Homework 1

1 Problem 1

1.1 (a)

$$\begin{aligned} a &= \frac{\delta v}{\delta t} && (1)\\ &= \frac{60mph - 0mph}{5s} && (2)\\ &= \frac{26.8m/s}{5s} && (3)\\ &= 5.36m/s^2 && (4) \end{aligned}$$

1.2 (b)

$$Ratio = \frac{10720N}{3.6 * 10^{22}N} = 2.978 * 10^{-19}$$

Given that the ratio of order of 10^{-19}, it needs this single "random" event to the gravitational force to the accuracy of 19 decimal places.

2 Problem 2

2.1 (a)

The meaning is that some new territory is not developed in Kolmogorov system. The new territory is the application applied to the real world that is conditional on our state of knowledge about the world. But in Kolmogorov system, this kind of knowledge is assumed arbitrarily at the beginning of the problem.

2.2 (b)

Both scientific and probabilistic model require infinite many measurement. In both cases, we are taking the lmit to validate the models, and both models converges to the solution when we take infinite steps.

2.3 (c)

Analytic mechanics often refers to employing mathematical structures to describe motion and forces. Probability aims to offer a frame work for understanding uncertainty. Both are fundamental discipline to solve complex problems. In mechanics, masses is the fundamental measure to calculate forces. Similarly, probability gives us a measure of likelihood of different outcomes.

In my opinion, "us" here signify people like us build model based on probability, people who emphasize their perspective on foundamental importance of probability, similar to how a phycist views mechanics.

3 Problem 3

3.1 (a)

If $\alpha = \frac{1}{5}$, $X_n = sin^2(\frac{2^n\pi}{5}) = sin^2(\frac{\pi}{5})$ or $sin^2(\frac{-\pi}{5}) = \frac{10-2\sqrt{5}}{16} = \frac{5-\sqrt{5}}{8}$

3.2 (b)

Given $\theta = arcsin(\sqrt{X_n})$,

$$\begin{aligned}\frac{\delta\theta}{\delta X} &= \frac{1}{\sqrt{1-X_n}} * \frac{1}{2}X^{-\frac{1}{2}} &(5)\\ &= \frac{1}{\sqrt{1-Xn}} * \frac{1}{2\sqrt{X_n}} &(6)\\ &= \frac{1}{2\sqrt{X_n(1-X_n)}} &(7)\\ & &(8)\end{aligned}$$

$$\begin{aligned}\rho_\theta(\theta) &= \rho(X)|\frac{\delta X}{\delta\theta}| &(9)\\ &= \rho(X) * 2\sqrt{X(1-X)} &(10)\\ &= \frac{1}{\pi\sqrt{X(1-X)}}2\sqrt{X(1-X)} &(11)\\ &= \frac{2}{\pi} &(12)\end{aligned}$$

Therefore, the distribution of θ is $\frac{2}{\pi}$, which is uniformly distributed between 0 and $\frac{\pi}{2}$.

3.3 (c)

When α is rational, since $sin(2\pi)$ is periodic in nature, the sequence X_n will eventually become periodic. The function $\alpha 2^n\pi$ will eventually cycle through a set of value as n increase. On the other hand. if α is irrational, $\alpha 2^n\pi$ is not able to repeat a set of value as n increase, then X will not be periodic.

4 Problem 4

4.1 (a)

Take the second order derivative of X(t),

$$x(t) = Asin(\omega t + \phi) \tag{13}$$
$$\frac{\delta x}{\delta t} = Acos(\omega t + \phi)\omega \tag{14}$$
$$\frac{\delta^2 x}{\delta t^2} = -Asin(\omega t + \phi)\omega \tag{15}$$
$$= -\omega^2 x(t) \tag{16}$$

Since, $\frac{\delta^2 x}{\delta t^2} = -\frac{k}{m}x$, $\omega^2 = \frac{k}{m}$, $\omega = \sqrt{\frac{k}{m}}$.

4.2 (b)

Given $x(t) = Asin(\omega t + \phi)$ and $T = \frac{2\pi}{\omega}$, we write x(t) as the function t(x).

$$t = \frac{arcsin(\frac{x}{A})}{\omega} - \phi$$

$$\frac{\delta t}{\delta x} = \frac{1}{\sqrt{1 - (\frac{X}{A})^2}} * \frac{1}{\omega * A} \tag{17}$$
$$= \frac{1}{\omega * A\sqrt{1 - \frac{x^2}{A^2}}} \tag{18}$$

$\rho(x)$ is related to uniform distribution ρ,

$$\rho(x) = \rho(t)\frac{\delta t}{\delta x} \tag{19}$$
$$= \frac{\rho(t)}{\sqrt{1 - \frac{x^2}{A^2}}} \tag{20}$$

Based on the function $\rho(x)$, as x $\rightarrow$ A, $\rho(x)$ increase, as x $\rightarrow$ 0, $\rho(x)$ decrease. Therefore, value x is more dense as x gets closer to -A or A.