# Computational Optimal Transport

A Preprint

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**Abstract**

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## 1 Introduction

Optimal transport (OT) distance, known as the Earth Mover's distance, is a classic metric that plays a fundamental role in the probability simplex. Compared with Kullback-Leibler or Jensen-Shannon divergence, OT distance is the unique one that needs to be parameterized. Though the support sets of two distributions do not overlap with each other, the OT distance can still measure the relation between the distributions. [1]

Wasserstain distance

Barycenter unsupervised learning

Sinkhorn algorithm

## 2 Background

The OT distance is originally proposed by French Mathematics Monge. Given two random variables $X \sim \mu$ and $Y \sim \nu$, and a loss function

$$c : (X, Y) \in \mathscr{X} \times \mathscr{Y} \mapsto c(X, Y) \in \mathbb{R}^{+}. \tag{1}$$

The goal of the OT problem is to minimize the transport cost by finding an optimal mapping $T : \mathscr{X} \to \mathscr{Y}$, i.e.

$$\inf_{T} \mathbb{E}_{X\sim\mu} \left[ c\left(X, T(X)\right) \right] \quad s.t.\ T(X) \sim Y \tag{2}$$

Furthermore, Kantorovich relaxed the Monge problem into a minimization problem over couplings $(X, Y) \sim \pi$ instead of map $T : \mathscr{X} \to \mathscr{Y}$, where the marginal distributions of $\pi$ equal respectively with $\mu$ and $\nu$. The kantorovich relaxiation permits a given point $X \in \mathrm{Supp}(\mu)$ to be delivered into many different targets $Y \in \mathrm{Supp}(\nu)$. By appending a entropic regularization term $H(\cdot)$, i.e.

$$\begin{aligned} \inf_{\pi} \mathbb{E}_{(X,Y)\sim\pi} \left[ c\left(X, T(X)\right) \right] + \lambda H(\pi) \\ s.t.\ X \sim \mu, Y \sim \nu, (X, Y) \sim \pi \\ H(\pi) = \mathrm{KL}(\pi \mid \mu \otimes \nu) \end{aligned} \tag{3}$$

, the computation of OT problem can be accelerated.

## 3 Model

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## 4 Experiment

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## 5 Related Work

[2] [3] [4]

## 6 Conclusion

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## References


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