



$$a=2 \quad \begin{array}{c} \textcircled{1} \\ \diagdown \\ \textcircled{2} \end{array} \quad 2^n \quad T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

Master Theorem

$$d \triangleq \log_b a$$

① $f(n) \in O(n^{d-\varepsilon}) \Rightarrow T(n) \in \Theta(n^d)$
 # of branches
 ② $f(n) \in \Theta(n^d) \Rightarrow T(n) \in \Theta(n^d)$
 ③ $f(n) \in \Omega(n^{d+\varepsilon}) \Rightarrow T(n) \in \Theta(f(n))$

epsilon

extension $f(n) \in \Theta(n^d \log^k n)$

$$\Rightarrow T(n) \in \Theta(n^d \log^{k+1} n)$$

$$\underline{O(n^d \log n)} \leq O(\underline{n^{d+\varepsilon}})$$

$$\begin{aligned}
 Q1: T(n) &= 4T\left(\frac{n}{4}\right) + \frac{n}{\log n} \\
 &= 4\left(4 \cdot T\left(\frac{n}{4^2}\right) + \frac{\frac{n}{4}}{\log \frac{n}{4}}\right) + \frac{n}{\log n} \\
 &= 4^2 \cdot T\left(\frac{n}{4^2}\right) + \frac{n}{\log \frac{n}{4}} + \frac{n}{\log n} \\
 &\vdots \\
 &= 4^i \cdot T\left(\frac{n}{4^i}\right) + n \left(\frac{1}{\log \frac{n}{4^i}} + \frac{i}{\log n} \right)
 \end{aligned}$$

$$\begin{aligned}
 Q2: T(n) &= 5 \cdot T\left(\frac{n}{3}\right) + n \quad \text{不符}
 \end{aligned}$$

$n \log_3 5 > n^1$ option 3

$$\begin{aligned}
 Q3: n^{\log_3 9} &\quad \frac{n^3}{\checkmark} \quad \text{option 4}
 \end{aligned}$$

$$\begin{aligned}
 Q4: n^2 &\quad n^2 \log n \quad T(n) = \underline{\underline{n^2 \log^2 n}}
 \end{aligned}$$

$$\begin{aligned}
 \Theta(n^2 \log^{k-1} n) &\quad n^2 \log \quad \text{option (2)}
 \end{aligned}$$

Q5: substitution

$$T(n) = 4T\left(\frac{n}{2}\right) + \sqrt{n}$$

guess $T(n)$??

To prove $T(n) \in \Theta(n^2)$

hypothesis $T(n) \leq c_1 n^2$

$$T(1) ?? \quad T\left(\frac{n}{2}\right) \leq c_1 \frac{n^2}{4}$$

$$T(n) = 4T\left(\frac{n}{2}\right) + \sqrt{n} \leq c_1 n^2 + \sqrt{n}$$

$$T\left(\frac{n}{2}\right) \leq c_1 \frac{n^2}{4} + k_1 \left(\frac{n}{2}\right)^{k_2} \quad (k_2 < 2)$$

$$T(n) \leq c_1 n^2 + 4k_1 \left(\frac{n}{2}\right)^{k_2} + \sqrt{n}$$

$\therefore k_2 = \frac{1}{2}$, find k_1 s.t.

$$k_1 - \frac{4}{\sqrt{2}} k_1 + 1$$

$$T(n) \leq Cn^2 + k_1 n^{k_2} \quad (k_2 < 2)$$

use $T\left(\frac{n}{2}\right) \leq C \frac{n^2}{4} + k_1 \left(\frac{n}{2}\right)^{k_2}$



$$T(n) = 4T\left(\frac{n}{2}\right) + \sqrt{n}$$

$$\leq Cn^2 + \frac{4k_1}{2^{k_2}} n^{k_2} + \sqrt{n}$$



$$C=1, \quad k_2=\frac{1}{2}$$

$$k_1 = \frac{4k_1}{\sqrt{2}} + 1 ? \Rightarrow k_1 = -\frac{1}{2\sqrt{2}-1}$$

$$\text{Loc 2} \quad T(n) \leq B_1 n^2 + B_2 n$$

$$T\left(\frac{n}{2}\right) \leq B_1 \frac{n^2}{4} + B_2 \frac{n}{2}$$

$$T(n) \leq B_1 n^2 + 2B_2 n + h$$

$$2B_2 + 1 \equiv B_2$$

B_1 係々 值

$$B_2 = -1$$

$$\beta_2 = -1$$

$$T(1) \leq B_1 n^2 - n \Big|_{n=1} = B_1 - 1$$

$$\beta_1 \geq c + 1$$

Q6: k: n:

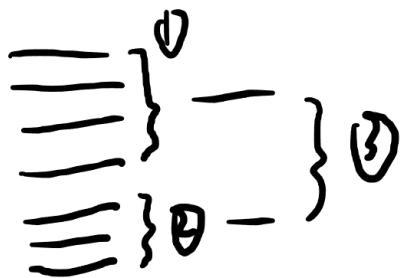
1

$$T(k,n) = T(\lceil \frac{k}{2} \rceil, n) + T(\lfloor \frac{k}{2} \rfloor, n)$$

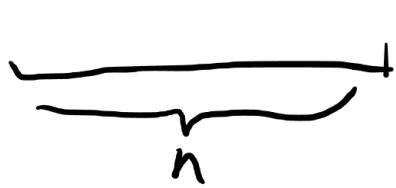
$$\lceil \lceil \text{ceiling}(\lceil 2.5 \rceil) \rceil = 3 + O(kn)$$

LJ floor LJ.5) = 2

A diagram consisting of several horizontal lines. On the left, there are four parallel horizontal lines. To their right is a brace (a vertical line with curly ends) enclosing the first two lines, with a circled '0' positioned above the brace. Further to the right is a single horizontal line. Below this line is another brace enclosing the last two lines, with a circled '0' positioned below the brace.



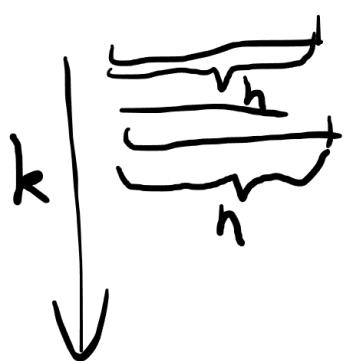
$$T(n) = 2T\left(\frac{n}{2}\right) + O(n) \quad n:$$



$$T(n) = O(n \log n)$$

$$T(k, n) \leq 2T\left(\lceil \frac{k}{2} \rceil, n\right) + O(kn)$$

$n=1$



$$O(nk \log k)$$

$$T(k, n) \leq 2T\left(\lceil \frac{k}{2} \rceil, n\right) + O(kn)$$

$$\begin{aligned} &\leq 4T\left(\lceil \frac{k}{4} \rceil, n\right) + 2O\left(\lceil \frac{k}{2} \rceil n\right) \\ &\quad + O(kn) \end{aligned}$$

