理 例题讲解:

1. $\frac{dy}{dt} = 1-\sin t$ $\qquad$ $\frac{dx}{dt}\big|_{t=0} = 1$

$e^y \frac{dy}{dt} + y + t\frac{dy}{dt} + \cos t = 0 \Rightarrow \frac{dy}{dt} = -1$

$x = 1$

$e^y = 1 \quad (\Rightarrow y=0)$

2. $\frac{x-1}{1} = \frac{y-0}{-1}$

2. $x = 1+t$, $y = 0$, $z = 2+2t$

$d^2 = [2-(1+t)]^2 + 1^2 + [-3-(2+2t)]^2$

$\Rightarrow \cdots$

选 B

3. $F(x,y,z) = x^2 + \cos(xy) + yz + x = 0$

$\nabla F = (2x - y\sin(xy)+1, -x\sin xy + z, y)$

$\nabla F(0,1,-1) = (1, -1, 1)$.

故选 A

4. $\begin{vmatrix} i & j & k \\ 2 & -1 & 0 \\ 0 & 1 & 2 \end{vmatrix} = (-2, -4, 2)$ $\qquad$ $F(x,y,z) = x^2 + 2y^2 + \frac{z^2}{4} - z$

$\nabla F = (2x, 4y, -1)$

$\Rightarrow 2x = 2 \quad 4y = 2 \Rightarrow x = \frac{1}{2} \quad y = \frac{1}{2}$

$\Rightarrow z = \frac{3}{2}$

$\Rightarrow x + 2y - z = 0$

5. 猜位: $\Delta f = \frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r} + \frac{1}{r^2}\frac{\partial f}{\partial \theta^2}$

$\Delta g = \frac{\partial^2 f(\frac{1}{r})}{\partial r^2} + \frac{1}{r}\frac{\partial f(\frac{1}{r})}{\partial r} = \left(-f'(\frac{1}{r})\frac{1}{r^2}\right)' + \frac{1}{r}f'(\frac{1}{r})(-\frac{1}{r^2})$

$= r^{-3} f'(\frac{1}{r}) + r^{-4} f''(\frac{1}{r})$

6. 全题: $f(x,y) = 2x - y + 2 \cdot o(\sqrt{x^2+(y-1)^2}) \Rightarrow dz\big|_{(0,1)} = 2dx - dy$

7.

A 反例： $f(x,y)=|x|+|y|$

除 C、D，取 $f(x,y)=x$，因为 $\lim\limits_{\substack{x\to 0 \\ y\to 0}}\dfrac{f(x,y)}{|x|+|y|}$ 与 $\lim\dfrac{f(x,y)}{x^2+y^2}$ 都不存在。

B: 考察定义。 $\checkmark$

故选 B

8. 选 A （基本的运算题）

9. $\dfrac{\partial u}{\partial x}=y^2z^3$, $\quad \dfrac{\partial u}{\partial y}=2xyz^3$ $\quad\quad \dfrac{\partial u}{\partial z}=3xy^2z^2$

$\quad =1 \quad\quad\quad\quad\quad =2. \quad\quad\quad\quad\quad =3$

$\cos\alpha=\cos\beta=\dfrac{2}{3}$, $\quad \cos\gamma=-\dfrac{1}{3}$

则 $\dfrac{\partial u}{\partial n}=\dfrac{2}{3}\times 1+\dfrac{2}{3}\times 2+(-\dfrac{1}{3})\times 3=1$

10. 答案选 D