Constants and Functions of Mathematics: A Selection of Numerical Values[††]

$\pi \approx$ `3.1415926535897932384626433832795028841971693993751058209749445923078164062862089986280348253421170679821480865132823...`$^{(\clubsuit)}$

$e \approx$ `2.7182818284590452353602874713526624977572470936999595749669676277240766303535475945713821785251664274274663919320031...`$^{(\clubsuit)}$

$\gamma \approx$ `0.5772156649015328606065120900824024310421593359399235988057672348848677267776646709369470632917467495146314472498070 7...`$^{(\clubsuit)}$

$\sqrt{2} \approx$ `1.4142135623730950488016887242096980785696718753769480731766797379907324784621070388503875343276415727350138462309 1...`$^{(\spadesuit)}$

$\sqrt[11]{1,234,577} \approx$ `3.5791045114591548768153212105350630962775453348188234400399444094013620033960426669499611436651400500240979...`$^{(\spadesuit)}$

$\left(2^{37,156,667} - 1\right) \approx$ `2.02254406890977335534188152263156829946846602582743182989551057360547514579758125084672139009590...` $\times$ `10`$^{11,185,271\,(\heartsuit)}$

$\log(2) \approx$ `0.693147180559945309417232121458176568075500134360255254120680009493393621969694715605863326996418687542001481021...`$^{(\diamondsuit)}$

$\tan\left(\frac{41}{47}\right) \approx$ `1.19096924080642343385499535864391655166210224629673547857304600594374722030068682468661891601859189079094775669...`$^{(\diamondsuit)}$

$K\left(\frac{223}{227}\right) \approx$ `3.416318583595656700122075675276685023954045462894077431616065338794050923737892895565858867201067103024955751973...`$^{(\bullet)}$

$\Gamma\left(\frac{1993}{733}\right) \approx$ `1.56832829296510092932380417039280885375999457346886819067779819680920995389510328229092820783838846775884559403...`$^{(*)}$

$\psi^{(7)}\left(\frac{137}{103}\right) \approx$ `520.65928649004380134607822408858033860038226293423827493394689280966492211945702069826815235341695100104580921...`$^{(\star)}$

$\zeta(3) \approx$ `1.20205690315959428539973816151144999076498629234049888179227155534183820578631309018645587360933525814619915779526...`$^{(\ddagger)}$

$J_2\left(\frac{977}{97}\right) \approx$ `0.25345997611996908988882205817748010929828990811653582213814484514265655593603922525970423851794541649313648170...`$^{(\triangle)}$

$Y_{(3,000,029/3)}(1,000,033) \approx$ `0.006057862333738562607197426093372544077837320090945630562121845891121678461440056918759527940326059...`$^{(\triangle)}$

$P_{5/7}^{1/3}\left(\frac{467}{509}\right) \approx$ `1.1520435204127438817677190892228920418838167987147992211469539055545525550854745344108100895750442202154099869...`$^{(\circ)}$

Calculations use high-performance multiple precision floating point programs designed for numbers with about $10^2 ... 10^7$ decimal digits of precision. Certain integer functions and coefficients use symbolic math. **Multiplication** uses $O\left(N^2\right)$ traditional and $O\left(N^{\log_2(3)}\right)$ Karatsuba as well as $O\left(N \log_2(N)\right)$ Schönhage-Strassen FFT algorithms. **Fast Fourier Transforms** use the "Fastest Fourier Transform in the West" (FFTW) with the calculations distributed among 2^N CPU cores using parallel threads. **Integer Power** $^{(\heartsuit)}$ uses exponentiation by squaring. **Integer Root** $^{(\spadesuit)}$ uses quadratically convergent Newton iteration. **Constants** $^{(\clubsuit)}$ such as π, e and γ use Gauss arithmetic-geometric algorithms and binary splitting for computation of up to 32 million decimal digits. The calculation of one million decimal digits of π takes less than 10 seconds on a modern dual-core system. **Elementary Transcendental Functions** $^{(\diamondsuit)}$ use Taylor series, argument scaling, recursion and Newton iteration. **Orthogonal Polynomials** use generating functions and recursion. **Primes** and **Prime Factorization** use sieves and divide-and-conquer. **Elliptic Integrals** $^{(\bullet)}$ use arithmetic-geometric methods. **Gamma** $^{(*)}$ uses recursion and asymptotic series. **Polygamma** $^{(\star)}$ uses recursion, asymptotic series and Euler-Maclaurin summation. **Zeta** $^{(\ddagger)}$ uses the product over all primes, an accelerated sum of reciprocal powers, and Euler-Maclaurin summation. **Airy** uses Taylor series, asymptotic series and Bessel function representation. **Bessel** $^{(\triangle)}$ uses Taylor series, recursion, asymptotic hypergeometric series and uniform asymptotic Airy type expansions. **Hypergeometric** including **Legendre** $^{(\circ)}$ uses Taylor series, recursion and various asymptotic series. **Software design** uses Microsoft® Visual Studio® 2008, GNU Compiler Collection (GCC) 4.3.3, GNUmake 3.81, Intel® C++ 11.0.066, Mathematica® 7.0.1, the C++ programs `e_float` and `mp_cpp` (`e_float@yahoo.com`), GNU Multiple Precision (GMP) 4.2.4, and FFTW 2.15. **Visualization** uses LaTeX.

[††] π Archimedes' constant; e the natural logarithm base; γ the Euler-Mascheroni constant; $\sqrt{2}$ Pythagoras' constant; $\sqrt[11]{1,234,577}$ an integer root of a random prime number; $\left(2^{37,156,667} - 1\right)$ a huge integer power of 2 minus 1 expressing the 46^{th} Mersenne prime number; $\log(2)$ the natural logarithm of 2; $\tan\left(\frac{41}{47}\right)$ the tangent of a rational number; $K\left(\frac{223}{227}\right)$ the complete elliptic integral of a rational number; $\Gamma\left(\frac{1993}{733}\right)$ the Gamma function of a rational number; $\psi^{(7)}\left(\frac{137}{103}\right)$ the 7^{th} order Polygamma function of a rational number; $\zeta(3)$ the Zeta function of the prime number 3, also known as Apéry's constant; $J_2\left(\frac{977}{97}\right)$ the second order cylindrical Bessel coefficient of a rational number; $Y_{(3,000,029/3)}(1,000,033)$ a cylindrical Neumann function with a very high rational order evaluated for a prime-numbered argument in the transition region; $P_{5/7}^{1/3}\left(\frac{467}{509}\right)$ a hypergeometric Legendre function with rational degree and rational order evaluated for a rational argument.