Amath482

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1 Introduction

A submarine is moving through Puget Sound, emitting an unknown acoustic frequency that must be detected to track its location. In this report, a Fourier Transform is applied to convert noisy acoustic data from the time domain to the frequency domain, enabling the identification of relevant frequency components. To further refine the data, a Gaussian Filter is used to denoise the signal by reducing irrelevant noise while preserving the critical acoustic signature. Finally, these techniques detect the submarine's acoustic frequency for effective tracking.

2 Theoretical Background

2.1 Fourier Series
The Fourier Series is a mathematical method that combines a periodic function with sine and cosine waves with varying frequencies, amplitudes, and phases.
Fourier Series is written as:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos(nx) + b_n \sin(nx)\right), \quad x \in (-\pi, \pi]. \quad [1]$$

2.2 Fourier Transform
The Fourier Transform is a mathematical technique that transforms a signal from its original time domain into the frequency domain.
Fourier Transform and Inverse Fourier Transform are written as:

$$F(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ikx} f(x)\, dx \quad [2]$$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ikx} F(k)\, dk \quad [3]$$

2.3 Fast Fourier Transform (FFT)
The Fast Fourier Transform (FFT) is an efficient algorithm for quickly calculating the Discrete Fourier Transform (DFT) and its inverse.

2.4 Gaussian Filter

A Gaussian Filter is a mathematical technique that reduces noises and amplifies specific frequencies. It is written as:

$$F(k) = \exp\left(-\tau(k - k_0)^2\right) \quad [4]$$

3 Algorithm Implementation and Development

- Load the data from subdata.npy, which is a matrix of size 262144 x 49, a flattened matrix of a $64 \times 64 \times 64 \times 49$ matrix.
- The spatial domain spans [-10,10] in each direction with 64...
- Construct a 3D meshgrid to create coordinate arrays (`xx`, `yy`, `zz`) for computations
- Reshape the signal data into 3D by the function *np.reshape* **Part One**
- Compute Fourier Transform of the signals from spatial domain to frequency domain using the function *np.fftn*
- Move the lower frequency at the center using *np.fftshift*
- Sum up the Fourier transform of all signals, divide by the total number of signals, and normalize to find out the average Fourier Transform.

Part Two

- Find out $\sigma = 10, \tau = \frac{1}{2\sigma^2}$, and value of k, using *np.fft.fftfreq* to generate frequency values.
- Calculate the Gaussian Filter for 3 dimensions respectively
- Multiply each Fourier transformed signals with the Gaussian filter in the frequency domain.
- Utilize Inverse FFT to reconstruct filtered signal

Part Three

- Eliminate z from the 3-dimension
- Find the index of the largest values from 3 dimensions.
- Map the x and y index on the grid

3D graphing

- Find the index of filtered frequency with maximum value.
- Construct Isosurface visualization by using the function *go.Isosurface*, with the inputs of x=xx.flatten(), y=yy.flatten(), z=zz.flatten(), value=filtered_signal.flatten() and the isomin = isomax = 0.7

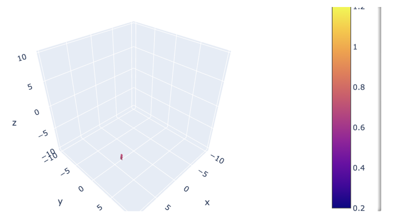

Figure 1: Averaging FT

4 Computational Results

The dataset contains multiple signals and noises components. By averaging the Fourier Transform, the random noises are canceled out, and the dominant frequencies are amplified. Throughout averaging Fourier Transform, the dominant frequency generated by the submarine is determined as (39, 49, 10). It is illustrated by figure 1.
Another way to eliminate noises is applying Gaussian Filter. By defining different sigma, the Gaussian Filter can filter out noises by different level. It has been illustrated in graphs that the smaller sigma might eliminate more noises. The dominant frequency generated by submarine is determined as (52, 15, 35). They are illustrated by Figure 2 and 3.
By plotting x and y coordinates of the submarine during the 24 hour period, we can determine the movement path of the submarine. The final position is [6.25 -5.3125], and it is illustrated by Figure 4

5 Summary and Conclusions

This study utilized Fourier Transforms to detect a submarine by identifying the dominant frequency, using techniques like averaging and Gaussian Filter. The dominant frequency used in the filtering method is (39, 49, 10). The dominant frequency utilized Gaussian Filtering is (52,15,35). The final position of the submarine's movement is [6.25,5.3125].

6 Acknowledgments

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7 References

[1][2][3][4]Kutz, J. N. (2013). *Data-driven modeling & scientific computation: Methods for integrating dynamics of complex systems and big data.* Oxford University Press.

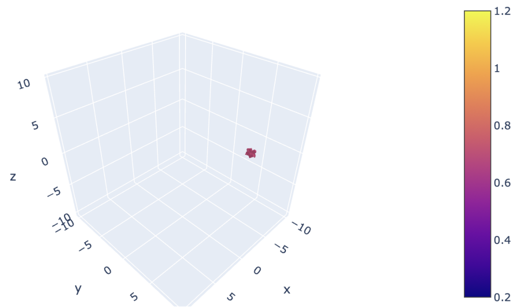

Figure 2: Sigma = 3

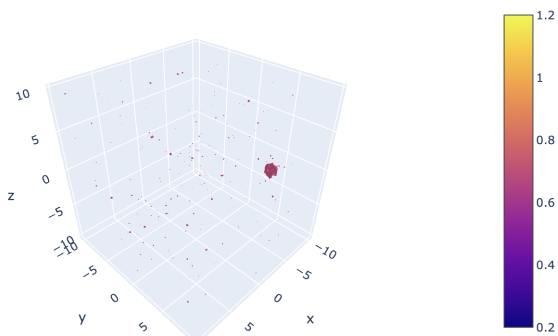

Figure 3: Sigma = 10

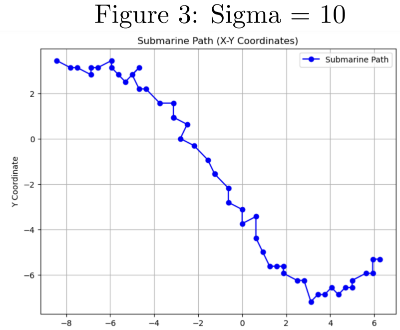

Figure 4: Submarine Path