1. Introduction

The purpose of this project is to establish a Robot-Advisor for investors. Our Robo Advisor provides three different portfolios (low, medium and high) for people with different risk appetite, which will be determined by the risk score in the IPS questionnaire. Each portfolio associated with different risk levels is constructed under different return and volatility requirements.
In this report, we summarized the performance of our portfolios with low, medium and high risk level. We divided our capital into both US market and Canadian market. Portfolio assets and weights are selected based on investors risk appetite and will be rebalanced every 6 months during the 5-year investment horizon. We use both nominal Mean-Variance Optimization (nominal MVO) and Risk Parity models to construct our portfolio. We will invest in Equity, Fixed Income, Real Estate and Commodity for high risk portfolio. For low and medium risk portfolio, we will invest in Equity, Fixed Income, Real Estate but not in Commodity, which is relatively more volatile. At last, each of these portfolios' performance will be compared with a specific benchmark.
Different performance and risk metrics of the portfolios are illustrated in the report including quarterly P&L, Value at Risk (VaR), Expected Shortfall, Sharpe Ratio, Risk Contribution, and some other risk measures. Furthermore, we also implemented scenario tests and stress test, which evaluates the performance of the portfolio under different economic scenarios.

2. Methodology

Data Preparation

We used only ETFs and FX options in our three portfolios.

Table 1: Data Set for Low and Mid Risk Portfolio

Ticker	ETF Name	Asset Class	Country
XLU US	Utilities Select Sector SPDR Fund	Equity	US
VDC US	Vanguard Consumer Staples ETF	Equity	US
VFH US	Vanguard Financials ETF	Equity	US
VIS US	Vanguard Industrials ETF	Equity	US
SHY US	iShares 1-3 Year Treasury Bond ETF	Fixed Income	US
LQD US	iShares iBoxx Investment Grade Corporate Bond ETF	Fixed Income	US
TIP US	iShares TIPS ETF	Fixed Income	US
VNQ US	Vanguard Real Estate ETF	REITs	US
XIU CN	iShares S&P/TSX 60 Index ETF	Equity	CA
XIC CN	iShares Core S&P/TSX Capped Composite Index ETF	Equity	CA
XCV CN	iShares Canadian Value Index ETF	Equity	CA
XGB CN	iShares Canadian Government Bond Index ETF	Fixed Income	CA
XSB CN	iShares Canadian Short Term Bond Index ETF	Fixed Income	CA

Low and mid risk portfolios contain the same ETFs. The ETFs are consisted of three asset classes: Equity, Fixed Income and REITs. The capital are split into US dollars and Canadian dollars. For Equity, ETFs are from four different industries: Utilities, Consumer Staples, Financial and Industries. Equities from these industries are well-known as relatively low risk assets. Our portfolios also include many Fixed Income ETFs, such as Short-Term Treasury Bond ETF and Highly-Rated Corporate Bond ETF. We used risk parity method to assign weights to these assets. Risk parity approach allocates assets such that they have similar risk contributions, which is usually measured in terms of volatility. Therefore, it will assign more weights to low risk assets and less to high risk assets, which is in line with our goal.

Table 2: Data Set for High Risk Portfolio

Ticker	ETF Name	Asset Class	Country
XLV US	Health Care Select Sector SPDR Fund	Equity	US
VCR US	Vanguard Consumer Discretionary ETF	Equity	US
VDE US	Vanguard Energy ETF	Equity	US
XLB US	Materials Select Sector SPDR Trust	Equity	US
VGT US	Vanguard Information Technology ETF	Equity	US
XOV US	Vanguard Communication Services ETF	Equity	US
VWO US	Vanguard FTSE Emerging Markets ETF	Equity	US
VYM US	Vanguard High Dividend Yield ETF	Equity	US
EMB US	iShares J.P. Morgan USD Emerging Markets Bond ETF	Fixed Income	US
HYG US	iShares iBoxx High Yield Corporate Bond ETF	Fixed Income	US
VNQ US	Vanguard Real Estate ETF	REITs	US
GLD US	SPDR Gold Shares	Commodity	US
USO US	United States Oil Fund LP	Commodity	US
DBA US	Invesco DB Agriculture Fund	Commodity	US
XIU CN	iShares S&P/TSX 60 Index ETF	Equity	CA
XCS CN	iShares S&P/TSX SmallCap Index ETF	Equity	CA
XIC CN	iShares Core S&P/TSX Capped Composite Index ETF	Equity	CA
XCV CN	iShares Canadian Value Index ETF	Equity	CA
XLB CN	iShares Core Canadian Long Term Bond Index ETF	Fixed Income	CA

The high risk portfolio contains ETFs from four asset classes: Equity, Fixed Income, REITs and Commodities. We added commodities because we pursue high returns and our risk tolerance is high. In addition, we are more aggressive in choosing equity ETFs, such as from technology, energy and materials sector. We also added an emerging markets ETF to the portfolio. For bond ETFs, we focused more on high yields rather than high credit rating. In terms of portfolio construction, we used nominal MVO.

Nominal Mean-Variance Optimization (MVO)

The nominal MVO seeks both to minimize variance and maximize return, yielding an optimal compromise based on a risk aversion coefficient $\lambda > 0$,

$$\begin{aligned} \min_x \quad & x^T Q x - \lambda \mu^T x \\ \text{s.t.} \quad & 1^T x = 1 \end{aligned} \tag{1}$$

where $\mu \in \mathbb{R}^n$ is the expected return vector and $Q \in \mathbb{R}^{n \times n}$ is covariance matrix which estimated from historical asset returns. In the later section, we tested different values of λ to study its effect in the resulting portfolio in the model training period.

Risk Parity

We will use the portfolio variance as the risk measure and we are allowed to short-sell the assets. Equally weighted risk contribution method is utilized.

$$\begin{aligned} \min_x \quad & \textstyle\sum_{i=1}^N [x_i - \frac{\sigma(x)^2}{(\sum x)_i N}]^2 \\ \text{s.t.} \quad & 1^T x = 1 \end{aligned} \tag{2}$$

where $\sigma(x) = \sum_{i=1}^N \sigma_i(x)$ and $\sigma_i(x)$ is total risk contribution of asset i ($\sigma_i(x) = \frac{x_i (\sum x)_i}{\sqrt{x^T Q x}}$). $x \in \mathbb{R}^n$ is the weights, $Q \in \mathbb{R}^n$ is the covariance matrix which estimated from historical asset returns.

Currency Hedging

We will long put option on currency pair USDCAD to Hedge the exposure of the foreign exchange risk in US market (US dollar depreciates). Option's margin is defined as the maximum expected loss. Since the maximum expected loss of longing an option is 0 (excluding initial premium), there is no initial margin requirement. Since there is no market price data available, we use the Black Scholes analytical formula to estimate the no-arbitrage FX put option price.

$$P_0 = K e^{-r_{us}(T-t)} N(-d_2) - S_0 e^{-r_{ca}(T-t)} N(-d_1) \tag{3}$$

where

$$\frac{\ln \frac{S_0}{K} + (r_{us} - r_{ca} + \frac{\sigma^2}{2})(T-t)}{\sigma \sqrt{T-t}}$$

and

$$d_2 = d_1 - \sigma \sqrt{T-t}$$

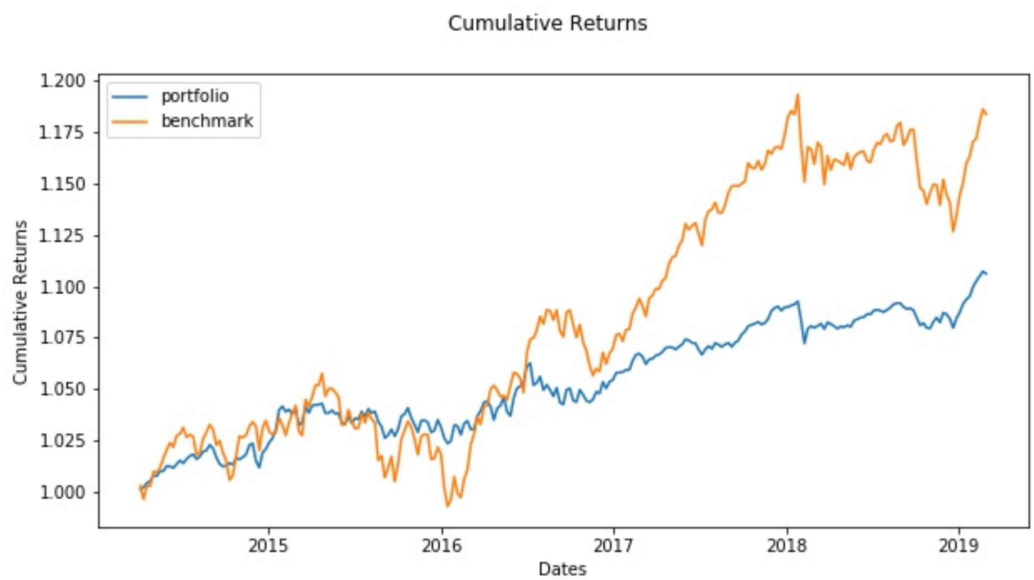
We want to buy an out-of-money put option with the market value on April 1st 2014 for $S_0 = 1.0981$ CAD/USD, K = 1 CAD/USD, T = 5 years,r_{us} = 0.8% and r_{ca} = 0.21%. In order to calculate the σ_{year}, we downloaded the weekly FX data and computed the weekly σ_{week}. Then, $\sigma_{year} \approx \sigma_{week} * \sqrt{52}$ = 9.3%. Based on the pricing formula, the put option price is \$0.036.

3. Portfolio Construction

Low Risk Portfolio

We utilized our low & mid risk asset data set with risk parity for portfolio optimization. As we allowed short-sell in this portfolio, we have to reserve some cash for the margin call. We reserved 150% of the short asset values as initial margin. We also ensured that the cash amount is 100% of the current market value of the short sale, along with at least 25% of the total market value of the securities in the margin account to meet the maintenance margin. For the low risk benchmark, we used iShares Core Conservative Allocation ETF (AOK).

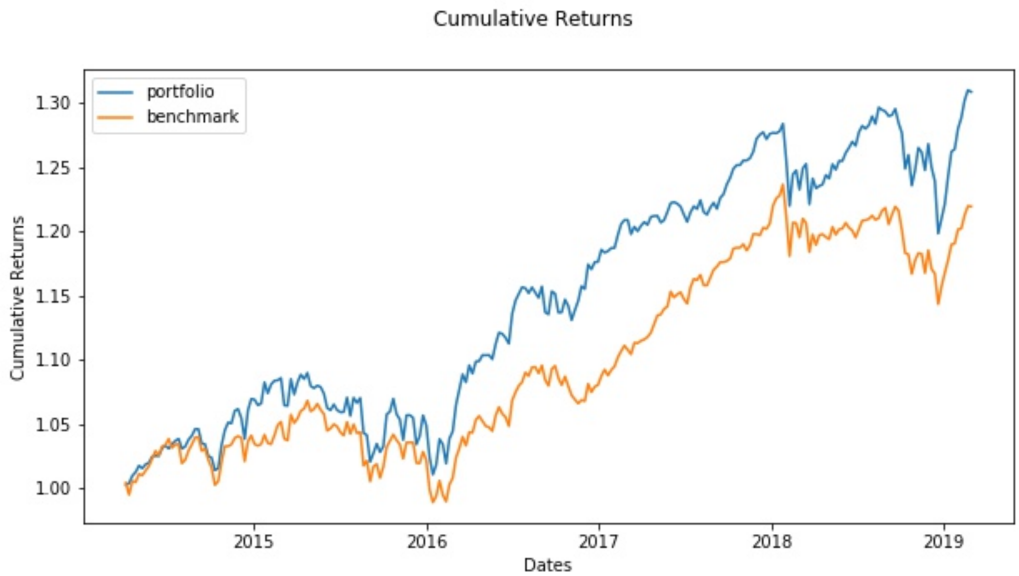
AOK is iShares' attempt at providing an all-encompassing conservative asset-allocation strategy to investors. The fund invests exclusively in iShares ETFs, targeting a conservative risk profile with an emphasis on fixed income ETFs. In general, the fund devotes around two-thirds of its portfolio in fixed income funds (like the iShares Core Total US Bond Market Fund IUSB) and around one-third to equity ETFs (like IVV).



Mid Risk Portfolio

We utilized our low & mid risk assets data set with nominal MVO for portfolio optimization. The parameter lambda was set to zero to reduce the expected returns effect in portfolio construction. For the mid risk benchmark, we used iShares Core Moderate Allocation ETF (AOM).

The fund invests exclusively in iShares ETFs aiming for exposure that is consistent with a moderate risk tolerance. The underlying ETFs are vanilla in nature, meaning market-cap-weighted for equity and market-value-weighted for fixed income, with the emphasis on top-line allocation and no security selection. Domestic fixed-income accounts for roughly half of AOM's portfolio, making it a more conservative play than one might expect for the level of risk being targeted.



High Risk Portfolio

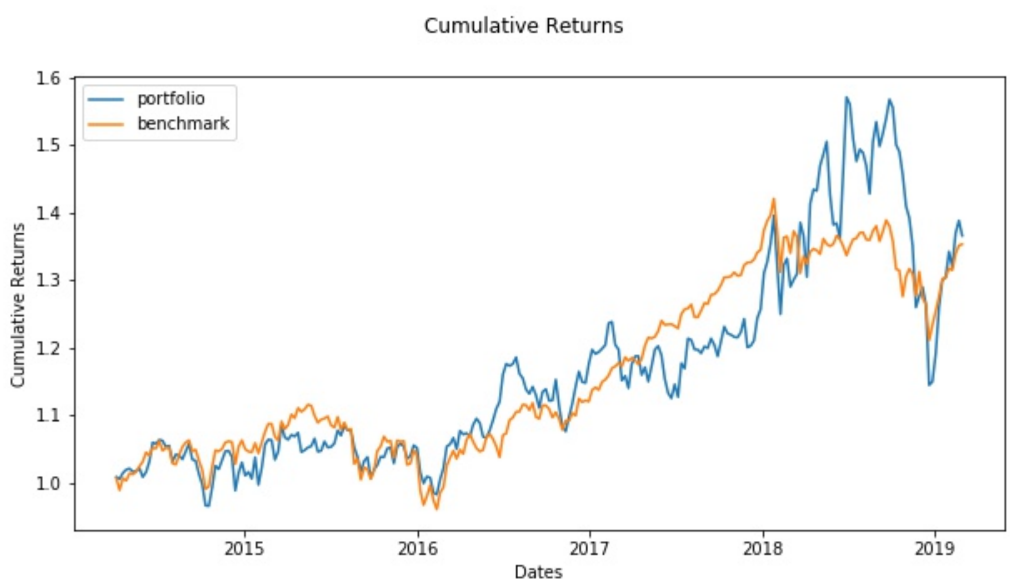
In our high risk portfolio, we pursue a more aggressive approach. Thus we used a different asset universe compared to low & mid risk portfolio. Moreover, we used nominal MVO and chose the parameter lambda to be 0.5. The parameter was chosen by analyzing the portfolio performance between 2009 and 2014. And since we disallowed short-sell in this portfolio, we do not have to consider problems of margin call.

For the benchmark, we used the iShares Core Aggressive Allocation ETF (AOA). AOA from iShares

Table 3: Portfolio Performance with different parameter λ

Lambda	Annualized Returns	Annualized Std	Sharpe Ratio
0	0.102	0.098	1.04
0.5	0.166	0.158	1.051
1	0.175	0.18	0.972
1.5	0.177	0.184	0.962
2	0.189	0.189	1.005

offers an all-encompassing aggressive asset-allocation strategy in a fund-of-funds wrapper. The fund invests across asset classes and subgroups using other iShares ETFs. The use of ETFs in the basket allows investors to quickly understand how the fund is positioned. It also keeps the focus on top-down allocation rather than bottom-up security selection. AOA plays it straight with large allocations to vanilla versions of major asset groups: US large- and midcaps, international equities and broad bonds. The fund also allocates in smaller proportions to emerging market equities, US small-caps, high-yield bonds, REITs and TIPS. What makes the fund "aggressive" is a higher allocation to equities than fixed income.



Margin Call Problems

As we allowed short-selling in the three portfolios, we have to consider the margin call problems. Whenever our model decided to short sell some assets, we should reserve 150% of the value of the short sale at the time the sale is initiated. What's more, we should keep at least 100% of the current market value of the short sale for the maintenance requirements, along with at least 25% of the total market value of the securities in the margin account.

Summary

Here is a summary for our portfolio performance.

	Low	Mid	High
Sharpe Ratio	0.955	0.862	0.469
Annualized Returns	2.1%	5.6%	7.6%
Annualized Volatility	2.2%	6.5%	16.2%
VaR using Normal Distribution (%)	5.7	16.3	34.3
CVaR (%)	0.6	1.7	4.5
Money Weighted Returns	3.8%	3.3%	1.5%
Time Weighted Returns	2.0%	5.4%	6.3%

4. Scenario Analysis

We picked 10 years of economic and market factors (2004 - 2014, quarterly) for both US and Canada, and fitted an ARMA-GARCH model for each of the factors. The economic factors are Real GDP (% change), Inflation (% change), Unemployment (%), Short-term Interest Rate, Long-term Interest Rate, S&P 500 Growth (%), Oil Price and FX (USDCAD, for US only). We chose US Treasury Yield, US Corporate High Yield Bond Index, US Investment Grade Corporate Bond Index, US Real Estate Index, Crude Oil Index, S&P500 Index and VIX Index as US market factors. We chose Canada Government Bond Yield, Gold Index, Crude Oil Index, TSX Index and FX Index as Canada market factors.

The ARMA-GARCH model is as below:

$$r_t = \mu + \sum_{i=1}^{p} a_i r_{t-i} + \sum_{j=1}^{q} b_i \epsilon_{t-j} + \epsilon_t$$

$$\epsilon_t = z_t \sigma_t, z_t \sim iid(0,1)$$

$$\sigma_t^2 = \omega + \sum_{k=1}^{m} \alpha_{t-k} \epsilon_{t-k}^2 + \sum_{l=1}^{n} \beta_{t-l} \sigma_{t-l}^2$$

For each factor's time series, we started with ARMA(p = 0, q = 0) and experimented with different p and q's. To prevent over-fitting, we also restricted p and q to be no larger than 3. We then used the Akaike information criterion (AIC) and picked the p-q pair that yields the lowest AIC. If ARMA(p = 0, q = 0) gives the lowest AIC, we would consider the factor as white noise and not use it.

For simplicity, we then fitted the residuals using GARCH(1, 1) model. The fitted residuals are then saved in a matrix.

To get the joint distribution between economic and market factors, we simulated 1000 scenarios with Monte Carlo and used ARMA-GARCH model to predict the next 4 consecutive quarters of outcomes. Instead of assuming that the residuals follow Gaussian distributions, we randomly sampled from our historical residual matrix. This is to account for codependence among factors as well non-Gaussian features (eg. fat-tails). Since we have 40 distinct historical residuals, we are able to generate 2560000 possible outcomes, which is sufficient to determine the joint distribution.

At the same time, we used linear regression to fit the prices of each asset in our portfolio with economic and market factors selected. We then passed the simulated factor values to these regression models and multiplied by the corresponding number of holdings in our initial portfolio (2014-04-01) to obtain the portfolio value. We did this using the third and fourth quarter's simulated factor values and calculated the unconditional distribution of our quarterly Profit-and-Loss (PnL).

We are interested in understanding how our portfolios perform under different economic scenarios. To do this, we again fitted the unconditional PnLs with economic factors using linear regression. We did this separately for US portfolio PnLs and Canada portfolio PnLs. Only US economic factors were used for US PnLs, whereas for Canada PnLs, only Canada economic factors were used. We selected Deloitte and OECD's projections for US and Canada's economic factors in 2015 (since we are investing in 2014) respectively and passed to our regression models to get the expected conditional PnL. We also looked at extreme scenarios such as the 2008 Financial Crisis.

US Scenario	UP	Base	Down	Stress Testing
Real GDP (% change)	3	2.3	2	-8.45
Inflation (% change)	0.8	0.6	0.5	-0.92
Unemployment (%)	5.4	5.4	5.5	5.8
IR Short (%)	0.23	0.21	0.16	2.79
IR Long (%)	2.34	2.33	2.31	3.86
S&P 500 (%)	4.54	1.13	3.04	-23.56
Oil ($/bbl, WTI)	54.45	50.9	46.22	133.37
FX (USD/CAD)	1.25	1.2	1.16	0.98

CA Scenario	UP	Base	Down	Stress Testing
Real GDP (% change)	2.6	2	1.7	-4.6
Inflation (% change)	1.5	1.4	1.2	-1.1
Unemployment (%)	6.8	7	7.1	6.1
IR Short (%)	0.81	0.8	0.74	3.4
IR Long (%)	1.32	1.3	1.28	3.64
S&P 500 (%)	4.54	1.13	3.04	-23.56
Oil ($/bbl, WTI)	54.45	50.9	46.22	133.37

Besides conditional PnLs, we also calculated 95% conditional Value-at-Risk (CVaR) under each of the scenarios.

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