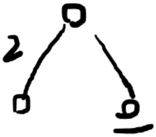
$a=2$  2^n

$$T(n) = a\,T\left(\frac{n}{b}\right) + f(n)$$

a: # of branches

Master Theorem

$d \triangleq \log_b a$

① $f(n) \in O(n^{d-\varepsilon}) \Rightarrow$ $f(n) \in o(n^d)$

$T(n) \in \Theta(n^d)$

② $f(n) \in \Theta(n^d) \Rightarrow T(n) \in \Theta(n^d \log n)$

③ $f(n) \in \Omega(n^{d+\varepsilon}) \Rightarrow$

$T(n) \in \Theta(f(n))$

epsilon

extension $f(n) \in \Theta(n^d \log^k n)$

$\Rightarrow T(n) \in \Theta(n^d \log^{k+1} n)$

$O(n^d \underline{\log n}) \subseteq O(\underline{n^{d+\varepsilon}})$

Q1: $T(n) = 4\,T(\frac{n}{4}) + \frac{n}{\log n}$

$= 4\left(4\cdot T(\frac{n}{4^2}) + \frac{\frac{n}{4}}{\log\frac{n}{4}}\right) + \frac{n}{\log n}$

$= 4^2\cdot T(\frac{n}{4^2}) + \frac{n}{\log\frac{n}{4}} + \frac{n}{\log n}$

$\vdots$

$= 4^i\cdot T(\frac{n}{4^i}) + n\left(\frac{1}{\log\frac{n}{4}} + \frac{4}{\log n}\right)$

待定

Q2: $T(n) = 5\cdot T(\frac{n}{3}) + n$

$n^{\log_3 5} > n^1$ option 3

Q3: $n^{\log_3 9}$ $\underline{n^3}$ ✓ option 4

Q4: n^2 $n^2\log n$ $T(n) = \Theta(n^2\log^2 n)$

② $n^2\log^{k-1} n$ $n^2\log$

option (2)

Q5: substitution

$$T(n) = 4T\left(\frac{n}{2}\right) + \sqrt{n}$$

guess $T(n)$??

To prove $T(n) \in \Theta(n^2)$

hypothesis $T(n) \le c_1 n^2$

$T(1)$?? $\quad T\left(\frac{n}{2}\right) \le c_1 \frac{n^2}{4}$

$$T(n) = 4T\left(\frac{n}{2}\right) + \sqrt{n} \le c_1 n^2 + \sqrt{n}$$

$$T\left(\frac{n}{2}\right) \le c_1 \frac{n^2}{4} + k\left(\frac{n}{2}\right)^{k_2} \quad (k_2 < 2)$$

$$T(n) \le c_1 n^2 + 4k_1\left(\frac{n}{2}\right)^{k_2} + \sqrt{n}$$

令 $k_2 = \frac{1}{2}$, find k_1 s.t.

$$k_1 = \frac{4}{\sqrt{2}} k_1 + 1$$

$$T(n) \leq cn^2 + k_1 n^{k_2} \quad (k_2 < 2)$$

use $T(\frac{n}{2}) \leq c\frac{n^2}{4} + k_1 (\frac{n}{2})^{k_2}$

$\downarrow$

$$T(n) = 4T(\frac{n}{2}) + \sqrt{n}$$

$$\leq cn^2 + \frac{4k_1}{2^{k_2}} n^{k_2} + \sqrt{n}$$

$c = 1, \quad k_2 = \frac{1}{2}$

$$k_1 = \frac{4k_1}{\sqrt{2}} + 1 \; ? \Rightarrow k_1 = -\frac{1}{2\sqrt{2}-1}$$

Lec 2 $\qquad T(n) \leq B_1 n^2 + B_2 n$

$$T(\frac{n}{2}) \leq B_1 \frac{n^2}{4} + B_2 \frac{n}{2}$$

$$T(n) \leq B_1 n^2 + 2B_2 n + n$$

$2B_2 + 1 = B_2$ $\qquad$ B_1 任意值

$B_2 = -1$

$B_2 = -1$

$$\underset{\overset{\|}{c}}{T(1)} \le B_1 n^2 - n \Big|_{n=1} = B_1 - 1$$

$B_1 \ge c + 1$

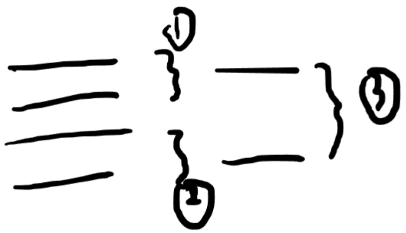
$\square$

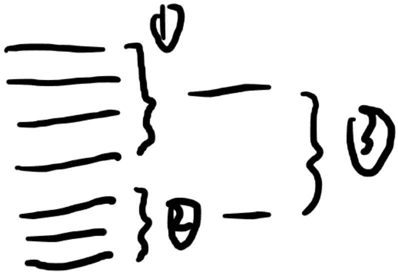
Q6: k: n:

$$T(k,n) = T\left(\left\lceil \frac{k}{2} \right\rceil, n\right) + T\left(\left\lfloor \frac{k}{2} \right\rfloor, n\right) + O(kn)$$

$\lceil\ \rceil$ ceiling $\lceil 2.5 \rceil = 3$

$\lfloor\ \rfloor$ floor $\lfloor 2.5 \rfloor = 2$





$$T(n) = 2T\left(\frac{n}{2}\right) + O(n)$$ n:

$$T(n) = O(n \log n)$$

$$T(k,n) \leq 2T\left(\left\lceil \frac{k}{2} \right\rceil, n\right) + O(kn)$$

$n=1$

$$O(nk\log k)$$

$$T(k,n) \leq 2T\left(\left\lceil \frac{k}{2} \right\rceil, n\right) + O(kn)$$
$$\leq 4T\left(\left\lceil \frac{k}{4} \right\rceil, n\right) + 2O\left(\left\lceil \frac{k}{2} \right\rceil n\right) + O(kn)$$