A Scalable Message-Passing Algorithm for Supply Chain Formation

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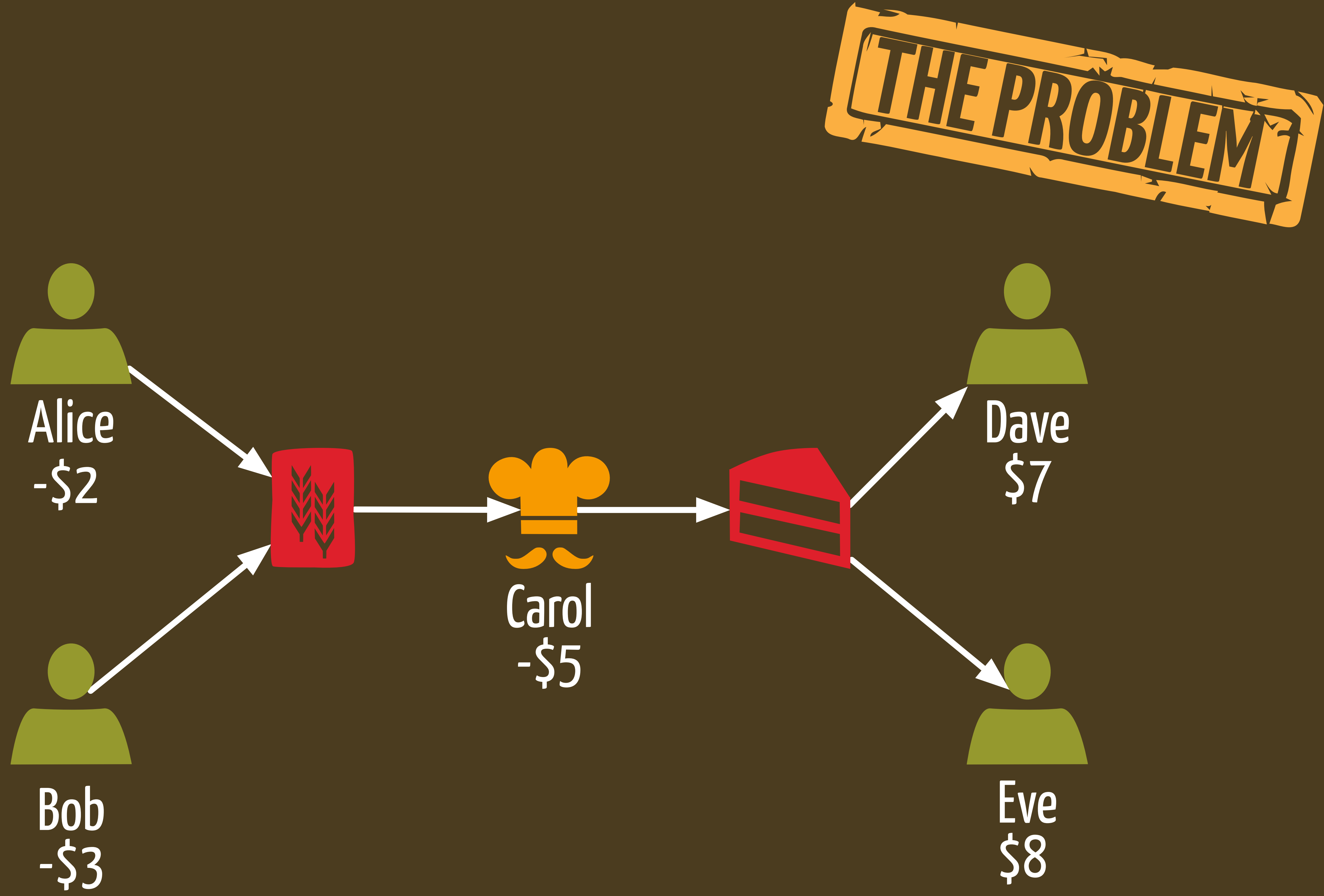
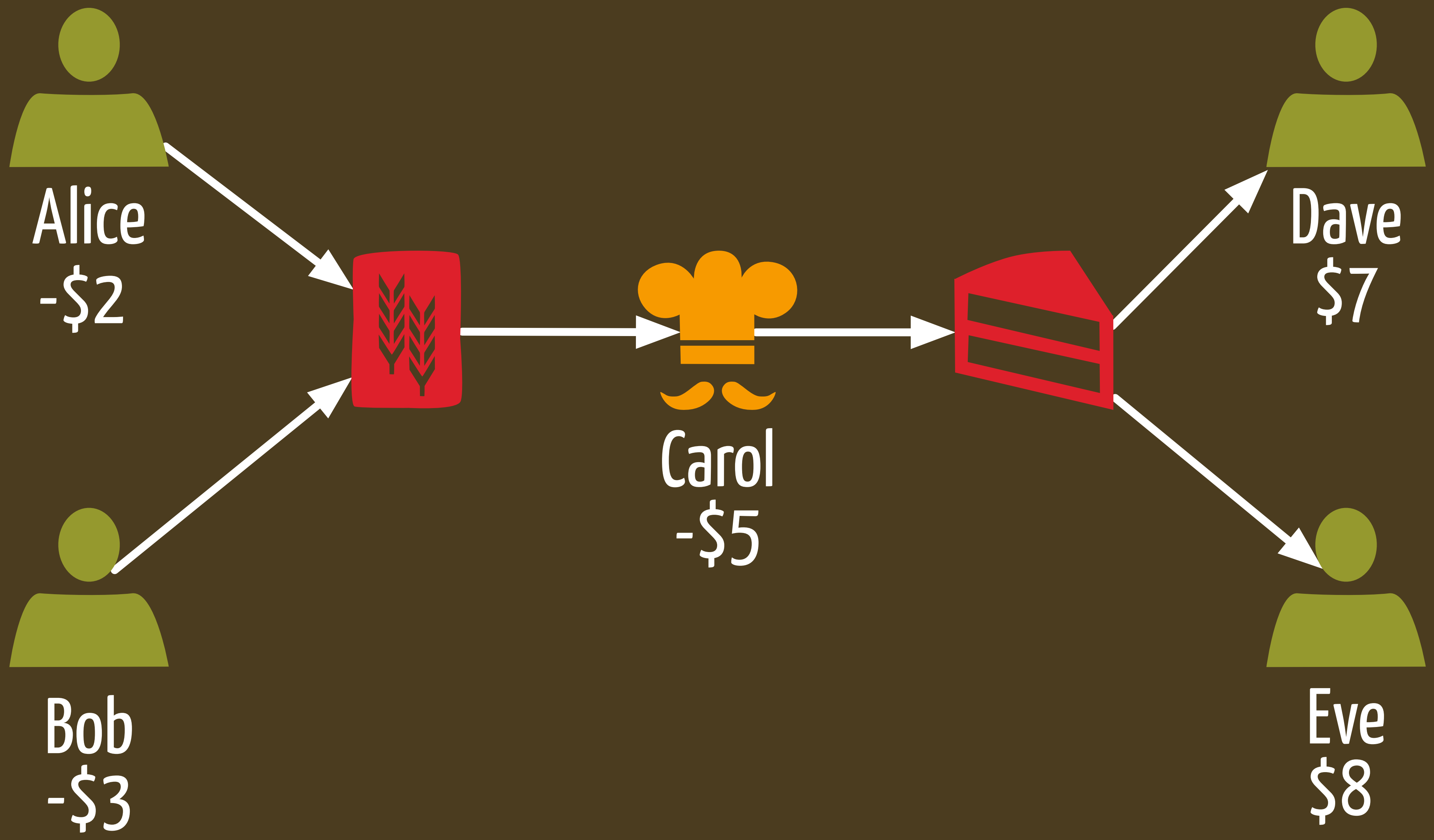
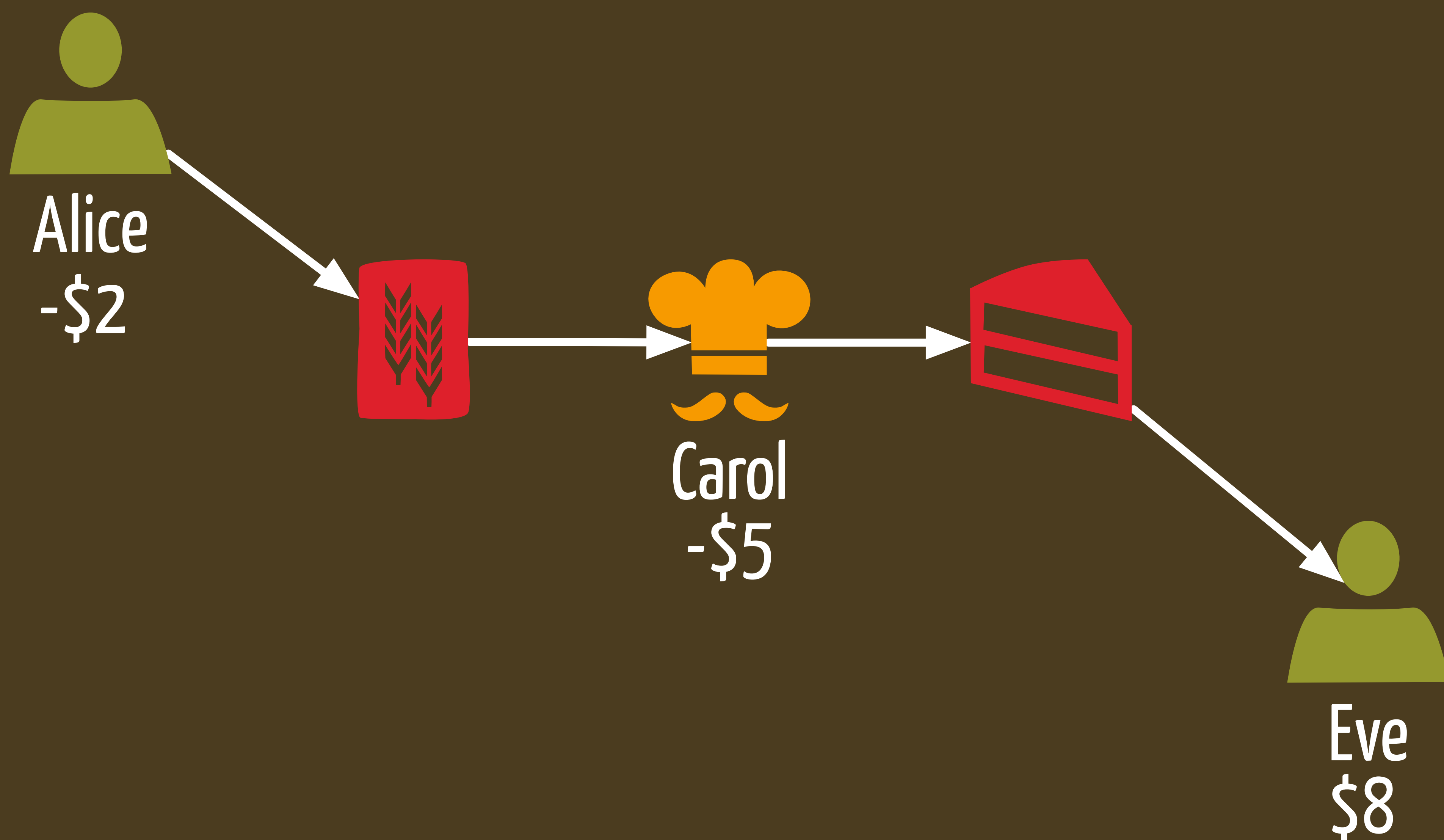
for the 26th AAAI Conference


THE PROBLEM
Alice
-$2
Bob
-$3
Carol
-$5
Dave
$7
Eve
$8



The Supply Chain Formation problem is that of finding the feasible configuration with maximum value.

THE PROBLEM



Supply Chain Value = $8 - $5 - $2 = $1

THE GOAL

To provide a scalable method for Supply Chain Formation in markets with high degrees of competition.

CONTRIBUTION

Encoding in a novel graphical model.

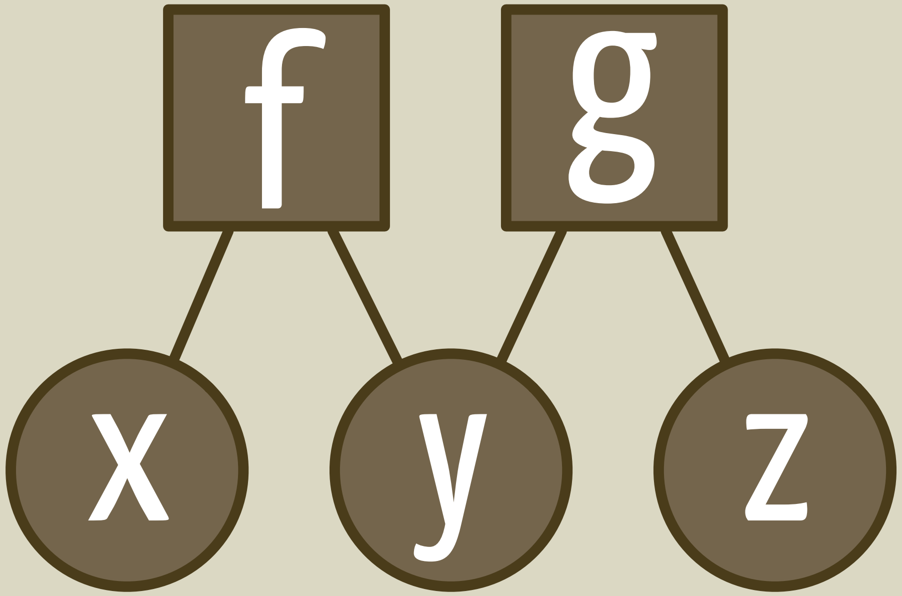
Optimized implementation of max-sum.

BENEFITS

Reduced memory, communication and computation requirements.

Higher quality solutions.

factor graph is a graphical model to represent functions.



$f(x,y) + g(y,z)$

max-sum is a message passing algorithm.
max-sum can efficiently approximate optimization problems.

Loopy Belief Propagation

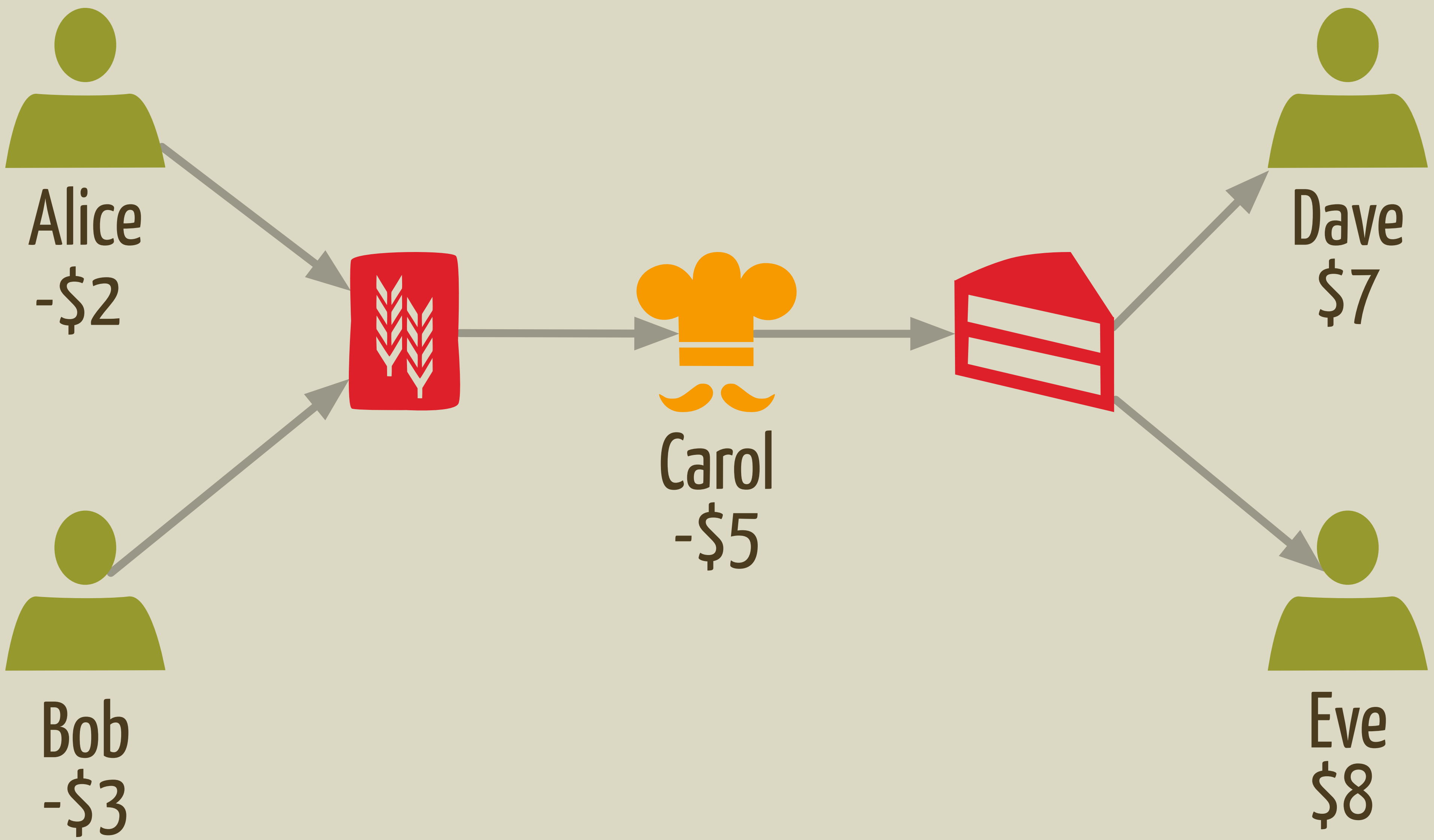
Michael Winsper
Maria Chli

Loopy Belief Propagation

map the Supply Chain Formation problem into a factor graph.

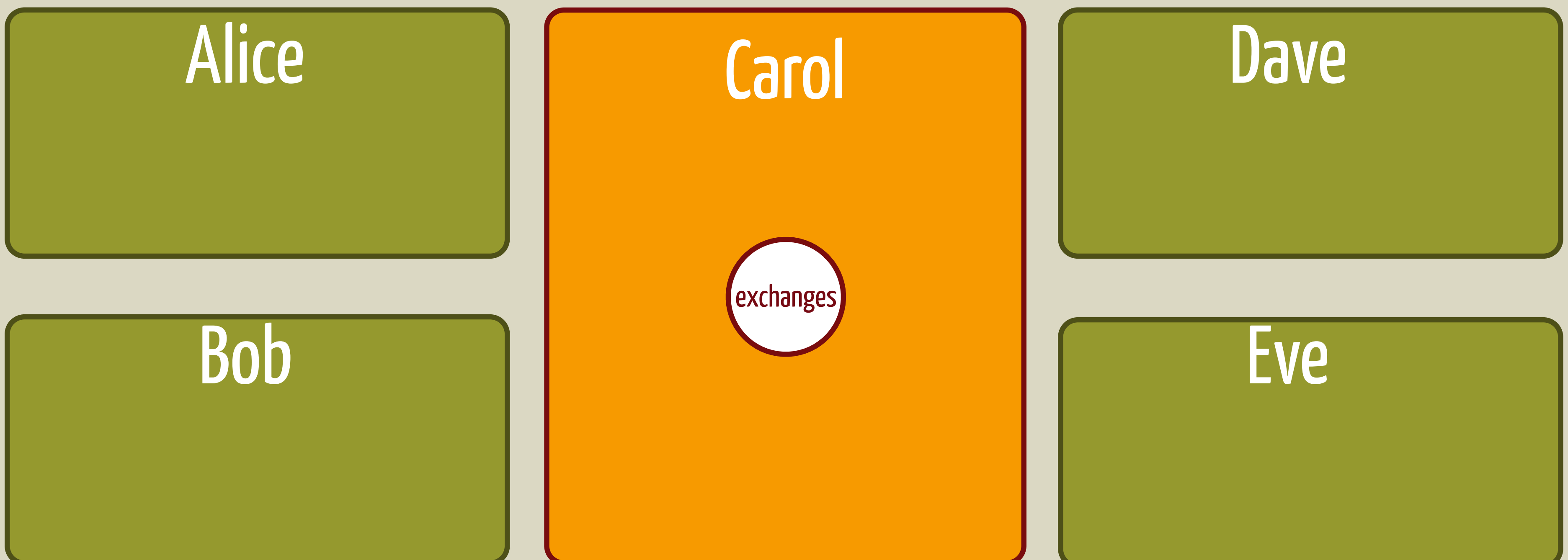
apply max-sum over the factor graph to obtain a solution.

Mapping into a Factor Graph



Factor Graph Mapping

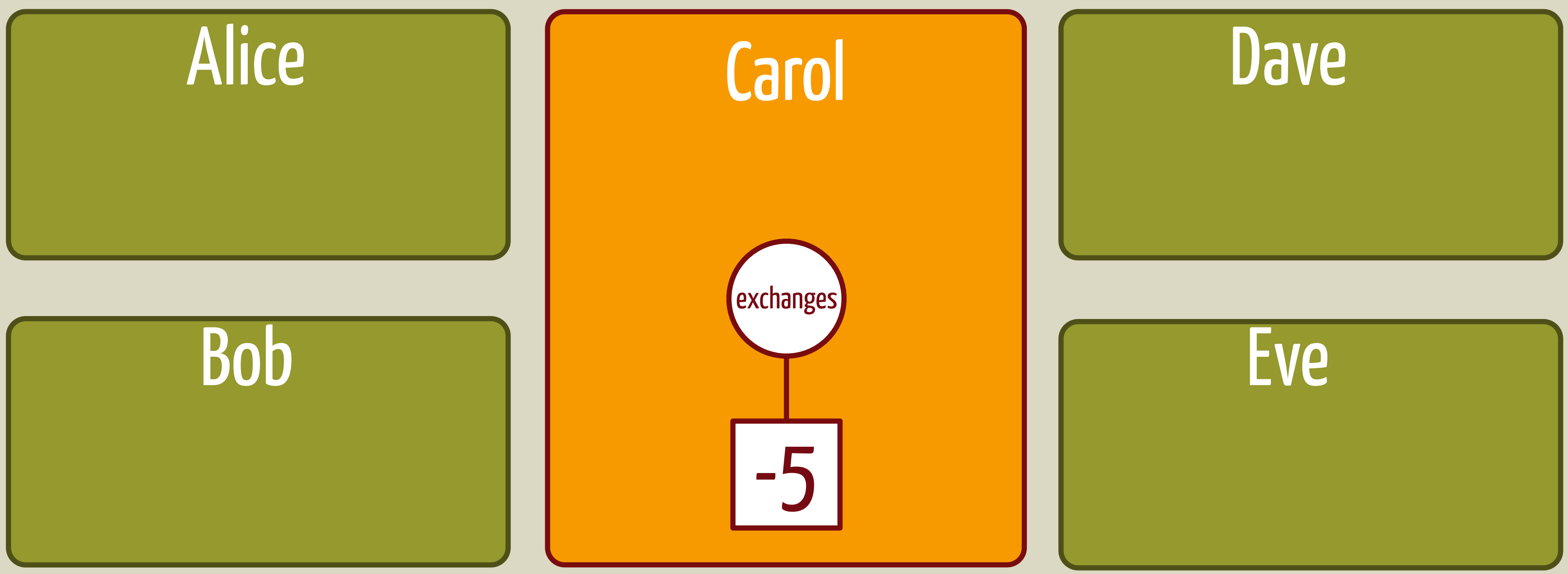
Factor Graph Mapping



0 do nothing
1 buy from Alice, sell to Dave
2 buy from Alice, sell to Eve
3 buy from Bob, sell to Dave
4 buy from Bob, sell to Eve

Agent variable encodes all of Carol’s possible exchanges.

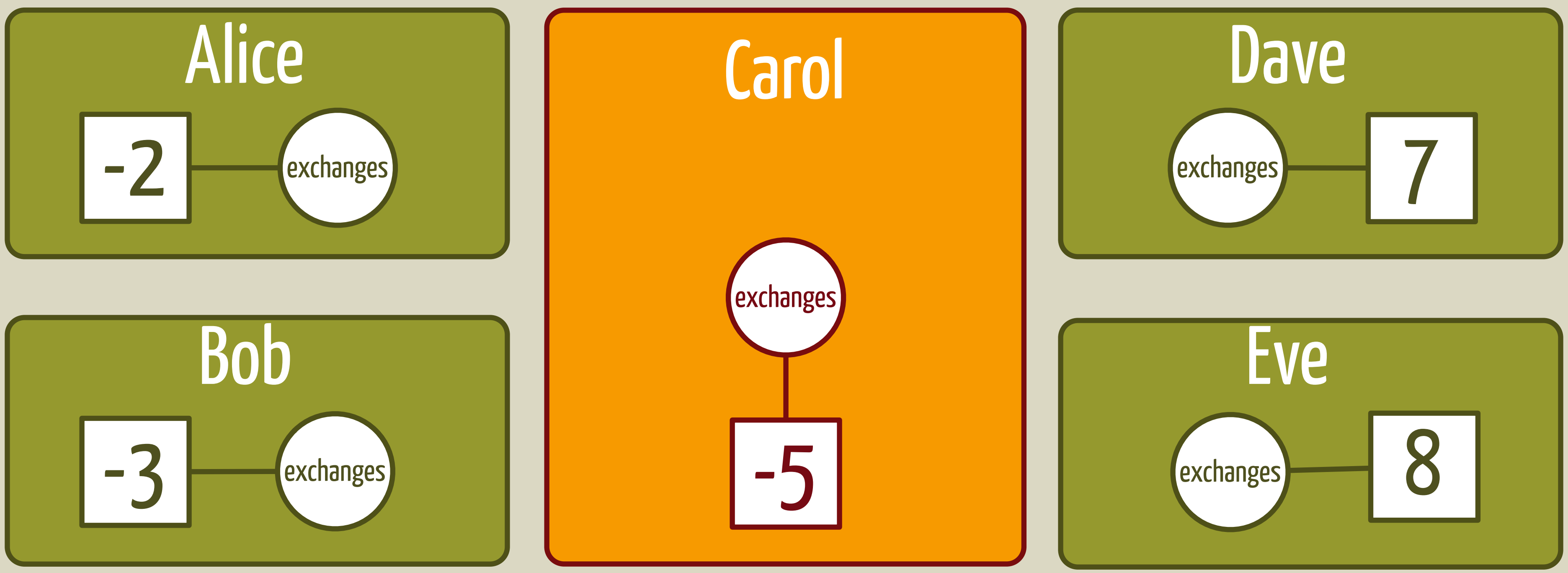
Factor Graph Mapping

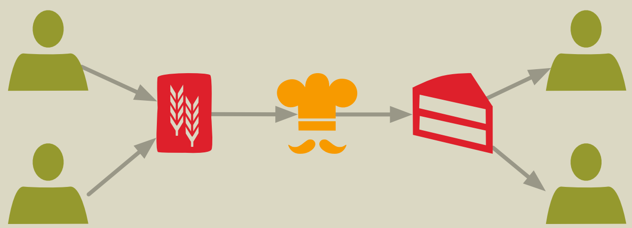


inactive	0
otherwise	-5

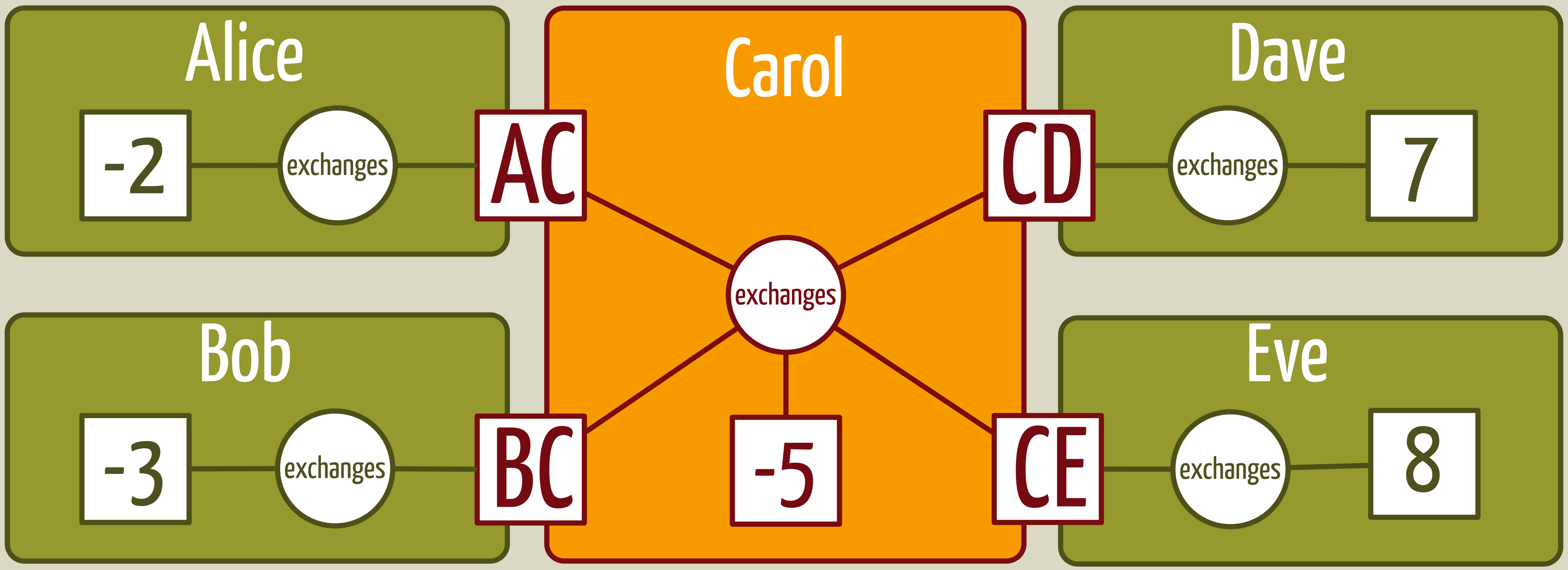
Activation factor encodes Carol's activation cost.

Factor Graph Mapping





Factor Graph Mapping



Compatibility factors encode compatibility between Carol's states and her neighbors'.

	LBP
memory per agent	$O(G \cdot A^{2G+1})$
bandwidth per agent	$O(G \cdot A^{G+1})$
computation time per agent	$O(G \cdot A^{2G+1})$

RB-LBP

Our Approach

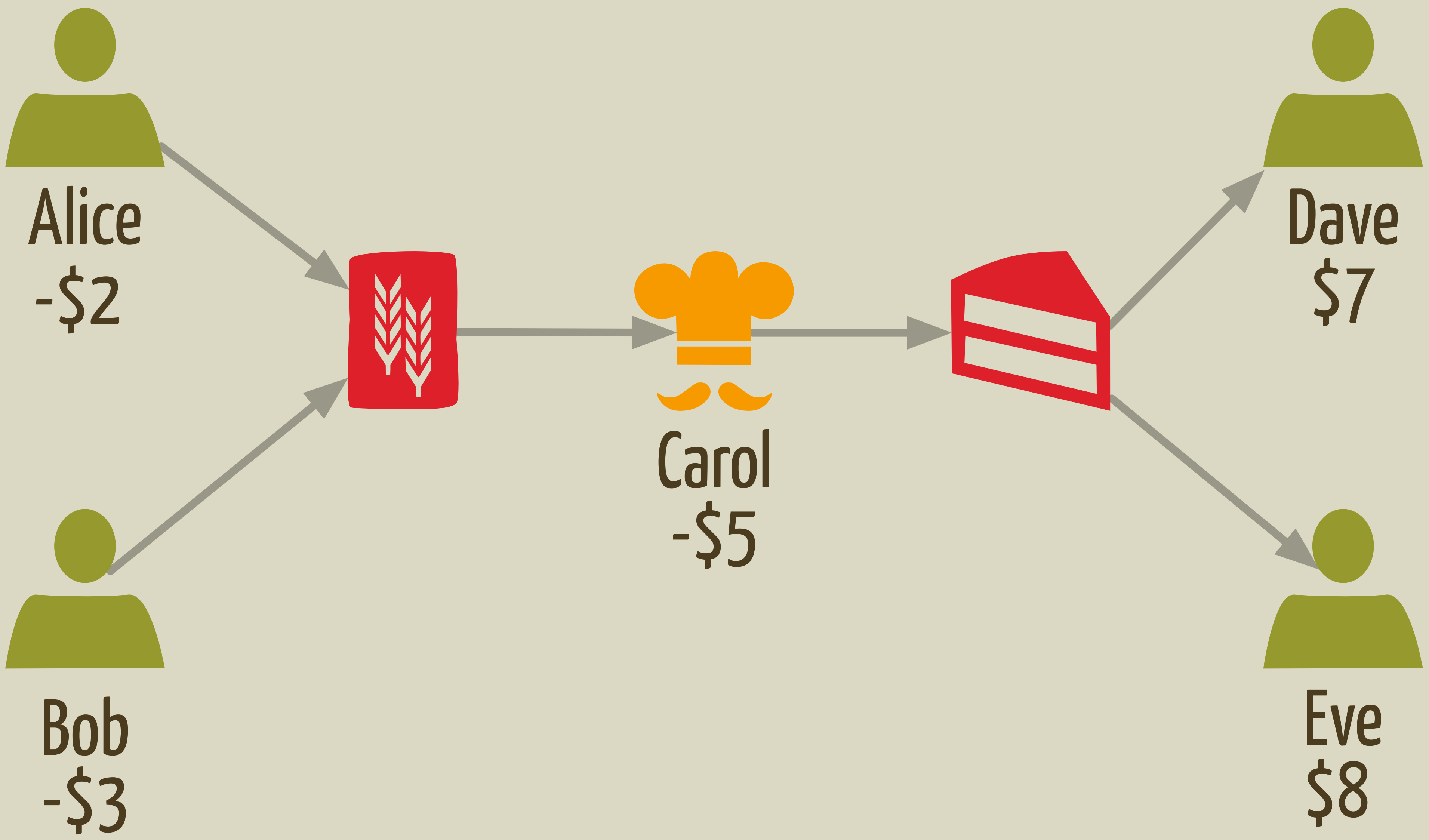
RB-LBP Our Approach

map problem into a binary factor graph containing binary variables and logical constraints.

optimized max-sum implementation.

apply max-sum over the factor graph to obtain a solution.

Mapping into a Binary Factor Graph



Binary Factor Graph Mapping

Alice

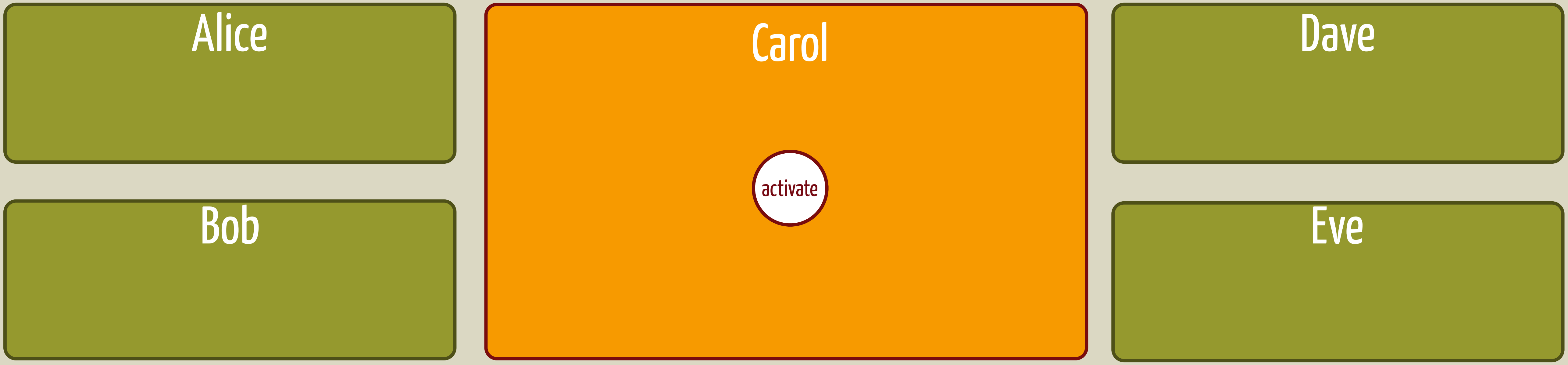
Bob

Carol

Dave

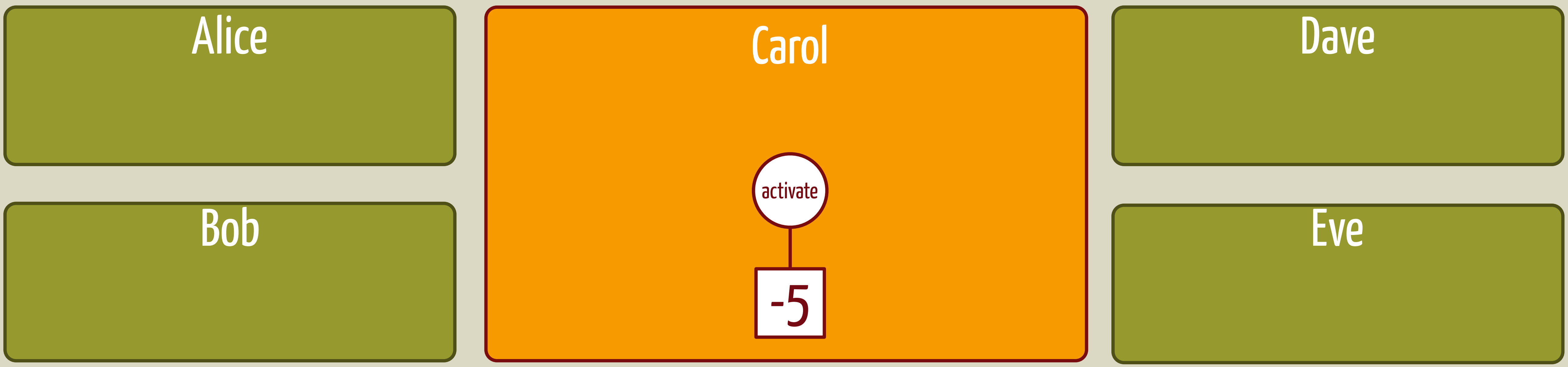
Eve

Binary Factor Graph Mapping

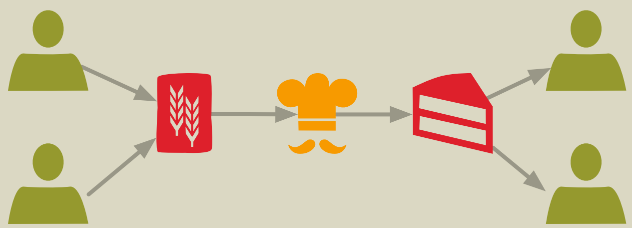


Activation variable encodes Carol's decision to be active.

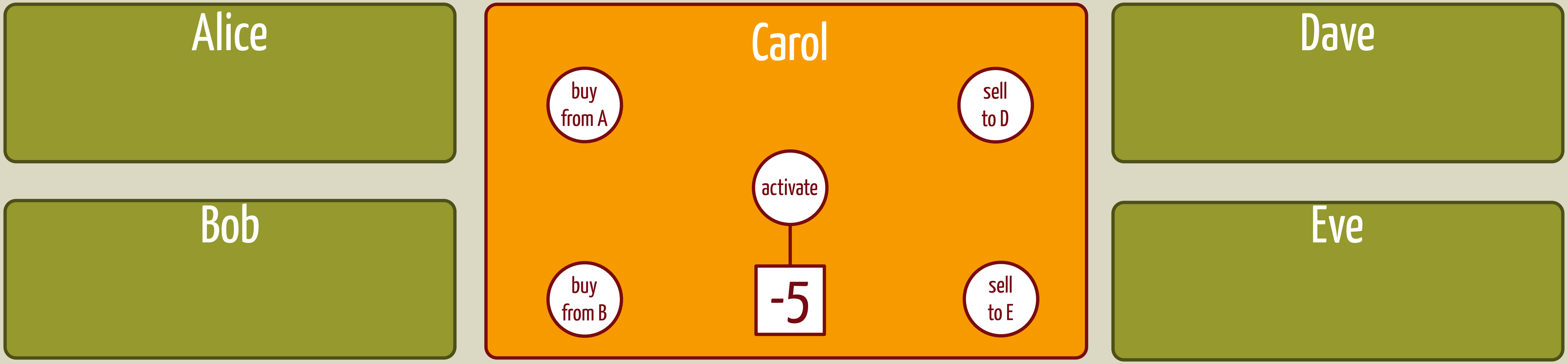
BINARY FACTOR GRAPH MAPPING



Activation factor encodes Carol's activation cost.

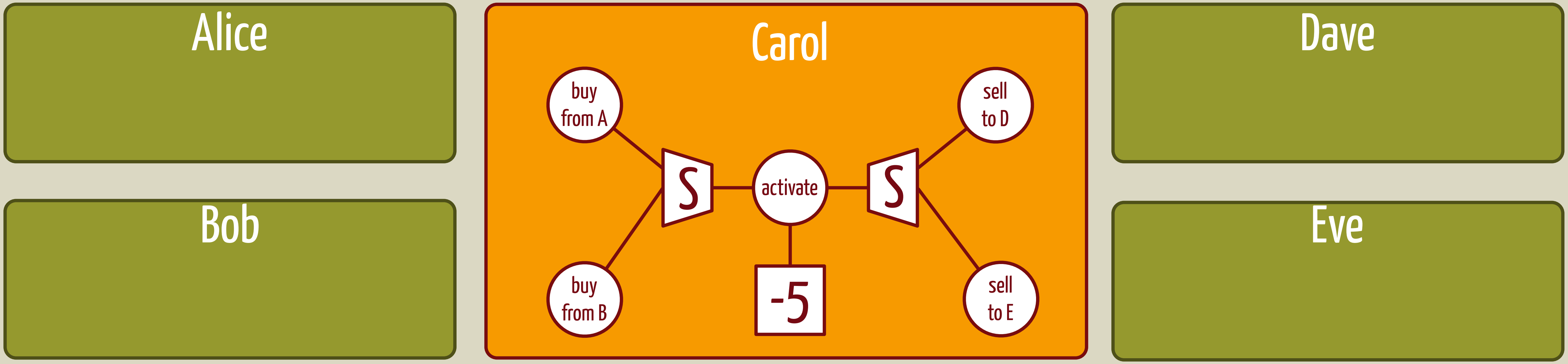


Binary Factor Graph Mapping



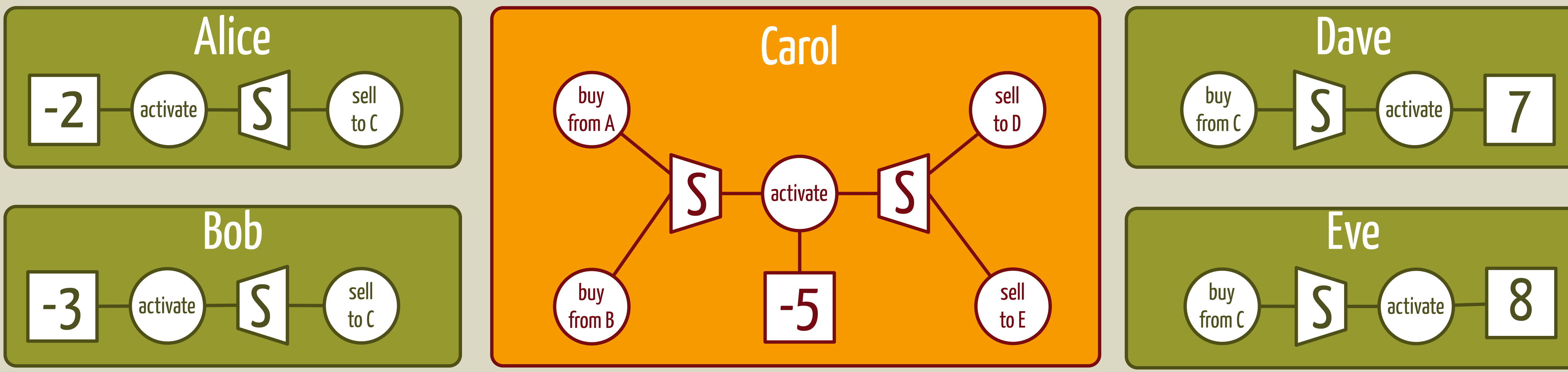
Option variables encode Carol's decision to trade each of her goods with each of her potential partners.

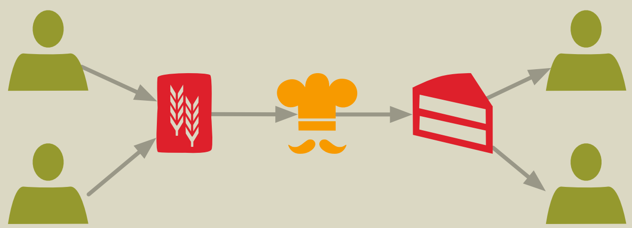
Binary Factor Graph Mapping



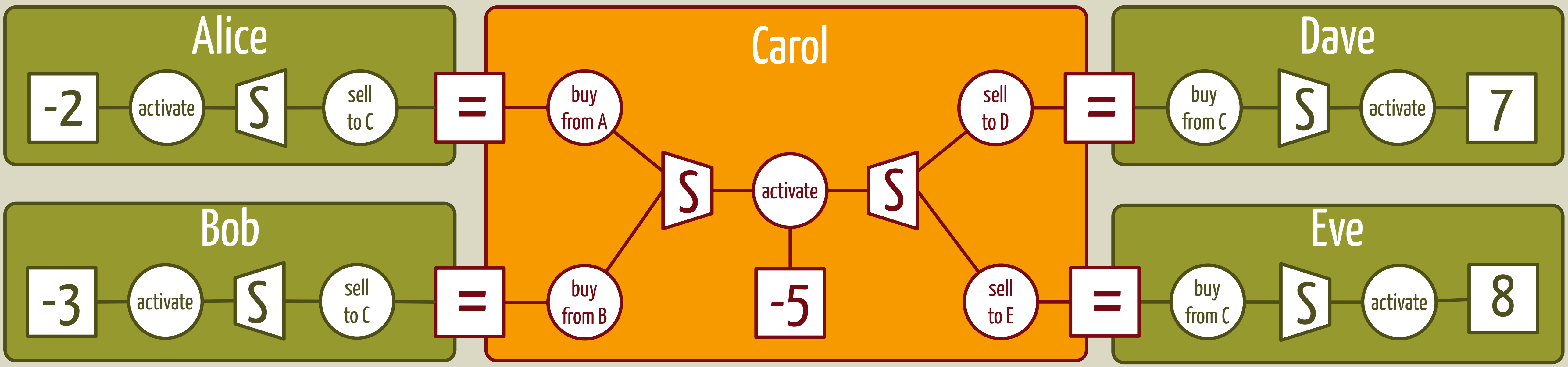
Selection factors guarantee that only one of the providers is selected for each good.

Binary Factor Graph Mapping





Binary Factor Graph Mapping



Equality factors guarantee that Carol takes coherent decisions with her neighbors'.

Factors as logical constraints

There is no need to store factors in memory.

Single-valued messages

Contain agents' willingness to collaborate with each other.

Simplified message calculation

	LBP	RB-LBP
memory per agent	$O(G \cdot A^{2G+1})$	$O(G \cdot A)$
bandwidth per agent	$O(G \cdot A^{G+1})$	$O(G \cdot A)$
computation time per agent	$O(G \cdot A^{2G+1})$	$O(G \cdot A^2)$

Experimental Analysis

Experimental Setup

Small networks [Walsh2003].

Large networks [Vinyals2008].

Measure memory, communication, time, and quality.

small networks

<20 goods

<40 agents

Small Networks

up to 13 times less memory

up to 5 times less bandwidth

same solution quality

negligible time differences

large networks

50 goods

40-500 agents

Large Networks

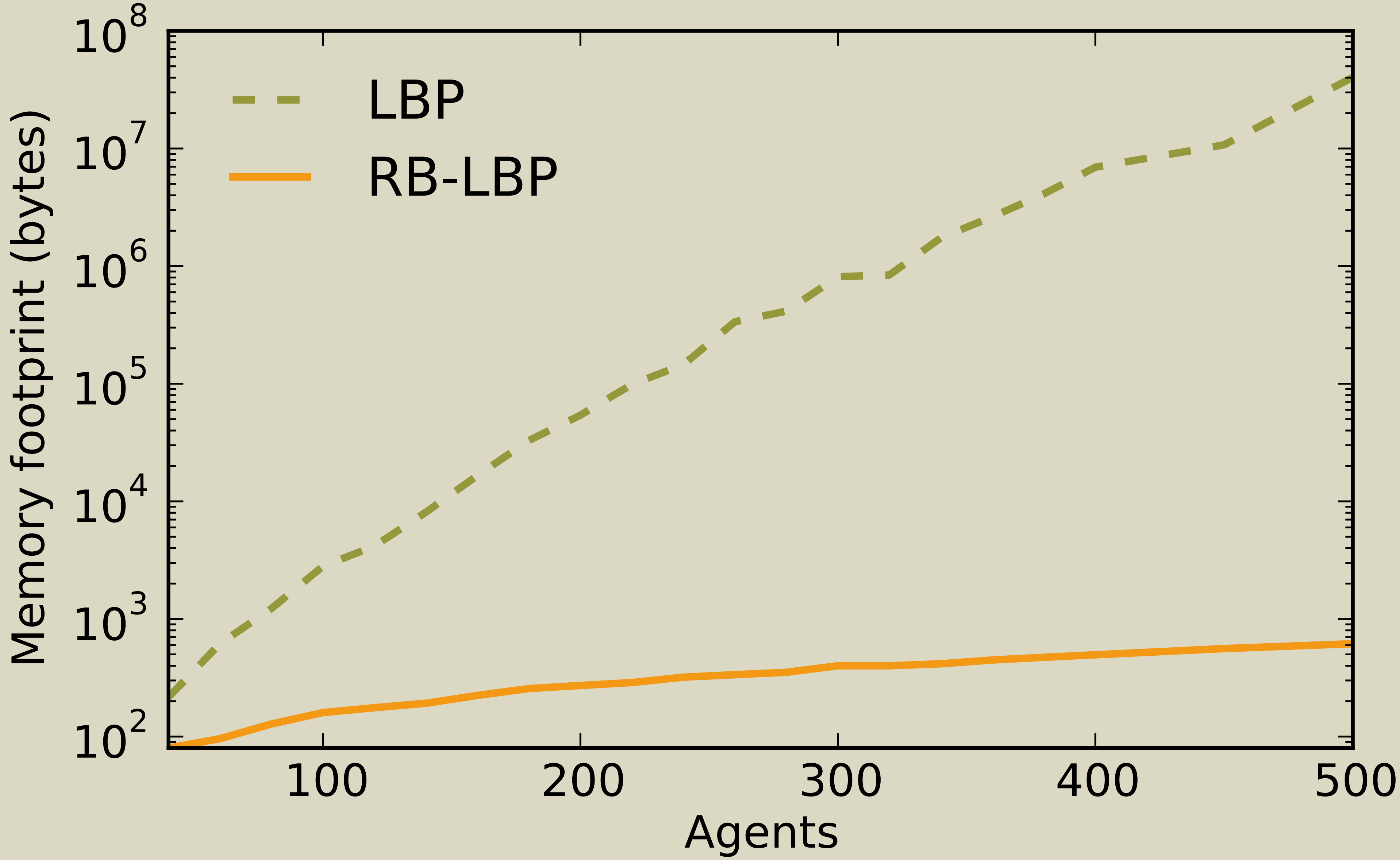
up to 10^5 times less memory

up to 787 times less bandwidth

up to 20 times faster

up to twice better solutions

Large Networks



up to 10^5 times less memory

FINAL RECAP

Encoding in a novel graphical model.

Optimized implementation of max-sum.

Reduced memory, communication and computation requirements.

Higher quality solutions.

Coming soon ...

Coming soon …

… Better valued solutions

Thank You

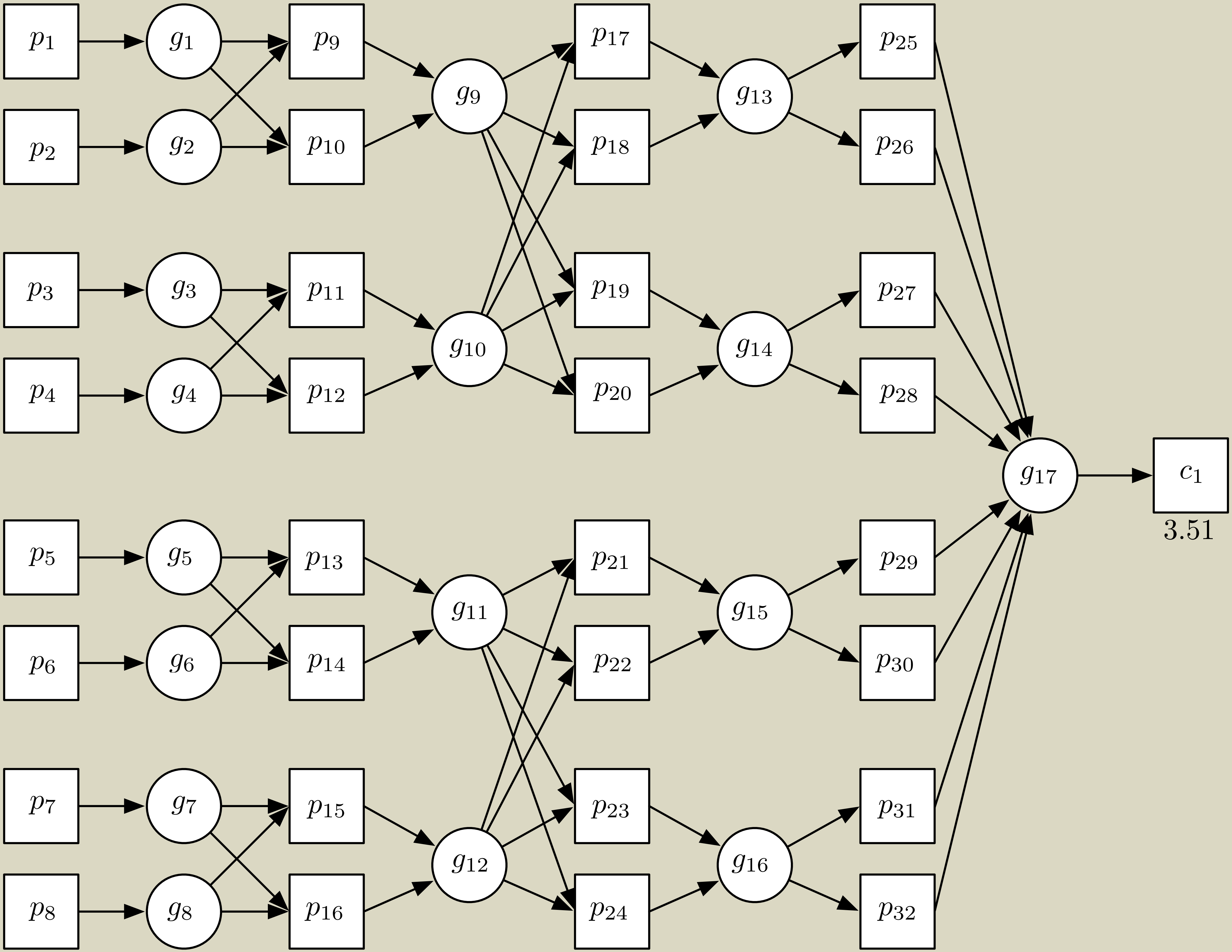
Questions?

[Winsper2003] M. Winsper and M. Chli, Decentralised Supply Chain Formation: A Belief Propagation-based Approach, Agent-Mediated Electronic Commerce, 2010.

[Walsh2003] W. E. Walsh and M. P. Wellman, Decentralized Supply Chain Formation : A Market Protocol and Competitive Equilibrium Analysis, Journal of Artificial intelligence Research (JAIR), vol. 19, pp. 513-567, 2003.

[Vinyals2008] M. Vinyals, A. Giovannucci, J. Cerquides, P. Meseguer, and J. A. Rodriguez-Aguilar, A test suite for the evaluation of mixed multi-unit combinatorial auctions, Journal of Algorithms, vol. 63, no. 1-3, pp. 130-150, 2008.

Walsh's networks



p_1
p_2
p_3
p_4
p_5
p_6
p_7
p_8
g_1
g_2
g_3
g_4
g_5
g_6
g_7
g_8
p_9
p_{10}
p_{11}
p_{12}
p_{13}
p_{14}
p_{15}
p_{16}
g_9
g_{10}
g_{11}
g_{12}
p_{17}
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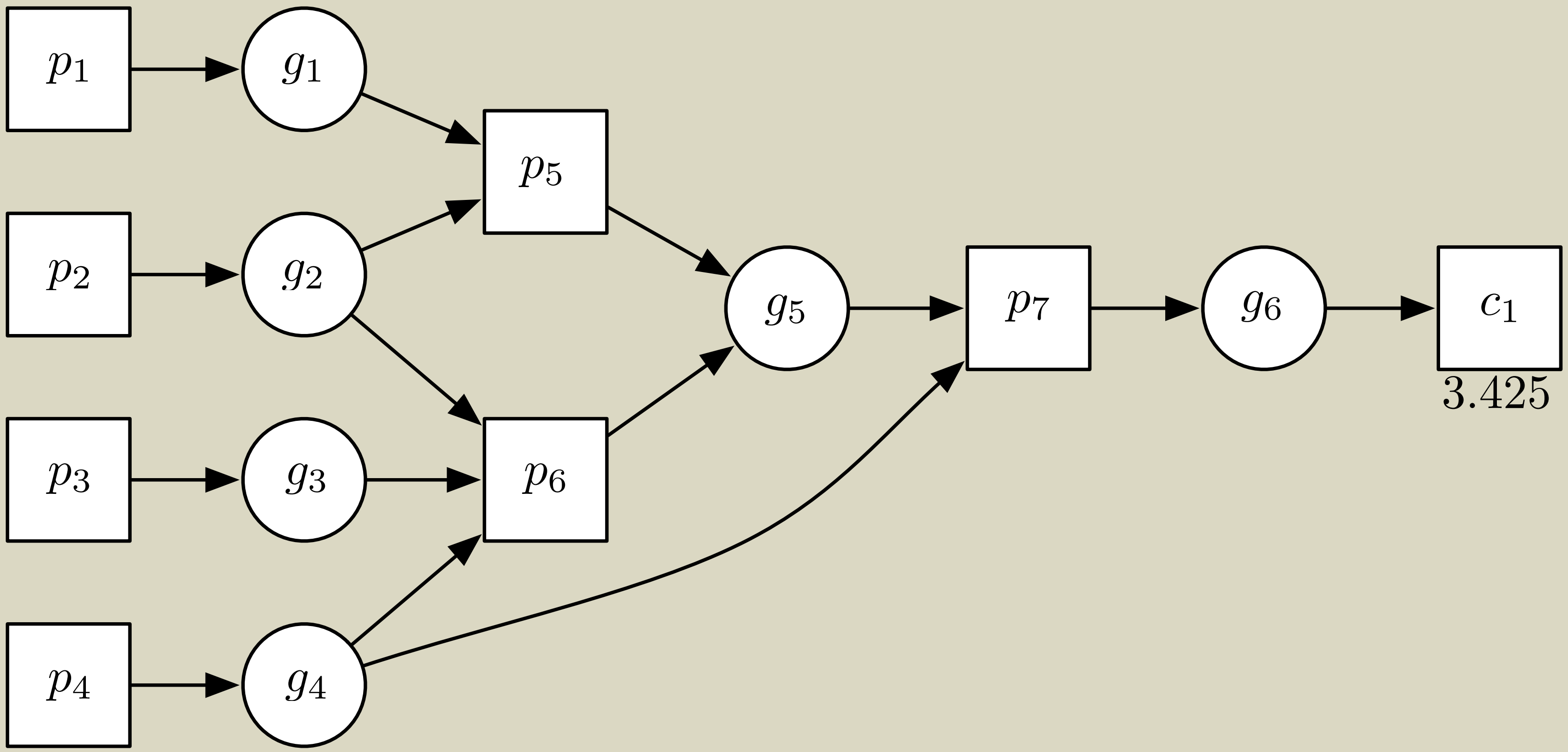
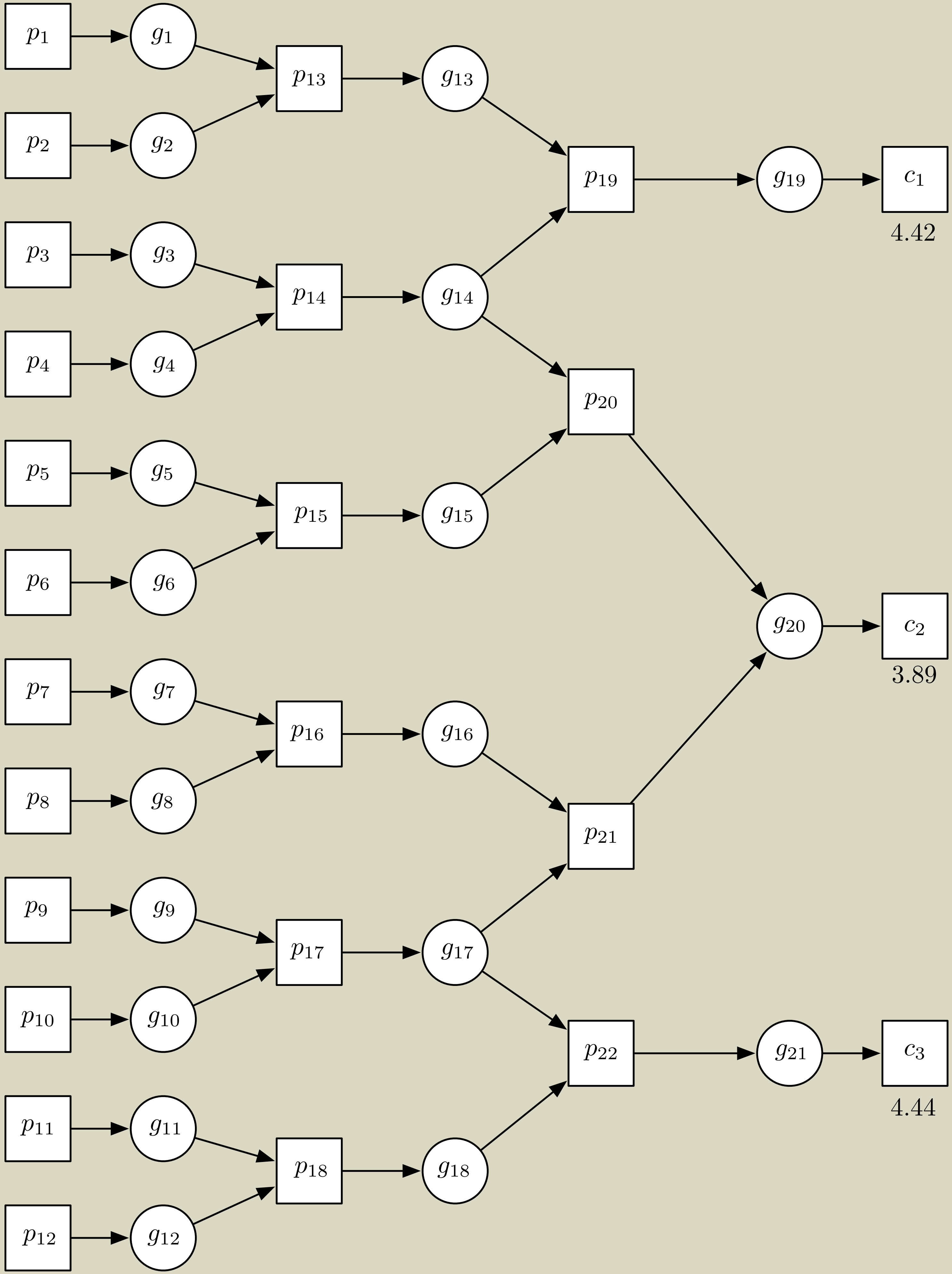
p_{22}
p_{23}
p_{24}
g_{13}
g_{14}
g_{15}
g_{16}
p_{25}
p_{26}
p_{27}
p_{28}
p_{29}
p_{30}
p_{31}
p_{32}
g_{17}
c_1
3.51



p_1
p_2
p_3
p_4
g_1
g_2
g_3
g_4
p_5
p_6
g_5
p_7
g_6
c_1
3.425


p_1
p_2
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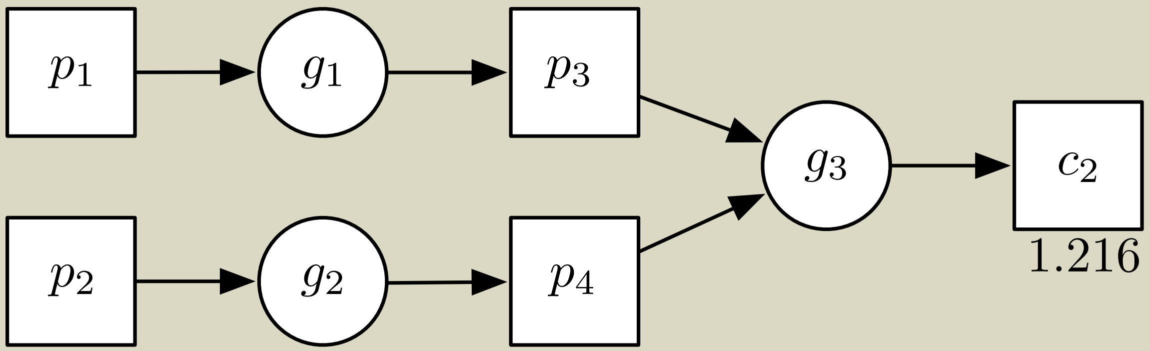
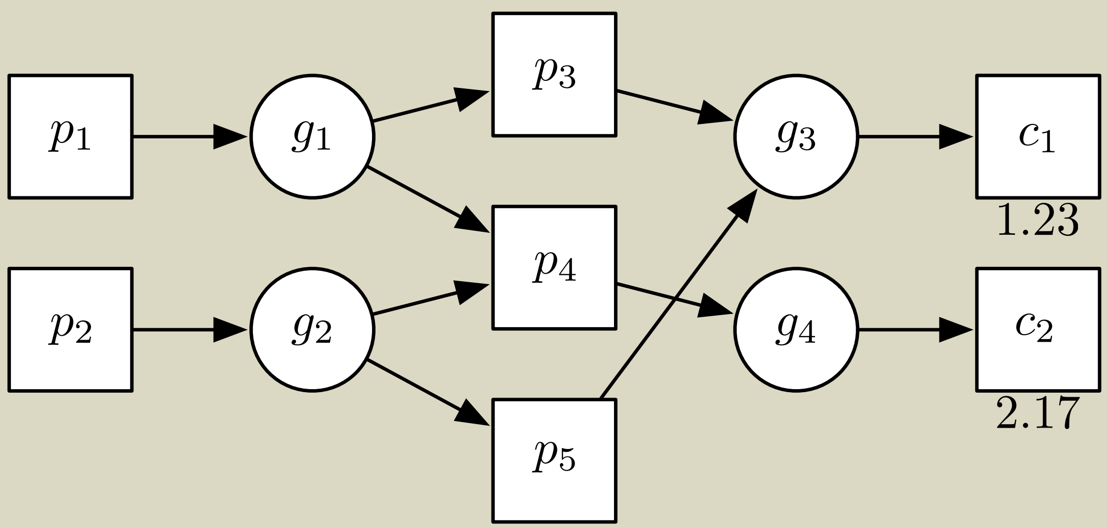
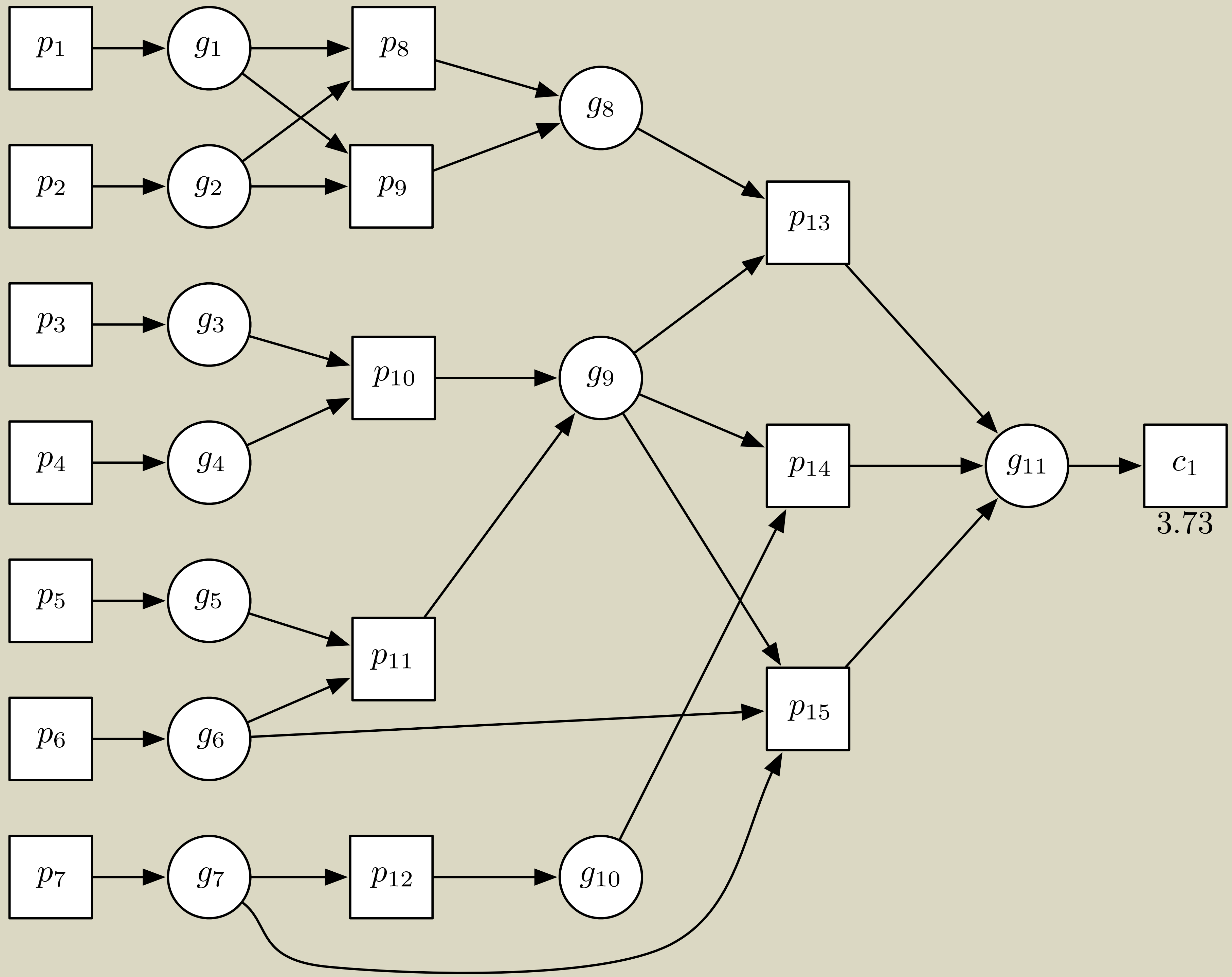
p_4
p_5
p_6
p_7
p_8
p_9
p_{10}
p_{11}
p_{12}
g_1
g_2
g_3
g_4
g_5
g_6
g_7
g_8
g_9
g_{10}
g_{11}
g_{12}
p_{13}
p_{14}
p_{15}
p_{16}
p_{17}
p_{18}
g_{13}
g_{14}
g_{15}
g_{16}
g_{17}
g_{18}
p_{19}
p_{20}
p_{21}
p_{22}
g_{19}
g_{20}
g_{21}
c_1
4.42
c_2
3.89
c_3
4.44


p_1
g_1
p_3
g_3
c_2
1.216
p_2
g_2
p_4

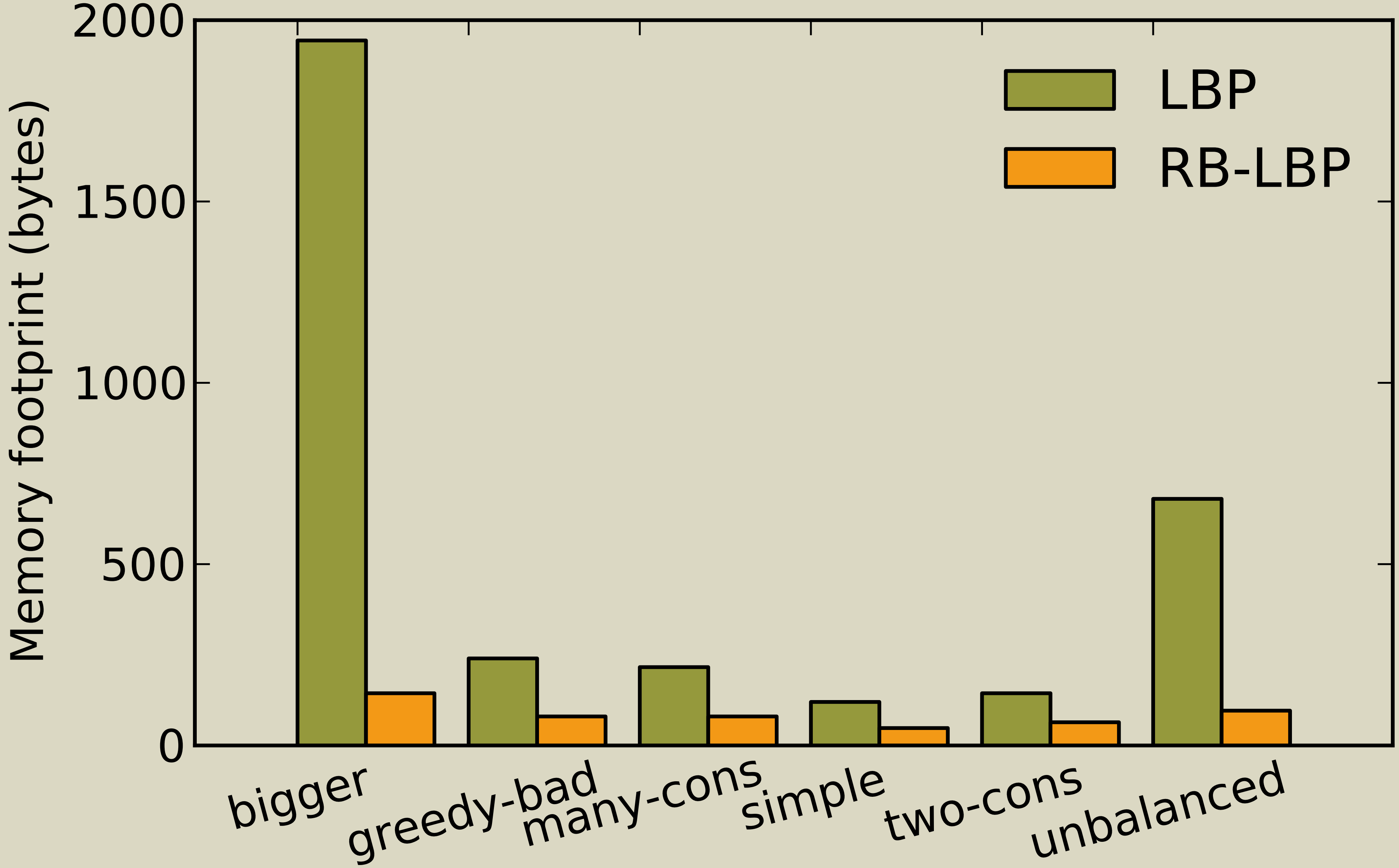


p_1
p_2
g_1
g_2
p_3
p_4
p_5
g_3
g_4
c_1
1.23
c_2
2.17

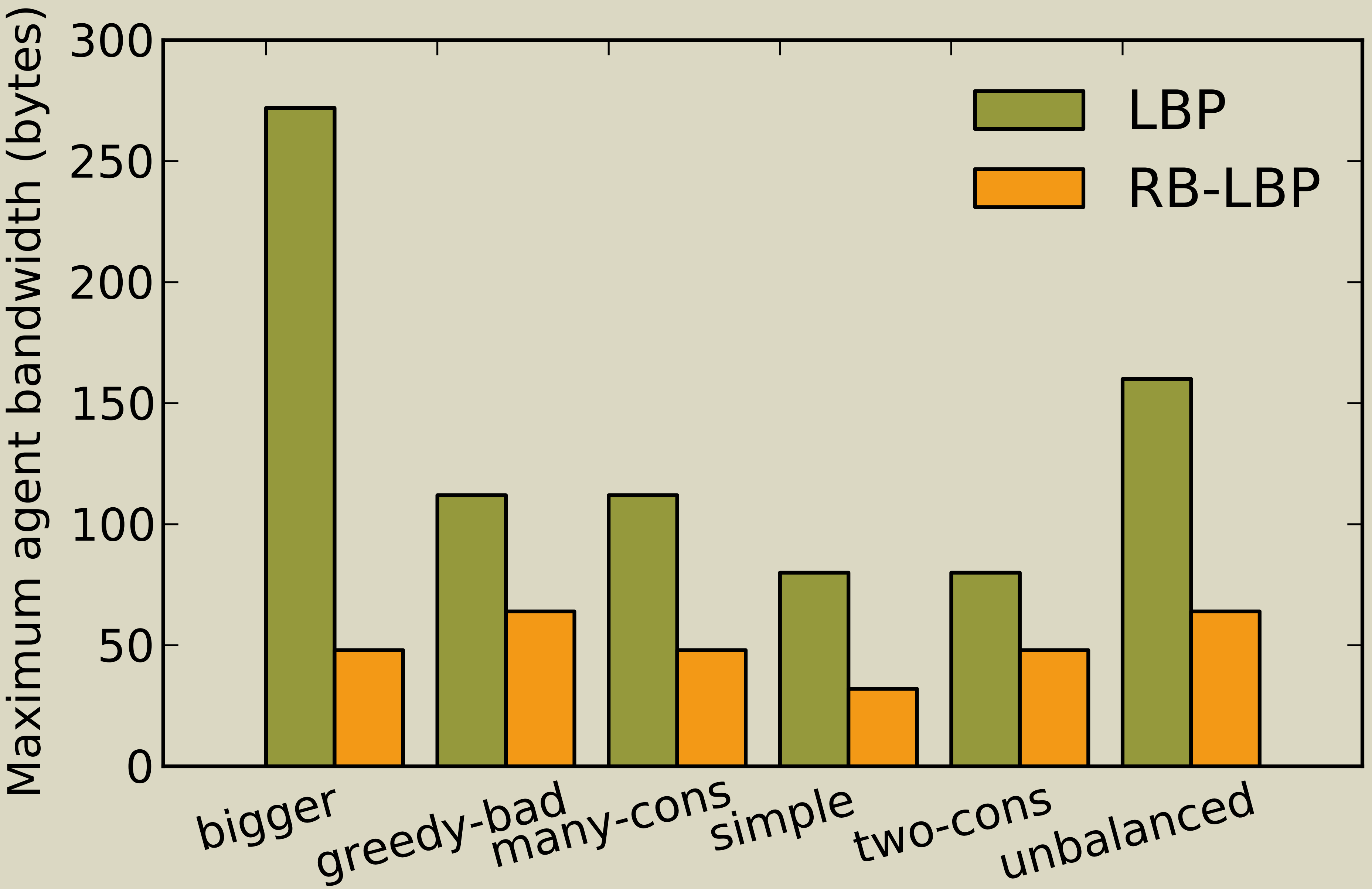


p_1
p_2
p_3
p_4
p_5
p_6
p_7
g_1
g_2
g_3
g_4
g_5
g_6
g_7
p_8
p_9
p_{10}
p_{11}
p_{12}
g_8
g_9
g_{10}
p_{13}
p_{14}
p_{15}
g_{11}
c_1
3.73

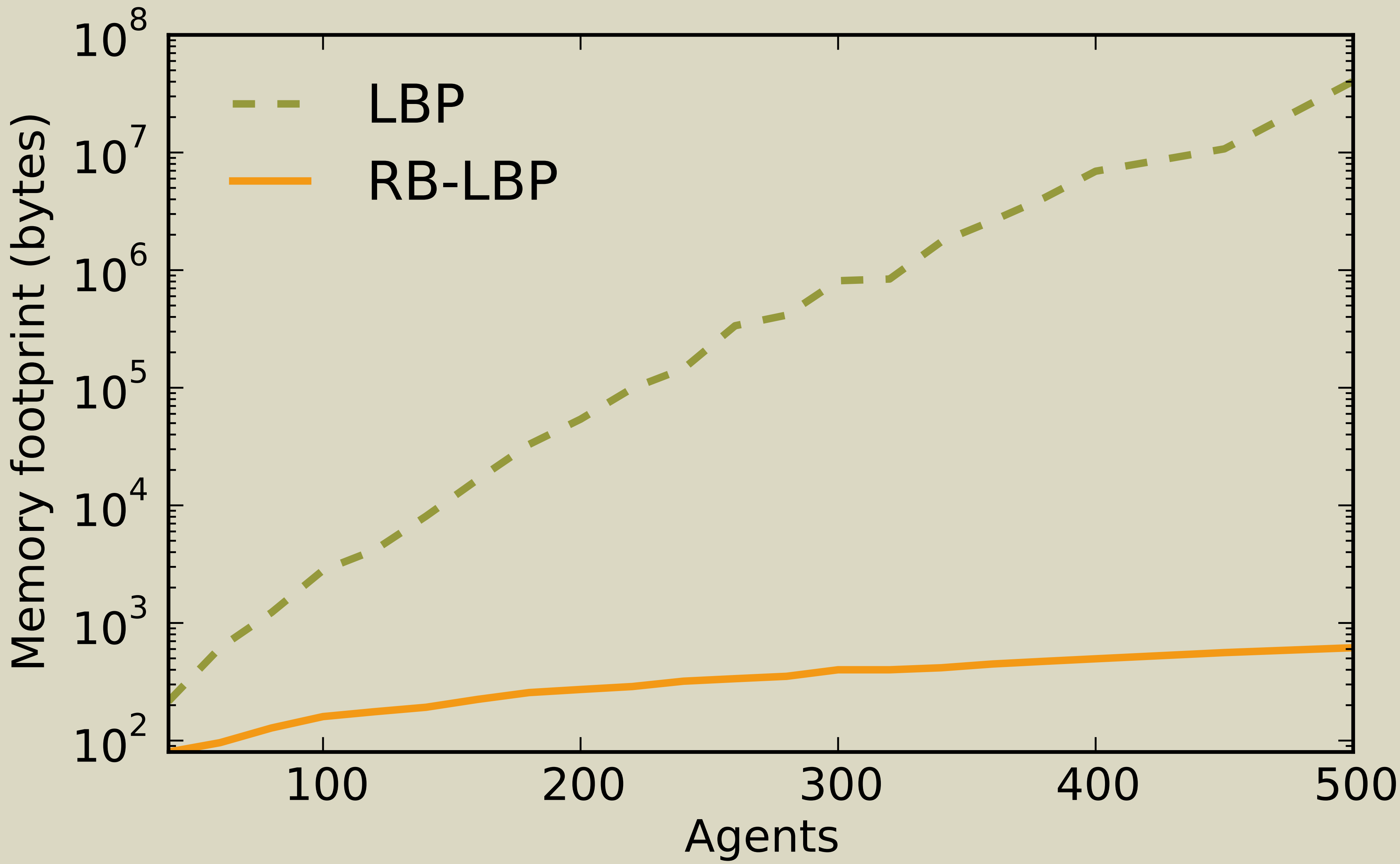
Plots



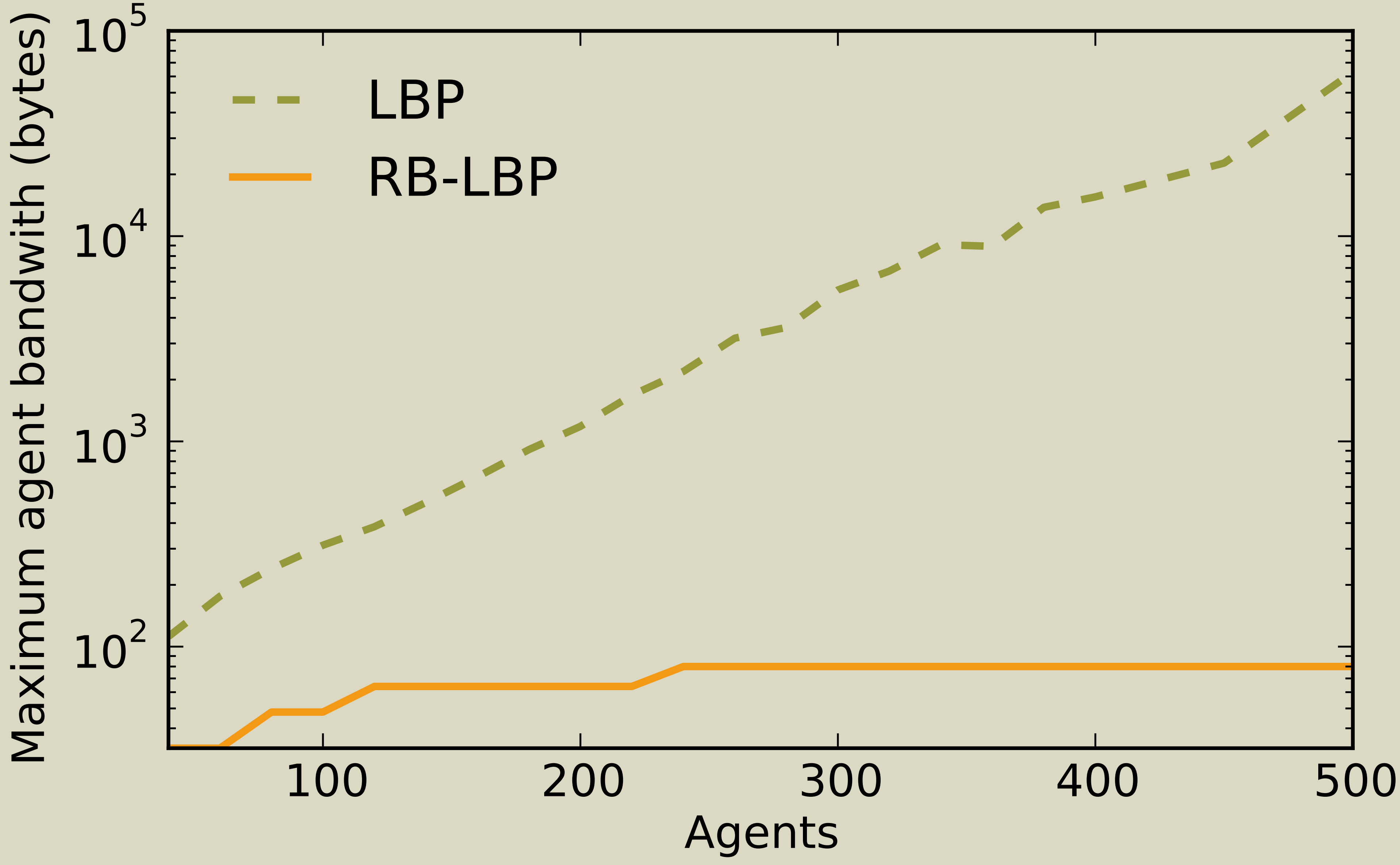
up to 13 times less memory



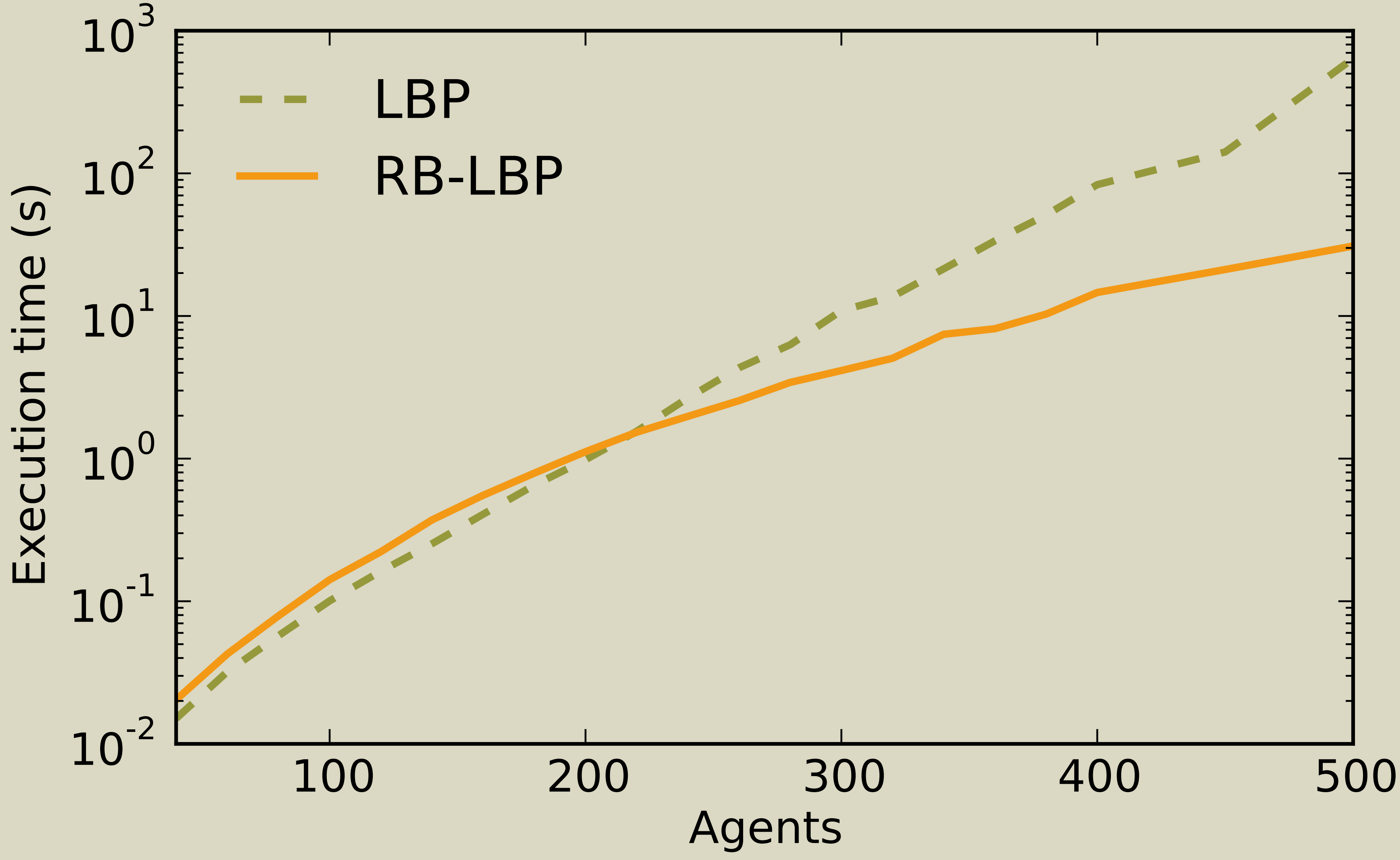
up to 5 times less bandwidth



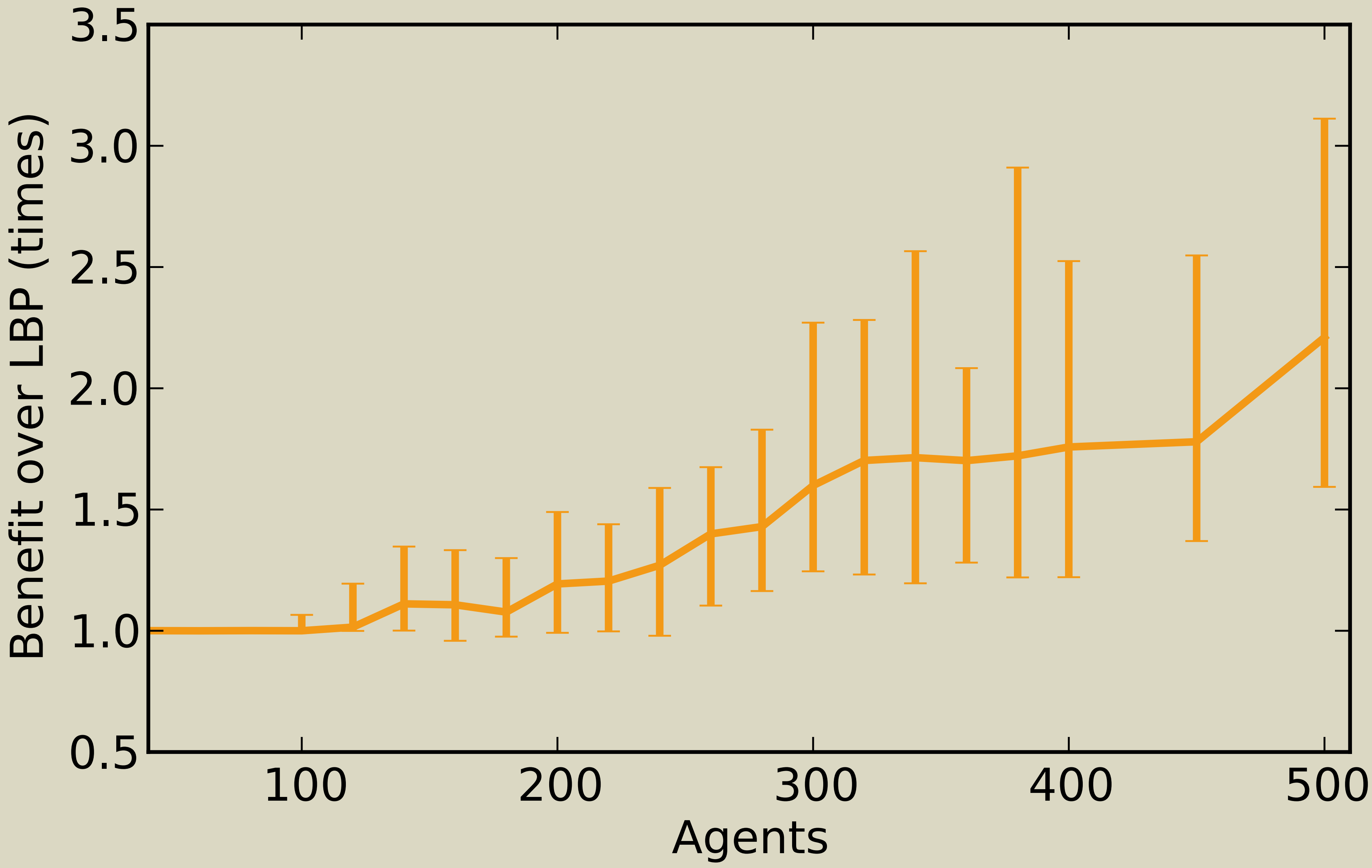
up to 10^5 times less memory



up to 787 times less bandwidth



up to 20 times faster



up to twice as good