Test-4-version A

No work = No credit!

1.Surface $\mathcal{S}$ is hemisphere $z=\sqrt{x^2+y^2}$, $1\le z\le 2$.(6+7+7=20pt)

a) Find the parametric equation $\overrightarrow{r}(\rho,\theta)$ of the surface with spherical coordinates and find the domain D for ρ and θ;

b) Find $\overrightarrow{r_\phi}$, $\overrightarrow{r_\theta}$ and $\overrightarrow{r_\rho}\times\overrightarrow{r_\theta}$;

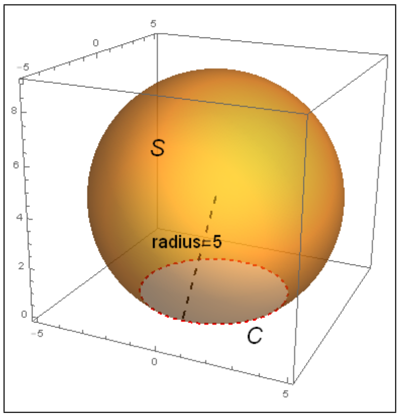
c) Find $\displaystyle\iint_{\mathcal{S}}(x^2+y^2+z^2)\,d\mathcal{S}$.

2.Surface $\mathcal{S}$ is part of the sphere $x^2+y^2+z^2=4$ that lies above $z=0$, upward oriented.(5+7+8=20pt)

a) Find the parametric equation $\overrightarrow{r}(\phi,\theta)$ of the surface;

b) Find $\overrightarrow{r_\phi}$, $\overrightarrow{r_\theta}$ and $\overrightarrow{r_\phi}\times\overrightarrow{r_\theta}$;

c) $\overrightarrow{F}=\langle x,y,z\rangle$, find $\displaystyle\iint_{\mathcal{S}}\overrightarrow{F}\cdot d\overrightarrow{\mathcal{S}}$.



3. Surface $\mathcal{S}$ is part of the sphere $x^2+y^2+(z-3)^2=25$, $z\ge 0$, outward oriented and $\overrightarrow{F}=\langle -y+e^{2017z}, x+\sin(z)\tan(z), xyz\rangle$. (10+8+12=30pt)

a) Find the parametric equation for the boundary $\mathcal{C}$ of $\mathcal{S}$;

b) Find the parametric equation for region enclosed by $\mathcal{C}$ on xy-plane;

c) Use Stoke's Theorem to find $\displaystyle\iint_{\mathcal{S}} curl\overrightarrow{F}\cdot d\overrightarrow{\mathcal{S}}$.

4. Curve $\mathcal{C}$ is the positively oriented boundary of surface $\mathcal{S}$: $z=1-x-y$ in the first octant and $\overrightarrow{F}=\langle xy,yz,zx\rangle$. (5+7+8=20pt)

a) Find $curl\overrightarrow{F}$;

b) Find the parametric equation for $\mathcal{S}$;

c) Use Stoke's Theorem to find $\displaystyle\oint_{\mathcal{C}}\overrightarrow{F}\cdot d\overrightarrow{r}$.

5. $\mathcal{S}$ is $z=2-x^2-y^2$, $z\ge 1$, upward oriented. $\overrightarrow{F}=\langle z\tan^{-1}(y^2), z^3 ln(1+x^2), z\rangle$. (7+13=20pt)

a) Find $div\overrightarrow{F}$;

b) Use Divergence Theorem to find $\displaystyle\iint_{\mathcal{S}}\overrightarrow{F}\cdot d\overrightarrow{\mathcal{S}}$.

Test-2-version B

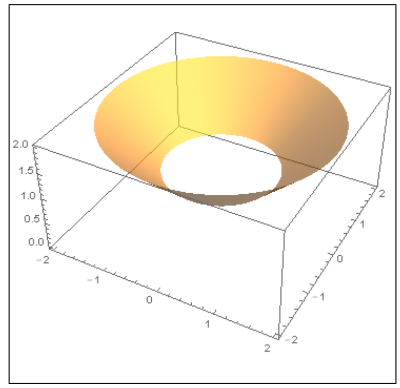
No work = No credit!

1.Surface $\mathcal{S}$ is $z = 4 - x^2 - y^2$, $z \geq 0$.(6+7+7=20pt)

a) Find the parametric equation $\overrightarrow{r}(\phi, \theta)$ of the surface with spherical coordinates and find the domain D for ϕ and θ;

b) Find $\overrightarrow{r_\phi}$, $\overrightarrow{r_\theta}$ and $\overrightarrow{r_\phi} \times \overrightarrow{r_\theta}$;

c) Find $\displaystyle\iint_{\mathcal{S}} (y^2 z + x^2 z)\, d\mathcal{S}$.

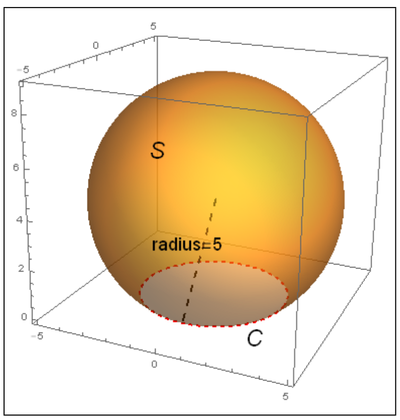


2.Surface $\mathcal{S}$ is part of the cone $z = \sqrt{x^2 + y^2}$ that lies between $z = 1$ and $z = 2$, upward oriented.(5+7+8=20pt)

a) Find the parametric equation $\overrightarrow{r}(u, v)$ of the surface with polar coordinates $x = u\cos(v)$ and $y = u\sin(v)$ and find the domain D for u and v;

b) Find $\overrightarrow{r_u}$, $\overrightarrow{r_v}$ and $\overrightarrow{r_u} \times \overrightarrow{r_v}$;

c) $\overrightarrow{F} = \langle x, y, -z^3 \rangle$, find $\displaystyle\iint_{\mathcal{S}} \overrightarrow{F} \cdot d\overrightarrow{\mathcal{S}}$.

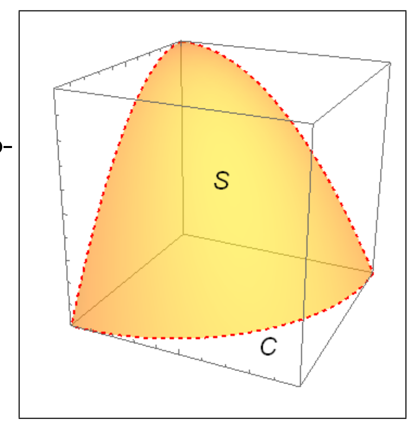


3. Surface $\mathcal{S}$ is part of the sphere $x^2 + y^2 + (z-4)^2 = 25$, $z \geq 0$, outward oriented and $\overrightarrow{F} = \langle y + \tan(z), -x + \sin(z), -xyz \rangle$. (10+8+12=30pt)

a) Find the parametric equation for the boundary $\mathcal{C}$ of $\mathcal{S}$;

b) Find the parametric equation for region enclosed by $\mathcal{C}$ on xy-plane;

c) Use Stoke's Theorem to find $\displaystyle\iint_{\mathcal{S}} curl\,\overrightarrow{F} \cdot d\overrightarrow{\mathcal{S}}$.



4. Curve $\mathcal{C}$ is the positively oriented boundary of surface $\mathcal{S}$: $z = 1 - x^2 - y^2$ in the first octant and $\overrightarrow{F} = \langle -xy, -yz, -zx \rangle$. (5+7+8=20pt)

a) Find $curl\,\overrightarrow{F}$;

b) Find the parametric equation for $\mathcal{S}$;

c) Use Stoke's Theorem to find $\displaystyle\oint_{\mathcal{C}} \overrightarrow{F} \cdot d\overrightarrow{r}$.

5. $\mathcal{S}$ is the outward oriented surface of solid ball **E** centered at origin with radius 2 and $\overrightarrow{F} = \langle x^3 + y^3, y^3 + z^3, z^3 + x^3 \rangle$. (5+7+8=20pt)

a) Find $div\,\overrightarrow{F}$;

b) Use spherical coordinates (ρ, θ, ϕ) to describe region **E**

c) Use Divergence Theorem to find $\displaystyle\iint_{\mathcal{S}} \overrightarrow{F} \cdot d\overrightarrow{\mathcal{S}}$.