Test-3 Version-A

No work = No credit!

1. Consider $y' = (1+y)y(1-y)$
   (a) Find the constant solution. (4pt)
   (b) Find the increasing and decreasing regions. (4pt×2)
   (c) Sketch the solutions with initial condition $y(0) = 2$, -2, 0.5, or -0.5 (2pt×4)

2. Solve $\begin{cases} (x^2+1)y' = 2x^3e^y \\ y(0) = 0 \end{cases}$

3. Suppose a population grows according to a logistic model with initial population 100 and carrying capacity 1,000. If the population grows to 250 after 10 year, what will the population be after 20 years, and when the population reaches 750?

4. Solve $\begin{cases} y'' + 6y' + 10y = 0 \\ y(0) = 1 \\ y'(0) = 1 \end{cases}$

5. Solve $\begin{cases} y'' - 4y' + 4y = 4x + 2e^{2x} \\ y(0) = 0 \\ y'(0) = 0 \end{cases}$

6. A series circuit contains a resistor with $R = 4$, an inductor with $L = 1H$, a capacitor with $C = \frac{1}{8}F$, and a $4V$ battery. The initial charge is 1 and the initial current is 0. Find the charge and current at time t.

Test-3 Version-B

No work = No credit!

1. Consider $y' = (2+y)y(2-y)$
   (a) Find the constant solution. (4pt)
   (b) Find the increasing and decreasing regions. (4pt×2)
   (c) Sketch the solutions with initial condition $y(0) = 1$, -1, 3, or -3 (2pt×4)

2. Solve $\begin{cases} (x^2+1)y' = 2x^3e^{-y} \\ y(0) = 0 \end{cases}$

3. Suppose a population grows according to a logistic model with initial population 100 and carrying capacity 1,000. If the population grows to 500 after 20 year, what will the population be after 30 years, and when the population reaches 250?

4. Solve $\begin{cases} y'' - 6y' + 10y = 0 \\ y(0) = 1 \\ y'(0) = 1 \end{cases}$

5. Solve $\begin{cases} y'' - 4y' + 4y = 4x - 2e^{2x} \\ y(0) = 0 \\ y'(0) = 0 \end{cases}$

6. A spring with a $1kg$ mass (damping constant $c = 4$) has natural length $10m$ and spring constant $k = 8$. If the spring is stretched to a length of $11m$ and then released with zero velocity, and an external force $4N$ is acting on the mass. Find the position and velocity of the mass at any time t.