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### **Abstract**

It is proposed to add to the C++ standard **optional floating-point typedefs having specified widths**. The optional typedefs include float16\_t, float32\_t, float64\_t, float128\_t, their corresponding least and fast types, and the corresponding maximum-width type. These are to conform with the corresponding specifications of binary16, binary32, binary64, and binary128 in IEEE\_floating-point format.

The optional floating-point typedefs having specified widths are to be contained in a **new standard library header <cstdfloat>**.

They will be defined in the std namespace.

New C-style macros to facilitate initialization of the optional floating-point typedefs having specified widths from floating-point literal constants are proposed.

It is not proposed to make any mandatory changes to <cmath>, special functions, <limits>, or <complex>.

The main objectives of this proposal are to:

- Extend the benefits of specified-width typedefs for integer types to floating-point types.
- Improve floating-point safety and reliability by providing standardized typedefs that behave identically on all platforms.
- Optionally extend the range of floating-point to lower and to higher precision.
- Provide a Standard way of specifying 128-bit precision.



### Introduction

Since the inceptions of C and C++, the built-in types float, double, and long double have provided a strong basis for floating-point calculations. Optional compiler conformance with IEEE\_ floating-point format has generally led to a relatively reliable and portable environment for floating-point calculations in the programming community.

Support for mathematical facilities and specialized number types in C++ is progressing rapidly. Currently, C++11 supports floating-point calculations with its built-in types float, double, and long double as well as implementations of numerous elementary and transcendental functions.

A variety of higher transcendental functions of pure and applied mathematics were added to the C++11 libraries via technical report TR1. It is now proposed to fix these into the next C++1Y standard.<sup>1</sup>

Other mathematical special functions are also now proposed, for example, A proposal to add special mathematical functions according to the ISO/IEC 80000-2:2009 standard Document number: N3494 Version: 1.0 Date: 2012-12-19

It is, however, emphasized that floating-point adherence to IEEE\_ floating-point format is not mandated by the current C++ language standard. Nor does the standard specify the widths, precisions and layouts of its built-in types float, double, and long double. This can lead to portability problems, introduce poor efficiency on cost-sensitive microcontroller architectures, and reduce reliability and safety.

This situation reveals a need for a standard way to specify floating-point precision in C++.

Providing optional floating-point typedefs having specified widths is expected to significantly improve portability, reliability, and safety of floating-point calculations in C++. Analogous improvements for integer calculations were recently achieved via standardization of integer types having specified widths such as int8\_t, int16\_t, int32\_t, and int64\_t.



<sup>&</sup>lt;sup>1</sup> Conditionally-supported Special Math Functions for C++14, N3584, Walter E. Brown

### The proposed typedefs and potential extensions

The core of this proposal is based on the optional floating-point typedefs float16\_t, float32\_t, float64\_t, float128\_t, their corresponding least and fast types, and the corresponding maximum-width type.

In particular,

```
// Sample partial synopsis of <cstdfloat>
namespace std
{
  typedef float      float32_t;
  typedef double      float64_t;
  typedef long double float128_t;
  typedef float128_t floatmax_t;

  // ... and the corresponding least and fast types.
}
```

These proposed optional floating-point typedefs are to conform with the corresponding specifications of binary16, binary32, binary64, and binary128 in IEEE\_floating-point format. In particular, float16\_t, float32\_t, float64\_t, and float128\_t correspond to floating-point types with 11, 24, 53, and 113 binary significand digits, respectively. These are defined in IEEE\_floating-point format, and there are more detailed descriptions of each type at IEEE half-precision floating-point format, IEEE single-precision floating-point format, IEEE double-precision floating-point format, Quadruple-precision floating-point format, and IEEE 754 extended precision formats and x86 80-bit Extended Precision Format.

Here, we specifically mean equivalence of the following.

```
float16_t == binary16;
float32_t == binary32;
float64_t == binary64;
float128_t == binary128;
```

This equivalence results in far-reaching benefits.

It means that floating-point software written with float16\_t, float32\_t, float64\_t, and float128\_t should behave identically when used on any platform with any implementation that correctly supports the typedefs.

It also creates the opportunity to implement quadruple-precision (Quadruple-precision floating-point format) in a specified, and therefore portable, fashion.

One could envision two ways to name the proposed optional floating-point typedefs having specified widths:

```
float11_t, float24_t, float53_t, float113_t, ...float16_t, float32_t, float64_t, float128_t, ...
```

The first set above is intuitively coined from IEE754:2008. It is also consistent with the gist of integer types having specified widths such as int64\_t, in so far as the number of binary digits of *significand* precision is contained within the name of the data type.

On the other hand, the second set with the size of the *whole type* contained within the name may be more intuitive to users. Here, we prefer the latter naming scheme.

No matter what naming scheme is used, the exact layout and number of significand and exponent bits can be confirmed as IEEE754 by checking std::numeric\_limits<type>::is\_iec559 == true, and the byte order. Little-endian IEEE754 architectures now predominate.





#### Note

IEEE\_ floating-point format prescribes a method of precision extension, that allows for conforming types other than binary16, binary32, binary64, and binary128. This makes it possible to extend floating-point precision to both lower and higher precisions in a standardized way using implementation-specific typedefs that are not derived from float, double, and long double.



#### Note

Paragraph 3.7 in IEEE\_floating-point format states: Language standards should define mechanisms supporting extendable precision for each supported radix. This proposal embodies a potential way for C++ to adhere to this requirement.



#### **Note**

IEEE\_ floating-point format does not specify the byte order for floating-point storage (the so-called endianness). This is the same situation that prevails for integer storage in C++.

We will now consider various examples that show how implementations might introduce some of the optional floating-point typedefs having specified widths into the std namespace.

An implementation has float and double corresponding to IEEE754 binary32, binary64, respectively. This implementation could introduce float32\_t, float64\_t, and floatmax\_t into the std namespace as shown below.

```
// In <cstdfloat>
namespace std
{
  typedef float    float32_t;
  typedef double    float64_t;
  typedef float64_t floatmax_t;
}
```

There may be a need for octuple-precision float, in other words an extension to float256\_t with about 240 binary significand digits of precision. In addition, a float512\_t type with even more precision may be considered as an option. Beyond these, there may be potential extension to multiprecision types, or even arbitrary precision, in the future.

Consider an implementation for a supercomputer. This platform has float, double, and long double corresponding to IEEE754 binary32, binary64, and binary128, respectively. In addition, this implementation has floating-point types with octuple-precision and hextuple-precision. The implementation for this supercomputer could introduce its optional floating-point typedefs having specified widths into the std namespace as shown below.



A cost-sensitive 8-bit microcontroller platform without an FPU does not have sufficient resources to support the eight-byte, 64-bit binary64 type in a feasible fashion. An implementation for this platform can, however, support half-precision float16\_t and single-precision float32\_t. This implementation could introduce its optional floating-point typedefs having specified widths into the std namespace as shown below.

```
// In <cstdfloat>

namespace std
{
  typedef floating-point type float16_t;
  typedef float float32_t;
  typedef float32_t floatmax_t;
}
```

The popular Intel X8087 chipset architecture supports a 10-byte floating-point format. It may be useful to extend the optional support to float80\_t. Several implementations using x86 Extended Precision Format already exist in practice.

Consider an implementation that supports single-precision float, double-precision double, and 10-byte long double. This implementation could introduce its optional typedefs float32\_t, float64\_t, float80\_t, and floatmax\_t into the std namespace as shown below.

```
// In <cstdfloat>

namespace std
{
  typedef float     float32_t;
  typedef double     float64_t;
  typedef long double float80_t;
  typedef float80_t     floatmax_t;
}
```



# **Handling floating-point literals**

We will now examine how to use floating-point literal constants in combination with the optional floating-point typedefs having specified widths. This will be done in a manner analogous to the mechanism specified for integer types having specified widths, in other words using C-style macros.

The header <cstdfloat> should contain all necessary C-style function macros in the form shown below.

```
FLOAT{16 32 64 80 128 256 512 MAX}_C
```

The code below, for example, initializes a constant float128\_t value using one of these macros.

```
#include <cstdfloat>
constexpr std::float128_t euler = FLOAT128_C(0.57721566490153286060651209008240243104216);
```

The following code initializes a constant float16\_t value using another one of these macros.

```
#include <cstdfloat>
constexpr std::float16_t euler = FLOAT16_C(0.577216);
```

In addition, the header <cstdfloat> should contain all necessary macros of the form:

```
FLOAT_[FAST LEAST]{16 32 64 80 128 256 512}_MIN
FLOAT_[FAST LEAST]{16 32 64 80 128 256 512}_MAX
FLOATMAX_MIN
FLOATMAX_MAX
```

These macros can be used to query the ranges of the optional floating-point typedefs having specified widths at compile-time. For example,



### Place in the standard

The proper place for defining the optional floating-point typedefs having specified widths should be oriented along the lines of the current standard. Consider the existing specification of integer typedefs having specified widths in C++11. A partial synopsis is shown below.

18.4 Integer types [cstdint]

18.4.1 Header <cstdint> synopsis [cstdint.syn]

```
namespace std
{
  typedef signed integer type int8_t; // optional
  typedef signed integer type int16_t; // optional
  typedef signed integer type int32_t; // optional
  typedef signed integer type int64_t; // optional
}

// ... and the corresponding least and fast types.
```

It is not immediately obvious where the optional floating-point typedefs having specified widths should reside. One potential place is <cstdint>. The int, however, implies integer types. Here, we prefer the proposed new header <cstdfloat>.

We propose to add a new header <cstdfloat> to the standard library. The header <cstdfloat> should contain all optional floating-point typedefs having specified widths included in the implementation and the corresponding C-style macros shown above.

Section 18.4 could be extended as shown below.

18.4? Integer and Floating-Point Types Having Specified Widths

18.4.1 Header <cstdint> synopsis [cstdint.syn]

18.4.2? Header <cstdfloat> synopsis [cstdfloat.syn]



```
namespace std
  typedef floating-point type float16_t;
                                           // optional.
  typedef floating-point type float32_t; // optional.
  typedef floating-point type float64_t; // optional.
  typedef floating-point type float80_t;
                                           // optional.
  typedef floating-point type float128_t; // optional.
  typedef floating-point type float256_t; // optional.
  typedef floating-point type float512_t; // optional.
  typedef floating-point type floatmax_t; // optional.
  typedef floating-point type float_least16_t; // optional.
  typedef floating-point type float_least32_t; // optional.
typedef floating-point type float_least64_t; // optional.
  typedef floating-point type float_least80_t; // optional.
  typedef floating-point type float_least128_t; // optional.
  typedef floating-point type float_least256_t; // optional.
  typedef floating-point type float_least512_t; // optional.
  typedef floating-point type float_fast16_t; // optional.
  typedef floating-point type float_fast32_t; // optional.
  typedef floating-point type float_fast64_t; // optional.
  typedef floating-point type float_fast80_t; // optional.
  typedef floating-point type float_fast128_t; // optional.
  typedef floating-point type float_fast256_t; // optional.
  typedef floating-point type float_fast512_t; // optional.
```



## Interoperation with <cmath> and special functions

It is not proposed to make any mandatory changes to <cmath> or special functions.

Any of the optional floating-point typedefs having specified widths that are typedefed from the built-in types float, double, and long double should automatically be supported by the implementation's existing <cmath> and special functions.

Implementation-specific optional floating-point typedefs having specified widths that are not derived from float, double, and long double can optionally be supported by <cmath> and special functions. This is considered an implementation detail.



#### **Note**

Support of elementary functions (and possibly some special functions, even where only optional) can be very useful for real-life computational regimes.



### Interoperation with imits>

It is not proposed to make any mandatory changes to imits>.

Any of the optional floating-point typedefs having specified widths that are typedefed from the built-in types float, double, and long double should automatically be supported by the implementation's existing limits>.

Implementation-specific optional floating-point typedefs having specified widths that are not derived from float, double, and long double can optionally be supported by imits>. This is considered an implementation detail.



#### Note

Support for states, even where optional, can be very useful, especially for portability. This allows programs to query the floating-point limits at compile-time and use, among other things, std::numeric\_limits<>::is\_iec559 to verify conformance with IEEE\_ floating-point format.



#### **Note**

Each of the optional floating-point typedefs having specified widths can only have true for the value of std::numeric\_limits<>::is\_iec559 if its underlying type (be it float, double, long double or an implementation-dependent type) conforms with one of binary16, binary32, binary64, or binary128, or the prescribed method of precision extension in IEEE\_ floating-point format.



# Interoperation with <complex>

It is not proposed to make any mandatory changes to <complex>.

Any of the optional floating-point typedefs having specified widths that are typedefed from the built-in types float, double, and long double should automatically be supported by the implementation's existing <complex>.

Implementation-specific optional floating-point typedefs having specified widths that are *not derived from float*, *double*, *and long double* can optionally be supported by <complex>. This is considered an implementation detail.



# **Specifying 128-bit precision**

The proposed typedef float128\_t provides a standardized way to specify quadruple-precision (Quadruple-precision floating-point format) in C++.

On powerful PCs and workstations, implementation-specific versions of long double as well as various floating-point extensions to 128-bit have been treated in a variety of ways. This has resulted in numerous portability problems.

The Intel X8087 chipset is capable of performing calculations with internal 80-bit registers. This increases the width of the significand from 53 to 63 bits, thereby gaining about 3 decimal digits precision and extending it from 18 and 21. If an implementation has a type that uses all 80 bits from this chipset to calculate Extended precision, it could could use an optional typedef of this type to float80\_t.

Some hardware, for example Sparc, provides a full 128-bit quadruple-precision floating-point chip. An implementation for this kind of architecture might already have a built-in type corresponding to binary128, and this type could be optionally typedefed to float128\_t.

GCC has recently developed quadruple-precision support on a variety of platforms using GCC libquadmath. However, the implementation-specific type \_\_float128 is used rather than long double. These implementations could optionally typedef \_\_float128 to float128\_t in addition to any other optional typedefs.

Darwin long double uses a double-double format developed first by Keith Briggs. This gives about 106-bits of precision (about 33 decimal digits) but has rather odd behavior at the extremes making implementation of std::numeric\_limits<>::epsilon() problematic.

It may be useful if future implementations for powerful PCs and workstations strive to make implementation-specific extensions to 128-bit floating-point or the built-in type long double equivalent to binary128, and to include the corresponding typedef to float128\_t.

Some architectures have hardware support for this. Those lacking direct hardware support can use software emulation.

#### Survey of extended-precision types

- 1. GNU C supports additional floating types, \_\_float80 and \_\_float128 to support 80-bit (XFmode) and 128-bit (TFmode) floating types.
- 2. Intel C++ provides an internal 128-bit floating-point type called \_Quad. When the -Qoption, cpp, --extended\_float\_type command line option is supplied, it supports what appears to be an undocumented data type \_Quad. This type is equivalent to GCC's \_\_float128.
- 3. Intel FORTRAN REAL\*16 is an actual 128-bit IEEE quad, emulated in software. But "I don't know of any plan to implement full C support for 128-bit IEEE format, although evidently ifort has support libraries." This is equivalent to the proposed float128\_t type.
- 4. The 360/85 and follow-on System/370 added support for a 128-bit "extended" IBM extended precision formats. These formats are still supported in the current design, where they are now called the "hexadecimal floating point" (HFP) formats.



# **Extending to lower precision**

Some implementations for cost-sensitive microcontroller platforms support float, double, and long double, and some of these are compliant with IEEE\_floating-point format. Some of these implementations treat double exactly as float, and even treat long double exactly as double. This is permitted by the standard which does not prescribe the precision for any floating-point (or integer) types, leaving them to be implementation-defined. On these platforms, the existing floating-point types could optionally be type-defined to float32\_t. Optional support for an extension to float16\_t could provide a very useful and efficient floating-point type with half-precision, but reduced range.

Some implementations for cost-sensitive microcontroller platforms also support a 24-bit floating-point type. Here, an extension of the optional floating-point typedefs with specified widths could include float24\_t. This would be equivalent to three-quarter precision floating-point, the layout of which should adhere to the method of precision extension specified in IEEE\_ floating-point format.

Some embedded graphics systems use an 8-bit floating-point representation, primarily for storage of pixel information. Here, an extension of the optional floating-point typedefs with specified widths could include float8\_t. This would be equivalent to one-quarter precision floating-point, the layout of which should adhere to the method of precision extension specified in IEEE\_ floating-point format.

These potential embedded extensions for cost-sensitive microcontroller platforms are shown in the code sample below



# The context among existing implementations

Many existing implementations already support float, double, and long double. In addition, some of these either are or strive to be compliant with IEEE\_floating-point format. In these cases, it will be straightforward to support (at least) a subset of the proposed optional floating-point typedefs having specified widths by adding any desired optional type definitions and the corresponding macro definitions.



### References

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- 5. lists binary 16, 32, 64 and 128 (and also decimal 32, 64, and 128)
- 6. IEEE Std 754-2008
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