# CS3230 Semester 2 2025/2026

# Assignment 01
# Introduction and Asymptotic Analysis

Due: Sunday, 25th Jan 2026, 11:59 pm SGT.

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**Instructions:**

- Canvas Assignment Submission page: `Assignments/Assignment 1`.
- Please upload PDFs containing your solutions (hand-written & scanned, or typed) by the due date.
- Name the file **Assignment1_SID.pdf**, where SID should be replaced by your student ID.
- You may discuss the problems with your classmates at a high level only. You should write up your solutions on your own (any copying from your co-students or usage of Internet or AI tools is not allowed). Please note the names of your collaborators or any other sources in your submission; failure to do so would be considered plagiarism.
- Question listed as "graded for correctness" (worth 6 points) requires complete answers. Other questions (worth 1 point each) will be graded only based on reasonable attempts. However, you should still do these questions, as they are practice questions, which would be useful for exams as well as for your knowledge.
- Notation: The following notation is used for all the assignments.

  $\ln n$ denotes natural logarithm of $n$.

  $\log_b n$ denotes base $b$ logarithm of $n$.

  When base is not given, as in $\log n$, then it denotes base 2 logarithm.

  $n!$ denotes $1 \cdot 2 \cdot 3 \cdot \ldots \cdot n$.

  $\mathbb{Z}$ denotes the set of integers.

  $\mathbb{Z}_{\geq 0}$ denotes the set of non-negative integers, $\{0, 1, 2, \ldots\}$.

1. (6 points; graded for correctness)

   (a) (3 points) Consider the following eight functions of $n$.

   $$4^{2n}, \qquad n^n, \qquad n^{1.5}, \quad n^3 - n^2 \log n,$$

   $$9^{\log_3 n}, \quad \log_{10} 2^{(n^3)}, \qquad n!, \qquad \sqrt{n}$$

   Order the above functions on the basis of nondecreasing order from smallest to largest, where $f(n)$ is considered smaller than $g(n)$ if $f(n) \in O(g(n))$ but $g(n) \notin O(f(n))$. If $f(n) \in \Theta(g(n))$, then either can come earlier in the order. Give proof/arguments on why your order is correct.

   (b) (3 points) Suppose $g$ is a function which is defined recursively as follows for $n \geq 3$ being power of 3:

   $g(3) = 3$, $g(n) = 3g(n/3) + n$.

   Find a closed-form solution for $g(n)$, for $n \geq 3$ being power of 3.

   (**Hint:** Try to find $g(n)$ for $n$ being $3, 3^2, 3^3, 3^4, \ldots$. Do you see any pattern?)

2. (1 point) Recall that the Fibonacci numbers are defined as follows: $F_0 = 0$, $F_1 = 1$, and for $n \geq 2$, $F_n = F_{n-1} + F_{n-2}$.

   Prove that every non-negative integer $m$ can be expressed as a sum of a finite set of Fibonacci numbers, no two of which are the same or consecutive. That is, every non-negative integer $m$ can be written as $m = F_{i_1} + F_{i_2} + \ldots + F_{i_k}$, where (a) $i_1 < i_2 < \cdots < i_k$, and (b) for $1 \leq j < k$, $i_j + 1 < i_{j+1}$.

3. (1 point) Suppose that $f(n), g(n) : \mathbb{Z}_{\geq 0} \to \mathbb{Z}_{\geq 0}$ are increasing functions such that $f(n) \in o(g(n))$. Must it be true that $\lceil \log(f(n)) \rceil \in o(\lceil \log(g(n)) \rceil)$?

   (**Note:** For a real number $x$, the ceiling $\lceil x \rceil$ denotes the smallest integer at least $x$.)