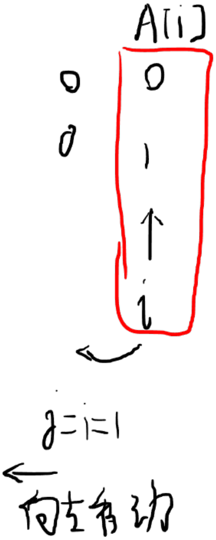
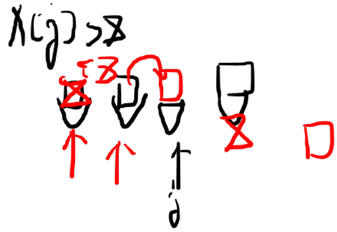
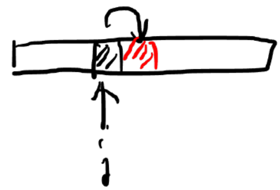
Q1: index start from 0





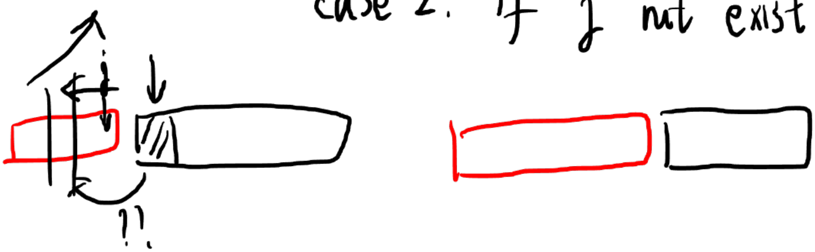
inner loop until j ←



case 1: $A[j] \le X$ 插入点...

then break

case 2: if j not exist, i.e. $A[0] > X$



(a) for each loop i, at the start of iteration prefix $A[0, i-1]$ is sorted in increasing order and is a permutation of the original elements in index $0 \dots i-1$.

3 1 2 4 | T_0 3 1 2 4 |

1 3 2 4 | T_1 1 4 3 2 |

1 2 3 4 | T_2

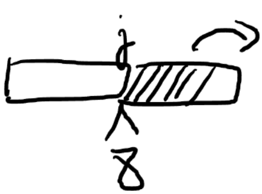
$\vdots$

T_n

(b) initialization: $i = 1$, check $A[0,0] = A[0]$
it holds. the above invariant;

maintenance: at iteration i, the invariant holds.
$A[0, i-1]$ sorted and $A[0, i-1]$ permutation of $(i-1)$-prefix.

Check at the end of iteration i, $A[0, i]$ has the condition ?



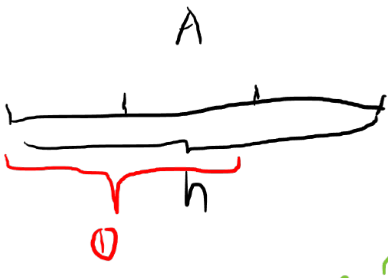
右移动一格

Hence, $A[0, i]$ sorted and is permutation of i-prefix.

termination: at iteration $N-1$, it follows that $A[0, N-1]$ sorted and ... □

1 5 2 6

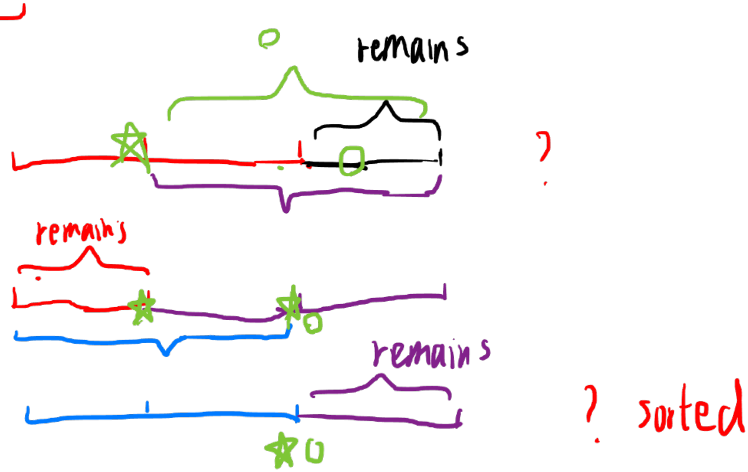
Q2:



after line 5

after line 6

after line 7



3 2 1

2 3 1

2 1 3

1 2 3 V sorted

(a) proof of correctness

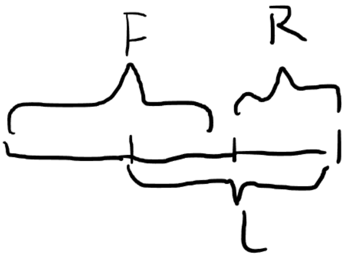
base case: $n=1, n=2$ holds

$n=3$ holds

inductive hypothesis: holds for length $< n$, to prove holds for n.

let $k = \lceil \frac{2n}{3} \rceil$. $m = n-k$ (剩余k后)

$F := A[0, k-1]$ $L := A[m, n-1]$



$(n-1)-m+1 = k$

$R := A[k, n-1]$

$|R| = m$

(1) sort F; (2) sort L; (3) sort F;

By induction hypothesis, (1) (2) (3) outputs are sorted

After step 2, R is sorted.

L is sorted.

Hence, $\max(A[m, k-1]) \leq \min(R)$

$\quad\quad\quad\quad\quad\quad \| \quad\quad\quad\quad\quad \|$

$\quad\quad\quad\quad\quad A[k-1] \quad\quad A[k]$

After step 3. sort $F = A[0, k-1]$ again

$R = A[k, n-1]$ remains and is sorted.

$A[k-1]$ last element of $F \in [m, k-1]$

$A[k-1] \leq \max(A[m, k-1])$

use previous condition

some case

$$A[k-1] \leq \min(R)$$
$$\|$$
$$\max F$$

Hence, $F + R$ is sorted.

(b) $T(n) = 3T(\frac{2n}{3}) + O(1)$

$T(n) = \Theta(n^{\log_{\frac{3}{2}} 3} \approx 2.70)$

Q3 existence: maximum value

Q4: $m \times n$ $T(m,n) = 2T(m, \frac{n}{2}) + \Theta(m)$

$\Rightarrow T(m,n) = \Theta(mn)$

Q5: existence: such peak always exist.

remove line 9 or line 10

(只找一侧)

$T(m,n) = \cancel{2}\, T(m, \frac{n}{2}) + \Theta(m)$

$\Rightarrow T(m,n) = \Theta(m \log n)$.