Exploring how the geometry of the representation space UNIVERSITE Influences curiosity-based exploration





Nils Ruet, Tonglin Yan, Dimitri Ognibene, Kenneth Williford, David Rudrauf, Grégoire Sergeant-Perthuis

How does geometry influence exploration behaviors?

In human spatial awareness...

- 3D Projective geometry structures information integration and planning
- Subjective perspectives can be taken on this internal representation space

Methodology

- Agent a looks for an object O using observations $y_o \in Y$.
- ullet Agent's "internal world model" space X is a group structured space, where a group acts on it. We compare spaces structured by Euclidean Geometry vs Projective Geometry.
- lacktriangleright The agent plans its actions m by taking perspectives on its internal G-space X, corresponding to the choice of a group action.
- Beliefs P_X about the position of object O =probability measure on the internal space.
- Uncertainty of observations = Markov Kernel $P_{Y|X}$. Updated beliefs $P_{X|y_o}$ are computed using Bayes' rule.

Epistemic value and exploration algorithm

ullet Epistemic value/curiosity $C={
m divergence}\ H$ between prior and posterior beliefs (from Active Inference)

$$C(P_X) := \mathbb{E}_{P_Y} \left[H(P_{X|Y}|P_X) \right] \tag{1}$$

- A broader belief distribution yields a greater epistemic value
- The agent maximizes at each step the epistemic value of the posterior beliefs.

Setup of toy model **Euclidean Case Projective Case**

Approaching a point in projective geometry magnifies the surrounding space.

Results & discussion

I. Geometry can induce a drive to approach the object

Euclidean case: the agent stays still

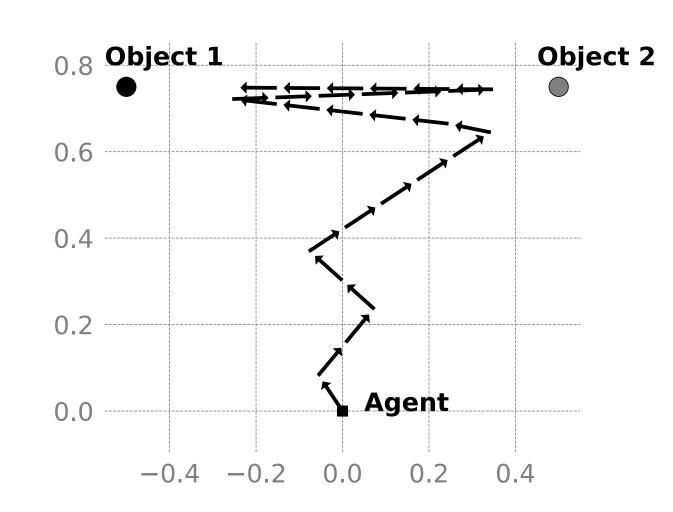
Viewpoint doesn't affect the broadness of beliefs: epistemic value is constant w.r.t. movement

Projective case: the agent approaches the object Getting closer magnifies the belief distribution = broader beliefs = epistemic value increases as evidence becomes more valuable

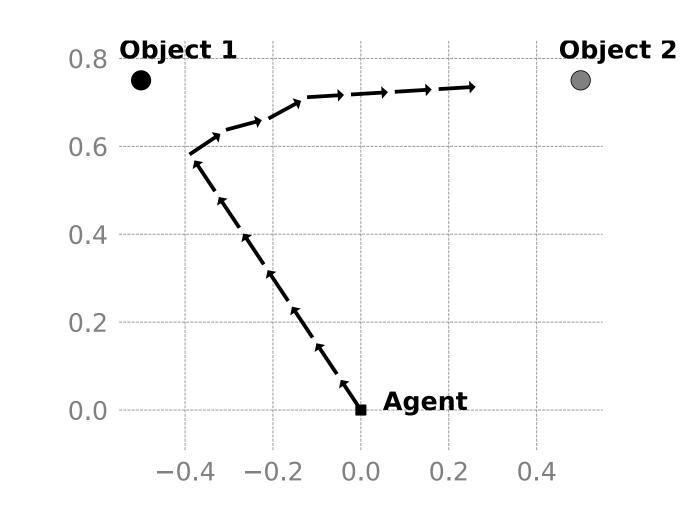
object agent agent

II. Exploration behaviors in multi-object projective settings

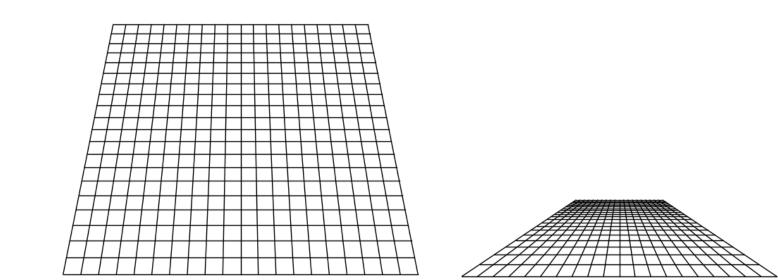
- Observing an object weakens the drive to approach it as the epistemic value decreases (the belief distribution narrows with evidence)
- Projective geometry: approaching the object increases the epistemic value (surrounding) space is magnified = broader beliefs)
- Strong projective deformation = amplified magnifying = higher drive to approach. This drive may compensate the decrease in epistemic value from the observations.
- Altering the magnitude of the projective effect results in different exploration behaviors



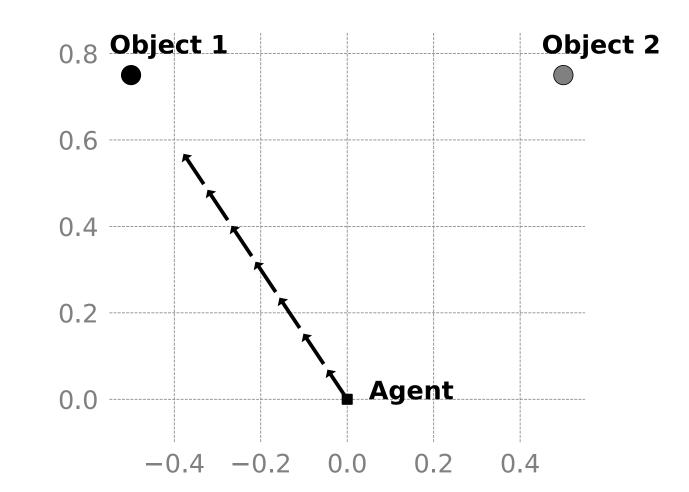
Weak projective deformation = lower drive to approach = quick oscillation between objects



Moderate projective deformation = lowfrequency oscillation



Weak vs strong projective deformation of the euclidean grid. The magnifying effect on approach is amplified with greater deformations.



Strong projective deformation = the **drive to** approach prevails over the decrease in epistemic value (very low-frequency)

Conclusion

- Projective geometry generates an epistemic drive to approach an object
- The magnitude of projective deformations can be interpreted as modulating the agent's focus on a target.

^{3.} Williford, Bennequin, and Rudrauf, "Pre-Reflective Self-Consciousness & Projective Geometry", 2022 4. Rudrauf, Bennequin, and Williford, "The Moon Illusion Explained by the Projective Consciousness Model", 2020