ACM e-Energy 2020, Virtual

# Real-time Flexibility Feedback

*for Closed-loop* ***Aggregator*** *and System* ***Operator*** *Coordination*

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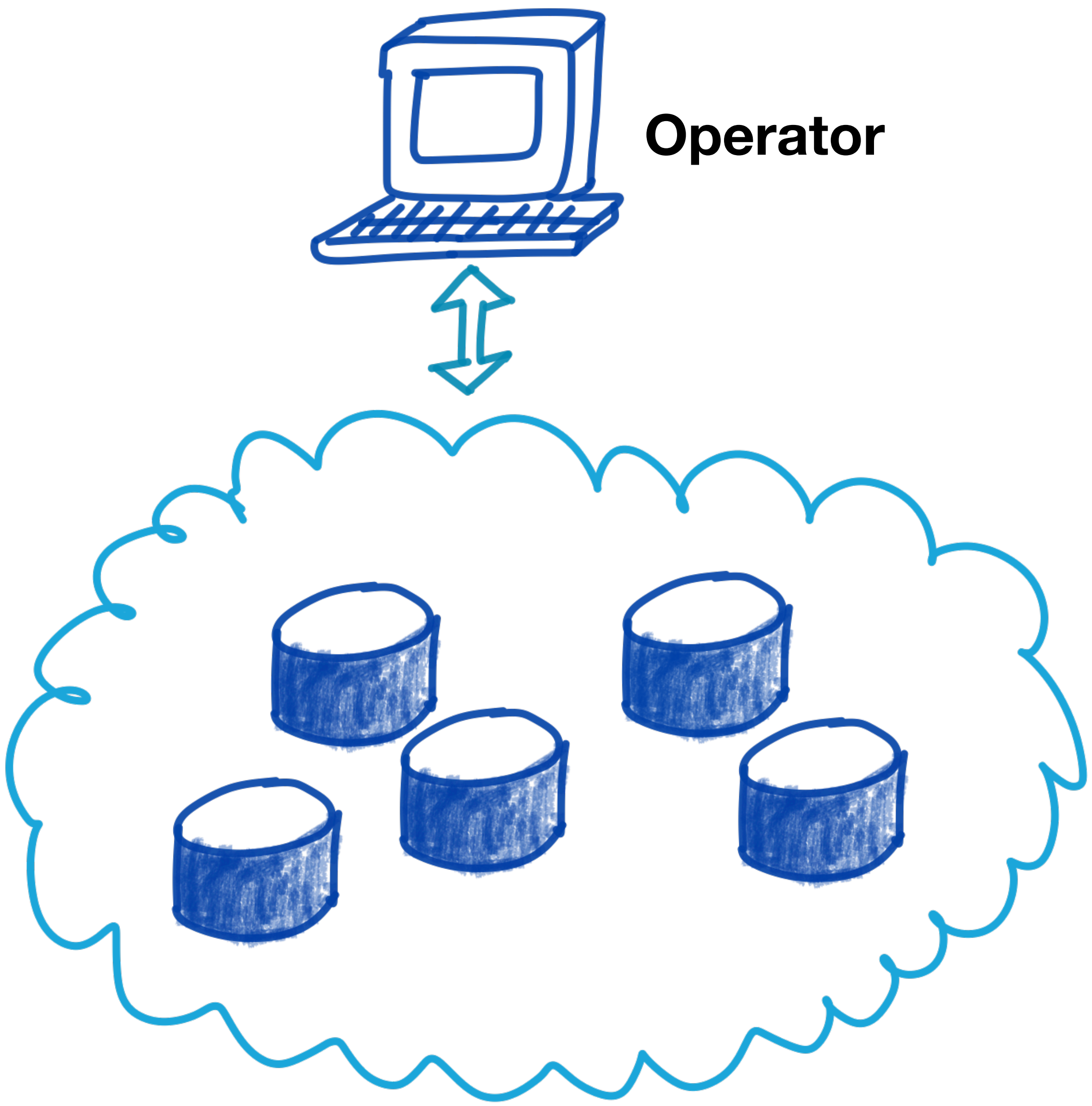
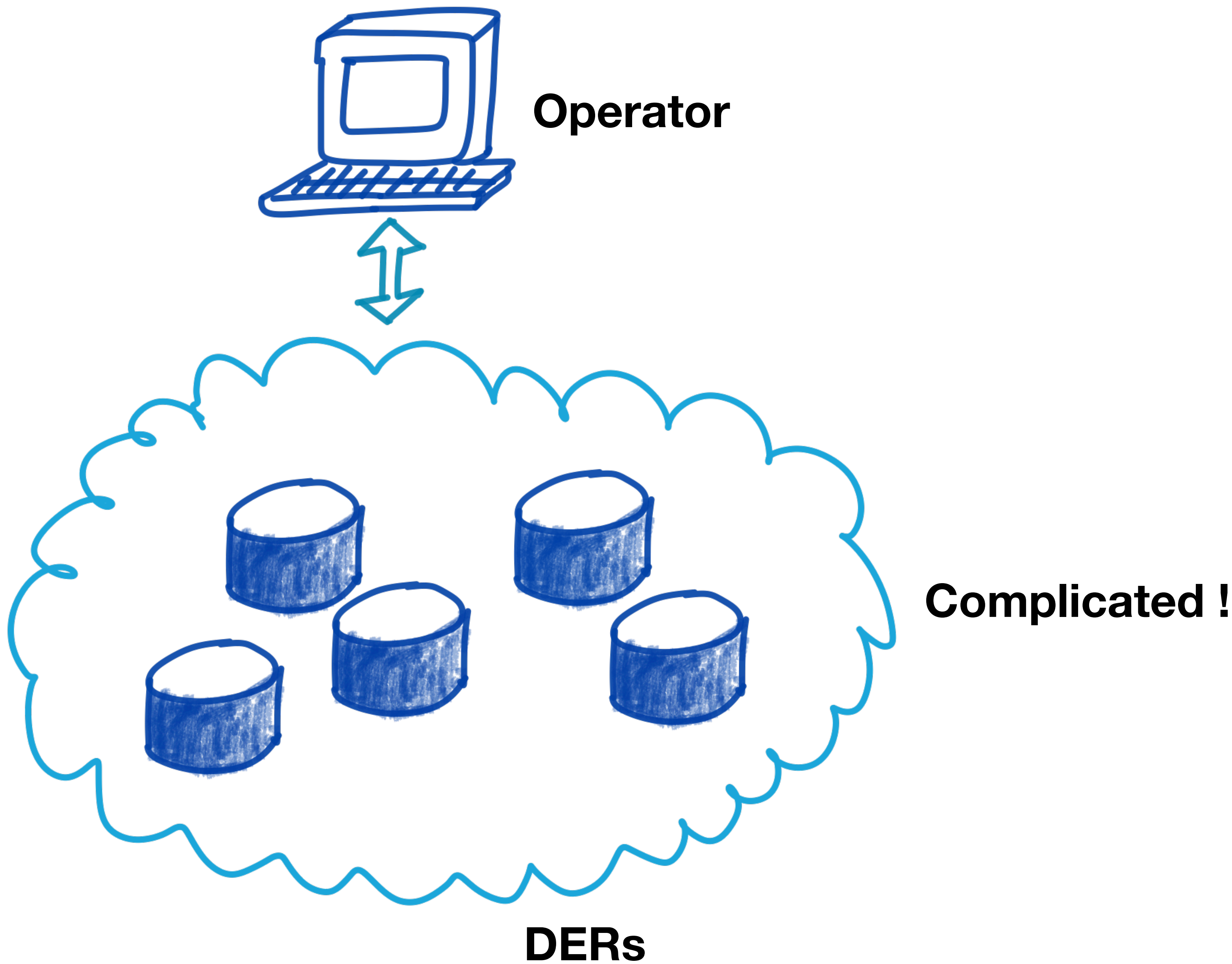
**Why need information aggregation?**



## Why need information aggregation?



## Challenges

| | | |
|---|---|---|
| **Issues:** | • States of the DERs are hard to be transmitted | **Communication complexity** |
| | • Operator's optimization is hard to solve | **Computational complexity** |
| | • Constraints cannot be transmitted exactly | **Privacy** |

…

## Challenges

**Issues:**

- States of the DERs are hard to be transmitted — **Communication complexity**
- Operator's optimization is hard to solve — **Computational complexity**
- Constraints cannot be transmitted exactly — **Privacy**

…

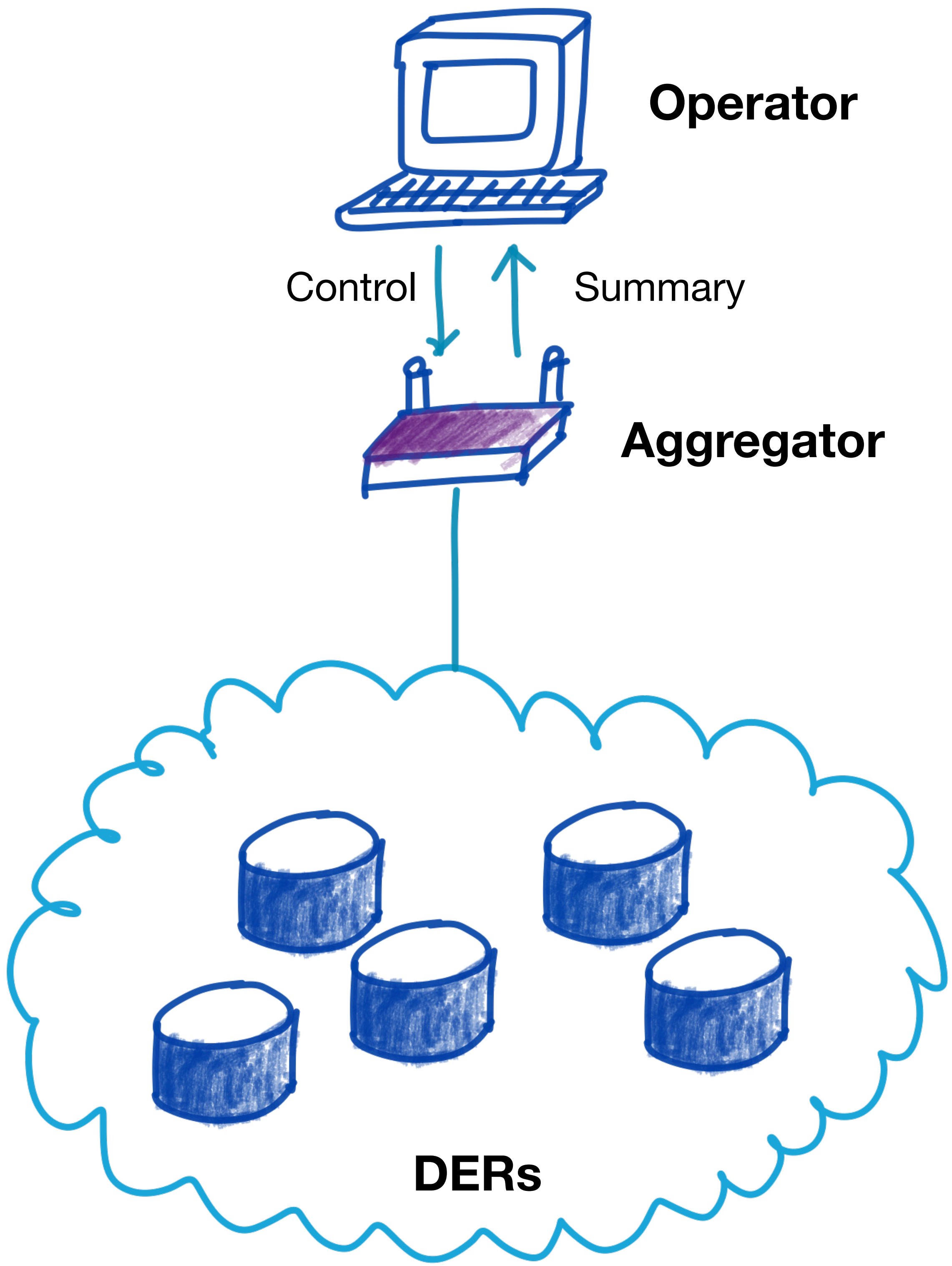
**Aggregate flexibility** ⟷ Power systems

Cluster scheduling

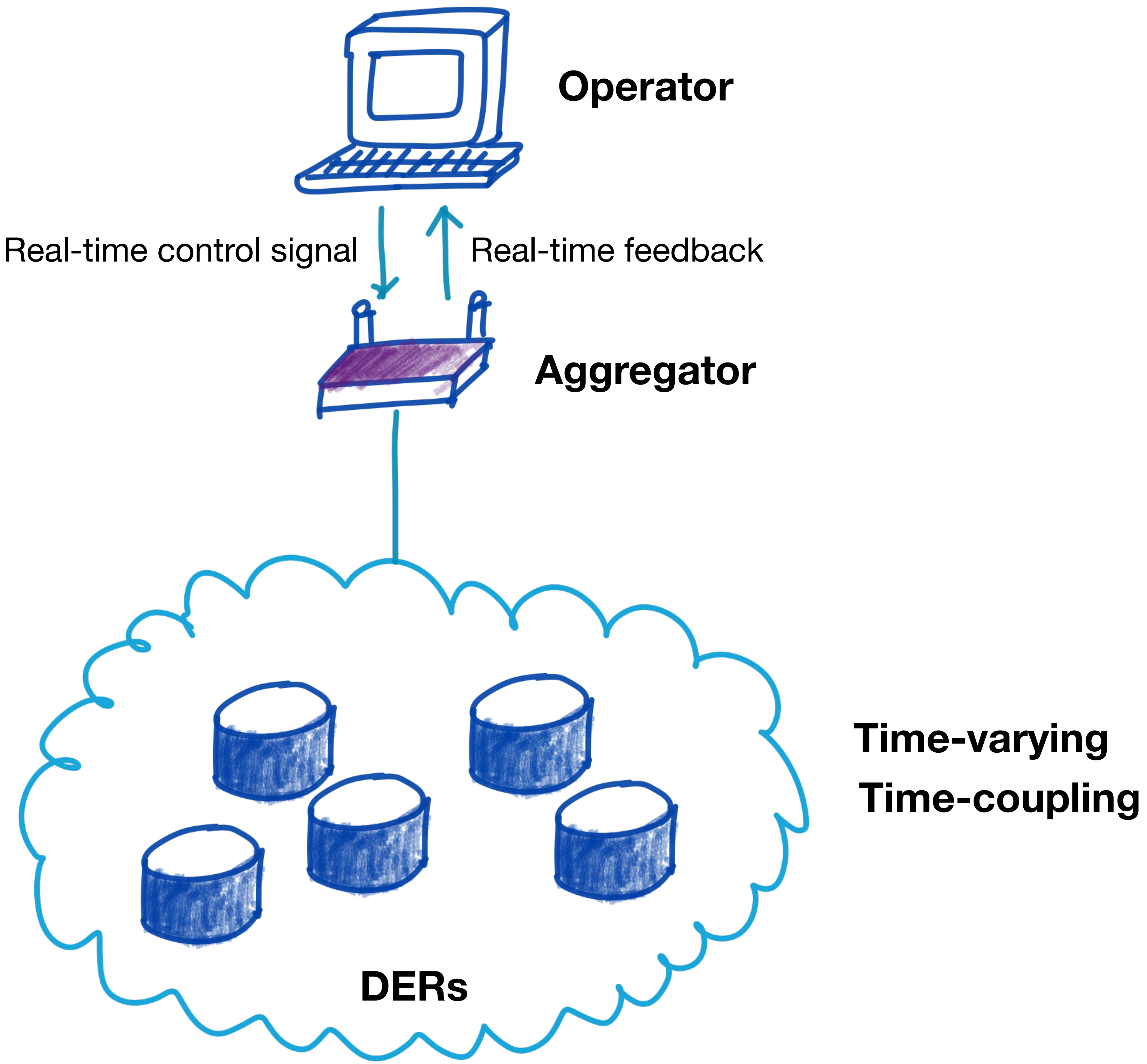
Fog computing

…

## Why need information aggregation?



## In this talk…


Operator
Real-time control signal
Real-time feedback
Aggregator
Time-varying
Time-coupling
DERs

**Why need information aggregation?**

**Existing approaches**

(offline approximations)

A. Virtual battery model
*[Hao et al. 2014] [Zhao et al. 2017] [Madjidian et al. 2018]*

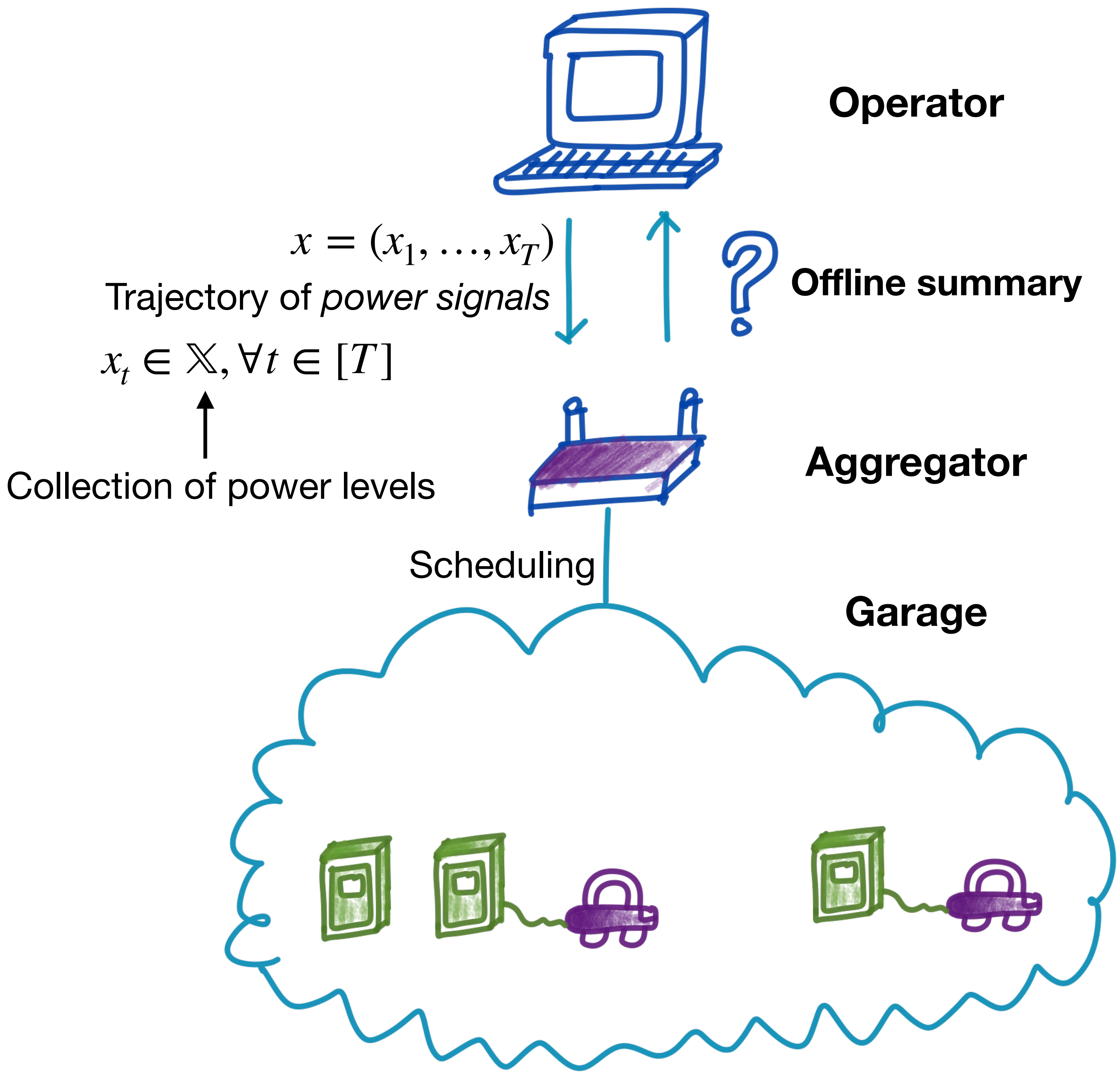
B. Hyper-rectangular approximation
*[Chen et al. 2018]*

C. Minkowski sum of polytopes
*[Zhao et al. 2017]*

...

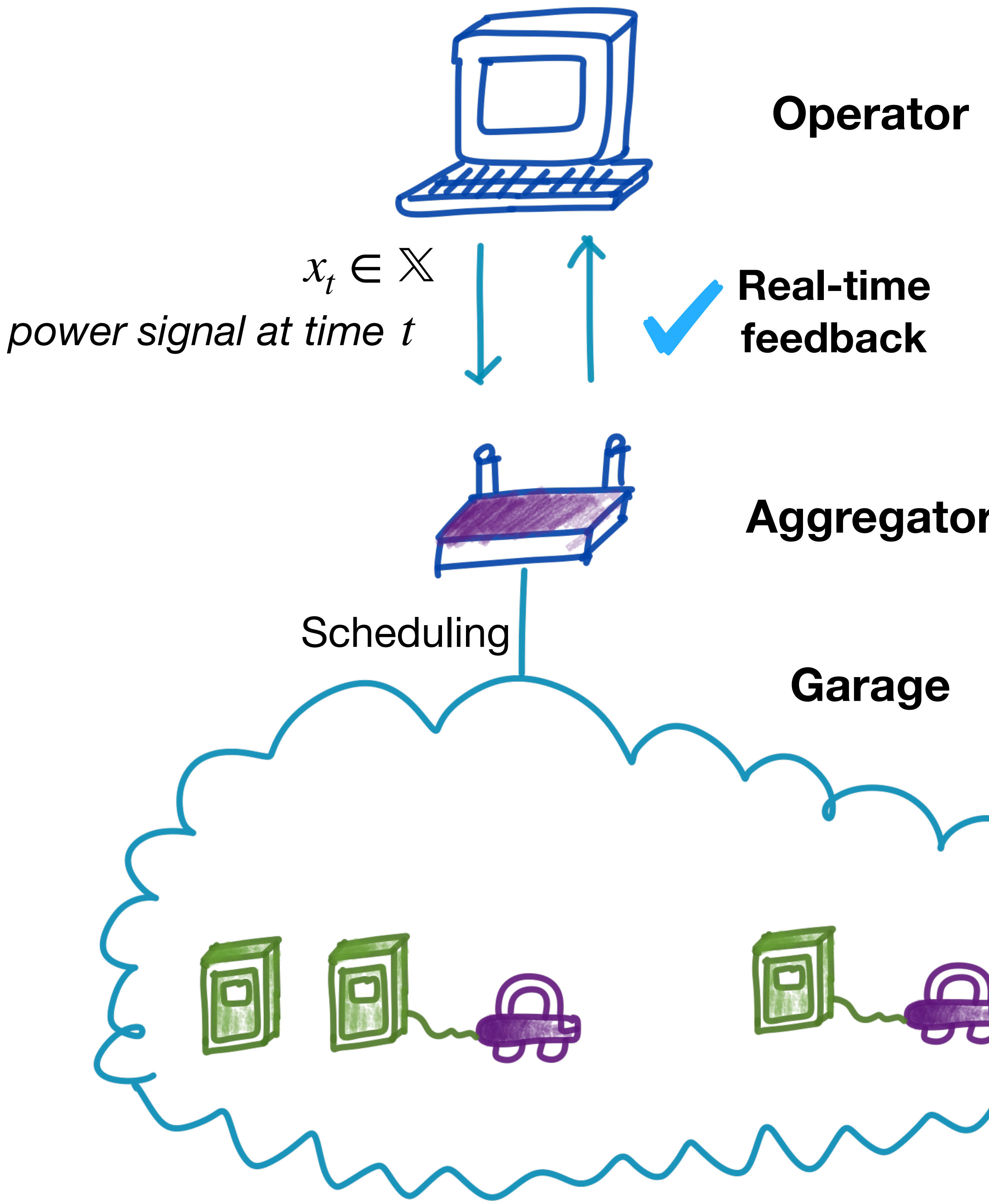
(Online approximations) Concise + General + Theoretical Guarantees

# An Example



Operator
$x = (x_1, \ldots, x_T)$
Trajectory of *power signals*
Offline summary
$x_t \in \mathbb{X}, \forall t \in [T]$
Collection of power levels
Aggregator
Scheduling
Garage

# An Example




Q: How to design it?

Q: How do we use it?

## Revisit the Issues …

| Issues | Aggregate Flexibility<br>Offline |
|---|---|
| • Constraints are hard to be transmitted | ✓ |
| • Optimization is hard to solve | ✓ |
| • Constraints cannot be transmitted exactly | ✓ |

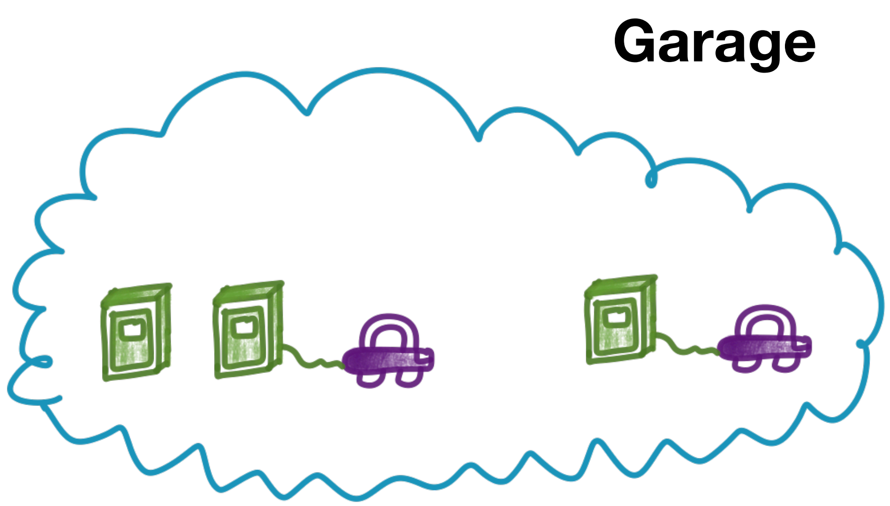
## Revisit the Issues …

| Issues | Aggregate Flexibility: Offline | Aggregate Flexibility: Online |
|---|---|---|
| • Constraints are hard to be transmitted | ✓ | ✓ |
| • Optimization is hard to solve | ✓ | ✓ |
| • Constraints cannot be transmitted exactly | ✓ | ✓ |
| • Offline forecasts of constraints are hard to obtain | | ✓ |
| • Real-time electricity market | | ✓ |

## More Challenges

| Issues: | | |
|---|---|---|
| | • States of the DERs are hard to be transmitted | **Communication complexity** |
| | • Operator's optimization is hard to solve | **Computational complexity** |
| | • Constraints cannot be transmitted exactly | **Privacy** |
| | • Constraints and objectives are hard to predict | **Real-time Implementation** |

## An Example




| | |
|---|---|
| $\phi_t(j)$ | Scheduling |
| $a(j)$ | Arriving time |
| $d(j)$ | Departure time |
| $e(j)$ | Energy to be delivered |
| $r(j)$ | Charging rate limit |
| $\mathbb{X}$ | Set of power levels |

$(x_t \in \mathbb{X}, \forall t \in [T])$

### EV Constraints

***EV arrives***

$$\phi_t(j) = 0 \ , \ t < a(j), \ j = 1, \ldots, n,$$

$$\phi_t(j) = 0 \ , \ t > d(j), \ j = 1, \ldots, n,$$

***Assigning***

$$\sum_{j=1}^{n} \phi_t(j) = x_t, \ t = 1, \ldots, T,$$

***Charging***
*(Linear and lossless)*

$$\sum_{t=1}^{T} \phi_t(j) = e(j), \ j = 1, \ldots, n,$$

***Rate Limit***

$$0 \leq \phi_t(j) \leq r(j), \ t = 1, \ldots, T$$

# Set of admissible trajectories

**Garage**

**Set of admissible trajectories**

$$\mathcal{S}(\phi,\xi) := \left\{x \in \mathbb{X}^T : x \text{ satisfies } (1)\right\}$$

Set of power levels

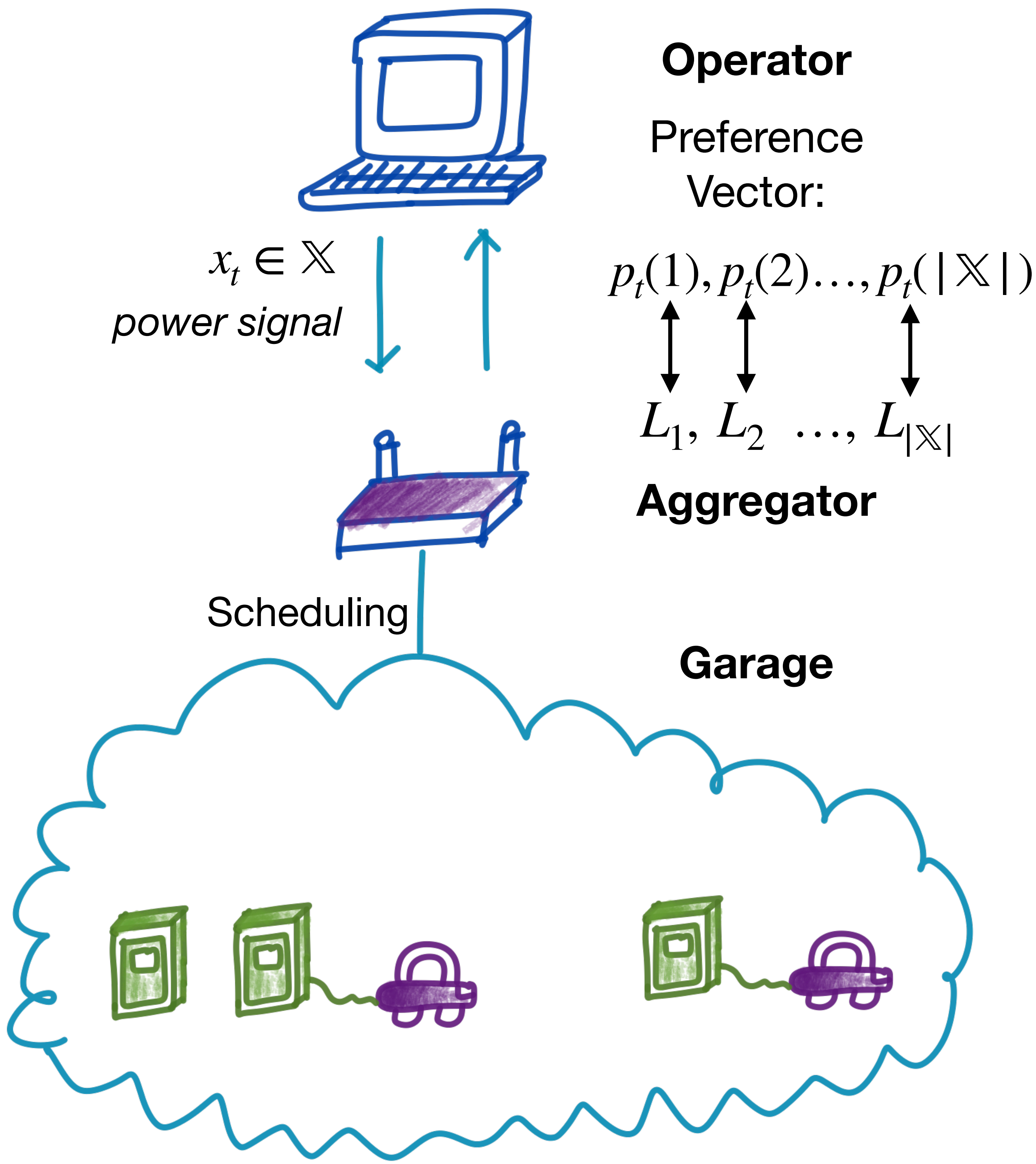
$\{a(j), d(j), e(j), r(j) : j \in [n]\}$

Scheduling algorithm

**EV Constraints**

$$\left.\begin{aligned}
\phi_t(j) &= 0\ ,\ t < a(j),\ j = 1,\ldots,n,\\
\phi_t(j) &= 0\ ,\ t > d(j),\ j = 1,\ldots,n,\\
\sum_{j=1}^{n} \phi_t(j) &= x_t,\ \ t = 1,\ldots,T,\\
\sum_{t=1}^{T} \phi_t(j) &= e(j),\ j = 1,\ldots,n,\\
0 \le \phi_t(j) &\le r(j),\ \ t = 1,\ldots,T
\end{aligned}\right\} \quad (1)$$

# Real-time flexibility feedback



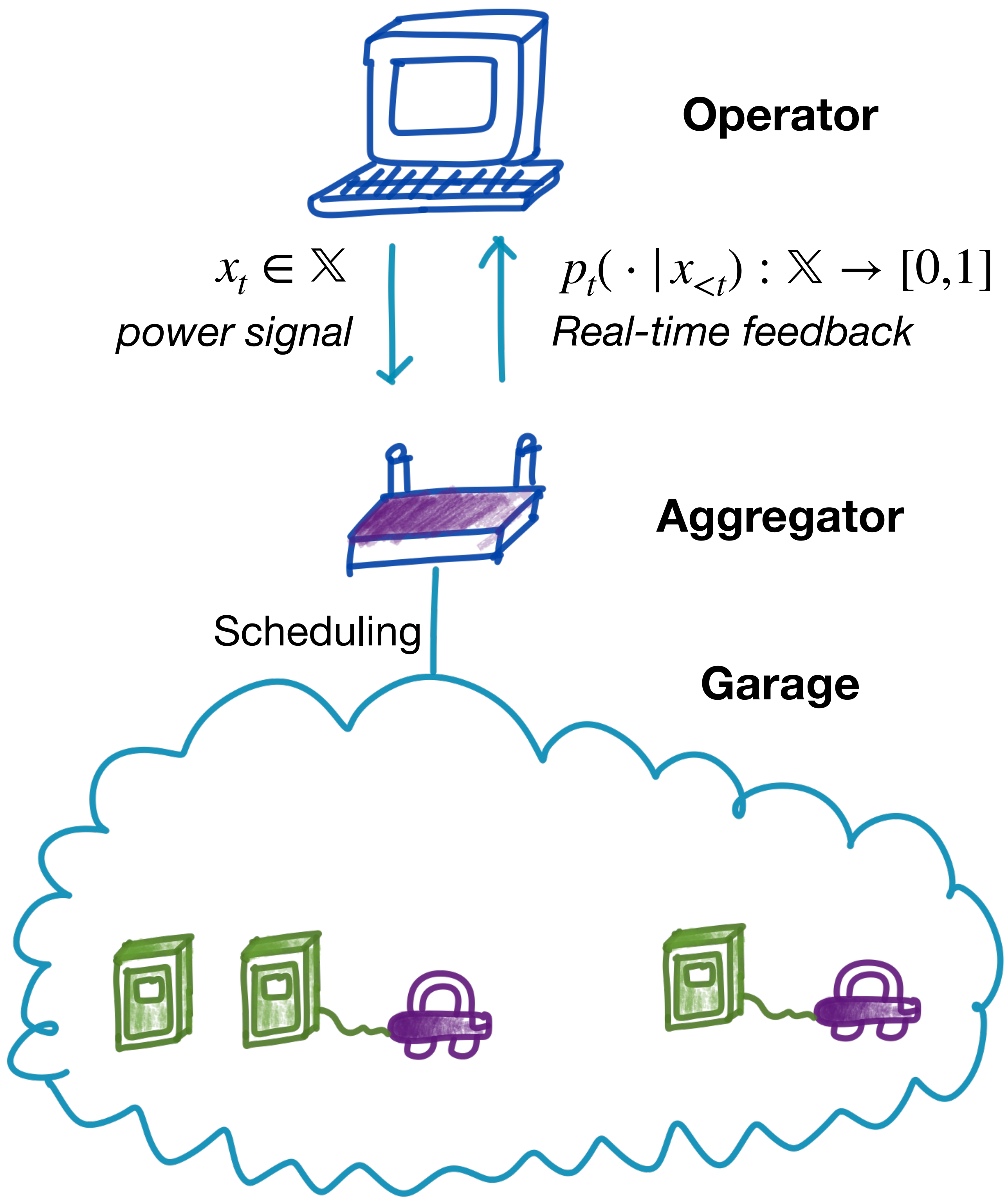
## Real-time flexibility feedback

Preference Vector: $p_t(1), p_t(2) \ldots, p_t(|\mathbb{X}|)$

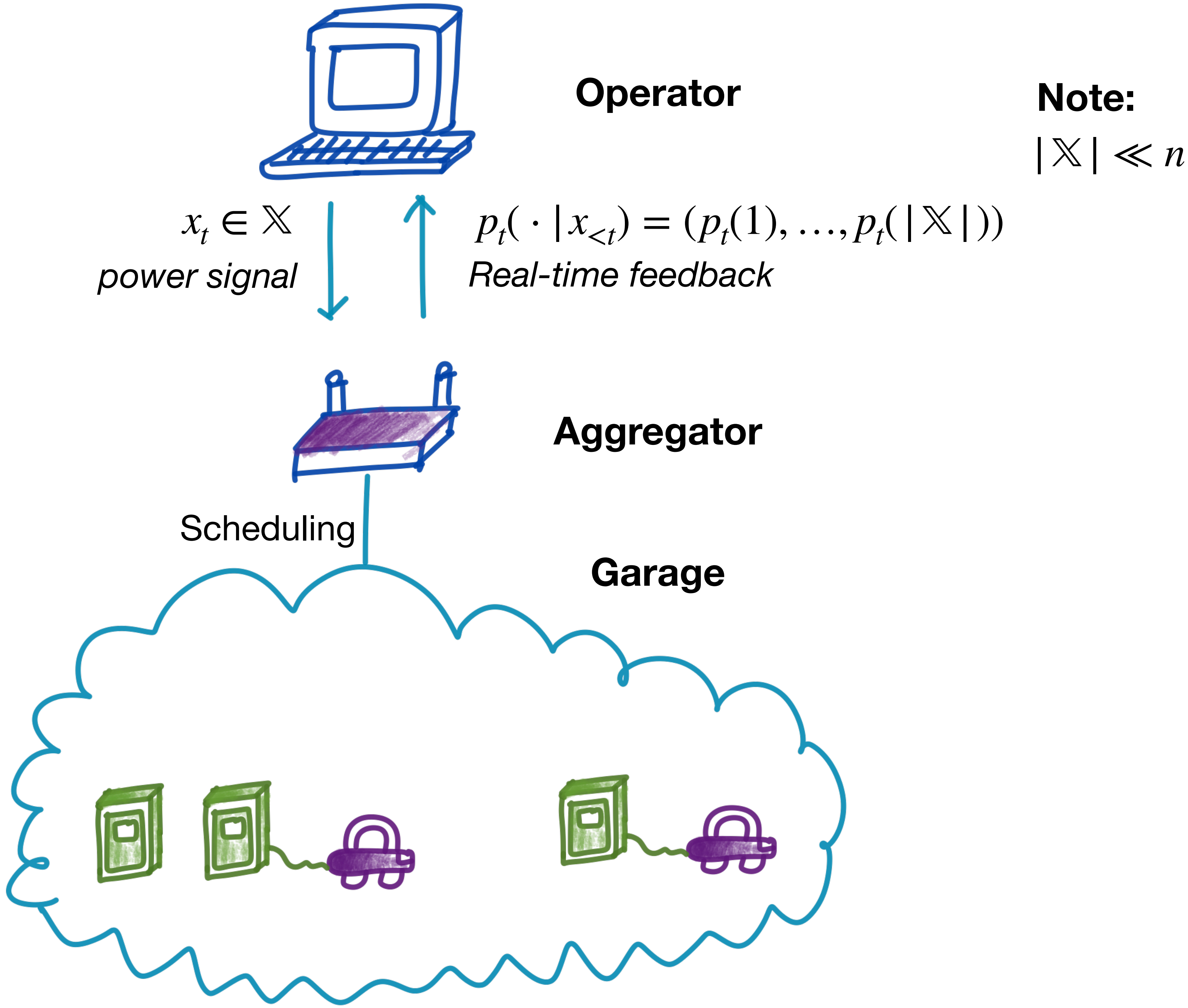
Power Levels: $L_1,\ L_2\ \ \ldots,\ L_{|\mathbb{X}|}$

$\forall i = 1, \ldots, |\mathbb{X}|$ $\boxed{p_t(i) \uparrow}$ $\Longleftrightarrow$ $\boxed{\text{“Flexibility”} \uparrow \quad \text{if } x_t = L_i}$

# Real-time flexibility feedback



## Real-time flexibility feedback



## Real-time flexibility feedback

$$\mathcal{S}(\phi, \xi) \xrightarrow{\textit{Decomposition}} p_1, \ldots, p_T$$

**Want:**

*1. Be informative*

*2. Guarantees feasibility*

## Design

$$\mathcal{S}(\phi, \xi) \xrightarrow{\text{Decomposition}} p_1, \ldots, p_T$$

$$\max_{p_1,\ldots,p_T} \sum_{t=1}^{T} \mathbb{H}\left(X_t \,|\, X_{<t}\right)$$

$$\text{subject to } X \in \mathcal{S}(\phi, \xi)$$

- Maximize the information encapsulated in $p_1, \ldots, p_T$

- The variables are conditional distributions (feedback):

$$p_t \;:=\; p_t(\,\cdot\,|\,\cdot\,) := \mathbb{P}_{X_t|X_{<t}}(\,\cdot\,|\,\cdot\,), \qquad t \in [T]$$

- $X \in \mathbb{X}^T$ is a random variable distributed according to the joint distribution $\prod_{t=1}^{T} p_t$

- $\mathbb{H}\left(X_t \,|\, X_{<t}\right) := \sum_{x_1,\ldots,x_t \in \mathbb{X}} \left(-\prod_{\ell=1}^{t} p_\ell(x_\ell \,|\, x_{<\ell})\right) \log p_t(x_t \,|\, x_{<t})\,.$

## Properties of $p_1^*, \ldots, p_T^*$

$$\mathcal{S}(\phi, \xi) \xrightarrow{\textit{Decomposition}} p_1, \ldots, p_T$$

$$\max_{p_1,\ldots,p_T} \sum_{t=1}^{T} \mathbb{H}\left(X_t \mid X_{<t}\right) \quad \text{subject to } X \in \mathcal{S}(\phi, \xi) \tag{2}$$

**Definition (OFF)**

*Optimal flexibility feedback (OFF)* $p_1^*, \ldots, p_T^*$ if it is a unique optimal solution of $(2)$

## Properties of $p_1^*, \dots, p_T^*$

$$\mathcal{S}(\phi, \xi) \xrightarrow{\textit{Decomposition}} p_1, \dots, p_T$$

$$\max_{p_1, \dots, p_T} \sum_{t=1}^{T} \mathbb{H}\left(X_t \,|\, X_{<t}\right) \qquad (2)$$
$$\text{subject to } X \in \mathcal{S}(\phi, \xi)$$

**Definition (OFF)**

*Optimal flexibility feedback (OFF)* $p_1^*, \dots, p_T^*$ if it is a unique optimal solution of $(2)$

- Time decomposition of the set of feasible trajectories
- The variable $p_t^* = p_t^*(\,\cdot\,|\, x_{<t})$ is causal

## Properties of $p_1^*, \ldots, p_T^*$

**Theorem**

The *optimal flexibility feedback (OFF)* is given by

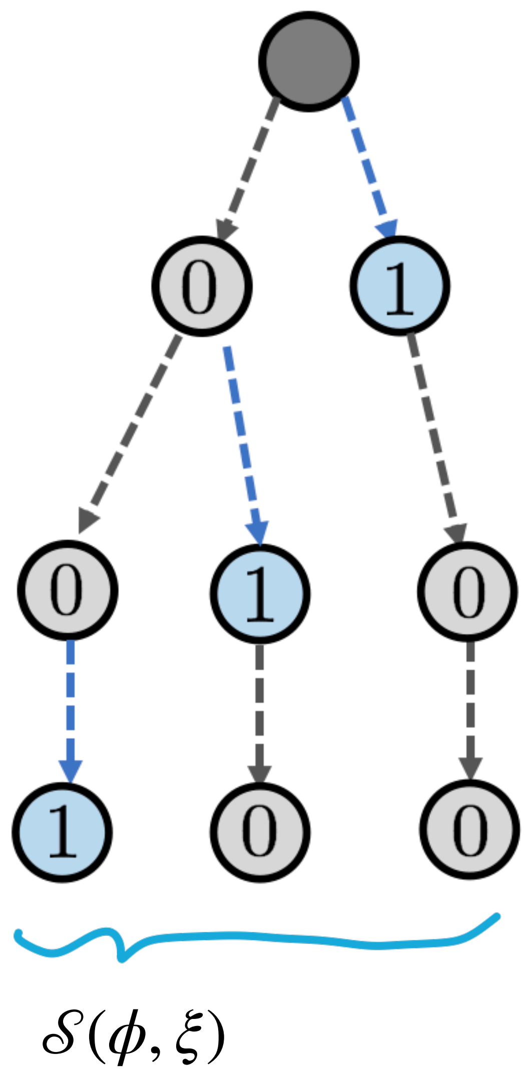
$$p_t^*(x_t \,|\, x_{<t}) \;=\; \frac{\left|\mathcal{S}\left(\phi, \xi \,|\, (x_{<t}, x_t)\right)\right|}{\left|\mathcal{S}(\phi, \xi \,|\, x_{<t})\right|}, \quad \forall (x_{<t}, x_t) \in \mathbb{X}^t$$

where

$$\mathcal{S}(\phi, \xi \,|\, x_{\le t}) \;:=\; \left\{ x_{>t} \in \mathbb{X}^{T-t} : x \in \mathcal{S}(\phi, \xi) \right\}$$

**Proof:** By induction on $t \in [T]$

## Properties of $p_1^*, \ldots, p_T^*$




$$p_1^*(x_1) = \begin{cases} \frac{2}{3} \text{ if } x_1 = 0 \\ \frac{1}{3} \text{ if } x_1 = 1 \end{cases}$$

$$p_2^*(x_2|x_1) = \begin{cases} \frac{1}{2}, & \text{if } (x_1, x_2) \in \{(0,0), (0,1)\} \\ 1, & \text{if } (x_1, x_2) = (1, 0) \\ 0, & \text{otherwise} \end{cases}$$

$$p_3^*(x_3|x_1, x_2) = \begin{cases} 1 & \text{if } (x_1, x_2, x_3) \in \{(0,0,1), (0,1,0), (1,0,0)\} \\ 0 & \text{otherwise} \end{cases}$$

## Properties of $p_1^*, \ldots, p_T^*$

**Corollary**

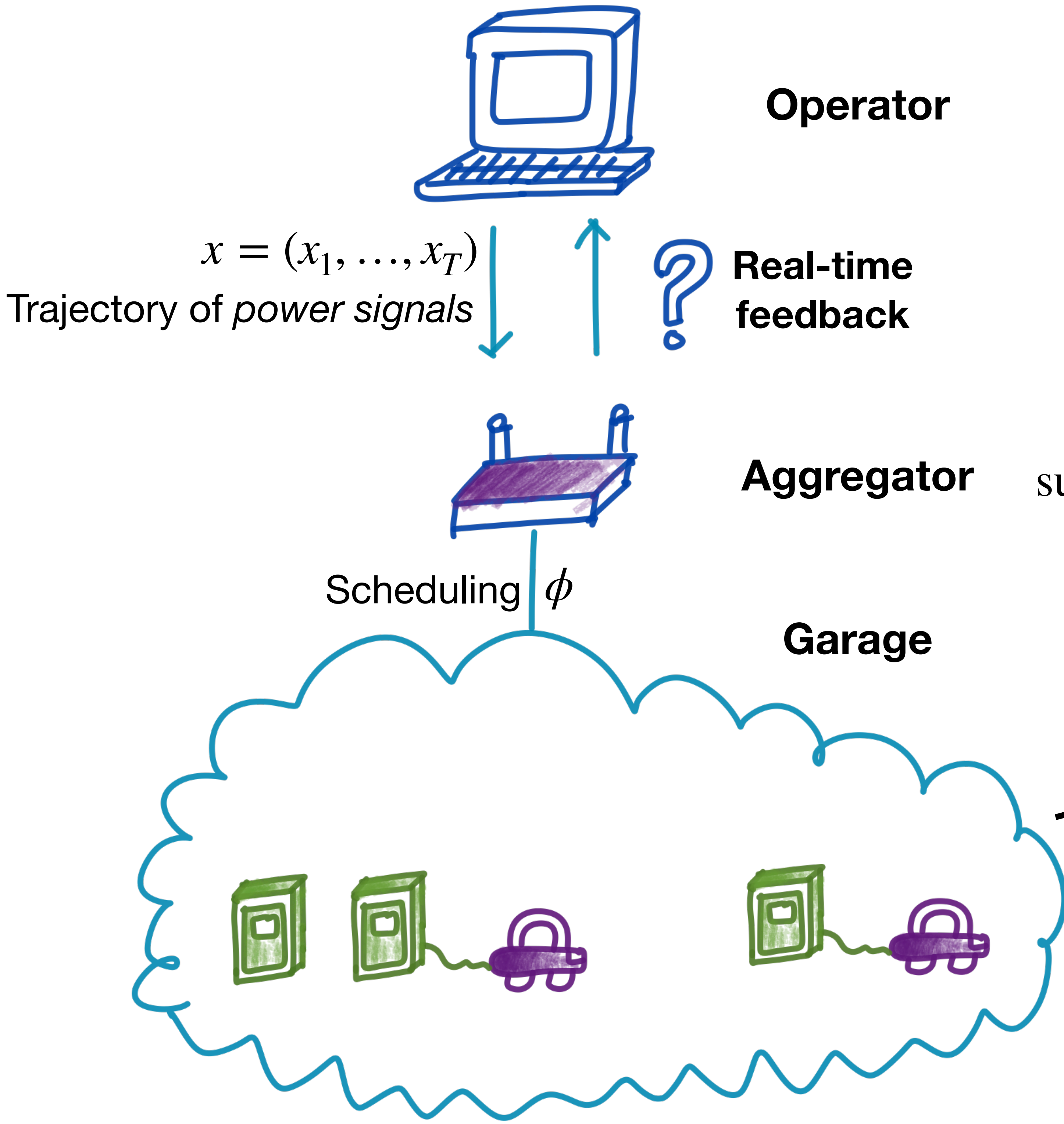
- For any trajectory $x = (x_1, \ldots, x_T)$ , if $p_t^*(x_t \mid x_{<t}) > 0, \forall t \in [T]$, then $x \in \mathcal{S}(\phi, \xi)$
- For all $x_t, x_t' \in \mathbb{X}$ at each time $t$, if $p_t^*(x_t \mid x_{<t}) \geq p_t^*(x_t' \mid x_{<t})$, then

$$|\mathcal{S}(\phi, \xi \mid (x_{<t}, x_t))| \geq |\mathcal{S}(\phi, \xi \mid (x_{<t}, x_t'))|$$

Feasibility

Interpretability

## Application 1: Cost minimization




**Online cost minimization**

electricity price

$$\min_x \sum_{t=1}^{T} c_t x_t$$

$$\text{subject to} \quad x \in \mathcal{S}(\phi, \xi)$$

## Application 1: Cost minimization

**Theorem**

$$\min_x \sum_{t=1}^{T} c_t x_t \quad \text{subject to} \quad x \in \mathcal{S}(\phi, \xi) \iff \min_x \sum_{\tau=1}^{T} c_\tau x_\tau \underbrace{-\overbrace{\beta}^{\text{Tuning parameter} > 0} \log p_\tau^*(x_\tau | x_{<\tau})}_{\text{Penalty}}$$

Sketch of proof:

$$\min_x \sum_{\tau=1}^{T} c_\tau x_\tau - \beta \log p_\tau^*(x_\tau | x_{<\tau}) = \sum_{\tau=1}^{T} c_\tau x_\tau - \beta \log \left( \prod_{t=1}^{T} p_t^*(x_t | x_{<t}) \right)$$

$$= \sum_{\tau=1}^{T} c_\tau x_\tau - \beta \log q(x)$$

$$q(x) := \begin{cases} 1/\left|\mathcal{S}(\phi, \xi)\right| & \text{if } x \in \mathcal{S}(\phi, \xi) \\ 0 & \text{otherwise} \end{cases}$$

## Application 1: Cost minimization

**PPC**: **P**enalized **P**redictive **C**ontrol (MPC + $p_1^*, \ldots, p_T^*$ )

$$\min_x \sum_{t=1}^{T} c_t x_t \quad \longrightarrow \quad \forall\ t \in [T] \quad \min_{x_{t:t+w}} \sum_{\tau=t}^{t+w} c_\tau x_\tau \ -\beta \log p_\tau^*(x_\tau \mid x_{<\tau})$$

$$\text{subject to} \quad x \in \mathcal{S}(\phi, \xi)$$

$w :=$ prediction window size

## Application 1: Cost minimization

**Theorem** $[x \in \mathcal{S}(\phi, \xi)]$

The trajectory $x = (x_1, \ldots, x_T)$ generated by the PPC scheme is always feasible

## Application 1: Cost minimization

**Theorem** $[x \in \mathcal{S}(\phi, \xi)]$

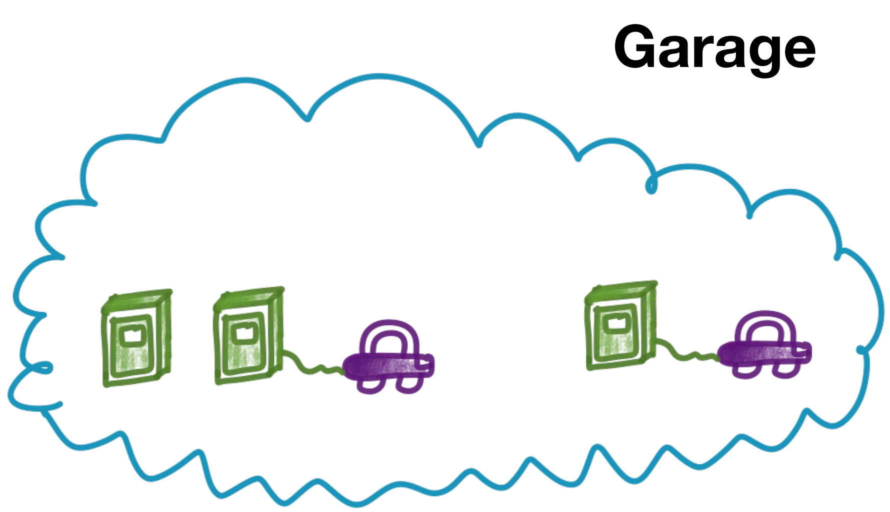
The trajectory $x = (x_1, \ldots, x_T)$ generated by the PPC scheme is always feasible

**Theorem** [Informal]

Under assumptions, if the prediction window size $w$ is $\omega(1)$, then the dynamic regret is $o(T)$

# Application 1: Cost minimization

## Experiments



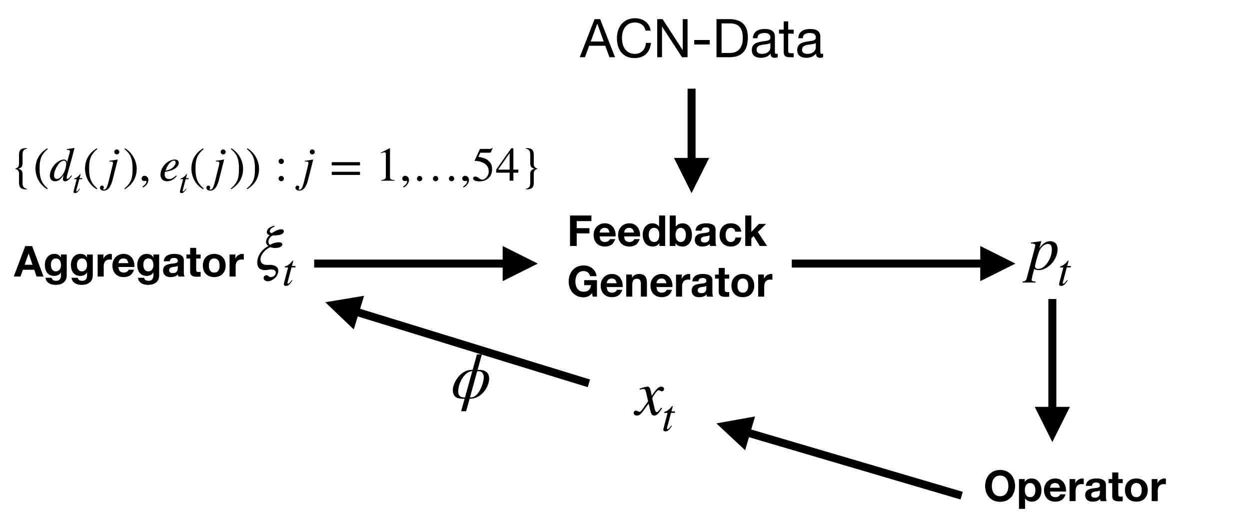

ACN-Data

[Lee et al. 2019]

| Parameter | Value |
|---|---|
| Number of power levels $\lvert \mathbb{X} \rvert$ | 10 |
| Number of EVSEs | 52 (JPL)/ 54 (Caltech) |
| State space | $\mathbb{R}_+^{104}$ / $\mathbb{R}_+^{108}$ |
| Action space | $[0,1]^{\lvert \mathbb{X} \rvert}$ |
| Time interval | 6 min |
| Data | ACN-Data |
| Scheduling algorithm | Least laxity first |
| Learning algorithm | Soft actor-critic |
| Cost functions | Average CAISO LMPs |
| Peak power limit | 150 kWh |

# Application 1: Cost minimization

## Experiments




$d_t(j)$ Remaining time

$e_t(j)$ Remaining energy

$$\min_{x_t} c_t x_t - \beta \log p_t^*(x_t \mid x_{<t})$$

## Reward function

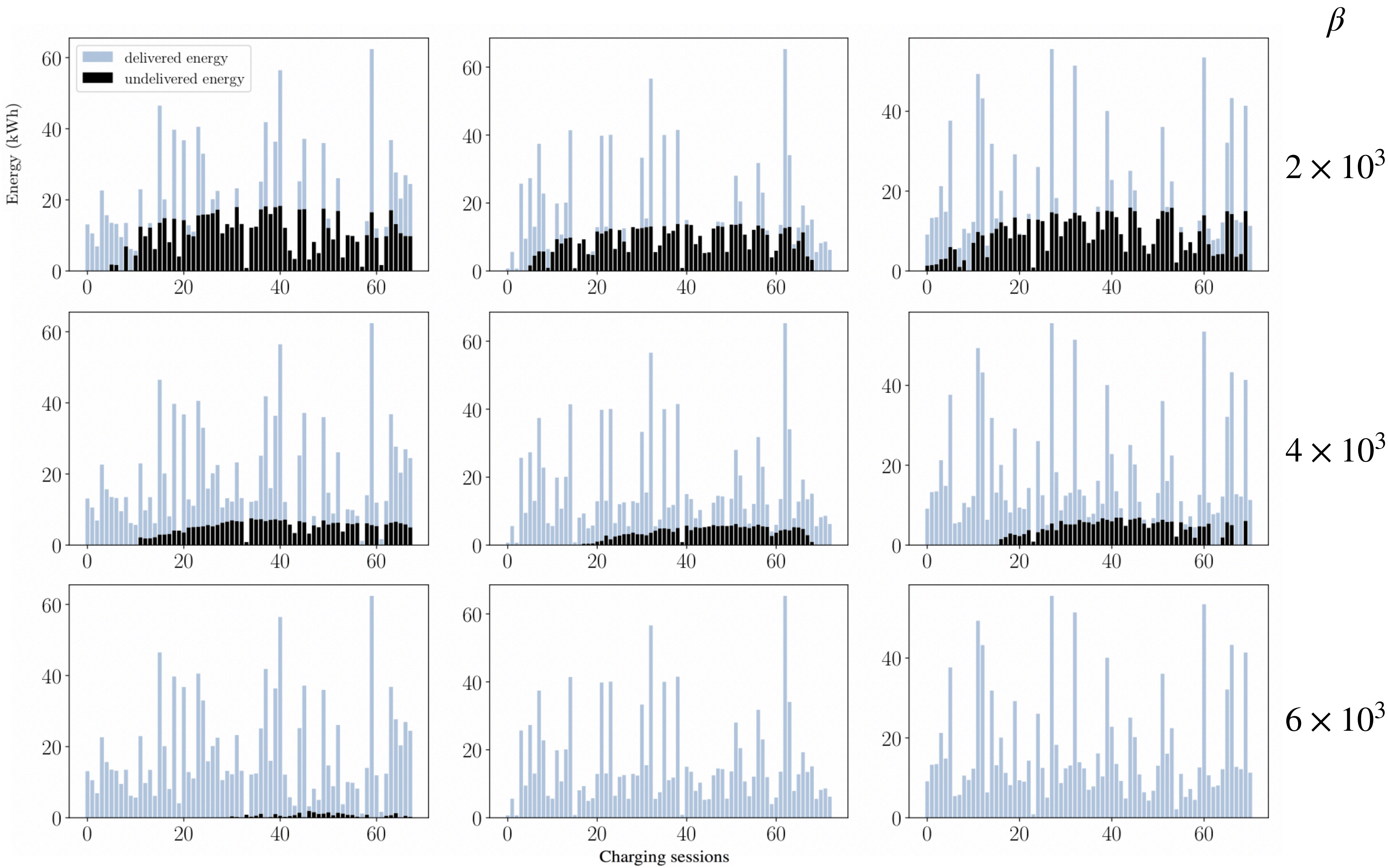
$$r_{\mathrm{EV}}(\xi, p_t) = \mathbb{H}(p_t) + \sigma_1 \left| \left| \phi_t(\xi_t, x_t) \right| \right|_2 - \sigma_2 \mathbb{I}(t < d_t(j) \leq t+1) \left[ e(j) - e_t(j) \right]_+ - \sigma_3 \left| x_t - \sum_{j=1}^{n} \phi_t(j) \right|$$

Definition

Charging performance

Tracking performance

# Application 1: Cost minimization



## Application 1: Cost minimization

### Flexibility vs Optimality

$\beta$

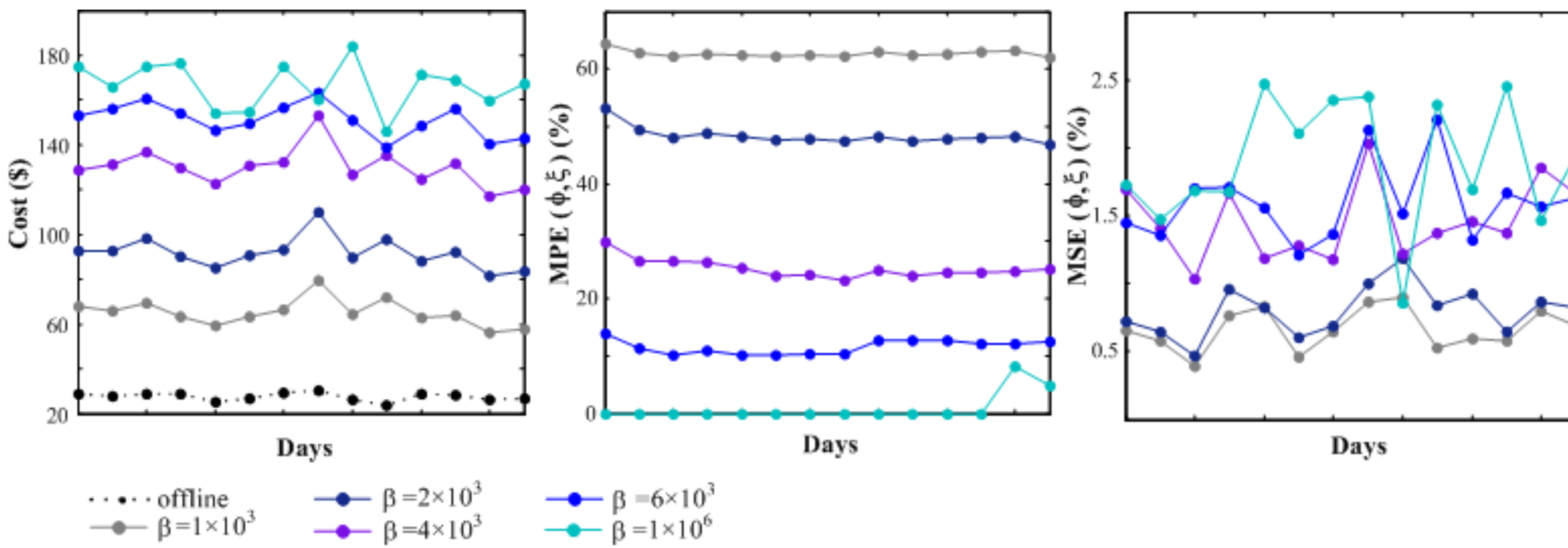
Large

Small

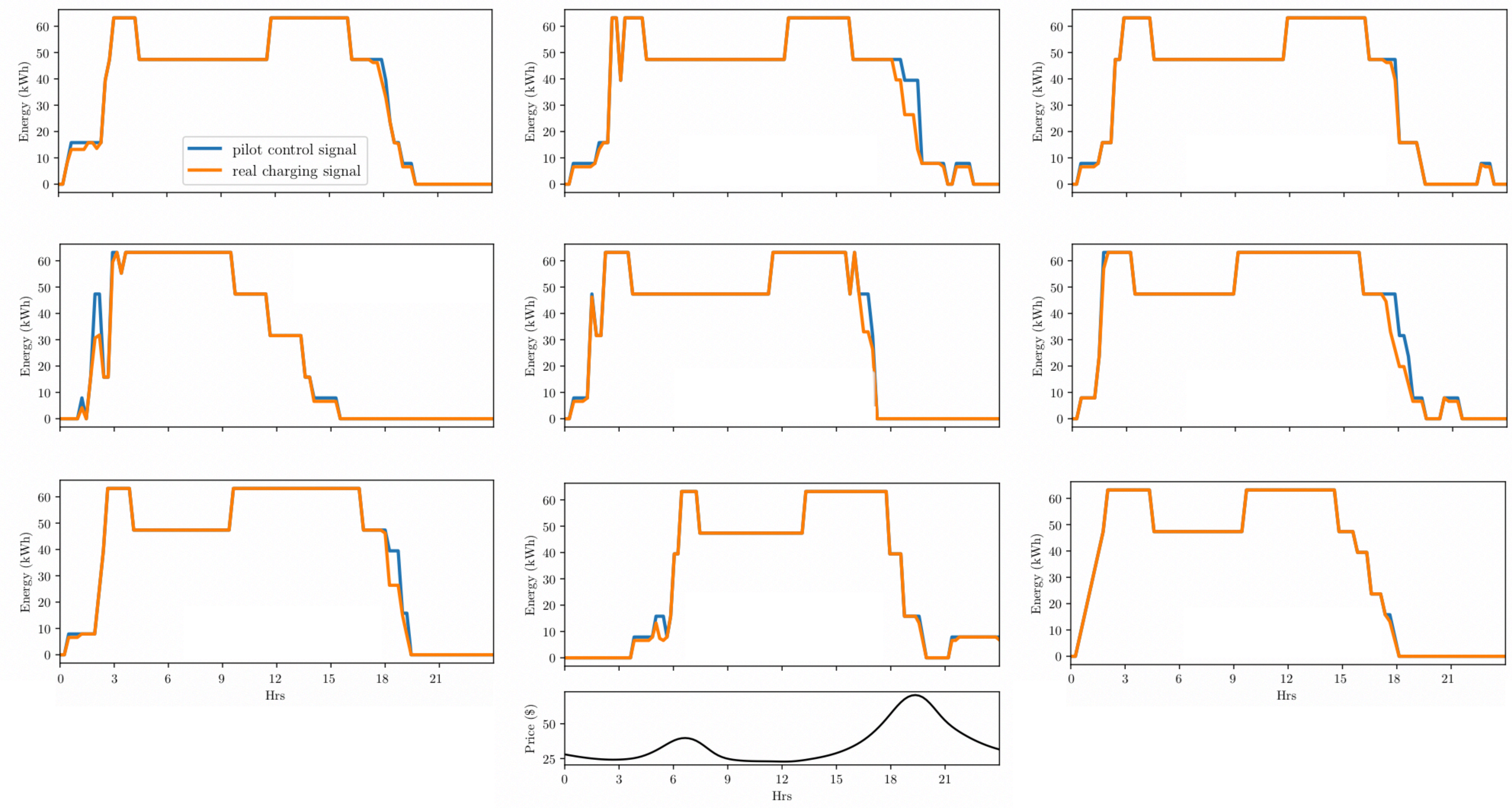
Cost

Charging

Tracking



# Application 1: Cost minimization



# Application 2: Capacity Estimation

Garage

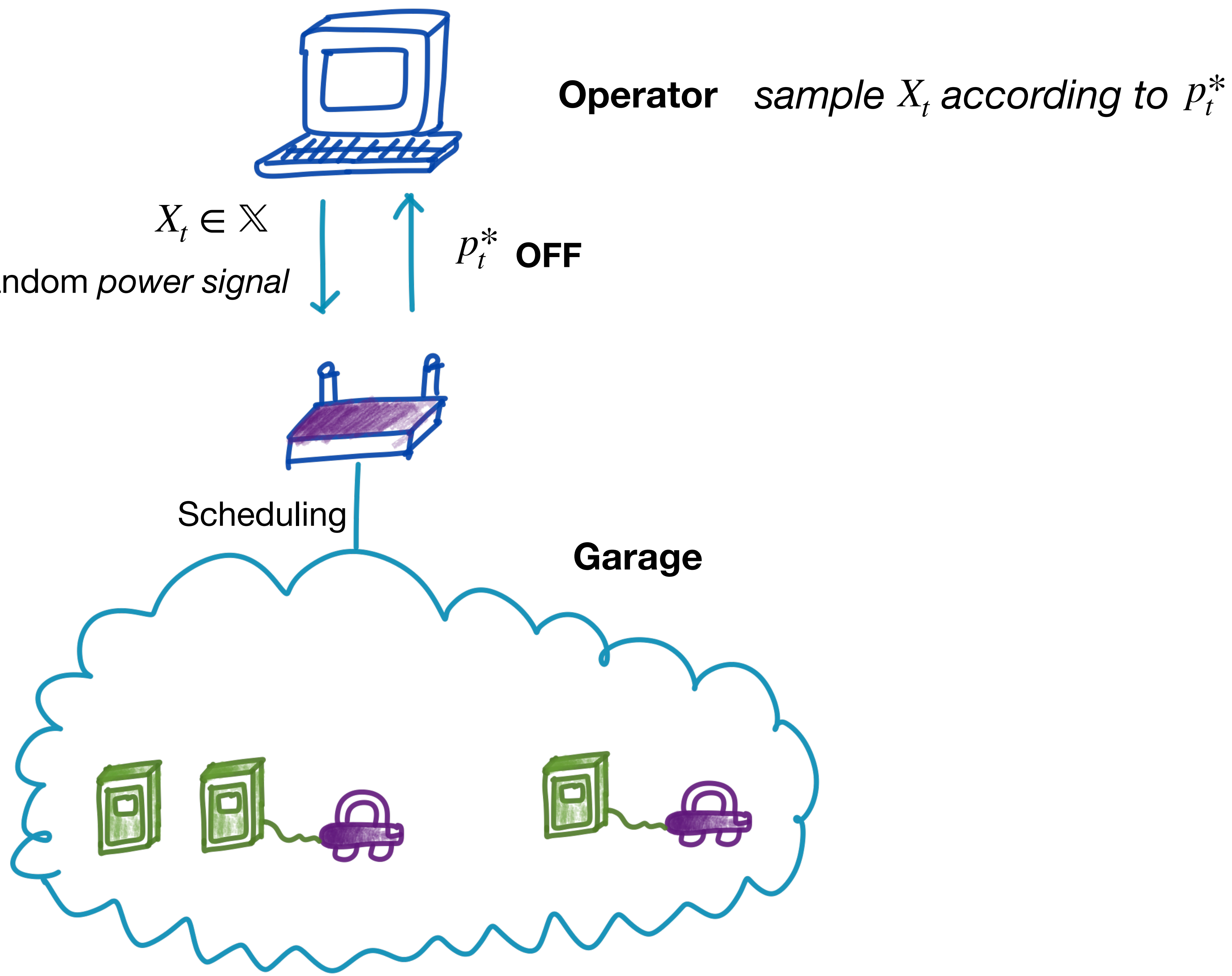
**Set of admissible trajectories**

$$\mathcal{S}(\phi, \xi) := \left\{ x \in \mathbb{X}^T : x \text{ is admissible} \right\}$$

System Capacity

**What is** $|\mathcal{S}(\phi, \xi)|$ **?**

## Application 2: Capacity Estimation



## Application 2: Capacity Estimation

**Theorem**

Consider $N$ trajectories $x(1), \ldots, x(N)$ generated i.i.d. by sampling according to $p_1^*, \cdots, p_T^*$

Then

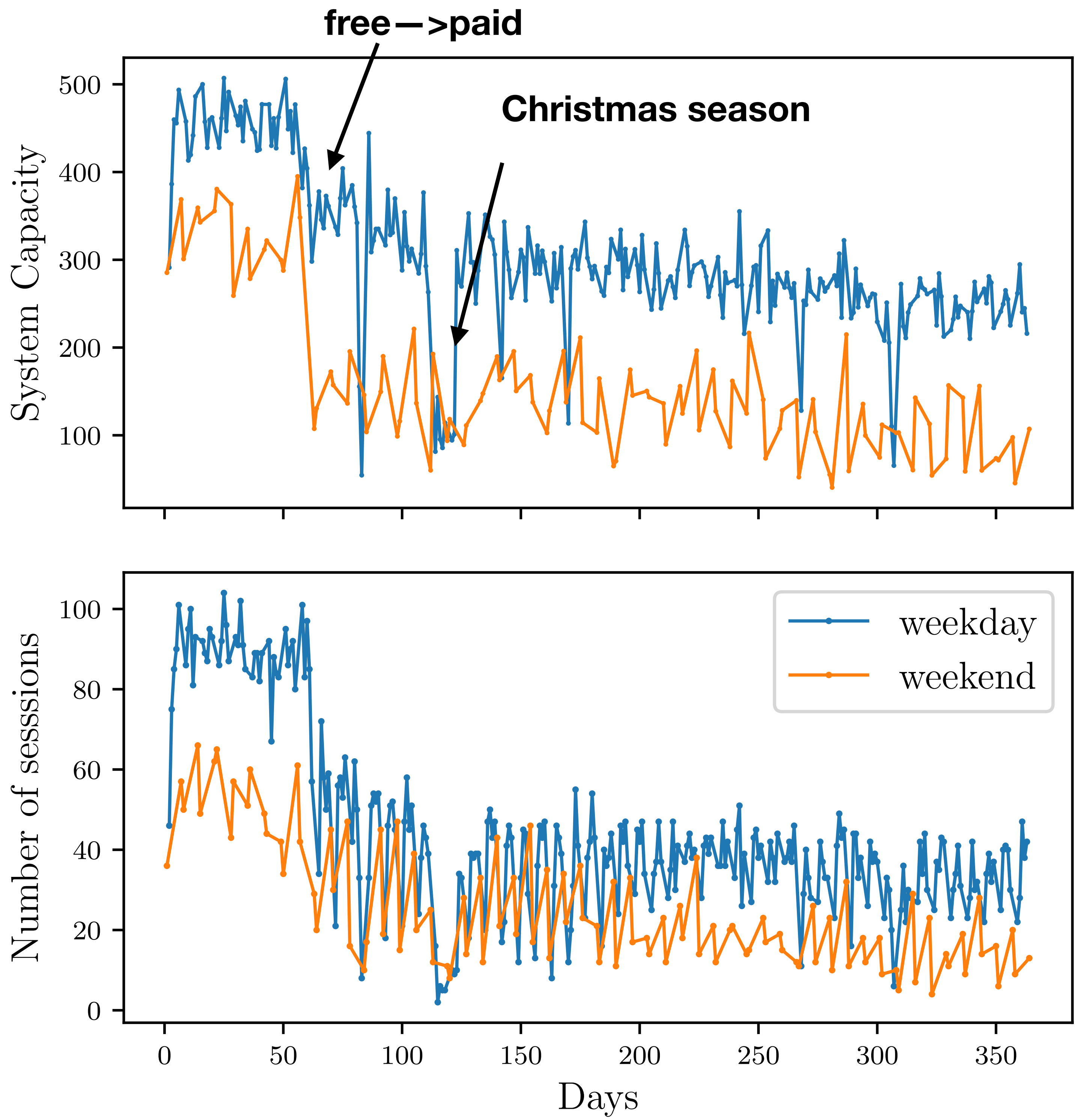
$$\frac{1}{N}\sum_{\ell=1}^{N}\sum_{t=1}^{T}\mathbb{H}\left(p_t^*(\,\cdot\,|\,x_{<t}(\ell))\right) \xrightarrow{a.s.} \log|\,\mathcal{S}(\phi,\xi)\,| \text{ as } N \to \infty\,.$$

Idea of Proof: Law of large numbers + $\log|\,\mathcal{S}(\phi,\xi)\,| = \max_{p_1,\ldots,p_T} \sum_{t=1}^{T}\mathbb{H}\left(X_t\,|\,X_{<t}\right)$

$\text{subject to } X \in \mathcal{S}(\phi,\xi)$

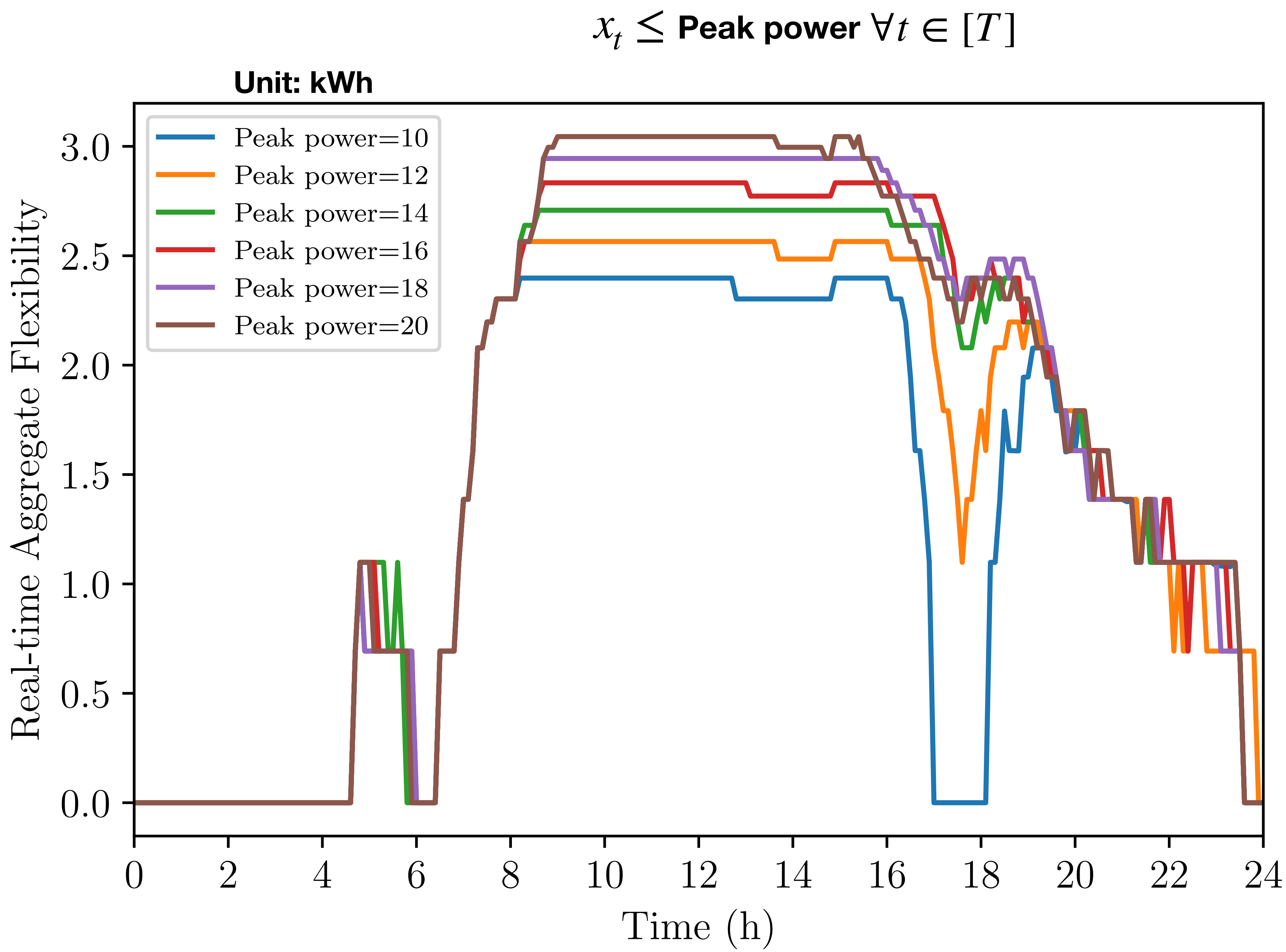
## Application 2: Capacity Estimation

### Experiments: Capacity of the Caltech Garage



# Application 2: Capacity Estimation

## Experiments: Capacity vs operational constraints



## Application 2: Capacity Estimation

### Experiments: Heatmap