

TU DRESDEN

ADVANCED PRACTICAL COURSE

LAB REPORT

---

# Nuclear Magnetic Resonance

---

*Authors:*

Toni EHMCKE  
Christian SIEGEL

*Supervisor:*

Rajib SARKAR

Dresden, December 7, 2015

Date of experimental procedure: November 19th, 2015

# Contents

|          |  |          |
|----------|--|----------|
| <b>1</b> | <b>Introduction</b>  | <b>2</b> |
| 1.1      | Motivation . . . . .   | 2        |
| 1.2      | Nuclear Zeeman-Effect . . . . .  | 2        |
| 1.3      | Hahn-spinecho . . . . .  | 3        |
| 1.4      | Spin-Lattice Relaxation . . . . .  | 3        |
| <b>2</b> | <b>Experimental procedure</b>  | <b>4</b> |
| 2.1      | Preparation of a high frequency resonant circuit . . . . .                       | 4        |
| 2.2      | Finding of the nuclear spin resonance by varying the pulse frequency . . . . .   | 5        |
| 2.3      | Optimization of the pulse sequence by recording a rotation angle curve . . . . . | 6        |
| 2.4      | Determination of the relaxation constants $T_1$ and $T_2$ . . . . .              | 7        |
| <b>3</b> | <b>Data Analysis</b>   | <b>7</b> |
| <b>4</b> | <b>Discussion and conclusions</b>  | <b>7</b> |

# 1 Introduction

## 1.1 Motivation

*Nuclear Magnetic Resonance* is a physical phenomenon that can be observed while placing an ensemble of nuclei into a static magnetic field and stimulate it with a high-frequent alternating field. A necessary condition for this effect is that the atoms of the sample have a *nuclear spin* different from zero. It is the central concept that is used for *NMR-Spectroscopy*, a standard methodology for the investigation of the structure and interaction of complex molecules and solid state bodies by measuring local magnetic fields, and the *magnetic resonance tomography* which is an imaging technique used in clinical diagnostics for describing the morphologic and physiologic build-up of tissues and organs. For all of those applications some important parameters of particular physical compensation-processes, the so called *relaxation times*  $T_1$  and  $T_2$  need to be quantified. In the following experiment exactly those material-characteristic observables are determined for an ensemble of  $^{57}\text{Fe}$ -nuclei. But at first some basic knowledge.

## 1.2 Nuclear Zeeman-Effect

Every quantum mechanic angular momentum - especially every spin - is correlated with a magnetic moment  $\mu$ . The proportionality factor is called the *gyromagnetic ratio*  $\gamma$ . So the intrinsic magnetic momentum matching to the nuclear spin  $\vec{I}$  considered in the experiment is given by:

$$\vec{\mu} = \gamma \vec{I} \quad (1)$$

$$\gamma = g_I \frac{\mu_N}{\hbar} \stackrel{^{57}\text{Fe}}{=} 0.8661 \cdot 10^7 \text{ T}^{-1} \text{ s}^{-1} \quad (2)$$

Where  $\mu_N$  is the *nuclear magneton* and  $g_I$  is the Landé-factor which both are core-specific parameters. If those magnetic moments are placed in a static magnetic field  $\vec{B} = B\vec{e}_z$  then the Hamiltonian of the system and its Eigenvalues to the Eigenstates of the spin-operator  $|I, m_I\rangle$  are given by:

$$\mathcal{H} = -\vec{\mu}\vec{B} \stackrel{(1)}{=} -\gamma \mathcal{I}_z B \quad (3)$$

$$\langle \mathcal{H} \rangle = \langle I, m_I | \mathcal{H} | I, m_I \rangle = -\gamma B \langle I, m_I | \mathcal{I}_z | I, m_I \rangle = -\gamma B \hbar m_I \equiv E_{m_I} \quad (4)$$

Where the Eigenvalues of the Spin-operator for given spin-quantumnumber  $I$  and magnetic quantum number  $m_I = -I, \dots, I$  were used. So the outer magnetic field annuls the  $2I + 1$ -fold degeneration of the energystates. The nuclear spin-quantum-number of  $^{57}\text{Fe}$  is  $I = 1/2$  so there are two additional energystates with a energydifference:

$$\Delta E = \hbar \gamma B = \hbar \omega_L \quad (5)$$

As equation (5) suggests, there may occur optical transitions between the terms which lead to an emission of photons with the angular frequency  $\omega_L$ . One finds that this frequency is equivalent to the *Larmor-Frequency* that describes the precession of a magnetic moment around the z-axis caused by the torsional moment  $\vec{M} = \vec{\mu} \times \vec{B}$  in a magnetic field. The classical description of this process leads to the same result as quantum mechanics do. If we consider the ensemble of  $N$  nuclei as canonical, then the number of spins in the state  $|s, m_I\rangle$  in the thermodynamic equilibrium is given by the Boltzman-statistics:

$$N(m_I) = N \cdot \frac{e^{-\frac{E_{m_I}}{k_B T}}}{Z} = N \cdot \frac{e^{-\frac{\hbar \gamma B m_I}{k_B T}}}{Z} \quad (6)$$

Where  $Z = \text{const.}$  is the canonical partition function and  $T$  is the absolute temperature of the environment. This implies that the spins prefer to be polarised not uniformly but in the direction

of the B-field so this leads to a mean magnetic moment  $\langle \vec{\mu} \rangle \neq 0$ . This leads to an observable macroscopic magnetisation in the volume  $V$ :

$$\vec{M} = \frac{d\vec{\mu}}{dV} \cong \frac{N}{V} \langle \vec{\mu} \rangle \neq 0 \quad (7)$$

These changes in magnetisation are used to induce voltages that can be measured.

### 1.3 Hahn-spinecho

If the spins are just inside of a static magnetic field along the z-axis then they precess around this axis with an angular frequency of  $-\omega_L$ . In the next step an alternating magnetic field  $\vec{B}_{RF} = B_{RF} \sin(\omega t) \vec{e}_x$  is applied to the spins additionally. If we consider *resonance* of the Larmor-precession and the radio-frequency field, i.e.  $\omega = \omega_L$ , then we can rotate the magnetisation around the x-axis by an angle of  $\alpha = \gamma B_{RF} t$  where t is the time the RF-field is applied. With this effect we can now apply different *magnetic resonance sequences* to investigate the interaction of the spins with each other and the external magnetic fields.

The first sequence is a really simple one: the 90°-pulse where the spins are flipped in the xy-plane, i.e.  $\vec{M} \perp B\vec{e}_z$ . Because they now rotate around the z-axis in the laboratory system the periodic change of the magnetic field within the coil that is part of the HF-circuit leads to the induction of an alternating voltage with frequency  $\omega_L$ . Because the spin-moments interact with each other their angular frequency differs from spin to spin. This Dephasing leads to the decrease of the mean magnetisation in xy-plane and so the measured voltage fades away. This effect is called *Free Induction Decay*. By applying an 180°-pulse after a time  $\tau$  the spins rephase and the voltage increases again (*Hahn-Spinecho*). Because of irreversible effects during de- and refocussing the polarisation decreases and the mean magnetisation in xy-plane and the measured voltage have a lower amplitude. The correlation between the dephasing time  $\tau$  and the mean magnetisation in xy-plane can be described by an exponentially decaying function:

$$U_{ind}(\tau) \propto M_{xy}(\tau) = M_{xy}(\tau = 0) \cdot e^{-\frac{2\tau}{T_2}} \quad (8)$$

The decay of the reversible nuclear magnetisation is called *spin-spin-relaxation* and the corresponding time constant is the *spin-spin-relaxationtime*  $T_2$ . In the experiment it will be measured by varying  $\tau$  and measuring the alternating voltage  $U_{ind}$ . Figure (1) depicts the Hahn-Spinecho.

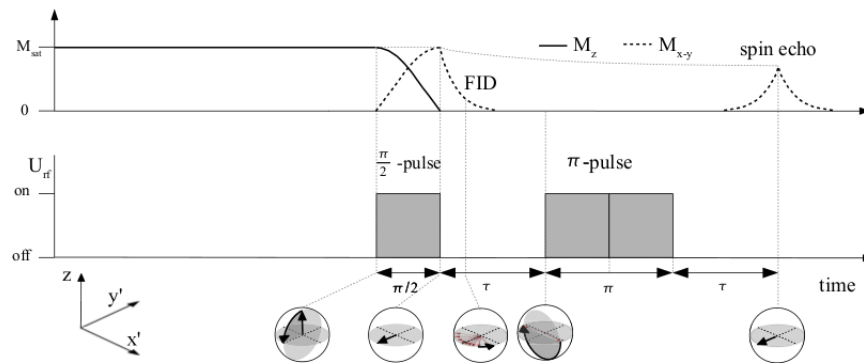


Figure 1: Hahn-Spinecho sequence for measuring  $T_2$  relaxation time

### 1.4 Spin-Lattice Relaxation

Because the energy of an ensemble of spins is minimal when they are polarised longitudinally along the magnetic field  $B\vec{e}_z$  they are into they will relax to this equilibrium state when it is perturbed,

e.g. by a  $90^\circ$ -pulse. After applying this pulse and waiting for a time  $\Delta t$  the z-component of the mean magnetic moment  $\langle \mu_z \rangle$  increases. After  $\Delta t$  a Hahn-Spin-echo sequence mentioned in section (1.3) is applied. Because the measure signal of this sequence is proportional to the initial magnetisation we will induce an echo-voltage in the HF-coil that increases by waiting longer after the  $90^\circ$ -pulse such that more spins are relaxed back to the equilibrium state. The dependency between  $\Delta t$  and the measure signal can be described by:

$$U_{ind}(\Delta t) \propto M_{xy}(\Delta t) = M_{sat} \cdot [1 - e^{-\frac{\Delta t}{T_1}}] \quad (9)$$

The relaxation of the longitudinal component of the magnetisation is called *Spin-lattice relaxation*. The occurring time constant  $T_1$  is the *Spin-lattice-relaxation time* and corresponds to the time one has to wait between the pulses such that the magnetisation is approximately 63.21 % of the saturation value. This parameter will be measured by varying  $\Delta t$  and holding the time-constant of the Hahn-sequence  $\tau$  constant. Figure (2) depicts the sequence.

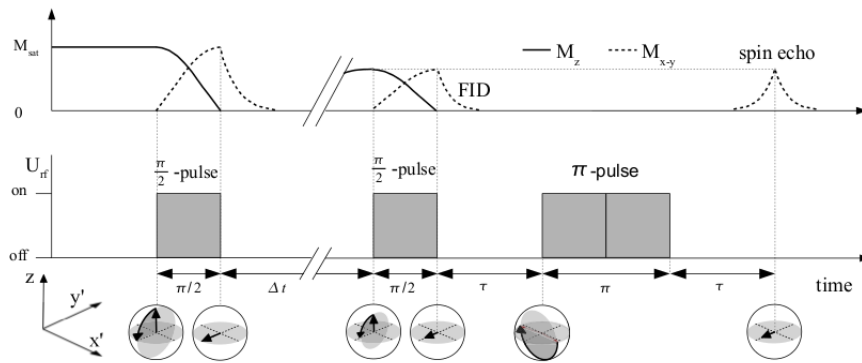


Figure 2: MR-sequence for measuring the spin-lattice relaxation time  $T_1$

## 2 Experimental procedure

The following task had to be done:

- 2.1:** Preparation of a radio frequency resonant circuit with the insertion of an iron powder probe in a cryostat
- 2.2:** Find the nuclear spin resonance by varying the NMR-pulse frequency while recording a spectrum of the  $^{57}\text{Fe}$  nuclear spin ensemble and calculation of the local magnetic field
- 2.3:** Optimization of the pulse sequence by recording a rotation angle curve
- 2.4:** Find the spin-spin- and spin-lattice relaxation constants  $T_2$  and  $T_1$ .

### 2.1 Preparation of a high frequency resonant circuit

First one has to prepare a copper coil with a diameter big enough to hold an iron powder assay. After the coil is wrapped one has to sold it onto the contacts of a stick, which provides a mechanism to tune the measured frequency to find the resonance frequency. While soldering one has to be careful to make sure, that one do not take to much tin, because the resistances of tin and copper are different. In the same way the soldered point should connect the contact and the coil directly, otherwise this could influence the results while tuning to the resonance frequency.

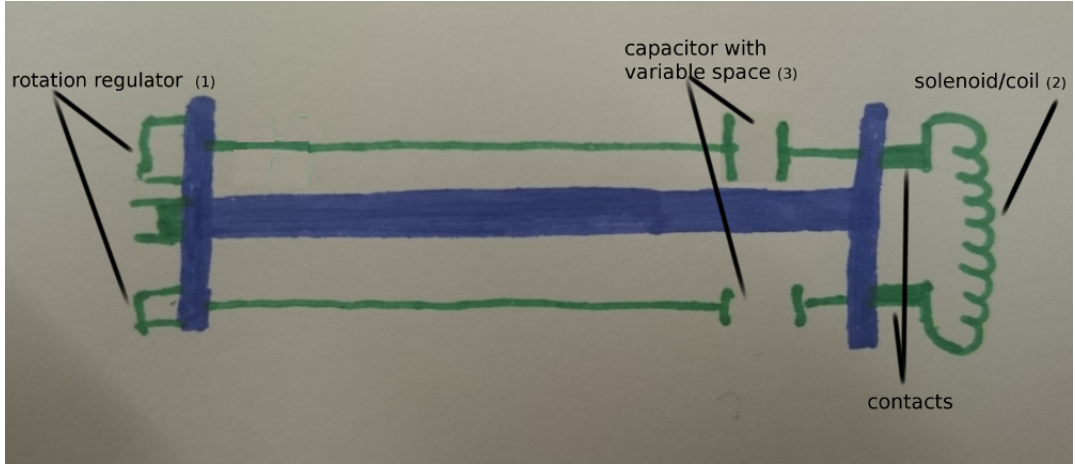
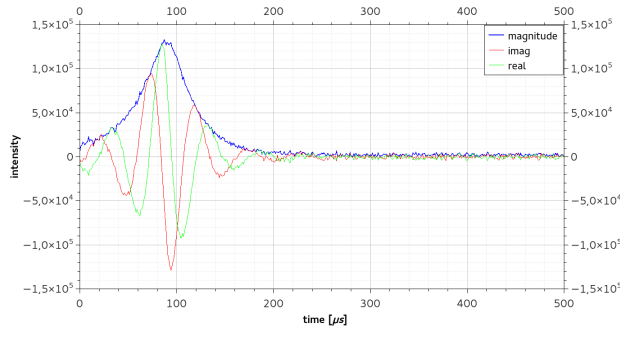


Figure 3: basic sketch of a probe with a coil on it

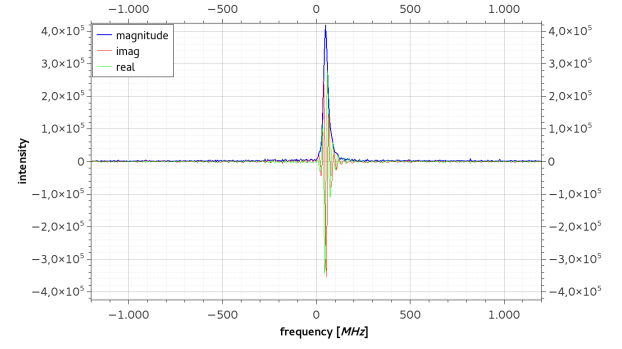
The rotation regulator (3.1) varies the space between the capacitor's plates (3.3) so one can tune the frequency of the solenoid (3.2). With an oscilloscope one finally can find the frequency between 42MHz up to 48MHz. The value which was found here is 45.01MHz.

## 2.2 Finding of the nuclear spin resonance by varying the pulse frequency

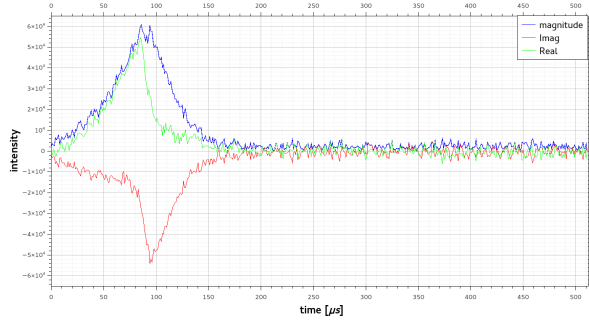
After the preparation of the solenoid and the insertion of the probe into the cryostat one has to mention what the different parameters built in the measurement-software do with the curves. The different parameters one could vary are  $pw$ ,  $\tau$ ,  $rd$  and  $ad$ .  $pw$  makes the magnitude look more gaussian and gives it a bigger value by make the imaginary and real parts more oscillating the higher it is. Varying  $\tau$  shows less oscillations and a lower magnitude the higher  $\tau$  is. At least one can also vary the parameter  $rd$ . The higher it is, the more oscillations can be found. These parameters are choosed such that the magnitude fits better to a gaussian. This means, one searches parameters with a low amount of oscillations, which also have not a big intensity. After the best parameters ( $pw = 2\mu s$ ,  $\tau = 60\mu s$ ,  $rd = 15\mu s$ ,  $ad = 10\mu s$ ) are found, one looks at the behavior of the curve at the resonance frequency, a little bit below and a bit above as shown in the figures 4(a) - 4(f).



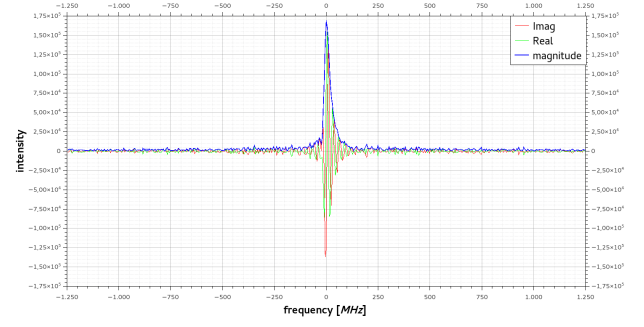
(a) intensity below resonance frequency in time domain



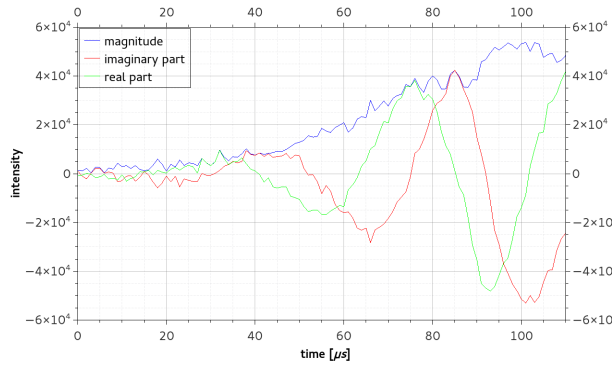
(b) intensity below resonance frequency in fourier space



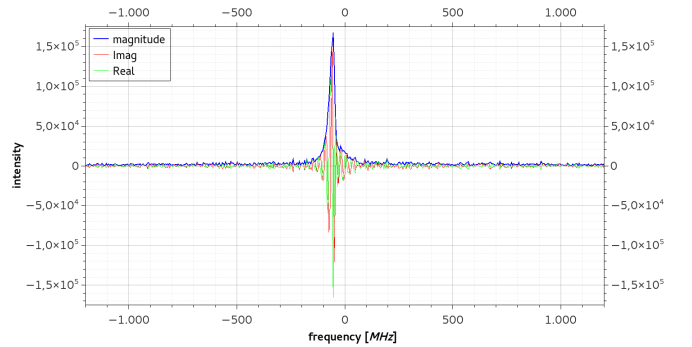
(c) intensity at resonance frequency in time domain



(d) intensity at resonance frequency in fourier space



(e) intensity below resonance frequency in time domain



(f) intensity below resonance frequency in fourier space

Figure 4: Different spectra below, at and above the resonance frequency

One can see that the number of oscillations in 4(e) and 4(a) is bigger than in 4(c). In figure 4(d) one recognizes a good gaussian peak. With that information the frequency is found at  $\omega_L = 45.47\text{MHz}$  (It was also given by the tutor). Using the  $\gamma$ -factor defined in (2) and the formula (5) one can calculate the magnetic field as  $B_z = \omega_L/\gamma = 0.525\text{ T}$ . This is a typical value for a magnetic flux density occuring in the bond of a solid state body.

### 2.3 Optimization of the pulse sequence by recording a rotation angle curve

Now one can configure the time intervalls how long the measurement device should wait until it takes a measurement. In both cases, the spin-spin- and the spin-lattice-relaxation one get different times for  $\tau$  as one can see in the diagrams. After this measurements the computer takes all magnitude maxima as data points one can analyze. One can recognize that 5(b) looks like an exponential growth and 5(a) looks similar to an exponential decay. To determine the relaxation constants one uses this information to make fits.

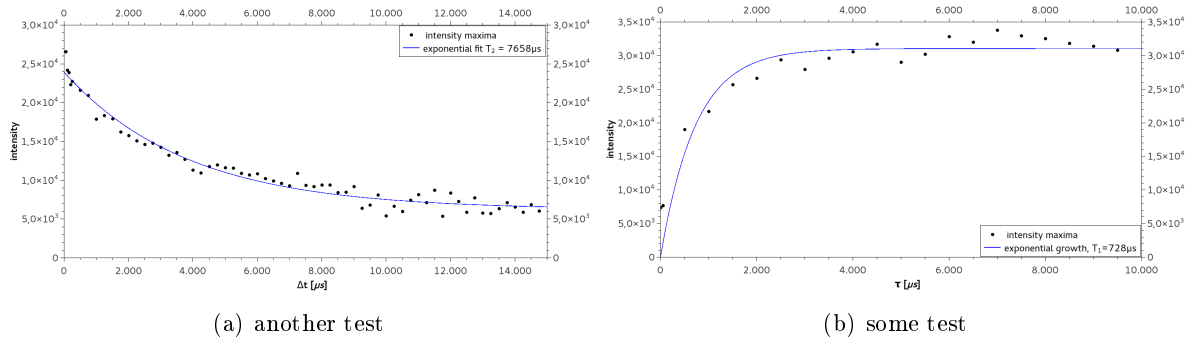


Figure 5: rotation angle curves for spin-spin- (a) and spin-lattice-relaxation (b)

## 2.4 Determination of the relaxation constants $T_1$ and $T_2$

The following fit functions are used;

$$y(\tau) = A \cdot (1 - e^{-\tau/T_1}) \quad (10)$$

$$y(\tau) = y_0 + Ae^{-2\tau/T_2} \quad (11)$$

With a tool called Qtiplot one makes a fit with these funktions a gets the blue line in 5(b) and 5(a). The parameters for 11 are:  $A = 17700 \pm 400$ ,  $y_0 = 6200 \pm 300$  and the spin-spin-relaxation constant  $T_2 = (7600 \pm 300)\mu s$  The parameters for the spin-lattice-relaxation are:

$$A = 31049.60435 \pm 0.00008, T_1 = (728.28187 \pm 0.00001)\mu s$$

## 3 Data Analysis

## 4 Discussion and conclusions

In the experiment the relaxation times  $T_1$  and  $T_2$  of an  $^{57}\text{Fe}$  spin ensemble were determined. The errors of the measurement are dominated by the deviation of the fits shown in the figures 5(b) and 5(a) that are because of the strong spreading relatively large. They could be reduced by taking several measurements for one constant value of the  $\tau$  and  $\Delta t$  time-variables. There also could be more abscissa-values for the fits all in all. The parameters one could choose in the measurement-software could also be improved.

In the experiment we learned something about the behaviour of nuclear spins interacting with each other. The measured relaxation time-constants are strongly dependant on the environment where the spins are built-in such that the measurement of the relaxation-times provide the approach to investigate chemical bonds between atoms. In medical applications this fact is used to distinguish between tissues in Magnetic Resonance Tomography. There the nuclear spin of the hydrogen-atoms that are found in almost all organic molecules we are made of is used. Different organs and tissues consist of different organic structures so that the interaction of the hydrogen-spins with their environment is strongly depends on their location. Damadian showed in 1970 that relaxation-times of hydrongen within tumour-tissue are significantly larger than in their healthy counterparts. One reason for that is the increased water-ratio in the diseased tissue.[02] Table (1) shows this effect.



| Tissue  | $T_{1,h}/s$ | $T_{1,t}/s$ |
|---------|-------------|-------------|
| skin    | 0.62        | 1.05        |
| lung    | 0.79        | 1.11        |
| bones   | 0.55        | 1.03        |
| stomach | 0.76        | 1.23        |
| liver   | 0.57        | 0.83        |

Table 1: Dependency of the  $T_1$ -relaxation time on different tissues. The index h stands for healthy tissue while t represents tumourous organs.[02]

These results can be transferred to the considered iron-nuclei. If we would consider the atoms within a different chemical bond the relaxation times would differ from those we just measured. These investigations could be object of further experiments or theoretical models.

## References

- [01] S. Diez. *Advanced practical course - Experiment MMC*, Dresden, 2015
- [02] R. Winter, F.Noll. *Methoden der Biophysikalischen Chemie*, Springer, 2013
- [03] <https://en.wikipedia.org/wiki/Kinesin> [November 22, 2015]
- [04] S. Ferbrugge et al. *Novel ways to determine kinesin-1's run length and randomness using fluorescence microscopy*. Amsterdam. October 2009
- [05] S. Klumpp, R. Lipowsky. *Cooperative cargo transport by several molecular motors*. Potsdam. November 2005