate on collision to maps of tuple value pairs in order to organize hierachical data and their aggregates.

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- Online documentation at http://www.herold-faulhaber.de/
 - Doxygen generated docs for (version 2.0.1) http://www.herold-faulhaber.de/itl/
 - Latest boost style documentation (version 3.1.0) http://www.herold-faulhaber.de/boost_itl/doc/libs/itl/doc/html/

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Availability

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 - Extended itl_xt: "histories" and cubes https://svn.boost.org/svn/boost/sandbox/itl/boost/itl_xt/ https://svn.boost.org/svn/boost/sandbox/itl/libs/itl_xt/
 - Validater LaBatea: Compiles with msvc-8.0 or newer, gcc-4.3.2 or newer https://svn.boost.org/svn/boost/sandbox/itl/boost/validate/ https://svn.boost.org/svn/boost/sandbox/itl/libs/validate/