CS3230 Semester 2 2025/2026

Assignment 03
Correctness and Divide-and-Conquer

Due: Sunday, 8th Feb 2026, 11:59 pm SGT.

Instructions:

- Canvas Assignment Submission page: `Assignments/Assignment 3`.
- Please upload PDFs containing your solutions (hand-written & scanned, or typed) by the due date.
- Name the file **Assignment3_SID.pdf**, where SID should be replaced by your student ID.
- You may discuss the problems with your classmates at a high level only. You should write up your solutions on your own (any copying from your co-students or usage of Internet or AI tools is not allowed). Please note the names of your collaborators or any other sources in your submission; failure to do so would be considered plagiarism.
- Question listed as "graded for correctness" (worth 6 points) requires complete answers. Other questions (worth 1 point each) will be graded only based on reasonable attempts. However, you should still do these questions, as they are practice questions, which would be useful for exams as well as for your knowledge.

1. (6 points; graded for correctness) Let $n \geq 1$ be an integer. An enemy fighter can hide in one of the n bunkers. Your government has developed a scanning device which can scan the bunkers in the following fashion: Given two numbers i, j in $[1..n]$, it can tell whether the fighter is in one of the bunkers $i, i+1, \ldots, j$. Your government has two such scanning devices. However, the devices can only be used at exactly 6AM each day, and only once a day (though both devices can be used simultaneously at 6AM).

   (a) (4 points) Given that the enemy fighter is hiding in one of the bunkers, design a method using the two scanning devices to determine, in the least number of days, the bunker in which the enemy fighter is hiding.

   (b) (2 points) Now suppose the enemy fighter can shift everyday at 7AM to one of the adjacent bunkers (from bunker i to bunker $i-1$ or $i+1$, as long as these are within $1..n$). For example, if $n = 10$, and the enemy fighter is in bunker 6, he moves to bunker 7 or 5 or stays in bunker 6 at 7AM. However, if he is in bunker 10, he moves to bunker 9 or stays in bunker 10. Since you have firepower to destroy only one bunker, devise an algorithm to find the bunker in which the fighter is hiding, using as few days as possible.

   For both parts, give a time analysis for your algorithm. Try not to use O notation, but give as accurate as possible number of days needed. Your grade depends on how good your algorithm's time bound is. Your answer being within an additive constant of optimal would not incur a penalty, but only being within a multiplicative constant factor would incur a penalty.

2. (1 point) Suppose you are given two sorted arrays A and B of size m and n respectively, such that all $m+n$ numbers are distinct, along with a number k with $1 \leq k \leq m+n$. Your task is to find the k-th smallest element among the $m+n$ numbers in A and B combined. Give an algorithm for the above problem. Your algorithm should run in time proportional to $\log k$.

3. (1 point) Consider the following algorithm for sorting:

   Input: Array $A[1..n]$ of integers.
   For $i = 1$ to $n-1$ Do
   Invariant 1
   For $j = 1$ to $n-i$ Do
   Invariant 2
   If $A[j] > A[j+1]$, then let $temp = A[j]; A[j] = A[j+1]; A[j+1] = temp$ Endif
   Endfor
   Endfor
   Output Array A.

   We need to prove that the algorithm sorts the array.

   a) Give an invariant for the array at the beginning of the first For loop (Invariant 1).

   b) Give an invariant for the array at the beginning of the second For loop (Invariant 2).

   c) Show that your invariants are true initially, are maintained by the algorithm, give the sorted array at the termination of the first For loop, and give the invariant for the next iteration of the first For loop at the termination of the second For loop.