Inventory Management Report

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March 2024

Contents

1 Introduction

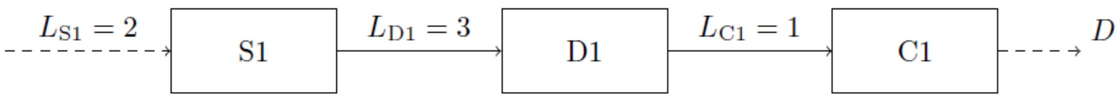

Figure 1: Serial Supply Chain of C1

We analyze a branch of the network in the dynamic setting by means of the stochastic-service model (SSM) framework in this project.

Customer C1 starts to operate its supply chain centrally in order to fulfill the external customer demand D. Consider the above serial supply chain of C1 and disregard all types of capacities, we know that the demand is stationary and follows a normal distribution, i.e., $D - N(20, 6^2)$. The backlog penalty is given by $b = 40$. The local holding costs for S1, D1 and C1 are given by 1, 4 and 8, respectively. In this periodic review system, $R = 1$, and the supply chain is governed by a centralized echelon base-stock policy.

2 Shang-Song Heuristic

We use the Shang-Song Heuristic to find the near-optimal echelon base-stock levels and corresponding safety stock levels.

Echelon holding cost	local holding cost	Echelon's Lead time	Local Lead time
$h_c = 4$	$h'_c = 8$	$L = 2$	$L_c = 1$
$h_d = 3$	$h_d = 4$	$L = 5$	$L_d = 3$
$h_S = 1$	$h_S = 1$	$L = 7$	$L_s = 2$

Table 1: Holding Cost and Lead Time for the calculations

Backlog penalty $b = 40$

Lead time demand:
$D_1 : N(20 * 2, (6^2) * 2) = N(40, 72)$
$D_2 : N(20 * 5, (6^2) * 5) = N(100, 180)$
$D_3 : N(20 * 7, (6^2) * 7) = N(140, 252)$

p	40		R	1																	
	C1	D1	S1				D~N(20, 6^2)														
Stage	1	2	3	stage	C1	D1	S1														
h	8	4	1	echelon holding cost	4	3	1														
L+R	2	5	7	L	1	3	2														
						1	2			1	2		Demand					1	2		
Stage 1	h	4	4		\alpha	0.91667	0.91667		z	1.38299	1.38299		mean	40	var	72	S	51.74	51.74		
C1	p	44	44												std	8.49	Echelon Base-	51.74		Local	51.74
													Demand					1	2		
Stage 2	h	3	7		\alpha	0.93182	0.85417		z	1.48947	1.05447		mean	100	var	180	S	119.98	114.15		
C1-D1	p	41	41												std.	13.42	Echelon Base-	117.07		Local	65.33
																	stock level				
													Demand					1	2		
Stage 3	h	1	8		\alpha	0.97561	0.83333		z	1.97051	0.96742		mean	140	var	252	S	171.28	155.36		
C1-D1-	p	40	40												std.	15.87451	Echelon Base-	163.32		Local	46.25
																	stock level				

Figure 2: Results from Shang-Song Heuristic

$$
\begin{array}{llll}
S_c^u = F_c^{-}1\frac{40+4}{40+8} = 0.91667 & S_c^u = 51.74 & S_c^l = F_c^{-}1\frac{40+4}{40+8} = 0.91667 & S_c^l = 51.74 \\
S_d^u = F_d^{-}1\frac{40+1}{40+4} = 0.93182 & S_d^u = 119.98 & S_d^l = F_d^{-}1\frac{40+1}{40+8} = 0.85417 & S_d^l = 114.15 \\
S_s^u = F_s^{-}1\frac{40+0}{40+1} = 0.97561 & S_s^u = 171.28 & S_s^l = F_s^{-}1\frac{40+0}{40+8} = 0.83333 & S_l^l = 155.36
\end{array}
$$

Table 2: Vectors of echelon base-stock levels

$$
\begin{array}{ccc}
S_c = \frac{1}{2}[51.74 + 51.74] = 51.74 & S_d = \frac{1}{2}[119.98 + 114.15] = 117.07 & S_s = \frac{1}{2}[171.28 + 155.36] = 163.32 \\
S_c' = 51.74 & S_d' = 117.07 - 51.74 = 65.33 & S_s' = 163.32 - 117.07 = 46.25 \\
S_c^* = 51.74 - 40 = 11.74 & S_d^* = 117.07 - 100 = 17.07 & S_s^* = 163.32 - 140 = 23.32
\end{array}
$$

Table 3: Echelon, Local base-stock, and safety stock levels

3 Simulation

Now, we create a simulation model to evaluate this network operating under the echelon base-stock policy. To compare the effect of random demands following a normal distribution and a heavy-tailed distribution, in Excel, we generated 100 random numbers from $\mathcal{N}(20, 6^2)$ and another 100 random numbers from the student T distribution using 6*T.inv(RAND(),5) + 20, which then has the same mean 20 and sd 6.

In this periodic review system $R = 1$, we plan to run 100 simulations. The simulation model will be based on these assumptions:

1. We assume the inventory position should go back to S after attempting to satisfy the backorder B from the previous. In the model, the backlog is an "extra demand/order" for the next period t, and thus every replenishment order needs to add it;
2. The holding and stockout costs are calculated based on the ending inventory level so we use local holding costs and local ending inventory levels. The ending EL at t is the

starting inventory level at $t+1$;

3. The first simulation starts at $t = 1$ and the initial parameters are: demand = backlog = 0, all in-transit inventory IT is 20 (based on the mean of the normal distribution), and thus the on-hand inventory OH levels for stage (C, D, S) are $(31.74, 5.33, 6.25)$.

The model is:

For echelon C:

$$d[t] = Demand[t] + Backlog[t-1]$$

$$OHC[t] = ELC[t-1] + ITC[t-1] \tag{1}$$

$$Backlog[t] = [d[t] - OHC[t]]^+$$

$$ITC[t] = \min((ELD[t-1] + ITD[t-3]), S_C - OHC[t] + Backlog[t-1]) \tag{2}$$

$$ELC[t] = [OHC[t] - d[t]]^+ \qquad \text{(ending IL for C)}$$

For echelon D:

$$OHD[t] = ELD[t-1] + ITD[t-3] \tag{3}$$

$$\begin{aligned} ITD[t] = \min((&ELS[t-1] + ITS[t-2]), \\ &S_D - OHC[t] - OHD[t] - ITD[t-1] - ITD[t-2] + Backlog[t-1]) \end{aligned} \tag{4}$$

$$ELD[t] = [OHD[t] - ITC[t]]^+ \qquad \text{(ending IL for D)}$$

For echelon S:

$$OHS[t] = ELS[t-1] + ITS[t-2] \tag{5}$$

$$\begin{aligned} ITS[t] = [&S_S - OHS[t] - ITS[t-1] - OHC[t] - OHD[t] \\ &- ITD[t] - ITD[t-2] + Backlog[t-1]]^+ \end{aligned} \tag{6}$$

$$ELS[t] = [OHS[t] - ITD[t]]^+ \qquad \text{(ending IL for S)}$$

$$Cost[t] = 8 \cdot ELC[t] + 4 \cdot ELD[t] + 1 \cdot ELS[t] + 40 \cdot Backlog[t]$$

Distribution	**Expected Cost**	**Type-1 Service Level**	**Type-2 Service Level**
Normal Distribution	178.059	0.820	0.936
Student-T distribution	227.536	0.680	0.843

Table 4: Table Showing Results from Different Distributions

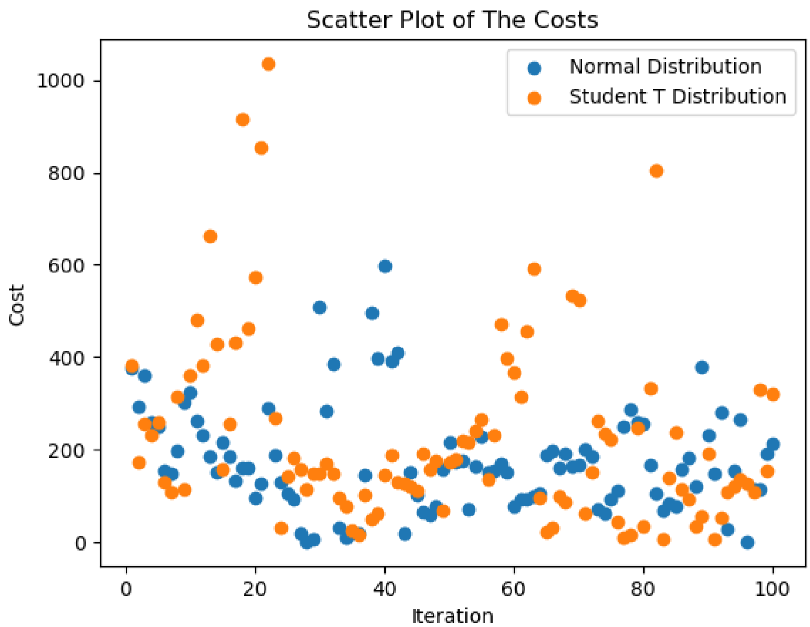

Figure 3: Results From Two Distributions

4 Results and Managerial Insights

From the table 4 above, the results from random demands from the normal distribution have higher Type-1 and -2 service levels. This means that almost 82% of periods did not experience stock-outs and almost 94.6% of demands were directly satisfied from the inventory. In comparison, random demands from the T distribution experienced 32 stockouts in 100 simulations. Besides, we also notice that in the graph below the results based on the T distribution have some very high costs (indicated by those yellow points at the top). This is mainly because heavy-tailed distributions can generate more outliers than the normal distribution, resulting in either high demand and consequently higher backlog costs, or lower demand and then higher holding costs.

As a result as seen in figure 3, in real-world scenarios, it is important to balance both holding costs and backlog costs simultaneously. Some strategic safety-stock policies, such as estimating demand based on new observations, reducing lead times, and arranging backup sourcing as well as optimizing protection interval periods, can all contribute to meeting the expected demand and buffering against demand uncertainty.

Further recommendations include introducing the capacity constraint across different nodes: supplier (S), distributor (D), and customer (C), to assess how these constraints affect the overall performance of the supply chain, particularly in terms of cost and service levels