

CS 581 Homework 6

Due on 02/22/2018

Problem 1.

Making Change: Suppose we want to make change for n cents and the only denominations allowed are 1, 10 and 25 cents (infinite amount).

- Find an example such that the greedy algorithm does not find the minimum number of coins required to make change for n cents (give a concrete counterexample).
- Is it possible to design a coin system (a set of possible denominations) such that a greedy algorithm yields an optimal solution? If so, find such an example with at least 3 denominations.

Problem 2.

What is an optimal Huffman code for the following set of frequencies, based on the first 8 Fibonacci numbers?

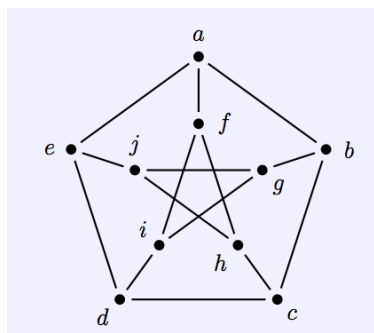
a:1 b:1 c:2 d:3 e:5 f:8 g:13 h:21

Can you generalize your answer to find the optimal code when the frequencies are the first n Fibonacci numbers?

Problem 3.

For the given graph below,

- Show the three representations of the graph according to how we discussed in class.
- Would you consider this graph sparse? If so, why? Justify your answer.



Problem 4.

Suppose we have n students in a class and a list of r student pairs to indicate the two people have dated before. A date only involves two people here. Give an $O(n + r)$ -time algorithm that determines whether it is possible to group students into two groups such that no one has ever dated someone from the same group.

Problem 5.

One can use DFS to determine whether or not a given undirected graph $G = (V, E)$ contains a cycle. Does the DFS here run in $O(|V| + |E|)$ time or just $O(|V|)$, independent of $|E|$ time?