# COMPUTATIONAL OPTIMAL TRANSPORT

A PREPRINT

**Tong Cheng**
Beijing Normal University

Li Cui
Beijing Normal University

Feb. 22, 2022

**ABSTRACT**

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

## 1 Introduction

Optimal transport (OT) distance, known as the Earth Mover's distance, is a classic metric that plays a fundamental role in the probability simplex. Compared with Kullback-Leibler or Jensen-Shannon divergence, OT distance is the unique one that needs to be parameterized. Though the support sets of two distributions do not overlap with each other, the OT distance can still measure the relation between the distributions. [1]

Wasserstain distance

$$W_p(\mu, \nu) = \left( \min_{\kappa \in K(\mu,\nu)} \int_{X \times Y} d(x,y)^p d\kappa(x,y) \right)^{1/p} \tag{1}$$

Barycenter unsupervised learning

Sinkhorn algorithm

Define trajactory $\tau$ and its distribution $p_\pi(\tau)$. Define initial state distribution $\mu(s_0)$.

$$\begin{aligned} R(s_t, a_t) \\ R(\tau) = \sum_{i=1}^{T} \gamma^i R(s_t, a_t) \\ \mathbb{E}_{\tau \sim p_\pi(\tau)} \left( R(\tau) \right) \\ \operatorname*{argmax}_{\pi} \mathbb{E}_{\tau \sim p_\pi(\tau)} \left( R(\tau) \right) \\ p_\pi \tau = \mu(s_0) \prod_{i=0}^{T} P(s_{i+1}|s_i, a_i) \cdot \pi(a_i|s_i) \end{aligned} \tag{2}$$

Wasserstein Robust Reinforcement Learning Objective

Define $\theta$ be the parameter of policy network and $\Phi$ be the parameter of environmental dynamic.

$$\max_{\theta} \left[ \min_{\Phi} \mathbb{E}_{\tau \sim p^{\theta}_{\Phi}(\tau)}[R(\tau)] \right] \quad \text{s.t.} \mathbb{E}_{(s,a) \sim ??}[W_2^2 \left( P_\Phi(\cdot|s,a), P_0(\cdot|s,a) \right)] \leq \varepsilon \tag{3}$$

## 2 Background

The OT distance is originally proposed by French Mathematics Monge. Given two random variables $X \sim \mu$ and $Y \sim \nu$, and a loss function

$$c : (X, Y) \in \mathscr{X} \times \mathscr{Y} \mapsto c(X, Y) \in \mathbb{R}^+. \tag{4}$$

The goal of the OT problem is to minimize the transport cost by finding an optimal mapping $T : \mathscr{X} \to \mathscr{Y}$, i.e.

$$\inf_T \mathbb{E}_{X \sim \mu} \left[ c\left(X, T(X)\right) \right] \quad s.t. \; T(X) \sim Y \tag{5}$$

Furthermore, Kantorovich relaxed the Monge problem into a minimization problem over couplings $(X, Y) \sim \pi$ instead of map $T : \mathscr{X} \to \mathscr{Y}$, where the marginal distributions of $\pi$ equal respectively with $\mu$ and $\nu$. The kantorovich relaxiation permits a given point $X \in \text{Supp}(\mu)$ to be delivered into many different targets $Y \in \text{Supp}(\nu)$. By appending a entropic regularization term $H(\cdot)$, i.e.

$$\begin{aligned} \inf_\pi \mathbb{E}_{(X,Y) \sim \pi} \left[ c\left(X, T(X)\right) \right] + \lambda H(\pi) \\ s.t. \; X \sim \mu, Y \sim \nu, (X, Y) \sim \pi \\ H(\pi) = \text{KL}(\pi \mid \mu \otimes \nu) \end{aligned} \tag{6}$$

, the computation of OT problem can be accelerated.

## 3 Model

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

## 4 Experiment

Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas. Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetuer.

## 5 Related Work

[2] [3] [4]

## 6 Conclusion

Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus.

## References


[1] Mengxue Li, Yi-Ming Zhai, You-Wei Luo, Peng-Fei Ge, and Chuan-Xian Ren. Enhanced transport distance for unsupervised domain adaptation. In *2020 IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR)*, pages 13933–13941, 2020.

[2] Gabriel Peyré and Marco Cuturi. Computational optimal transport, 2020.

[3] Victor M. Panaretos and Yoav Zemel. Statistical aspects of wasserstein distances. *Annual Review of Statistics and Its Application*, 6(1):405–431, Mar 2019.

[4] Na Lei, Dongsheng An, Yang Guo, Kehua Su, Shixia Liu, Zhongxuan Luo, Shing-Tung Yau, and Xianfeng Gu. A geometric understanding of deep learning. *Engineering*, 6(3):361–374, 2020.