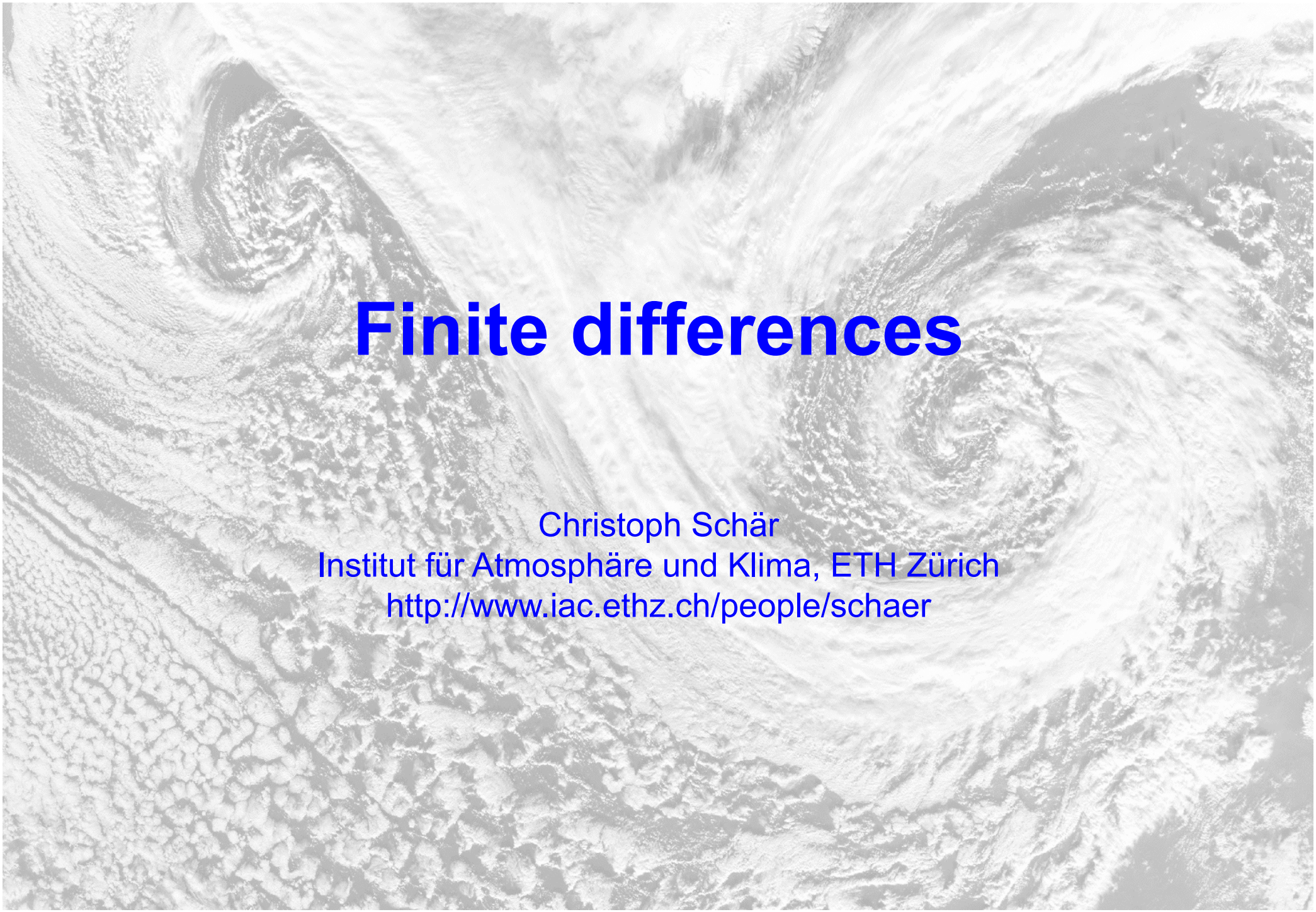

Finite differences

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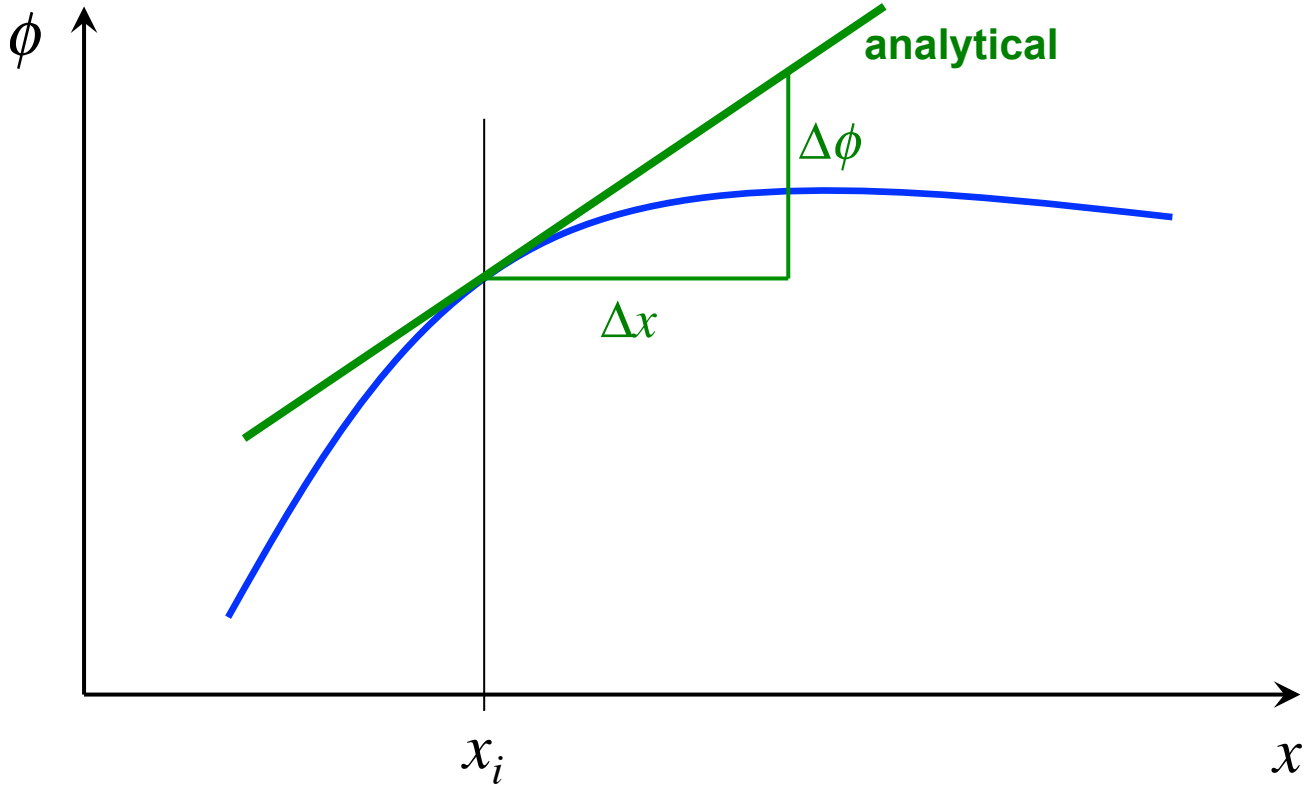
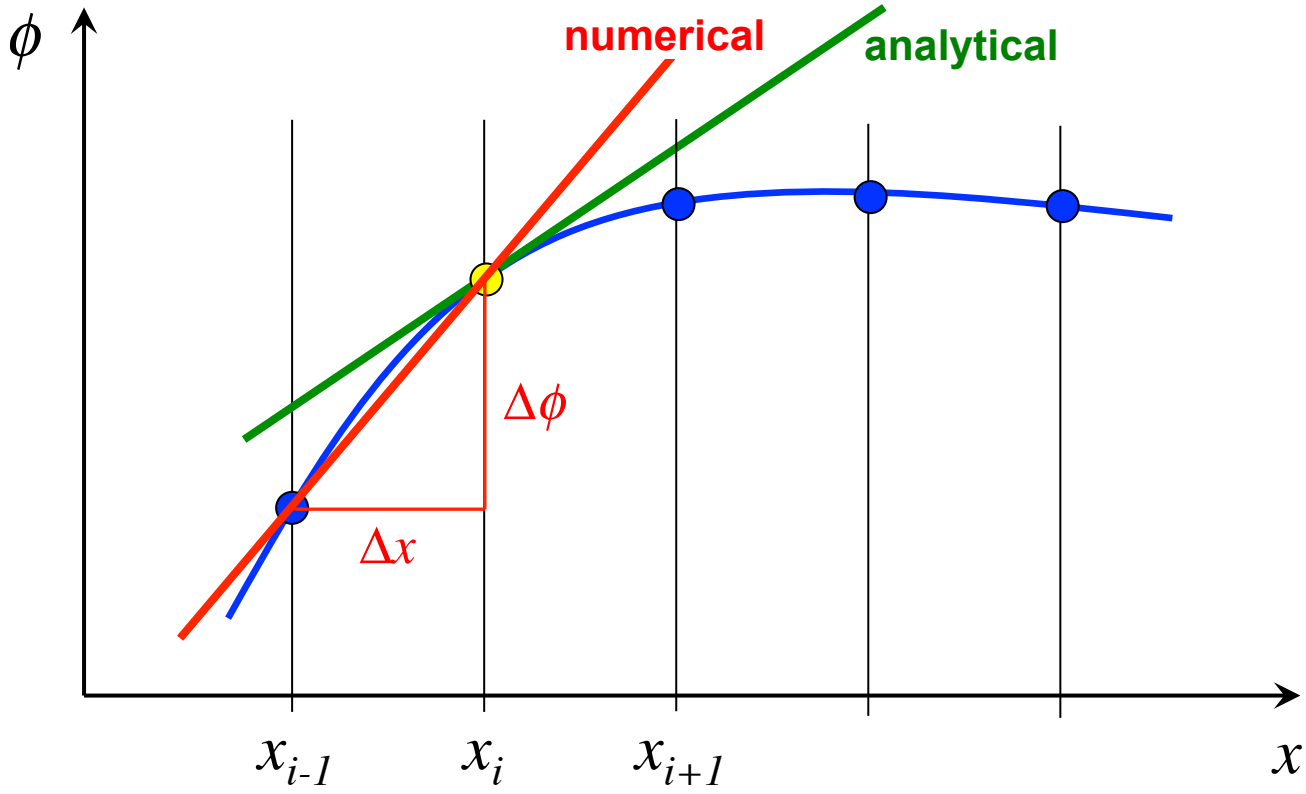
Analytical derivative

$$\left.\frac{\partial \phi}{\partial x}\right|_{x_i} \coloneqq \frac{\Delta \phi}{\Delta x}$$



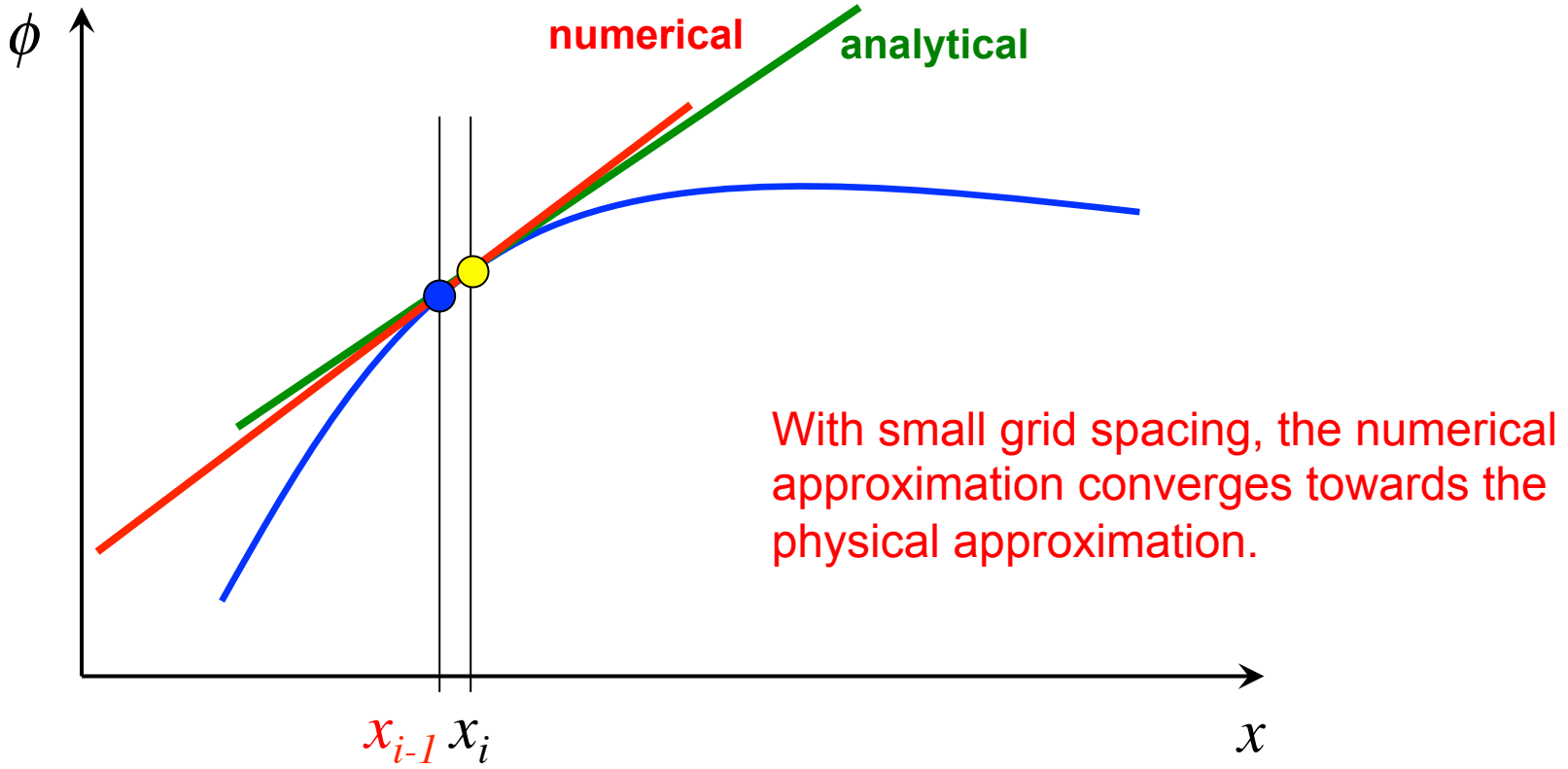
Derivative with finite differences

$$\left.\frac{\partial \phi}{\partial x}\right|_{x_i} \approx \frac{\phi_i - \phi_{i-1}}{\Delta x}$$



Derivative with finite differences

$$\left.\frac{\partial \phi}{\partial x}\right|_{x_i} \approx \frac{\phi_i - \phi_{i-1}}{\Delta x}$$

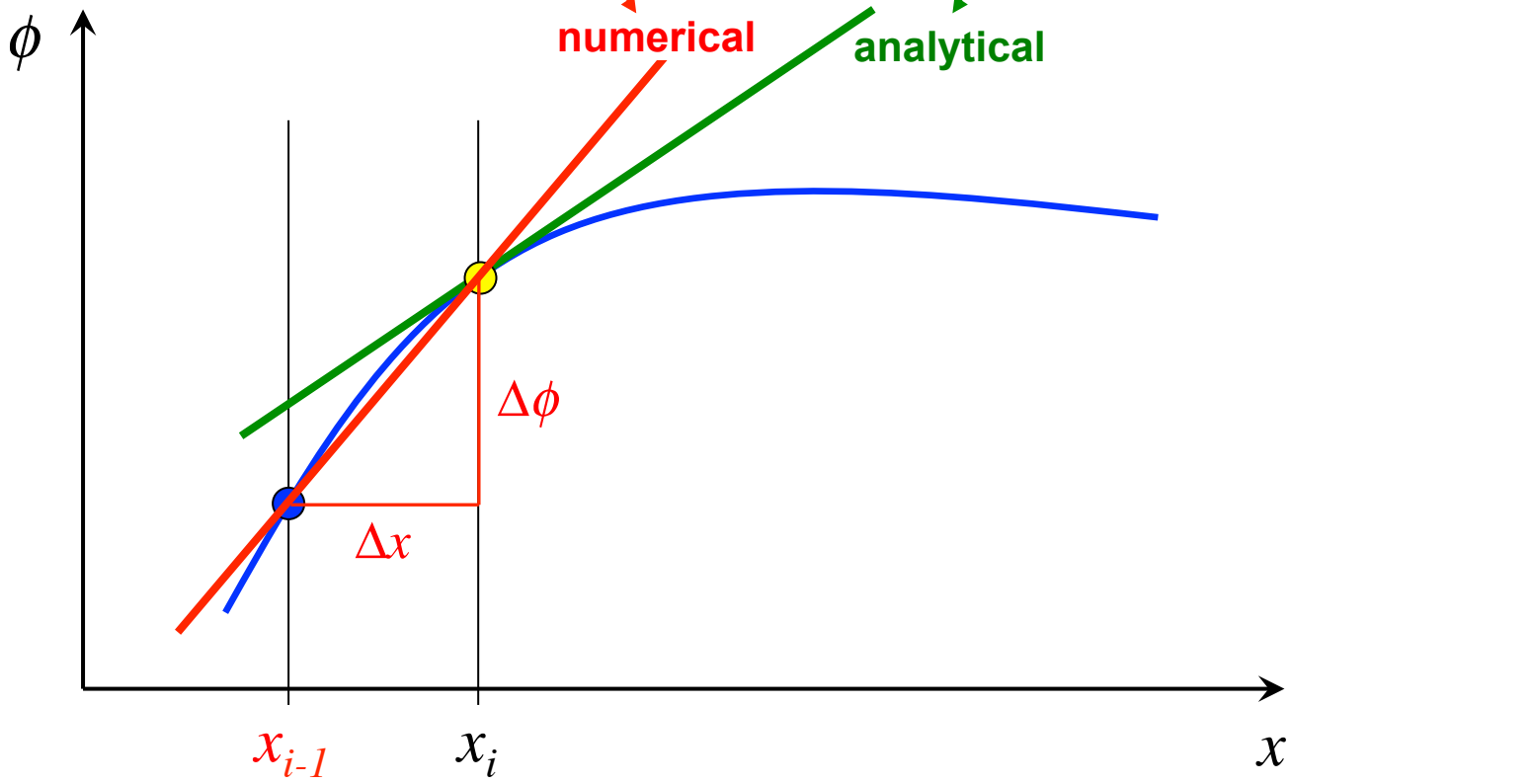


With small grid spacing, the numerical approximation converges towards the physical approximation.

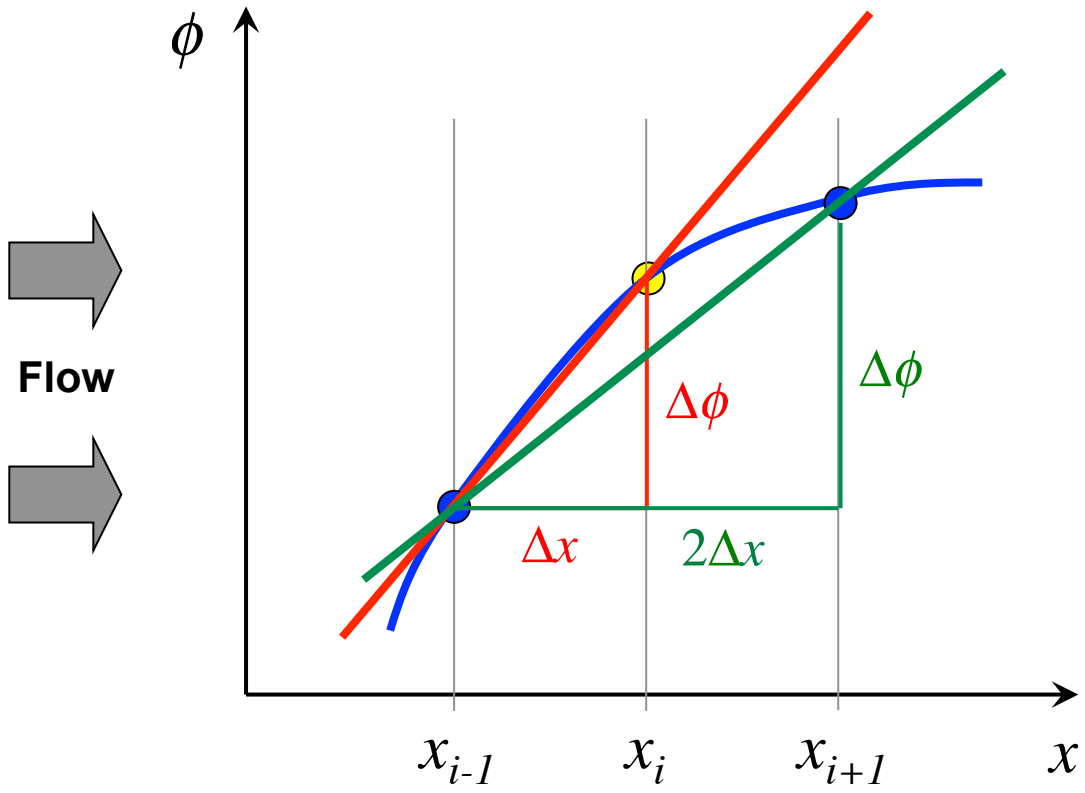
Derivative with finite differences

$$\left.\frac{\partial \phi}{\partial x}\right|_{x_i} \approx \frac{\phi_i - \phi_{i-1}}{\Delta x}$$

$$\left.\frac{\partial \phi}{\partial x}\right|_{x_i} := \lim_{\Delta x \to 0} \frac{\phi_i - \phi_{i-1}}{\Delta x}$$



Finite differences in space



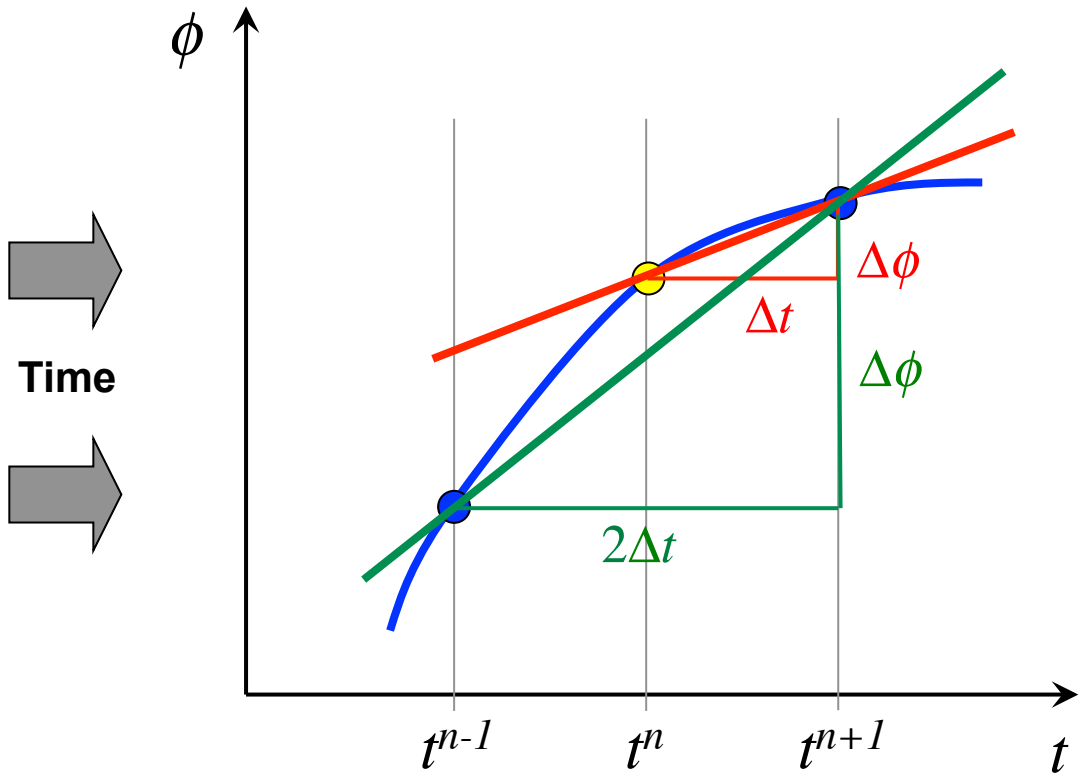
Upstream difference

$$\left.\frac{\partial \phi}{\partial x}\right|_{x_i} \approx \frac{\phi_i - \phi_{i-1}}{\Delta x}$$

Centered difference

$$\left.\frac{\partial \phi}{\partial x}\right|_{x_i} \approx \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x}$$

Finite differences in time



Forward step (Euler step)

$$\left.\frac{\partial\phi}{\partial t}\right|_{t^n} \approx \frac{\phi^{n+1}-\phi^n}{\Delta t}$$

Centered step (Leapfrog step)

$$\left.\frac{\partial\phi}{\partial t}\right|_{t^n} \approx \frac{\phi^{n+1}-\phi^{n-1}}{2\Delta t}$$