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An Introduction to the Interval Template Library

Lecture
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Lecture Outline

- Background and Motivation
- Characteristics of interval containers
- Examples
- Semantics
- Implementation
- Future Works
- Availability

- Interval containers simplified the implementation of date and time related tasks
  - Decomposing a stream of attributed events into segments with constant attributes.
  - Working with time grids, e.g. a grid of months.
  - Aggregations of values associated to date or time intervals.

- … that occurred frequently in programs like
  - Billing modules
  - Therapy scheduling programs
  - Hospital and controlling statistics

- Background is the date time problem domain ...
- … but the scope of the **Itl** as a generic library is more general:

an **interval_set** *implements a* **set**

via a set of intervals

an **interval_map** *implements a* **map**

via a map of interval value pairs

Aspects

- There are two aspects in the design of interval containers
  - Conceptual aspect

```
interval_set<int> mySet;
mySet.insert(42);
bool has_answer = mySet.contains(42);
```

  - On the conceptual aspect an interval_set can be used just as a set of elements
  - except for . . .
  - . . . ***iteration***
  - consider **`interval_set<double>`** or **`interval_set<string>`**

- Iterative aspect

```
// Switch on my favorite telecasts using an interval_set.
interval<seconds> news(make_seconds("20:00:00"),
                       make_seconds("20:15:00"));
interval<seconds> talk_show(make_seconds("22:45:30"),
                            make_seconds("23:30:50"));

interval_set<seconds> myTvProgram;
myTVProgram.add(news).add(talk_show);

// Iterating over elements (seconds) would be silly ...
for(interval_set<seconds>::iterator telecast =
    myTvProgram.begin();
    telecast != myTvProgram.end(); ++telecast)
  //...so this iterates over intervals
  TV.switch_on(*telecast);
```

Characteristics of Interval Containers

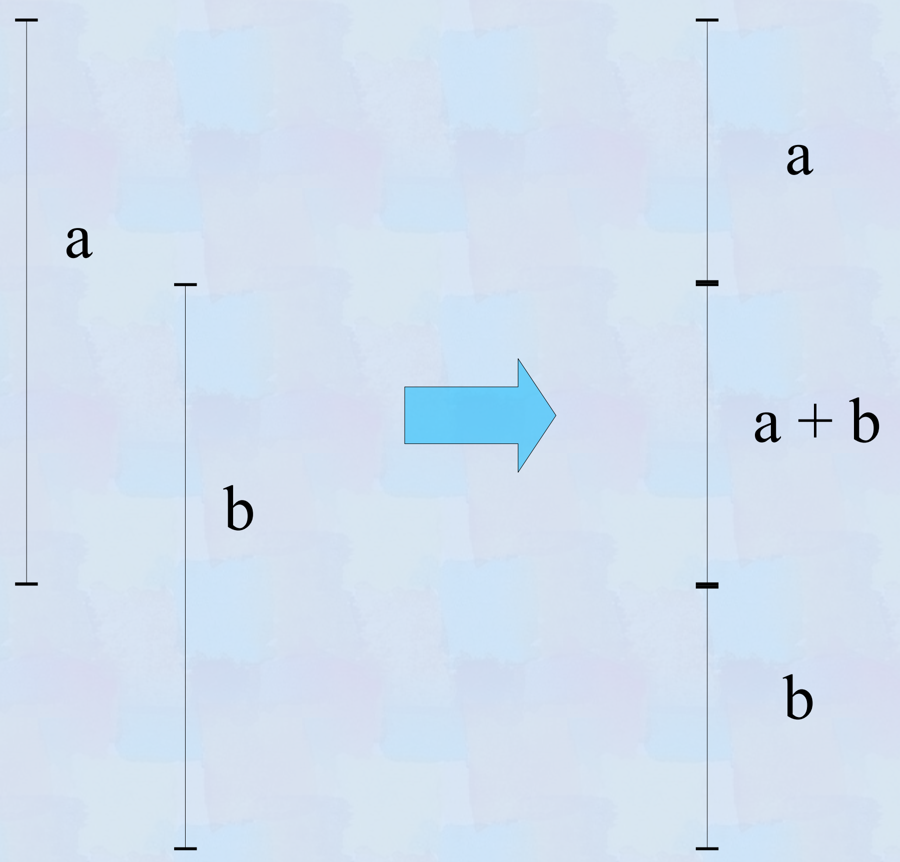
- Addability and Subtractability
  - All of itl's (interval) containers are *Addable* and *Subtractable*
  - They implement `operators +=, +, -=` and `-`

	`+=`	`-=`
sets	set union	set difference
maps	?	?

- A possible implementation for maps
  - Propagate addition/subtraction to the associated values
  - . . . or aggregate on overlap
  - . . . or aggregate on collision

Characteristics of Interval Containers

- Aggregate on overlap



- Decompositional effect on Intervals
- Accumulative effect on associated values

- Aggregate on overlap, a minimal example

```
typedef itl::set<string> guests;
interval_map<time, guests> party;

party += make_pair(
  interval<time>::rightopen(20:00, 22:00), guests("Mary"));

party += make_pair(
  interval<time>::rightopen(21:00, 23:00), guests("Harry"));


// party now contains
[20:00, 21:00)->{"Mary"}
[21:00, 22:00)->{"Harry","Mary"} //guest sets aggregated
[22:00, 23:00)->{"Harry"}
```

- The Itl's class templates

Granu -larity	Style	Sets	Maps
interval		`interval`	
	joining	`interval_set`	`interval_map`
	separating	`separate_interval_set`	
	splitting	`split_interval_set`	`split_interval_map`
element		`set`	`map`

- Interval Combining Styles: ***Joining***
  - Intervals are joined on overlap or on touch
  - . . . *for maps*, if associated values are equal
  - Keeps interval_maps and sets in a minimal form

interval_set

```
  {[1      3)          }
+       [2      4)
+                 [4 5)

= {[1           4)     }

= {[1                5)}
```

interval_map

```
  {[1      3)->1         }
+      [2      4)->1
+                [4 5)->1

={[1 2)[2 3)[3 4)        }
   ->1  ->2  ->1
={[1 2)[2 3)[3      5)   }
   ->1  ->2     ->1
```

- Interval Combining Styles: ***Splitting***
  - Intervals are split on overlap and kept separate on touch
  - All interval borders are preserved (insertion memory)

split_interval_set

```
  {[1      3)           }
+       [2      4)
+                 [4 5)

= {[1 2)[2 3)[3 4)      }

= {[1 2)[2 3)[3 4)[4 5)}
```

split_interval_map

```
  {[1      3)->1           }
+       [2      4)->1
+                 [4 5)->1

={[1 2)[2 3)[3 4)          }
   ->1  ->2  ->1
={[1 2)[2 3)[3 4)[4 5)     }
   ->1  ->2  ->1  ->1
```

- Interval Combining Styles: ***Separating***
  - Intervals are joined on overlap but kept separate on touch
  - Preserves borders that are never crossed (preserves a hidden grid).

```
separate_interval_set

  {[1      3)          }
+       [2      4)
+                 [4 5)

= {[1           4)     }

= {[1           4)[4 5)}
```

- A few instances of intervals (interval.cpp)

```
interval<int> int_interval = interval<int>::closed(3,7);

interval<double> sqrt_interval
  = interval<double>::rightopen(1/sqrt(2.0), sqrt(2.0));

interval<std::string> city_interval
  = interval<std::string>::leftopen("Barcelona", "Boston");

interval<boost::ptime> time_interval
  = interval<boost::ptime>::open(
                       time_from_string("2008-05-20 19:30"),
                       time_from_string("2008-05-20 23:00")
    );
```

- A way to iterate over months and weeks (`month_and_week_grid.cpp`)

```
#include <boost/itl/gregorian.hpp> //boost::gregorian plus adapter code
#include <boost/itl/split_interval_set.hpp>

// A split_interval_set of gregorian dates as date_grid.
typedef split_interval_set<boost::gregorian::date> date_grid;

// Compute a date_grid of months using boost::gregorian.
date_grid month_grid(const interval<date>& scope)
{
    date_grid month_grid;
    // Compute a date_grid of months using boost::gregorian.
    . . .
    return month_grid;
}

// Compute a date_grid of weeks using boost::gregorian.
date_grid week_grid(const interval<date>& scope)
{
    date_grid week_grid;
    // Compute a date_grid of weeks using boost::gregorian.
    . . .
    return week_grid;
}
```

- A way to iterate over months and weeks

```
void month_and_time_grid()
{
    date someday = day_clock::local_day();
    date thenday = someday + months(2);
    interval<date> scope = interval<date>::rightopen(someday, thenday);

    // An intersection of the month and week grids ...
    date_grid month_and_week_grid
        = month_grid(scope) & week_grid(scope);

    // ... allows to iterate months and weeks. Whenever a month
    // or a week changes there is a new interval.
    for(date_grid::iterator it = month_and_week_grid.begin();
        it != month_and_week_grid.end(); it++)
    {   . . .   }

    // We can also intersect the grid into an interval_map to make
    // shure that all intervals are within months and week bounds.
    interval_map<boost::gregorian::date, some_type> accrual;
    compute_some_result(accrual, scope);
    accrual &= month_and_week_grid;

}
```

- Aggregating with interval_maps (1) (`boost_party.cpp`)

```
typedef itl::set<string> guests;
interval_map<time, guests> party;

party += make_pair(
  interval<time>::rightopen(20:00, 22:00), guests("Mary"));

party += make_pair(
  interval<time>::rightopen(21:00, 23:00), guests("Harry"));


// party now contains
[20:00, 21:00)->{"Mary"}
[21:00, 22:00)->{"Harry","Mary"} //guest sets aggregated
[22:00, 23:00)->{"Harry"}
```

- Aggregating with interval_maps (2)
  - Computing averages via implementing **operator +=** (partys_guest_average.cpp)

```
class counted_sum
{
public:
    counted_sum():_sum(0),_count(0){}
    counted_sum(int sum):_sum(sum),_count(1){}

    int sum()const  {return _sum;}
    int count()const{return _count;}
    double average()const
     { return _count==0 ? 0.0 : _sum/static_cast<double>(_count); }

    counted_sum& operator += (const counted_sum& right)
    { _sum += right.sum(); _count += right.count(); return *this; }

private:
    int _sum;
    int _count;
};

bool operator == (const counted_sum& left, const counted_sum& right)
{ return left.sum()==right.sum() && left.count()==right.count(); }
```


- Aggregating with interval_maps (2)
  - Computing averages via implementing **operator +=**

```
void partys_height_average()
{
    interval_map<ptime, counted_sum> height_sums;

    height_sums += (
      make_pair(
        interval<ptime>::rightopen(
          time_from_string("2008-05-20 19:30"),
          time_from_string("2008-05-20 23:00")),
          counted_sum(165)) // Mary is 1,65 m tall.
    );

    // Add height of more pary guests . . .

    interval_map<ptime, counted_sum>::iterator height_sum_ =
        height_sums.begin();
    while(height_sum_ != height_sums.end())
    {
        interval<ptime> when = height_sum_->first;
        double height_average = (*height_sum_++).second.average();

        cout << "[" << when.first() << " - " << when.upper() << ")"
             << ": " << height_average << " cm" << endl;
    }
}
```

- Aggregating with interval_maps (3)
  - Computing maxima via instantiation of a template parameter (partys_tallest_guests.cpp)

```
typedef interval_map<ptime, int, partial_absorber, less, inplace_max>
    PartyHeightHistoryT;

void partys_height()
{
    PartyHeightHistoryT tallest_guest;

    tallest_guest += (
      make_pair(
        interval<ptime>::rightopen(
          time_from_string("2008-05-20 19:30"),
          time_from_string("2008-05-20 23:00")),
          180) // Max of Mary & Harry: Harry is 1,80 m tall.
    );

    . . .

}
```

- Interval containers allow to express a variety of date and time operations in an easy way.
  - Example `man_power.cpp ...`
  - Subtract weekends and holidays from an interval_set

```
worktime -= weekends(scope)
worktime -= german_reunification_day
```

  - Intersect an interval_map with an interval_set

```
claudias_working_hours &= worktime
```

  - Subtract and interval_set from an interval map

```
claudias_working_hours -= claudias_absense_times
```

  - Adding interval_maps

```
interval_map<date,int> manpower;
manpower += claudias_working_hours;
manpower += bodos_working_hours;
```

- Interval_maps can also be intersected
  Example **`user_groups.cpp`**

```
typedef boost::itl::set<string> MemberSetT;
typedef interval_map<date, MemberSetT> MembershipT;

void user_groups()
{
    . . .

    MembershipT med_users;
    // Compute membership of medical staff
    med_users += make_pair(member_interval_1, MemberSetT("Dr.Jekyll"));
    med_users += . . .

    MembershipT admin_users;
    // Compute membership of administation staff
    med_users += make_pair(member_interval_2, MemberSetT("Mr.Hyde"));
    . . .

    MembershipT all_users   = med_users + admin_users;

    MembershipT super_users = med_users & admin_users;
    . . .

}
```

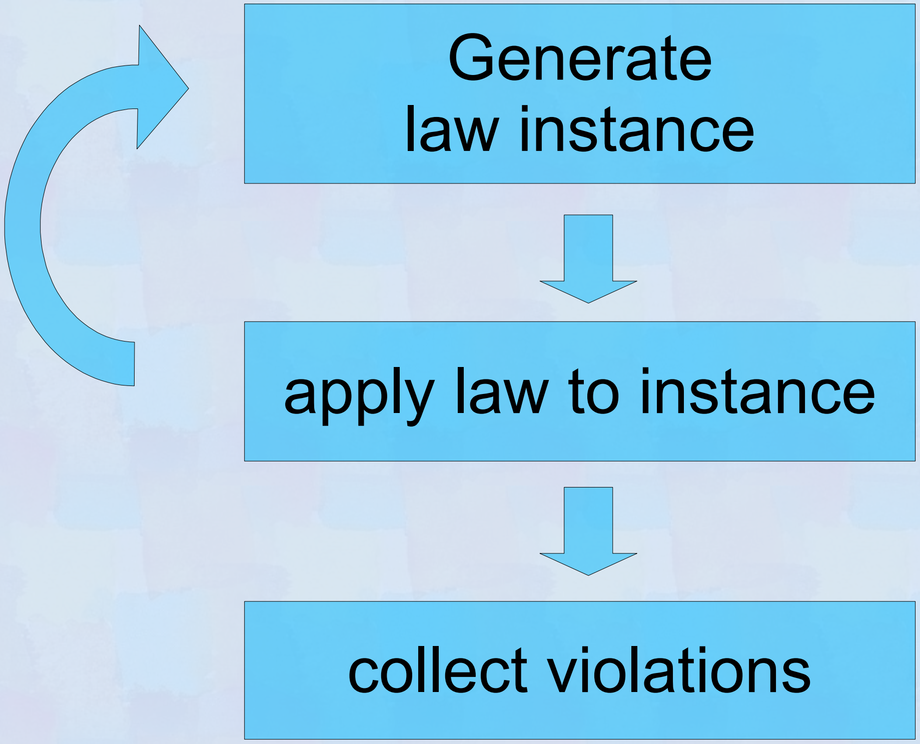
- The semantics of ***itl sets*** is based on a concept **`itl::Set`**
  - **`itl::set`**, **`interval_set`**, **`split_interval_set`** and **`separate_interval_set`** are models of concept **`itl::Set`**

```
// Abstract or conceptual part
empty set:          Set::Set()
subset relation:    bool Set::contained_in(const Set& s2)const
equality:           bool is_element_equal(const Set& s1, const Set& s2)
set union:          Set& operator += (Set& s1, const Set& s2)
                    Set  operator +  (const Set& s1, const Set& s2)
set difference:     Set& operator -= (Set& s1, const Set& s2)
                    Set  operator -  (const Set& s1, const Set& s2)
set intersection:   Set& operator &= (Set& s1, const Set& s2)
                    Set  operator &  (const Set& s1, const Set& s2)

// Part related to sequence and iteration
sorting order:      bool operator <  (const Set& s1, const Set& s2)
lexicographical equality:
                    bool operator == (const Set& s1, const Set& s2)
```

- The semantics of ***itl maps*** is based on a concept **`itl::Map`**
  - **`itl::map`**, **`interval_map`** and **`split_interval_map`** are models of concept **`itl::Map`**

```
// Abstract or conceptual part
empty map:         Map::Map()
submap relation:   bool Map::contained_in(const Map& m2)const
equality:          bool is_element_equal(const Map& m1, const Map& m2)
map union:         Map& operator += (Map& m1, const Map& m2)
                   Map  operator +  (const Map& m1, const Map& m2)
map difference:    Map& operator -= (Map& m1, const Map& m2)
                   Map  operator -  (const Map& m1, const Map& m2)
map intersection:  Map& operator &= (Map& m1, const Map& m2)
                   Map  operator &  (const Map& m1, const Map& m2)

// Part related to sequence and iteration
sorting order:     bool operator <  (const Map& m1, const Map& m2)
lexicographical equality:
                   bool operator == (const Map& m1, const Map& m2)
```

- Defining semantics of itl concepts via sets of laws
  - aka c++0x axioms
- Checking law sets via automatic testing:
  - A ***La***w ***Ba***sed ***Te***st ***A***utomaton ***LaBatea***



- Lexicographical Ordering and Equality
  - For all itl containers `operator <` implements a ***strict weak ordering***.
  - The ***induced equivalence*** of this ordering is ***lexicographical equality*** which is implemented as `operator ==`
  - This is in line with the semantics of SortedAssociativeContainers

- Subset Ordering and Element Equality
  - For all itl containers function `contained_in` implements a ***partial ordering***.
  - The ***induced equivalence*** of this ordering is ***equality of elements*** which is implemented as function `is_element_equal`.

- itl::Sets
- All itl sets implement a ***Set Algebra***, which is to say satisfy a "*classical*" set of laws . . .
  - . . . using `is_element_equal` as equality
  - Associativity, Neutrality, Commutativity
  - Distributivity, DeMorgan, Symmetric Difference
- Most of the itl sets satisfy the classical set of laws even if . . .
  - . . . lexicographical equality: `operator ==` is used
  - The differences reflect proper inequalities in sequence that occur for `separate_interval_set` and `split_interval_set`.

- itl::Maps
- Two major use cases
  - Maps of Sets: Collectors
  - Maps of Numbers: Quantifiers

- Map Traits:
  - Definedness
    - partial
    - total
  - Neutron semantics: Given a value pair `(k,0)`, the associated neutron value *codes*
    - *non existence*. `(k,0)` will be deleted: neutron absorber
    - *0-quantification*. `(k,0)` will **not** be deleted: neutron enricher

	neutron absorber	neutron enricher
partial	partial_absorber	partial_enricher
total	total_absorber	total_enricher

- Collectors: Maps of Sets . . .
  - Condition: `Trait == partial_absorber`.
    `interval_map<int, itl::set<int>, partial_absorber>`
  - *partial*: only value pairs exist that have been added
  - *neutron absorber*: If a value pair carries a neutron `(k,0)`, it is deleted (`0` codes *non existence).*
  - . . . then an itl Map of Sets is model of `itl::Set`
  - . . . it satisfies the same set of laws that **itl sets** do.

- Concepts induction / concept transition
  - Semantics of itl Maps appears to be dominated by the codomain type of the map
  - Itl Maps are mapping the semantics of the codomain type on themselves.

	is model of	if Trait	example
Map<D,Monoid>	Monoid		interval_map<int,string>
Map<D,Set>	Set	is_partial _aborber	interval_map<int,set<int>>
Map<D,CommutMonoid>	CommutMonoid		interval_map<int,unsigned>
Map<D,AbelianGroup>	AbelianGroup	is_total	interval_map<int,int>

- Itl conainters are implemented simply based on std::set and std::map
  - Basic operations like adding and subtracting intervals have a best case complexity of O(lg n), if the added or subtracted intervals are small.
  - Worst case complexity of adding or subtracting intervals for interval_set is O(n).
  - For all other interval containers adding or subtracting intervals has a worst case performance of O(n lg(n)).
  - There is a potential for optimization . . .

Implementation

- A segment_tree implementaion: A balanced tree, where . . .
  - an interval represents a perfectly balanced subtree
  - large intervals are rotated towards the root

- First results
  - much better worst case performance O(n) instead of O(n lg(n))
  - but slower for best case due to heavier bookkeeping and recursive algorithms.

Future Works

- Completing and optimizing the segment_tree implementation of interval containers
- Implementing interval_maps of sets more efficiently

- Revision of elements of the extended itl (itl_xt)
  - Decomposition of histories: k histories h_k with attribute types $A_1, \ldots, A_k$ are "*decomposed*" to a product history of attribute set tuples:
    $(h_1<T,A_1>,\ldots, h<T,A_k>) \rightarrow h<T, (set<A_1>,\ldots, set<A_k>)>$

  - Cubes (generalized crosstables): Applying *aggregate on collision* to maps of tuples in order to organize hierachical data and their aggregates.

Availability

- Itl project on sourceforge (version 2.0.1)
  http://sourceforge.net/projects/itl
- Latest version on boost vault/Containers (3.0.0)
  http://www.boostpro.com/vault/ → containers
  - itl.zip : Core itl in preparation for boost
  - itl_plus.zip : Extended itl including product histories, cubes and automatic validation (LaBatea).
- Online documentation at
  http://www.herold-faulhaber.de/
  - Doxygen generated docs for (version 2.0.1)
    http://www.herold-faulhaber.de/itl/
  - Latest boost style documentation (version 3.0.0)
    http://www.herold-faulhaber.de/boost_itl/doc/libs/itl/doc/html/