1. (a) 8项 2比较 $C_8^2 = 28$??

如何简化

$\longrightarrow$

$\sqrt{n} < n^{1.5}$ 3rd, 8th good observation

$\sqrt{n} = n^{1/2}$ 因为 $\frac{1}{2} < 1.5$

8项简化 n^α 形式

n, n^2, n^3, n^4 不需要 C_4^2

尝试去转化

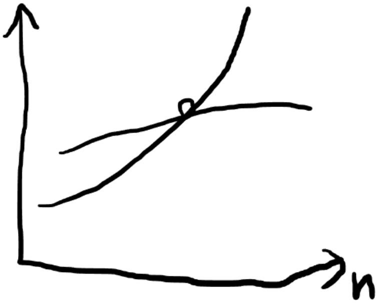
① $4^{2n} = 16^n$

exponential polynomial

α^n n^α

$O(\alpha^n) \overset{?}{\geq} O(n^\alpha)$ α is constant

$\alpha^n \notin O(n^\alpha)$ & $n^\alpha \in O(\alpha^n)$



写证明不要用

② $n^n \geq \alpha^n \geq 16^n$

③ $n^{1.5}$

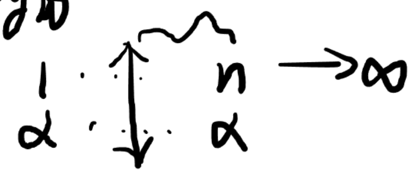
④ $n^3 - n^2 \log n \in O(n^3)$

⑤ $9^{\log_3 n} = n^2$

⑥ $\log_{10} 2^{(n^3)} = n^3 \log_{10} 2$

?

⑦ $n! \geq \alpha^n$



⑧ $n^{\frac{1}{2}}$

④ and ⑥ ④ $\simeq$ ⑥

Hence

$\sqrt{n} \leq n^{1.5} \leq 9^{\log_3 n} \leq$ ④ $\sim$ ⑥ $\leq 4^{2^n} \leq n! \leq n^n$

(b) $g(n) = 2n$ closed-form open

$g(n) = g(\frac{n}{2})$ recursive $x^2+y^2=1$

$g(3) = 3$, $g(n) = 3g(\frac{n}{3}) + n$

$g(5)$ × $g(6)$ ×

$n \in \{3^y : y \in \mathbb{Z}\}$

$g(3), g(9) = 9 + 9 = 18$

$g(27) = 3 \times 18 + 27 = 81$

思考. $g(n) = 3g(\frac{n}{3}) + n$

$= 3(3g(\frac{n}{9}) + n) + n$

$= 3^2 g(\frac{n}{3^2}) + 3n + n$

令 $n = 3^x$ $g(n) = g(3^x) = G(x) = 3g(3^{x-1}) + 3^x$

$= 3G(x-1) + 3^x$

$x \in \mathbb{Z}$

$$a_n = 3a_{n-1} + 3^n, \quad n=1, \quad G(1) = g(3) = 3$$

$$= 3(3a_{n-2} + 3^{n-1}) + 3^n$$

$$= 3^2 a_{n-2} + 2\cdot 3^n$$

$$\vdots$$

$$= 3^{n-1} a_1 + (n-1)\cdot 3^n$$

$$G(x) = 3^{x-1}\cdot 3 + (x-1)\cdot 3^x = x3^x$$

$$g(n) = n\log_3 n$$

$\square$

Made with Goodnotes

2. 0 1 1 2 3 5

$\underset{m}{5} = 2+3$? 不能连续 $5=5$

$\underset{m}{4} = 2+2$? 不能重复 $4 = 1+3$

任意 $m \geq 0$ 0 1 2 3 4 5

数学归纳法 A_0 ✓ induction

if A_{n-1} ✓, to prove A_n.

Hence, A_n holds for every n.

Assume $m = F_{i_1} + \cdots + F_{i_k}$ Induction

to prove $m+1$

也可以 Assume $\forall\, x<m$ holds,

to prove m Strong Induction

$\underbrace{\triangle \quad \triangle \quad \triangle \quad \triangle \quad \triangle}_{\text{holds}} \quad m$

思考: $m = F_i \cdots \qquad F \leq m$

Fibonacci $\quad F_0 \quad F_1 \quad \cdots \quad F_k \quad F_{k+1}$

$\leq m <$

$\exists k$, s.t. $F_k \leq m < F_{k+1}$.

$$F_{k+1} = F_k + F_{k-1} > m \iff \underbrace{m - F_k}_{\triangleq\, m^*} < F_{k-1}$$

$m = F_k + ?$

$m = F_k + m^* \qquad m^* < F_{k-1}$

hypothesis $\quad m^* = \sum_j F_{i_j}$ holds

$(F_{i_j}) + F_k$?

not consecutive

no duplicate

3. $f(n) = n$, $g(n) = n^2$

hint :

Extension: $f(n) \in O(g(n))$

is it true that $\log f(n) \in O(\log g(n))$?