CS3230 Semester 2 2025/2026

Design and Analysis of Algorithms

Tutorial 01
Introduction and Asymptotic Analysis
For Week 02

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1 Notes

CS3230 tutorial format is as follows: We will consider a few questions per tutorial. For **each question**, we may ask a student to solve it. A **reasonable** attempt for that question will earn the student participation points. TA will give each student at least two chances over the semester. As there are 11 tutorials (excluding the tutorial on Week 6, which will be a recorded video due to CNY), and around 5 questions per tutorial, each student should be able to get enough chances if they are coming regularly to the tutorials.

2 Lecture Review: Asymptotic Analysis

We say[1] that $f \in O(g)$ or $f = O(g)$ or $f(n) \in O(g(n))$ or $f(n) = O(g(n))$ if $\exists c, n_0 > 0$ such that $\forall n \geq n_0, 0 \leq f(n) \leq c \cdot g(n)$.

Informally, in words, the above says that (function) g is an upper bound on (function) f. This is the Big O worst-case time complexity analysis that you have learned since earlier courses, for example, from CS2040/C/S.

The four other asymptotic notations $\Omega, \Theta, o, \omega$ along with O, can be summarised as in the following table.

We say	if $\exists c, c_1, c_2, n_0 > 0$ such that $\forall n \geq n_0$	In other words
$f(n) \in O(g(n))$	$0 \leq f(n) \leq c \cdot g(n)$	g is an **upper** bound on f
$f(n) \in \Omega(g(n))$	$0 \leq c \cdot g(n) \leq f(n)$	g is a **lower** bound on f
$f(n) \in \Theta(g(n))$	$0 \leq c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$	g is a **tight** bound on f

[1]We are fine with either notation, although we prefer $f(n) \in O(g(n))$ notation.

We say	if $\forall c > 0$, $\exists n_0 > 0$ such that $\forall n \geq n_0$	In other words
$f(n) \in o(g(n))$	$0 \leq f(n) < c \cdot g(n)$	g is a **strict upper** bound on f
$f(n) \in \omega(g(n))$	$0 \leq c \cdot g(n) < f(n)$	g is a **strict lower** bound on f

3 Tutorial 01 Questions

Q1). Assume $f(n), g(n) > 0$, show:

(a) $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0 \Rightarrow f(n) \in o(g(n))$ — this has already been shown in lec01b.

(b) $\lim_{n\to\infty} \frac{f(n)}{g(n)} < \infty \Rightarrow f(n) \in O(g(n))$

(c) $0 < \lim_{n\to\infty} \frac{f(n)}{g(n)} < \infty \Rightarrow f(n) \in \Theta(g(n))$

(d) $\lim_{n\to\infty} \frac{f(n)}{g(n)} > 0 \Rightarrow f(n) \in \Omega(g(n))$

(e) $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty \Rightarrow f(n) \in \omega(g(n))$

Q2). Assume $f(n), g(n) > 0$, show:

(a) Reflexivity

- $f(n) \in O(f(n))$
- $f(n) \in \Omega(f(n))$
- $f(n) \in \Theta(f(n))$

(b) Transitivity

- $f(n) \in O(g(n))$ and $g(n) \in O(h(n))$ implies $f(n) \in O(h(n))$
- Do the same for Ω, Θ, o, ω

(c) Symmetry

- $f(n) \in \Theta(g(n))$ iff $g(n) \in \Theta(f(n))$

(d) Complementarity

- $f(n) \in O(g(n))$ iff $g(n) \in \Omega(f(n))$
- $f(n) \in o(g(n))$ iff $g(n) \in \omega(f(n))$

Q3). Which of the following statement(s) is/are True?

(a) $3^{n+1} \in O(3^n)$

(b) $4^n \in O(2^n)$

(c) $2^{\lfloor \log n \rfloor} \in \Theta(n)$ (we assume log is in base 2)

(d) For constants $i, a > 0$, we have $(n + a)^i \in O(n^i)$

Q4). Which of the following statement(s) is/are True?
$2^{\log_2 n} \in$

(a) $O(n)$

(b) $\Omega(n)$

(c) $\Theta(\sqrt{n})$

(d) $\omega(n)$

Q5). Rank the following functions by their order of growth.
(But if any two (or more) functions have the same order of growth, group them together).

- $f_1(n) = \log n$
- $f_2(n) = n!$
- $f_3(n) = 2^n + n$
- $f_4(n) = n^{2.3} + 16n + f_1(n)$
- $f_5(n) = \log(n^2)$
- $f_6(n) = \ln(n^{2n})$