Graphical description, dimensions, frames, and formulas for implementing differential drive robot

Aleksandar Haber

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Abstract

This document contains formulas for implementing differential drive robot. Please read the license associated with this document.

1 Robot dimensions and coordinate systems

The sketch of the robot and robot dimensions are shown in the Figure 1. The robot dimensions are parametrized by letters. The same letters are used in the URDF and XACRO files modeling the robot dynamics.

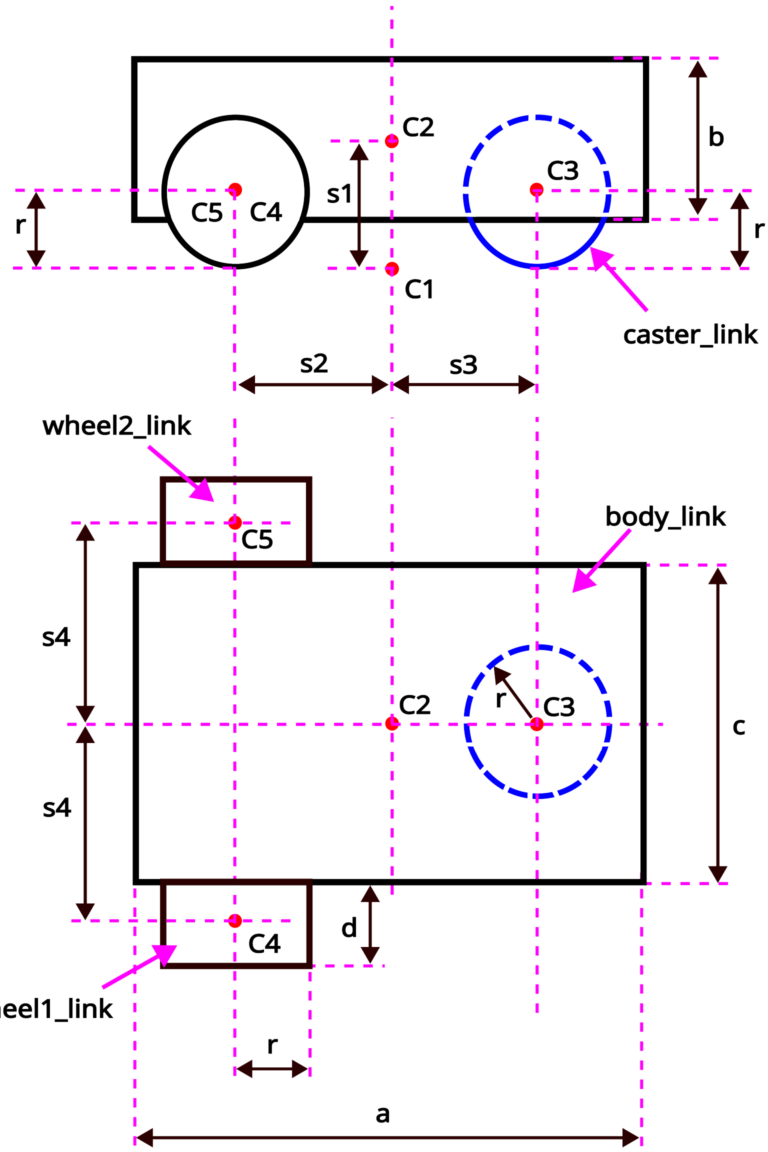

Figure 1: Representation of the body link

The robot consists of four links: "body_link", "wheel1_link", "wheel2_link", and "caster_link", and one fictious link necessary to implement the model in Gazebo called "base_footprint".

- "base_footprint" is a fictious link necessary to model the robot and to display it in Gazebo.
- "body_link" is used to model the robot body. This link is modeled as an URDF box.
- "wheel1_link", "wheel2_link" are used to model the robot wheels. These links are modeled as URDF cylinder links.

- "caster_link" is used to model the robot caster wheel. It is modeled as an URDF sphere. Here, we introduced a simplicifcation that the caster wheel is fixed. That is, the caster wheel is only sliding on the surface. This simplification will not affect the ability to control the robot since the caster wheel is a passive wheel, and it should only provide a support.

Four figures below show the frames located at the points C1, C2, C3, C4, and C5.

The figure below shows the coordinate systems (frames) at the points C1 and C2. The red color denotes the x axis, the light green is the y axis, and the light blue is the z axis. This is the standard convention in Gazebo.

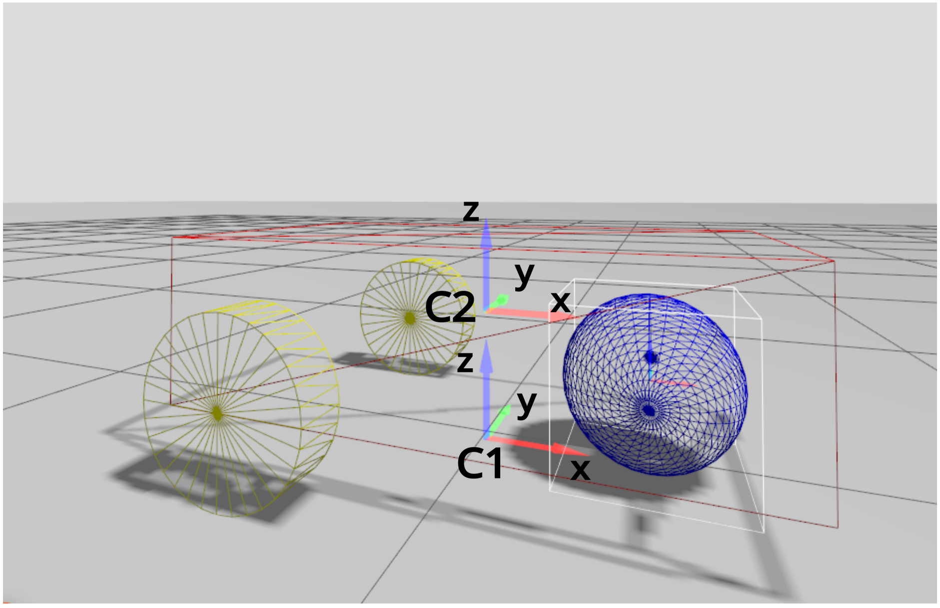

Figure 2: Frames at C1 and C2.

Here are the explanations

- The point C1 is the origin of two frames with the same orientation. The first frame is the link frame of the fictious link "base_footprint". The second frame is the link frame of the "body_link". "body_link" is the child link of the fictious link "base_footprint". The link frame of "body_link" is identical to the joint frame of the joint called "body_link_joint".
- The point C2 is the origin of the center of the mass frame of the link called "body_link".

Next, let us explain a frame whose origin is at the point C3. Consider the figure shown below.

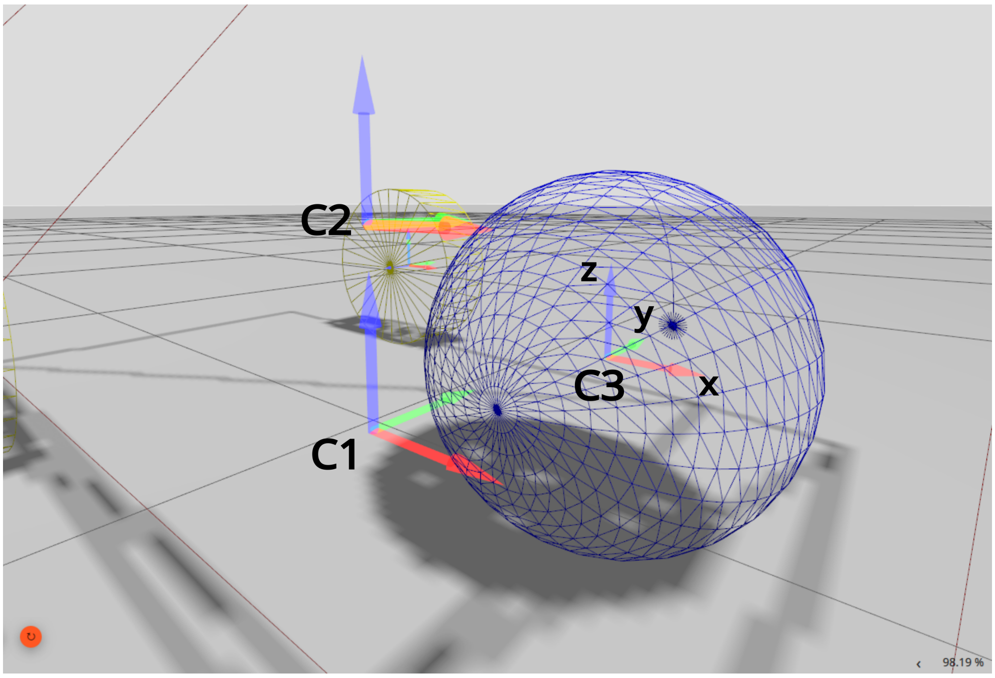

Figure 3: Frame at C3.

Two frames have origins at the point C3. The first frame is the link frame of the link called "caster_link". This is at the same time the joint frame of the fixed joint called "caster_joint". Here, for simplicity, we assumed that the "caster_joint" is fixed and that it only slides. The second frame is the center of the mass frame of the link "caster_link".

Figure below shows the frames located at the point C4.

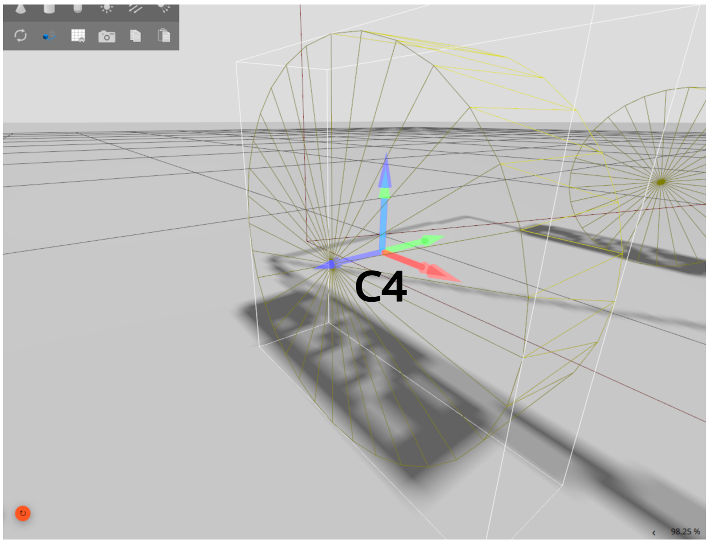

Figure 4: Frame at C4.

There are two frames locates at this point. The difference between these two frames is that they are rotated with respect to each other. That is, they have different orientations:

- The first frame is the link frame of the link called "wheel1_link". This frame is at the same time the joint frame of the joint "wheel1_joint". The axes of this frame are parallel to the frames located the points C1,C2 and C3.
- The second frame is the center of the mass frame of the link "wheel1_link". This frame is obtained by rotating the frame link of "wheel1_link" around x axis for 90 degrees. This rotation is performed in order to ensure that the z axis of the center of the mass frame is located such that we can properly define a cylindrical link.

Figure below shows the frames located at the point C5.

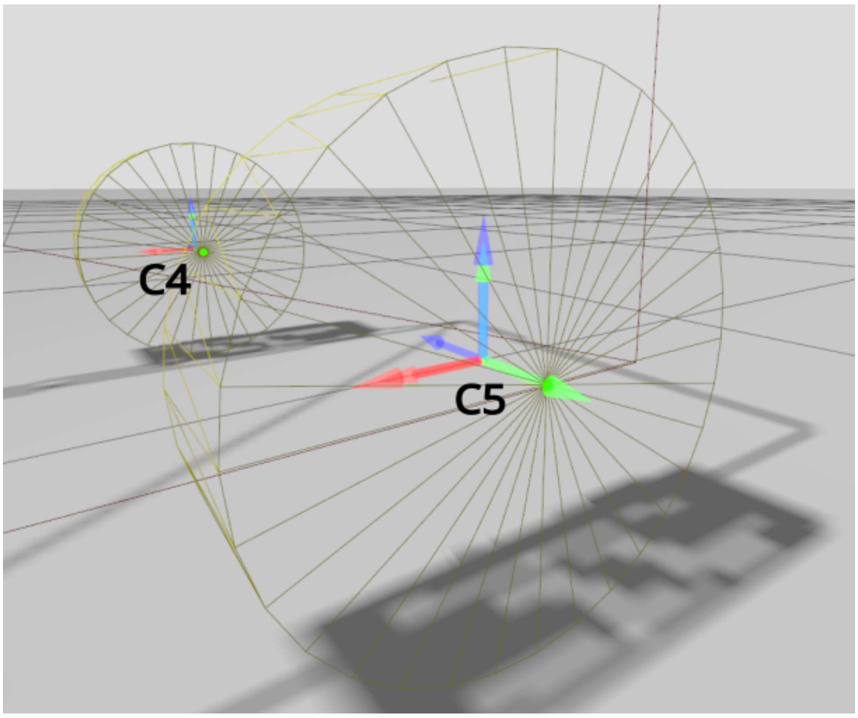

Figure 5: Frame at C5.

There are two frames locates at this point. The difference between these two frames is that they are rotated with respect to each other. That is, they have different orientations:

- The first frame is the link frame of the link called "wheel2_link". This frame is at the same time the joint frame of the joint "wheel2_joint". The axes of this frame are parallel to the frames located the points C1,C2 and C3.

- The second frame is the center of the mass frame of the link "wheel2_link". This frame is obtained by rotating the frame link of "wheel2_link" around x axis for 90 degrees. This rotation is performed in order to ensure that the z axis of the center of the mass frame is located such that we can properly define a cylindrical link.

2 Formulas for "body_link" - robot body

"body_link" is modeled as a box. The volume of "body_link" is

$$V1 = a \cdot b \cdot c \tag{1}$$

The mass is

$$m1 = \mathrm{d1} \cdot V = \mathrm{d1} \cdot a \cdot b \cdot c \tag{2}$$

where $d1$ is the density of the material of the body of the robot, and $m1$ is the mass of the body of the robot

The moments of inertia for the corresponding axes are

$$I_x = \frac{1}{12} \cdot m1 \cdot (b^2 + c^2) \tag{3}$$

$$I_y = \frac{1}{12} \cdot m1 \cdot (a^2 + b^2) \tag{4}$$

$$I_z = \frac{1}{12} \cdot m1 \cdot (a^2 + c^2) \tag{5}$$

3 Formulas for "wheel1_link" and "wheel2_link" - robot wheels

"wheel1_link" or "wheel2_link" are modeled as cylinders. The volume of "wheel1_link" or "wheel2_link" is

$$V2 = \pi \cdot r^2 \cdot d \tag{6}$$

The mass is

$$m2 = \mathrm{d2} \cdot V2 = \mathrm{d2} \cdot \pi \cdot r^2 \cdot d \tag{7}$$

where $d2$ is density and $m2$ is mass. The moments of inertia for the corresponding axes are

$$I_x = \frac{1}{12} \cdot m2 \cdot (3 \cdot r^2 + d^2) \tag{8}$$

$$I_y = \frac{1}{12} \cdot m2 \cdot (3 \cdot r^2 + d^2) \tag{9}$$

$$I_z = \frac{1}{2} \cdot m2 \cdot r^2 \tag{10}$$

4 Formulas for "caster_link" - caster

"caster_link" is modeled as a sphere. The volume is

$$V3 = \frac{4}{3} \cdot \pi \cdot r^3 \tag{11}$$

The mass is

$$m3 = \mathrm{d3} \cdot V3 = \mathrm{d3} \cdot \frac{4}{3} \cdot \pi \cdot r^3 \tag{12}$$

where d3 is the density and m3 is the mass. The moments of inertia for the corresponding axes are

$$I_x = \frac{2}{5} \cdot m3 \cdot r^2 \tag{13}$$

$$I_y = \frac{2}{5} \cdot m3 \cdot r^2 \tag{14}$$

$$I_z = \frac{2}{5} \cdot m3 \cdot r^2 \tag{15}$$