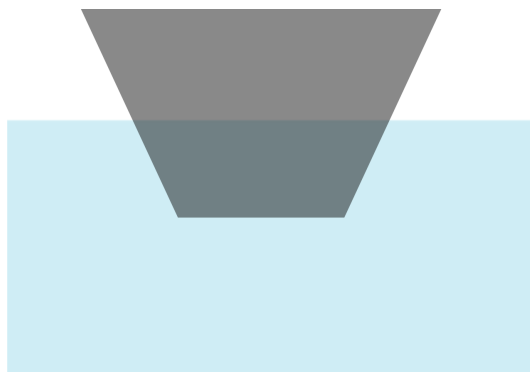


Test-1 Version A

**No work = No credit! The mass density of water is  $1000 \text{ kg/m}^3$  and for simplicity take gravity acceleration to be  $10 \text{ m/s}^2$ .**

1. Find the **EXACT** arc length of curve  $x = \sqrt{2}\sin^2(t)$ ,  $y = \sqrt{2}\cos^2(t)$ ,  $t \in (0, \pi/2)$ .
2.  $f(x) = x^2 - 2x + 2$ ,  $x \in [0, 3]$ . Find  $f_{ave}$  and  $c$  such that  $f(c) = f_{ave}$ .
3. The force to stretch a spring from its natural length 0.1m to 0.25m is 30N. How much work is done in stretching this spring from 0.2m to 0.3m?
4. A tank has the shape of a hemisphere (circular part at the top) with a radius of  $\sqrt{2}$  meters. If the tank is full of water, find the **EXACT** work required to empty the tank by pumping the water to the top of the tank.
5. A trapezoid with bases 5 m and 11 m is partially submerged vertically in water so that the top is 1 m above the surface and the bottom is 1 m below the surface (The figure below is not drawn to scale.) Express the hydrostatic force against one side of the plate as an integral and evaluate it Exactly.

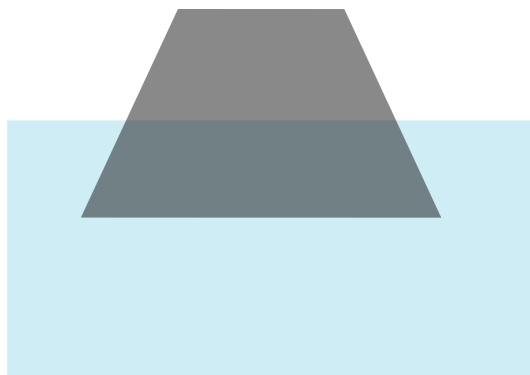


6. A plane lamina with a constant density  $\rho = 10$  is enclosed by  $x = 0$  from left,  $x = \pi/2$  from right,  $y = 0$  from below and  $y = \cos(x)$  from above. Find the **EXACT** values of moments  $M_x$ ,  $M_y$  and center of mass  $(\bar{x}, \bar{y})$ . (Property of  $\cos(x)$ :  $\cos^2(x) = \frac{1}{2}(\cos(2x) + 1)$ )

Test-1 Version B

**No work = No credit! The mass density of water is  $1000 \text{ kg/m}^3$  and for simplicity take gravity acceleration to be  $10 \text{ m/s}^2$ .**

1. Find the **EXACT** arc length of curve  $x = \cos^2(t)$ ,  $y = \sqrt{3}\sin^2(t)$ ,  $t \in (0, \pi/2)$ .
2.  $f(x) = x^2 - 4x + 5$ ,  $x \in [0, 3]$ . Find  $f_{ave}$  and  $c$  such that  $f(c) = f_{ave}$ .
3. The force to stretch a spring from its natural length 0.15m to 0.3m is 30N. How much work is done in stretching this spring from 0.25m to 0.35m?
4. A tank has the shape of a hemisphere (circular part at the top) with a radius of 2 meters. If the tank is full of water, find the **EXACT** work required to empty the tank by pumping the water to the top of the tank. (The mass density of water is  $1000 \text{ kg/m}^3$  and use gravity acceleration  $10 \text{ m/s}^2$ ).
5. A trapezoid with bases 11 m and 5 m is partially submerged vertically in water so that the top is 1 m above the surface and the bottom is 1 m below the surface (The figure below is not drawn to scale.) Express the hydrostatic force against one side of the plate as an integral and evaluate it Exactly.



6. A plane lamina with a constant density  $\rho = 10$  is enclosed by  $x = 0$  from left,  $x = \pi/2$  from right,  $y = 0$  from below and  $y = \sin(x)$  from above. Find the **EXACT** values of moments  $M_x$ ,  $M_y$  and center of mass  $(\bar{x}, \bar{y})$ . (Property of  $\sin(x)$ :  $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$ )