Constants and Functions of Mathematics: A Selection of Numerical Values††

$\pi \approx 3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706798214808651328230664709384460955058223172 54\ldots$ $^{(\clubsuit)}$

$e \approx 2.71828182845904523536028747135266249775724709369995957496696762772407663035354759457138217852516642742746639193200305992181741359662904357290 03\ldots$ $^{(\clubsuit)}$

$\gamma \approx 0.57721566490153286060651209008240243104215933593992359880576723488486772677766467093694706329174674951463144724980708248096050401448654283622 42\ldots$ $^{(\clubsuit)}$

$\sqrt{2} \approx 1.41421356237309504880168872420969807856967187537694807317667973799073247846210703885038753432764157273501384623091229702492483605585073721 26\ldots$ $^{(\spadesuit)}$

$\sqrt[11]{1,234,577} \approx 3.579104511459154876815321210535063096277545334818823440039944409401362003396042666949961143665140050024097912253868978181595908067 23\ldots$ $^{(\spadesuit)}$

$\left(2^{37,156,667}-1\right) \approx 2.022544068909773355341881522631568299468466025827431829895510573605475145797581250846721390095896345301420966744889977095\ldots \times 10^{11,185,271}$ $^{(\heartsuit)}$

$\log(2) \approx 0.69314718055994530941723212145817656807550013436025525412068000949339362196969471560586332699641868754200148102057068573368552023575813056\ldots$ $^{(\diamondsuit)}$

$\tan\left(\frac{41}{47}\right) \approx 1.190969240806423433854995358643916551662102246296735478573046005943747220300686824686618916018591890790947756688007135046795392252509 9063\ldots$ $^{(\diamondsuit)}$

$K\left(\frac{223}{227}\right) \approx 3.416318583595656700122075675276685023954045462894077431616065338794050923737892895565858867201067103024955751973263425216314376460659922 77\ldots$ $^{(\bullet)}$

$\Gamma\left(\frac{1993}{733}\right) \approx 1.568328292965100929323804170392808853759994573468868190677798196809209953895103282290928207838388467758845594031423562324052374907028592 16\ldots$ $^{(*)}$

$\psi^{(7)}\left(\frac{137}{103}\right) \approx 520.6592864900438013460782240885803386003822629342382749339468928096649221194570206982681523534169510010458092056599132278529509015522757\ldots$ $^{(\star)}$

$\zeta(3) \approx 1.202056903159594285399738161511449990764986292340498881792271555341838205786313090186455873609335258146199157795260719418491995998673283213 78\ldots$ $^{(\ddagger)}$

$J_2\left(\frac{977}{97}\right) \approx 0.253459976119969089888822058177480109298289908116535822138144845142656555936039225259704238517945416493136481697385497724468718612658378 37\ldots$ $^{(\triangle)}$

$Y_{(3,000,029/3)}(1,000,033) \approx -0.006057862333738562607197426093372544077837320090945630562121845891121678461440056918759527940326059047435459362229194575 76\ldots$ $^{(\triangle)}$

$P^{1/3}_{5/7}\left(\frac{467}{509}\right) \approx 1.152043520412743881767719089222892041883816798714799221146953905554552555085474534410810089575044220215409986872673618976971609478890845\ldots$ $^{(\circ)}$

Calculations use high-performance multiple precision floating point programs designed for numbers with about $10^2 \ldots 10^7$ decimal digits of precision. Certain integer functions and coefficients use symbolic math. **Multiplication** uses $O\left(N^2\right)$ traditional and $O\left(N^{\log_2(3)}\right)$ Karatsuba as well as $O\left(N \log_2(N)\right)$ Schönhage-Strassen FFT algorithms. **Fast Fourier Transforms** use the “Fastest Fourier Transform in the West” (FFTW) with the calculations distributed among 2^N CPU cores using parallel threads. **Integer Power** $^{(\heartsuit)}$ uses exponentiation by squaring. **Integer Root** $^{(\spadesuit)}$ uses quadratically convergent Newton iteration. **Constants** $^{(\clubsuit)}$ such as π, e and γ use Gauss arithmetic-geometric algorithms and binary splitting for computation of up to 32 million decimal digits. The calculation of one million decimal digits of π takes less than 10 seconds on a modern dual-core system. **Elementary Transcendental Functions** $^{(\diamondsuit)}$ use Taylor series, argument scaling, recursion and Newton iteration. **Orthogonal Polynomials** use generating functions and recursion. **Primes** and **Prime Factorization** use sieves and divide-and-conquer. **Elliptic Integrals** $^{(\bullet)}$ use arithmetic-geometric methods. **Gamma** $^{(*)}$ uses recursion and asymptotic series. **Polygamma** $^{(\star)}$ uses recursion, asymptotic series and Euler-Maclaurin summation. **Zeta** $^{(\ddagger)}$ uses the product over all primes, an accelerated sum of reciprocal powers, and Euler-Maclaurin summation. **Airy** uses Taylor series, asymptotic series and Bessel function representation. **Bessel** $^{(\triangle)}$ uses Taylor series, recursion, asymptotic hypergeometric series and uniform asymptotic Airy type expansions. **Hypergeometric** including **Legendre** $^{(\circ)}$ uses Taylor series, recursion and various asymptotic series. **Software design** uses Microsoft® Visual Studio® 2008, GNU Compiler Collection (GCC) 4.3.3, GNUmake 3.81, Intel® C++ 11.0.066, Mathematica® 7.0.1, the C++ programs `e_float` and `mp_cpp` (`e_float@yahoo.com`), GNU Multiple Precision (GMP) 4.2.4, and FFTW 2.15. **Visualization** uses LaTeX.

†† π Archimedes' constant; e the natural logarithm base; γ the Euler-Mascheroni constant; $\sqrt{2}$ Pythagoras' constant; $\sqrt[11]{1,234,577}$ an integer root of a random prime number; $\left(2^{37,156,667}-1\right)$ a huge integer power of 2 minus 1 expressing the 46^{th} Mersenne prime number; $\log(2)$ the natural logarithm of 2; $\tan\left(\frac{41}{47}\right)$ the tangent of a rational number; $K\left(\frac{223}{227}\right)$ the complete elliptic integral of a rational number; $\Gamma\left(\frac{1993}{733}\right)$ the Gamma function of a rational number; $\psi^{(7)}\left(\frac{137}{103}\right)$ the 7^{th} order Polygamma function of a rational number; $\zeta(3)$ the Zeta function of the prime number 3, also known as Apéry's constant; $J_2\left(\frac{977}{97}\right)$ the second order cylindrical Bessel coefficient of a rational number; $Y_{(3,000,029/3)}(1,000,033)$ a cylindrical Neumann function with a very high rational order evaluated for a prime-numbered argument in the transition region; $P^{1/3}_{5/7}\left(\frac{467}{509}\right)$ a hypergeometric Legendre function with rational degree and rational order evaluated for a rational argument.