AMATH 561 Homework Assignment #1
[Due online via Canvas: Monday 11:59pm, October 9, 2023]

1. A car, with a mass of $2,000$ kg, decelerates from 60 mph ($\approx$ 26.8 m/sec) to full stop in about 5 second.

(a) What is the acceleration? Assuming air resistance can be neglected, what is the force the car generates against the earth?

(b) In terms of the gravitational force between the earth and the sun ($\approx 3.6 \times 10^{22}$ newton), how accurate, *e.g.* number of decimal places, one needs in order to detect such an single "random" event as a perturbation to the gravitational force?

2. (a) What is the meaning of the statement that "half of probability theory is not present at all in the Kolmogorv system"?

(b) Discuss in parallel the following two statements: "an absolute verification of a scientific model for a real system requires measurements with infinite precision" and "a probabilistic model for a real phenomenon requires infinitely many repetitive measurements". Try to argue that in both cases a notion of "identical" (or same, or steady, etc.) is needed.

(c) Check out what ***analytic mechanics*** is. Then try to elaborate on the statements that "probability is a mathematical discipline with aims akin to analytic mechanics" and "probabilities play for us the same role as masses in mechanics". What does "us" signify here?

3. It is easy to verify that $x_n = \sin^2(\alpha 2^n \pi)$ satisfies the iterative, logistic equation $x_{n+1} = 4x_n(1-x_n)$, whose solution is known to exhibit complex behavior including chaos.

(a) When $\alpha = 1$, one has $x_n = \sin^2(2^n\pi) = 0$ for all n; when $\alpha = \frac{1}{3}$, $x_n = \sin^2(2^n\pi/3) = \sin^2(\pi/3)$ or $\sin^2(-\pi/3) = \frac{3}{4}$ for all n. What happens if $\alpha = \frac{1}{5}$?

(b) Knowing that the sequence x_n can be chaotic with invariant density function given as

$$\rho(x) = \frac{1}{\pi\sqrt{x(1-x)}} \text{ where } x \in (0,1),$$

what does this imply for the distribution of $\theta = \arcsin\sqrt{x_n}$ for all the positive integers n?

(c) Discuss the possibilities for sequence x_n as α being a rational number or being an irrational number.

4. Consider a harmonic oscillator, an idealized mechanical system, according to Newton's equation of motion, $\mathrm{d}^2x/\mathrm{d}t^2 = -(k/m)x$, where k is a spring constant and m is the

mass of a poinet mass:

$$x(t) = A\sin(\omega t + \phi).$$

(a) What is the value of ω?

(b) $x(t)$ is a periodic function of t with period $T = 2\pi/\omega$. As a function of t, the value of $x(t)$ is recurrent on interval $[-A, A]$. Let us assume that time t being uniform on $[0, T]$, what is the distribution of the value $x \in [-A, A]$?