4) For any node, the successor is the left most element of the right subtree. From the problem, we know the right subtree exists. Thus the successor is the left most child in the right subtree. However, the left most child cannot have a left child. (if node x has one left child, then node x is no more the left most).

Similarly, the predecessor is the right most child in the left subtree. The right most child cannot have a right child.

5) Two common self-balancing binary search trees are:

    AVL trees $O(\log n)$ for insertion
    Red-Black trees $O(\log n)$ for insertion

The red-black tree colors each node red or black. Then steps through 4 different cases to change color and thus position of the inserted node so the tree is balanced.

With AVL trees, you insert similar to binary search tree but then retrace up the tree and rotate nodes to balance the tree.

6) Stage 2, where I=4 for Roulette

s_4\d_4	0	1	2	3	4	n_4^*	d_4^*
[0-2)	0	—	—	—	—	0	0
[3-5)	0	8	—	—	—	8	1
[6-8)	0	8	14	—	—	14	2
[9-11)	0	8	14	18	—	18	3
[12	0	8	14	18	20	20	4

b)

Stage 2, where i=3 for Craps

$S_3 \backslash d_3$	0	1	2	3	4	r_3^x	d_3^x
(0-2)	0	-	-	-	-	0	0
[3-5)	8	-	-	-	-	8	0
[6-8]	14	11	-	-	-	14	0
[9-11]	18	19	-	-	-	19	1
[12-14]	20	25	21	-	-	25	1
[15-17]	20	29	29	-	-	29	2
[18-20)	20	31	35	30	-	35	2
[21-23]	20	31	39	38	-	39	3
[24-25]	20	31	41	44	38	44	3

Stage 3, where i=2 for Poker

$S_2 \backslash d_2$	0	1	2	3	4	r_2^x	d_2^x
0-3	0	-	-	-	-	0	0
4-5	8	9	-	-	-	9	0
6-7	14	9	-	-	-	14	0
8	14	17	-	-	-	17	1
9	19	17	-	-	-	19	1
10	19	17	19	-	-	19	1
11	19	23	18	-	-	23	1
12	25	23	18	-	-	25	0
13	25	23	26	-	-	26	2
14	25	28	26	-	-	28	1
15	29	28	26	26	-	29	0
16	29	28	32	26	-	32	2
17	29	34	32	26	-	34	1
18	35	34	32	34	-	35	0
19	35	34	37	34	-	37	2
20	35	38	37	34	34	38	1
21	39	38	37	40	34	40	3
22	39	38	43	40	34	43	2
23	39	44	43	40	42	44	1
24	44	44	43	45	42	45	3
25	44	44	47	45	42	47	2

6) Stage 4 $i = 1$ for Blackjack

S_4 \ d_4	0	1	2	3	4	η^x	$d,*$
25	47	50	51	47	41	51	2

We want 2 blackjack tables:

25 − 8 ; 17, which 1 table of poker is best,
25 − 8 − 5 ; 12, which 1 table of craps is best,
25 − 8 − 5 − 6 ; 6, which 2 tables of roulette is best

Our sequence is...

2 roulette, 1 craps, 1 poker, and 2 blackjack tables.