Test-3 Version-A

No work = No credit!
You may use the following facts:

$$\boxed{\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad \sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}}$$

$$\boxed{e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n, \quad \cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}}$$

1. Determine whether the following sequences converge or diverge. (12pt+8pt)
   (a) $a_n = \dfrac{1}{n\sec(n)}$
   (b) $a_n = n\sin\left(\dfrac{1}{n}\right)$ $\boxed{\text{Hint: L'Hopital Rule}}$

2. Determine whether $\displaystyle\sum_{n=1}^{\infty} a_n$ converge or diverge. (12pt+8pt)
   (a) $a_n = (-1)^n \dfrac{2n+3}{3n^2+2n}$
   (b) $a_n = \dfrac{\sqrt{n+1}}{n^2\sqrt{n+1}-n}$ $\boxed{\text{Hint: Comparison Test}}$

3. Find the interval of convergence and radius of convergence. (12pt+8pt)
   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^n$
   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-3)^n}{n^2 3^n}$

4. Find a power series representation for the following functions and use it to determine the radius of convergence. (12pt+8pt)
   (a) $f(x) = \ln(1+x^2)$ $\boxed{\text{Hint: } \left(\ln(1+x^2)\right)' = \dfrac{2x}{1+x^2}}$
   (b) $f(x) = \dfrac{2x^2}{(1-x)^3}$ $\boxed{\text{Hint: } \left(\dfrac{1}{1-x}\right)'' = \dfrac{2}{(1-x)^3}}$

5. Find the Taylor series and Taylor polynomial with degree 3 of the following functions. (12pt+8pt)
   (a) $f(x) = x\cos(2x), \quad a = 0$
   (b) $f(x) = \dfrac{1}{\sqrt{x}}, \quad a = 1$ $\boxed{\text{Hint: } f^{(n)}(x) = (-1)^n \dfrac{(2n-1)!}{2^n} x^{-\frac{2n+1}{2}}, \quad n \geq 1}$

6. Compute the exact values of following series. (6pt+6pt+8pt)
   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{2^{2n+1}(2n+1)!}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{3^{n+1} n!}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{3^n}$

Test-3 Version-B

No work = No credit!

You may use the following facts:

$$\boxed{\frac{1}{1-x}=\sum_{n=0}^{\infty} x^n, \quad \sin(x)=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{2n+1}}$$

$$\boxed{e^x=\sum_{n=0}^{\infty}\frac{1}{n!}x^n, \quad \cos(x)=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n}}$$

1. Determine whether the following sequences converge or diverge.(12pt+8pt)
   (a) $a_n=\dfrac{1}{\csc(n)n}$
   (b) $a_n=n\tan\left(\dfrac{1}{n}\right)$ $\boxed{\text{Hint: L'Hopital Rule}}$

2. Determine whether $\sum_{n=1}^{\infty} a_n$ converge or diverge.(12pt+8pt)
   (a) $a_n=(-1)^n\dfrac{3n+2}{2n^2+3n}$
   (b) $a_n=\dfrac{\sqrt{2n+1}}{n^2\sqrt{2n+1}-n}$ $\boxed{\text{Hint: Comparison Test}}$

3. Find the interval of convergence and radius of convergence.(12pt+8pt)
   (a) $\sum_{n=0}^{\infty}\dfrac{(-1)^n}{2n+1}x^n$
   (b) $\sum_{n=1}^{\infty}\dfrac{(x+3)^n}{n^2 3^n}$

4. Find a power series representation for the following functions and use it to determine the radius of convergence.(12pt+8pt)
   (a) $f(x)=\arctan(x^2)$ $\boxed{\text{Hint: } \left(\arctan(x^2)\right)'=\dfrac{2x}{1+x^4}}$
   (b) $f(x)=\dfrac{4x}{(1-x)^3}$ $\boxed{\text{Hint: } \left(\dfrac{1}{1-x}\right)''=\dfrac{2}{(1-x)^3}}$

5. Find the Taylor series and Taylor polynomial with degree 3 of the following functions. (12pt+8pt)
   (a) $f(x)=x\sin(2x), \quad a=0$
   (b) $f(x)=\dfrac{1}{\sqrt{x}}, \quad a=1$ $\boxed{\text{Hint: } f^{(n)}(x)=(-1)^n\dfrac{(2n-1)!}{2^n}x^{-\frac{2n+1}{2}}, \quad n\geq 1}$

6. Compute the exact values of following series.(6pt+6pt+8pt)
   (a) $\sum_{n=0}^{\infty}\dfrac{(-1)^n\pi^{2n}}{2^{2n}(2n)!}$ (b) $\sum_{n=0}^{\infty}\dfrac{2^{n+1}}{3^n n!}$ (c) $\sum_{n=1}^{\infty}\dfrac{n}{3^n}$