CS581 Homework 2 Solution

January 25, 2018

Problem 1.

Since $n! \sim \sqrt{2\pi n}(\frac{n}{e})^n$, then $\lim_{n\to\infty} \frac{n!}{\sqrt{2\pi n}(\frac{n}{e})^n} = 1$. Therefore, the difference of the two terms as n goes to infinity approaches 0, so the proportional error also must go to 0 as n goes to infinity. A more explicit definition is as follows:

$$\begin{aligned}
\lim_{n\to\infty} \frac{n! - \sqrt{2\pi n}(\frac{n}{e})^n}{n!} &= \lim_{n\to\infty}(1 - \frac{\sqrt{2\pi n}(\frac{n}{e})^n}{n!}) && (1)\\
&= (1-1) && (2)\\
&= 0 && (3)
\end{aligned}$$

Problem 2.

Let $n, c \in \mathbb{R}_{>0}$ be arbitrary. The equality of $n^{log(c)}$ and $c^{log(n)}$ will be shown by transforming one expression into the other through a series of equality preserving transformations.

$$\begin{aligned}
n^{log(c)} &= n^{log(c)} && \text{True by the symmetric property} && (4)\\
&= e^{log(n^{log(c)})} && \text{Basic property of logarithms} && (5)\\
&= e^{log(c)*log(n)} && \text{Basic property of logarithms} && (6)\\
&= (e^{log(c)})^{log(n)} && \text{Basic property of exponents} && (7)\\
&= c^{log(n)} && \text{Basic property of logarithms} && (8)
\end{aligned}$$

Since $n, c \in \mathbb{R}_{>0}$ were arbitrary, and since all of the above transformations preserve equality for positive real numbers, it follows that $n^{log(c)} = c^{log(n)}$ for all $n, c \in \mathbb{R}_{>0}$.

Problem 3.

X means true.

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$\log(n)$	n	X	X			
$\log(2n)$	n	X	X			
$\log^2(n)$	n	X	X			
n^2	$2n^2$	X		X	X	
$n + \log(n)$	n^2	X	X			
n	2^n	X	X			
$n^3 + 4n^2 + \log^4(n)$	n^3	X		X	X	X
$n + \log(n)$	n	X		X	X	X
$\sqrt{n}$	$n^{sin(n)}$			X		
$n!$	n^n	X	X			
$\log(n!)$	$\log(n^n)$	X		X	X	X
$25n\log(n) + 5n$	$\frac{1}{2}n\log(n)$	X		X	X	
$\sqrt{n} * \log(n)$	n	X	X			
$4^{\log(n)}$	$2n^2$	X		X	X	
$n\log(n)$	$n^{1.001}$	X	X			