„STEPS 1-6 FROM UNCERTAINTY BUDGET" ON RADIOMETRIC CALIBRATION EXAMPLE

LAURA MIHAI

WG4&WG1 - Training School "Traceability Chains, Uncertainty Propagation and Calibration/Validation"

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General terminology

Radiometry deals with the detection and measurement of electromagnetic energy across the entire spectral range.

The spectrometer output depends mainly on the **radiant energy** collected by its detector.

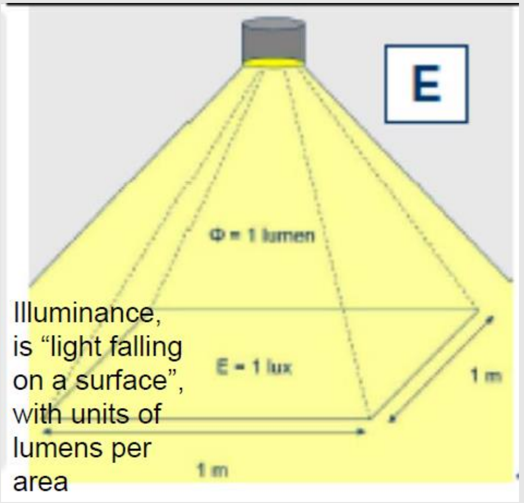
Radiant Energy is the is energy that travels by electromagnetic waves (having units[Joules]).

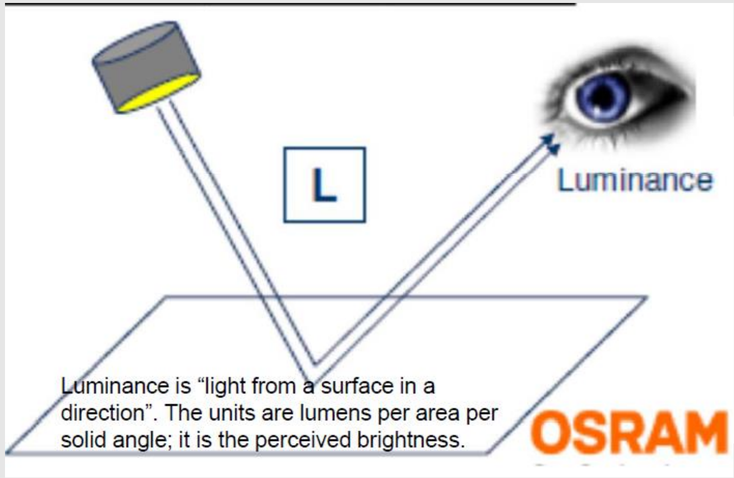
Radiant Flux Φ(units of [Watts or J/s]). is expressed as the total optical power of a light source.

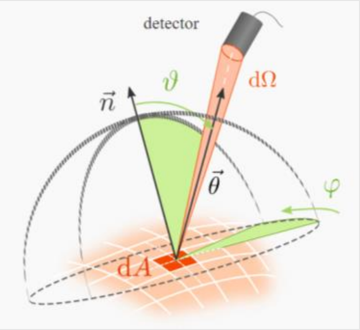
Radiant Intensity (**$I = d\Phi / d\Omega$**) is defined as the quantity of emitted flux through a known solid angle.

$E = \Phi / A$, **$E = E_0 \cos\theta$** units of $[W/m^2]$

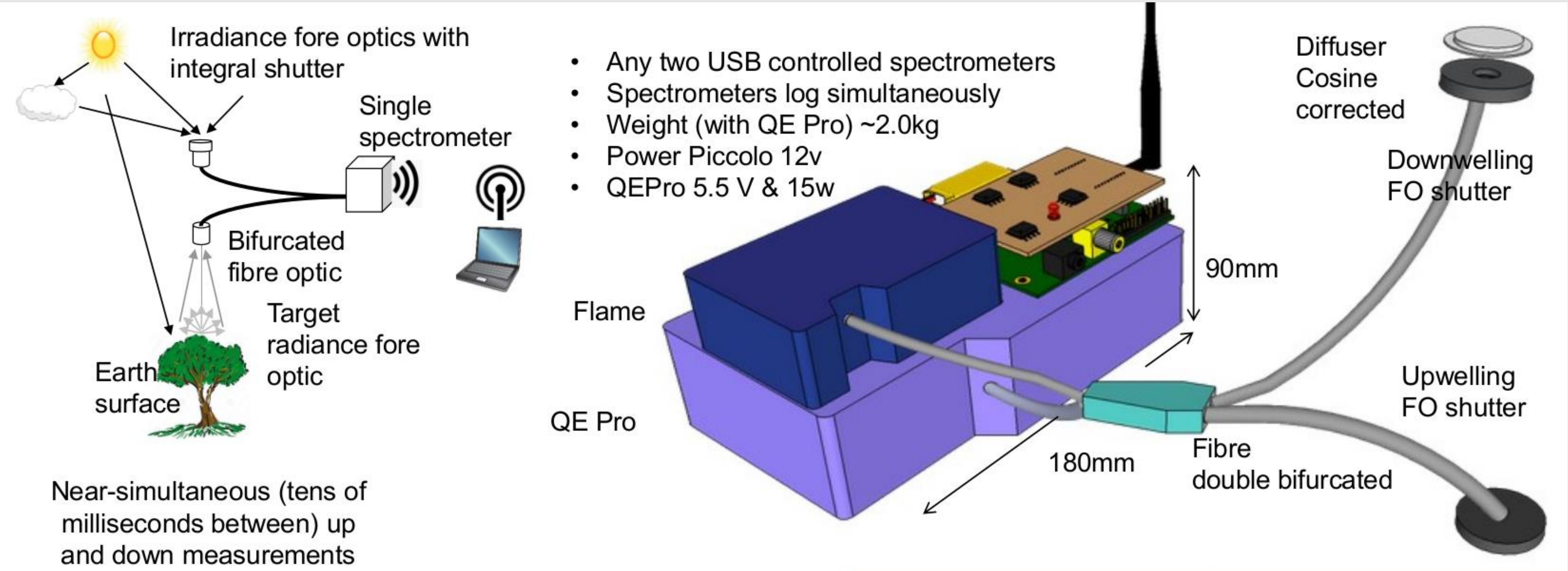
$L = dI / dA\cos\theta$, units $[W/m^2 * sr]$

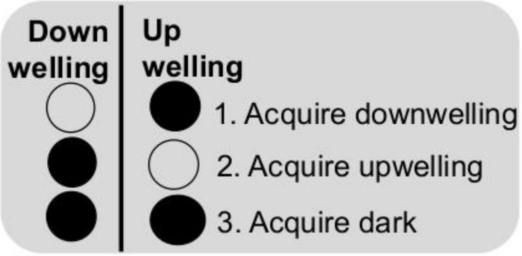






Piccolo System





Ocean Optics Flame 25 µm slit		QE Pro (cooled) 10µm slit)	
Range	~400 – 950nm	Range	~650 – 800nm
Sampling int.	~ 0.4nm	Sampling int.	~ 0.15nm
FWHM	1.3nm	FWHM	~0.36nm
Digitisation	16-bit	Digitisation	18-bit

The steps to uncertainty budget

Understanding the problem

- Step 1: Describing the traceability chain
- Step 2: Writing down the calculation equations
- Step 3: Considering the sources of uncertainty

Determining the formal relationships

- Step 4: Creating the measurement equation
- Step 5: Determining the sensitivity coefficients
- Step 6: Assigning uncertainties

Woolliams, E., 2014. Intermediate Uncertainty Analysis for Earth Observation.

Step 1: Describing the traceability chain

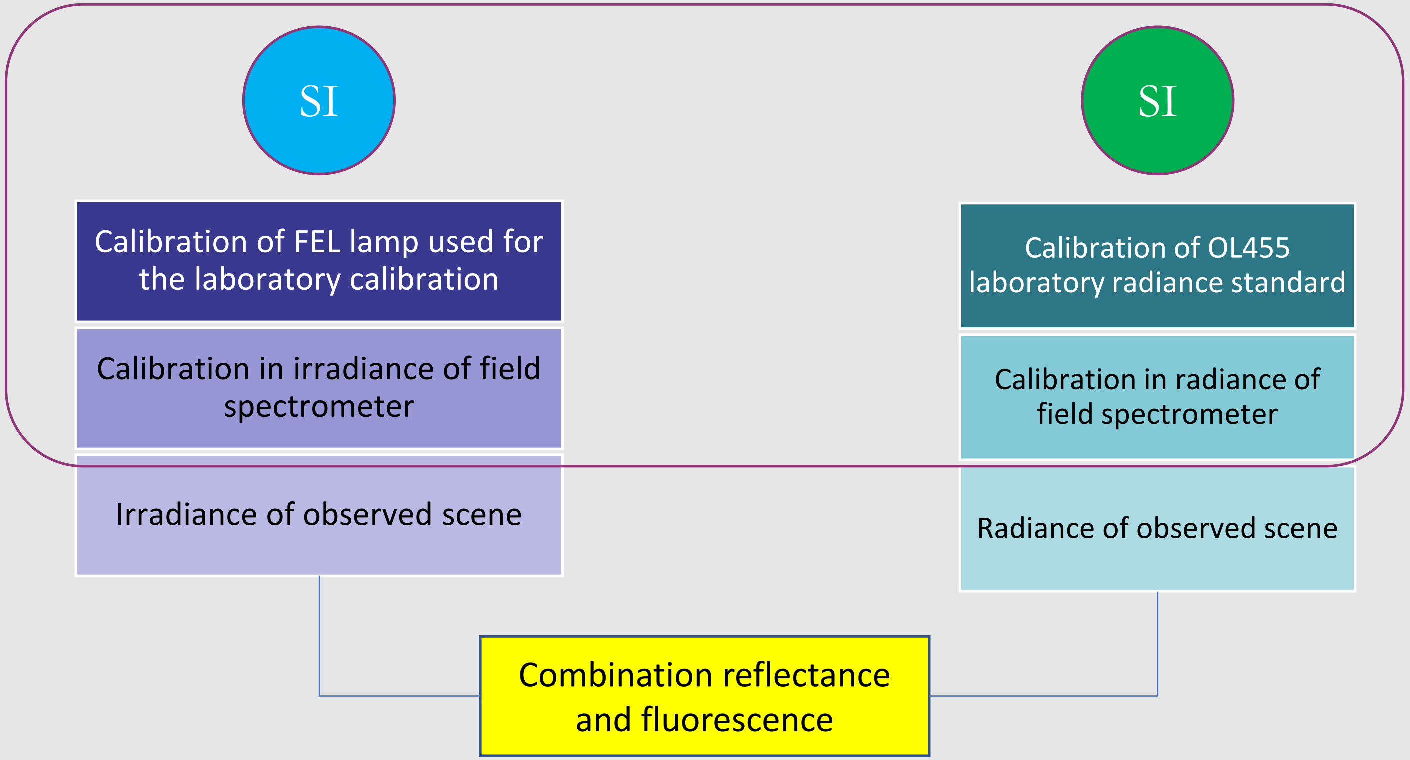
Questions to ask yourself:

- What is the traceability chain for my measurements?
- What references do I use? How were the references set up for the calibration?
- Am I relying on other secondary measurements (temperature, time, distance)?
- How are they calibrated? Are there intermediate steps – if I'm comparing two sources (lamps), what detector am I using?
- If I'm comparing two detectors, what source am I using?
- How far back do I need to go before I read the answer off a certificate?

Woolliams, E., 2014. Intermediate Uncertainty Analysis for Earth Observation.

Step 1: Describing the traceability chain

We consider calibrating a system that measures both: radiance and irradiance.



Step 2: Writing down the calculation equations

Questions to ask yourself:

- What is the equation used to calculate each box in my traceability chain?
- Is it a single step or multiple step process?
- Do I rely on other information (e.g. a distance measurement) that is not included in my traceability chain?
- At this stage – do I need to go back to step 1 and refine my traceability chain?

Woolliams, E., 2014. Intermediate Uncertainty Analysis for Earth Observation.

Step 2: Writing down the calculation equations

$$L_{scene} = \frac{S_{spectrom\_scene}}{C_{cal\_L}}$$

where $S_{spectrom\_Scene}$ is the signal, in digital numbers, recorded when field spectrometer views a given scene.

◦

$$C_{cal\_L} = \frac{S}{L}$$

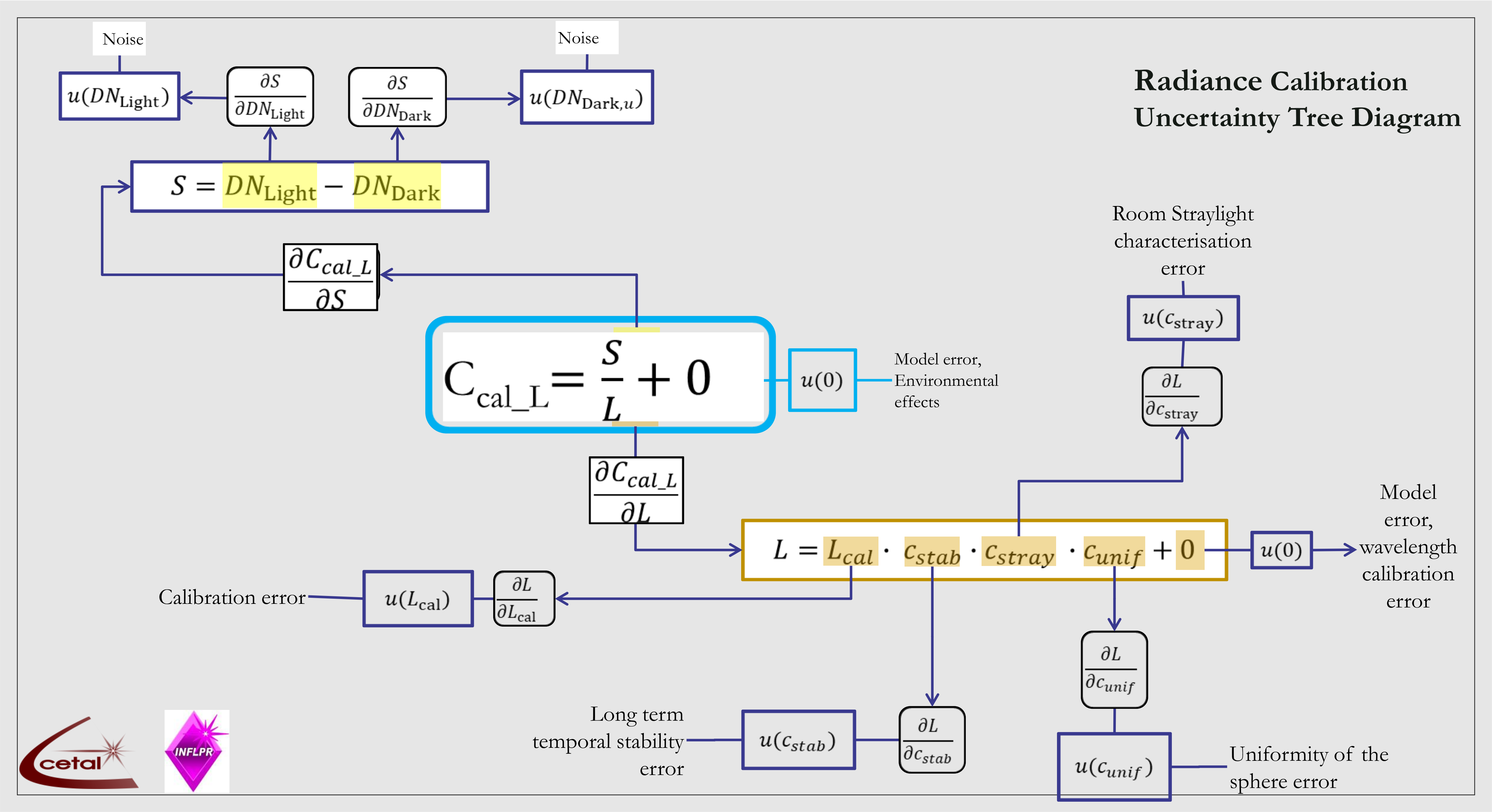
where C_{cal_L} is calibration coefficient, S is spectrometer signal in digital numbers recorded by field spectrometer when viewing the radiance standard integrating sphere OL455

($S = DN_{light} - DN_{dark}$), L is standard sphere radiance signal for one specific luminance level from calibration certificate.

Step 3: Considering the sources of uncertainty

Radiance Calibration

SOURCE OF UNCERTAINTY		OBSERVATION
spectral radiance of the sphere – L_{cal} *from certificate*	u_{Lcal}	for one luminance level - calibration certificate → has to be computed for all wavelengths
sphere calibration drift – stability	u_{cstab}	this can be estimated by the repeatability of multiple readings of the sphere alone over this time period (between two calibrations for the standard)
Room straylight	u_{ctray}	can be measured in front of the spectrometer using diffuse and non diffusing panel
sphere uniformity	u_{cunif}	generally mentioned in calibration certificate


Radiance Calibration Uncertainty Tree Diagram
Noise
$u(DN_{\mathrm{Light}})$
$\frac{\partial S}{\partial DN_{\mathrm{Light}}}$
$\frac{\partial S}{\partial DN_{\mathrm{Dark}}}$
Noise
$u(DN_{\mathrm{Dark},u})$
$S = DN_{\mathrm{Light}} - DN_{\mathrm{Dark}}$
$\frac{\partial C_{cal\_L}}{\partial S}$
$C_{cal\_L} = \frac{S}{L} + 0$
$u(0)$
Model error, Environmental effects
Room Straylight characterisation error
$u(c_{\mathrm{stray}})$
$\frac{\partial L}{\partial c_{\mathrm{stray}}}$
$\frac{\partial C_{cal\_L}}{\partial L}$
$L = L_{cal} \cdot c_{stab} \cdot c_{stray} \cdot c_{unif} + 0$
$u(0)$
Model error, wavelength calibration error
Calibration error
$u(L_{\mathrm{cal}})$
$\frac{\partial L}{\partial L_{\mathrm{cal}}}$
$\frac{\partial L}{\partial c_{unif}}$
Long term temporal stability error
$u(c_{stab})$
$\frac{\partial L}{\partial c_{stab}}$
$u(c_{unif})$
Uniformity of the sphere error

Step 4: Creating the measurement equation

Radiance Calibration

The measurement equation is an extended version of the calculation equation that also explicitly describes the other sources of uncertainty.

$$\mathbf{C}_{spectrom} = \frac{S_{spectrom}}{L_{sphere}}\mathbf{K}_{\mathrm{i}}$$

$$= \frac{DN_{light} - DN_{dark}}{L_{sphere}}\mathbf{K}_{\mathrm{i}}$$

$$= \frac{DN_{light} - DN_{dark}}{L_{sphere}}\mathbf{K}_{\mathrm{cstab}} * \mathbf{K}_{\mathrm{cstray}} * \mathbf{K}_{\mathrm{cunif}} * \mathbf{K}_{\mathrm{liniarity}} \ldots\ldots$$

K terms relate to the different effects

Step 5: Determining the sensitivity coefficients

The base eq. : $$f(S, L) = C_{cal\_L} = \frac{S}{L} \left(= \frac{DN_{light} - DN_{dark}}{L} = \frac{DN_{light}}{L} - \frac{DN_{dark}}{L}\right)$$

Sensitivity coefficient: $$\mathbf{c} = \frac{\partial C_{cal\_L}}{\partial S} + \frac{\partial C_{cal\_L}}{\partial L}$$

From the sensitivity coefficients we determine the uncertainty associated with the measurand (the answer calculated from the measurement equation) due to each component in turn. We will consider that all the components are independent of each other:

$$u^2(C_{cal_L}) = \left(\frac{\partial C_{cal_L}}{\partial S}\right)^2 u^2(S) + \left(\frac{\partial C_{cal_L}}{\partial L}\right)^2 u^2(L)$$

$$c_1 = \frac{\partial C_{cal_L}}{\partial S} = \frac{1}{L} = \frac{C_{cal\_L}}{S};$$

$$c_2 = \frac{\partial C_{cal_L}}{\partial L} = -\frac{S}{L^2} = -\frac{C_{cal_L} * L}{L^2} = -\frac{C_{cal\_L}}{L}$$

Step 5: Determining the sensitivity coefficients

- Then the eq. $$u^2(C_{cal_L}) = \left(\frac{\partial C_{cal_L}}{\partial S}\right)^2 u^2(S) + \left(\frac{\partial C_{cal_L}}{\partial L}\right)^2 u^2(L)$$

- Becomes: $$u^2(C_{cal_L}) = (\frac{C_{cal\_L}}{S})^2 u^2(S) + (-\frac{C_{cal_L}}{L})^2 u^2(L)$$

- The relative uncertainty: $\frac{u^2(C_{cal_L})}{C^2_{cal\_L}} = \frac{u^2(S)}{S^2} + \frac{u^2(L)}{L^2} \rightarrow u^2_{rel}(C_{cal_L}) = u^2_{rel}(S) + u^2_{rel}(L)$

But, for subtractions it is required to have absolute calculation of u:

$$u^2_{abs}(S) = (\frac{stdev(DN_{light})}{\sqrt{N}})^2 + (\frac{stdev(DN_{dark})}{\sqrt{N}})^2$$

$$u^2_{rel}(S) = \frac{u^2_{abs}(S)}{S^2} = \frac{u^2_{abs}(S)}{(DN_{light} - DN_{dark})^2}$$

$$u^2_{rel}(C_{cal_L}) = \frac{(\frac{stdev(DN_{light})}{\sqrt{N}})^2 + (\frac{stdev(DN_{dark})}{\sqrt{N}})^2}{(DN_{light} - DN_{dark})^2} + u^2_{rel}(L) + \ldots$$

To these eq. has to be added all the other sources of uncertainties: stability, linearity…

Step 6: Assigning uncertainties

Source of uncertainty	Uncertainty value		Probability distribution	Divisor	Relative Standard uncertainty	Sensitivity coefficient	Contribution to the combined standard uncertainty	U type
	Absolute	Relative			u(xi)/xi	ci	ciu(xi)/xi	
spectral radiance of the sphere		2.00%	normal	2	1.00%	1.00	1.0%	B
sphere calibration drift		0.50%	normal	2	0.25%	1.00	0.3%	B
straylight sphere			rectangular	1.73	0.00%	1.00	0.0%	B
sphere uniformity		1.00%	rectangular	1.73	0.58%	1.00	0.6%	B
sphere short term stability		0.50%	normal	2	0.25%	1.00	0.3%	A
spectrometer reproducibility - reposition			normal	1	0.00%	1.00	0.0%	A
spectrometer reproducibility - transportation			normal	1	0.00%	1.00		A
spectrometer reproducibility - ON/OFF			normal	1	0.00%	1.00		A
Spectrometer noise			normal	1	0.00%	1.00		A
Spectrometer dark			normal	1	0.00%	-1.00		A
spectrometer linearity			normal	1	0.00%	1.0%	1.0%	A
….								
						Combined standard uncertainty, uc(y)	0	
						Expanded uncertainty, U = kuc(y)	0	

Step 2: Writing down the calculation equations

$$C_{cal\_E} = \frac{S}{E_{FEL}}$$

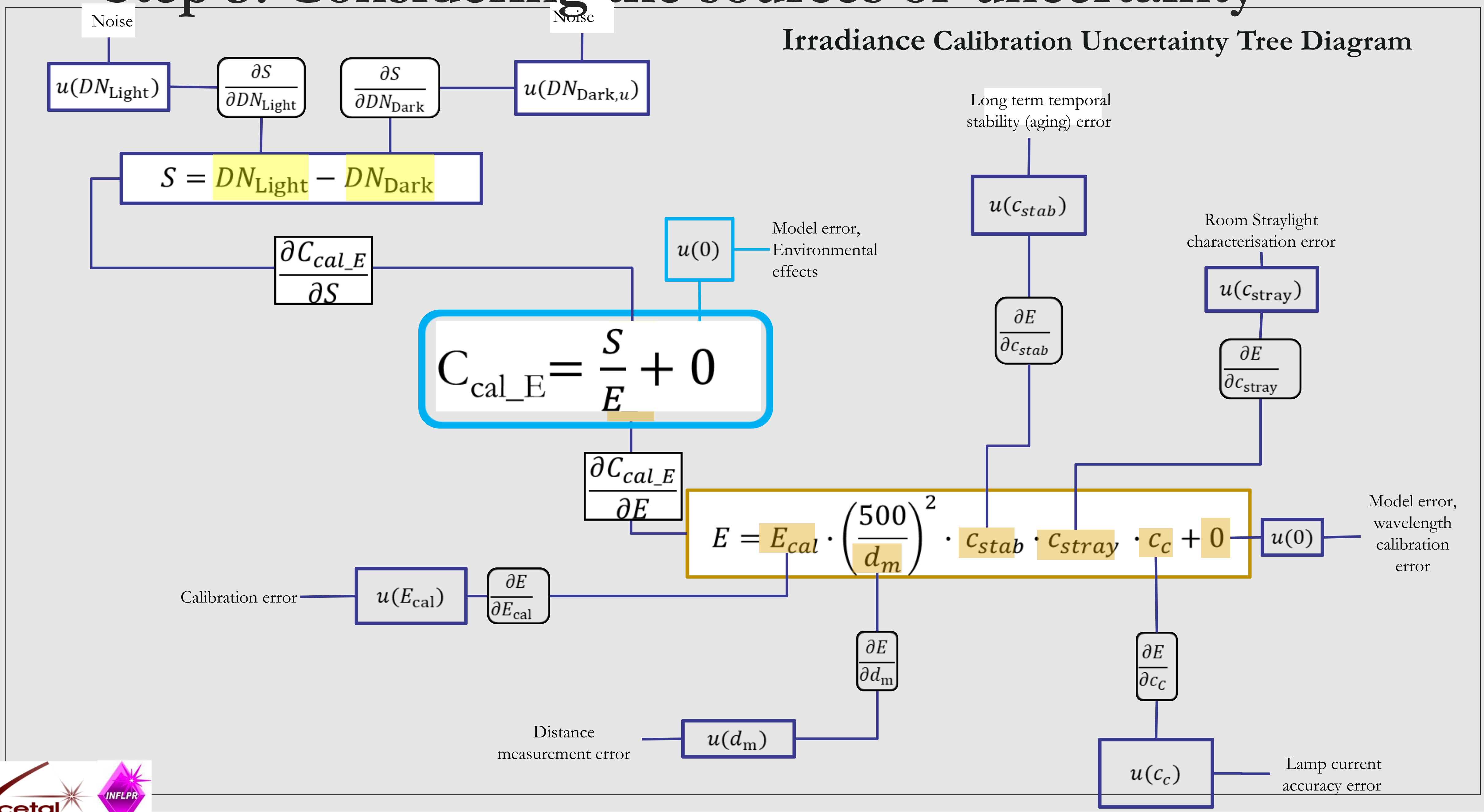
where C_{cal_E} is calibration coefficient, S is spectrometer signal: $S = DN_{light} - DN_{dark}$, E is standard irradiance signal for one specific current applied to the FEL lamp, corresponding to calibration certificate.

Step 3: Considering the sources of uncertainty

Irradiance Calibration

SOURCE OF UNCERTAINTY		OBSERVATION
spectral irradiance of the FEL – E_{cal} *from certificate*	u_{Ecal}	for one current calibration certificate → has to be computed for all wavelengths
FEL calibration stability - aging	u_{cstab}	this can be estimated by the repeatability of multiple readings of the FEL in time
Room straylight	u_{ctray}	can be measured in front of the spectrometer using diffuse and non diffusing panel
sphere uniformity	u_{cunif}	generally mentioned in calibration certificate

Step 3: Considering the sources of uncertainty



Step 4: Creating the measurement equation

The measurement equation is an extended version of the calculation equation that also explicitly describes the other sources of uncertainty.

$$C_{cal\_E} = \frac{S}{E} * K_i = \frac{S}{E_{cal} * \frac{d_{cal}}{d_{test}}} * \mathrm{K_i}$$

$$= \frac{DN_{light} - DN_{dark}}{E_{cal}} \frac{d_{test}}{d_{cal}} \mathrm{K_i}$$

$$= \frac{DN_{light} - DN_{dark}}{E_{cal}} * \frac{d_{test}}{d_{cal}} * \mathrm{K_{cstab}} * \mathrm{K_{cstray}} * \mathrm{K_{c\_curr}} \ldots\ldots$$

Step 5: Determining the sensitivity coefficients

The base eq. :

$$f(S,E) = C_{cal\_E} = \frac{S}{E}\left(= \frac{DN_{light} - DN_{dark}}{E} * \left(\frac{d_{test}}{d_{cal}}\right)^2 = \frac{DN_{light}}{E} * \left(\frac{d_{test}}{d_{cal}}\right)^2 - \frac{DN_{dark}}{E} * \left(\frac{d_{test}}{d_{cal}}\right)^2\right)$$

Sensitivity coefficient: $\mathbf{c} = \frac{\partial C_{cal\_E}}{\partial S} + \frac{\partial C_{cal\_E}}{\partial E}$

From the sensitivity coefficients we determine the uncertainty associated with the measurand (the answer calculated from the measurement equation) due to each component in turn. We will consider that all the components are independent of each other:

$$u^2(C_{cal_E}) = \left(\frac{\partial C_{cal_E}}{\partial S}\right)^2 u^2(S) + \left(\frac{\partial C_{cal_E}}{\partial E}\right)^2 u^2(E)$$

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Step 5: Determining the sensitivity coefficients

- Then the eq. $$u^2(C_{cal_E}) = \left(\frac{\partial C_{cal_E}}{\partial S}\right)^2 u^2(S) + \left(\frac{\partial C_{cal_E}}{\partial E}\right)^2 u^2(E)$$

- Becomes: $$u^2(C_{cal_E}) = \left(\frac{C_{cal\_E}}{S}\right)^2 u^2(S) + \left(-\frac{C_{cal_E}}{E}\right)^2 u^2(E)$$

- The relative uncertainty: $\frac{u^2(C_{cal_E})}{C^2_{cal\_E}} = \frac{u^2(S)}{S^2} + \frac{u^2(E)}{E^2} \rightarrow u^2_{rel}(C_{cal_E}) = u^2_{rel}(S) + u^2_{rel}(E)$

But, for subtractions it is required to have absolute calculation of u:

$$u^2_{abs}(S) = \left(\frac{stdev(DN_{light})}{\sqrt{N}}\right)^2 + \left(\frac{stdev(DN_{dark})}{\sqrt{N}}\right)^2$$

$$u^2_{rel}(S) = \frac{u^2_{abs}(S)}{S^2} = \frac{u^2_{abs}(S)}{(DN_{light} - DN_{dark})^2}$$

$$u^2_{rel}(C_{caE}) = \frac{\left(\frac{stdev(DN_{light})}{\sqrt{N}}\right)^2 + \left(\frac{stdev(DN_{dark})}{\sqrt{N}}\right)^2}{(DN_{light} - DN_{dark})^2} + u^2_{rel}(E)$$

To these eq. has to be added all the other sources of uncertainties: distance, stability, linearity…

Step 6: Assigning uncertainties

Source of uncertainty	Uncertainty value		Probability distribution	Divisor	Relative Standard uncertainty	Sensitivity coefficient	Contribution to the combined standard uncertainty	U type
	Absolute	Relative			u(xi)/xi	ci	ciu(xi)/xi	
spectral irradiance of FEL		1.50%	normal	2	0.75%	1.00	0.8%	B
FEL calibration drift			normal	1	0.00%		0.0%	B
straylight FEL			normal	1	0.00%	1.00	0.0%	B
FEL - ForeOpticcs distance	1mm	0.20%	rectangular	1.73	0.12%	2.00	0.2%	B
FEL alignment			rectangular	1.73	0.00%			A
FEL lamp current								A
						Combined standard uncertainty, uc(y)	0	
						Expanded uncertainty, U = kuc(y)	0	