

# Linguagens Formais e Autômatos

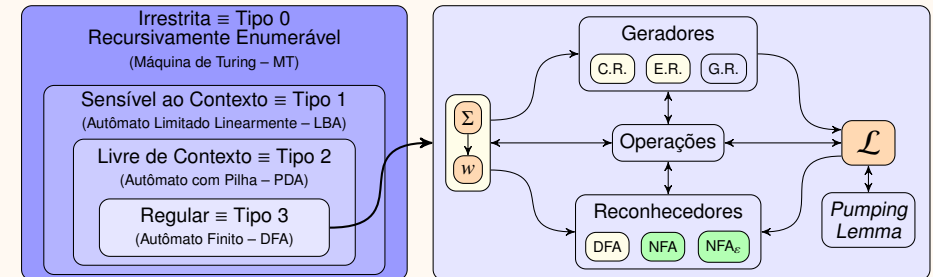
Humberto Longo

Instituto de Informática  
Universidade Federal de Goiás

Bacharelado em Ciência da Computação, 2021/1



## Roteiro



## Autômato finito não determinístico

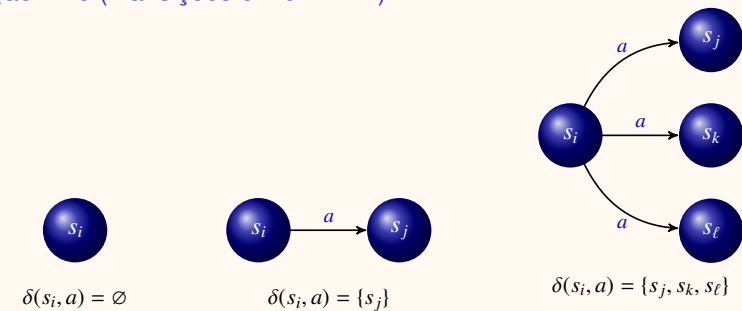
### Definição 1.44

- Um Autômato Finito Não Determinístico (NFA – *Nondeterministic Finite Automaton*) é uma quintupla  $N = \langle \Sigma, S, s_0, \delta, F \rangle$ , onde:
  - $\Sigma$  : alfabeto de entrada;
  - $S \neq \emptyset$  : conjunto finito de estados do modelo;
  - $s_0 \in S$  : estado inicial;
  - $\delta : S \times \Sigma \rightarrow \mathcal{P}(S)$  : função de transição de estados;
  - $F \subseteq S$  : conjunto de estados finais (ou de aceitação).



## Autômato finito não determinístico

### Definição 1.45 (Transições em um NFA)



## Linguagem de um NFA

### Definição 1.46

- ▶ A linguagem  $\mathcal{L}(N)$  de um NFA  $N = \langle \Sigma, S, s_0, \delta, F \rangle$  é o conjunto de cadeias em  $\Sigma^*$  aceitas por  $N$ .
- ▶ Dado o NFA  $N = \langle \Sigma, S, s_0, \delta, F \rangle$ , então  $\mathcal{L}(N) = \{w \mid [s_0, w] \xrightarrow{*} [s_i, \varepsilon] \text{ com } s_i \in F\}$ .



## Diagrama de estados

### Definição 1.47

- ▶ O diagrama de estados de um NFA  $N = \langle \Sigma, S, s_0, \delta, F \rangle$  é um grafo  $G$ , orientado e rotulado, definido pelas condições:
  1. os vértices de  $G$  são os elementos de  $S$ ;
  2. os rótulos dos arcos de  $G$  são os elementos de  $\Sigma$ ;
  3.  $s_0$  é o vértice inicial;
  4.  $F$  é o conjunto de vértices finais;
  5. existe um arco, rotulado de  $a$ , do vértice  $s_i$  ao  $s_j$  se  $s_j \in \delta(s_i, a)$ .



## Processamento em um NFA

- ▶ Uma cadeia  $w$  é aceita por um NFA  $N = \langle \Sigma, S, s_0, \delta, F \rangle$ , se  $w$  determina **pelo menos um caminho**  $p_w$  no diagrama de estados de  $N$  tal que  $\bar{\delta}(s_0, w) \in F$ .
- ▶ Uma cadeia pertence à linguagem de um NFA se existe um processamento que a aceita.
  - ▶ A existência de outros processamentos que não a aceitam é irrelevante!



## Exemplos de NFA's

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- ▶  $N_1 = \langle \{a, b\}, \{s_0, s_1, s_2\}, s_0, \delta, \{s_2\} \rangle$ , onde:

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- $\mathcal{L}(N_1) = (a \cup b)^*bb$

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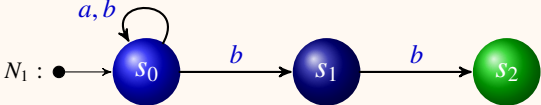


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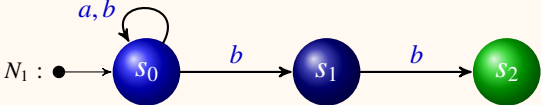


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- $[s_0, ababb]$

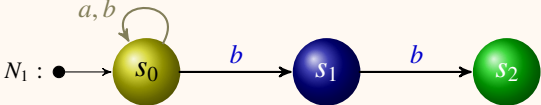


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- $[s_0, ababb] \mapsto [s_0, babb]$

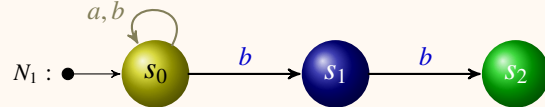


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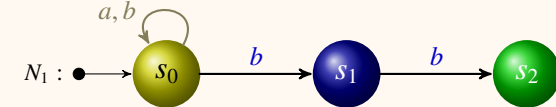


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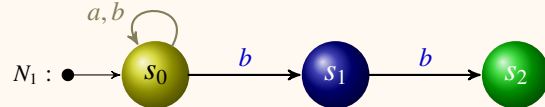


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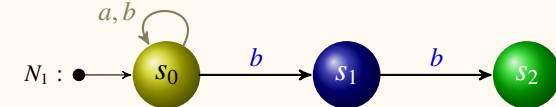


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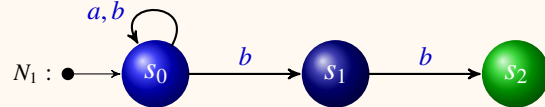
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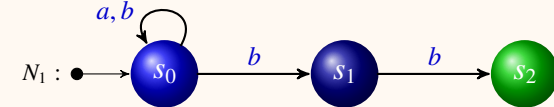
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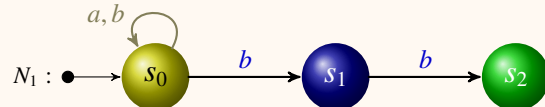
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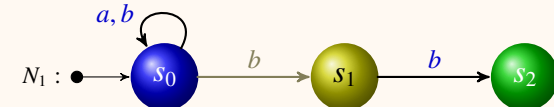
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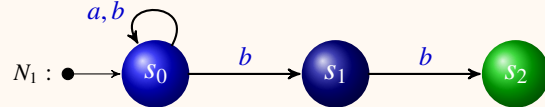
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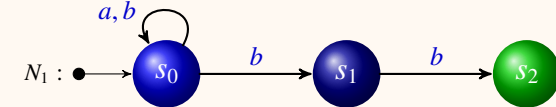
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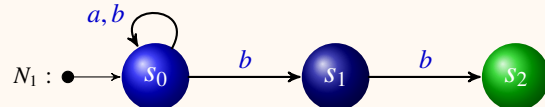
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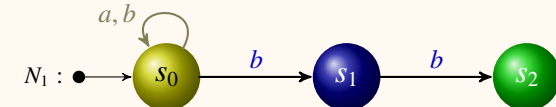
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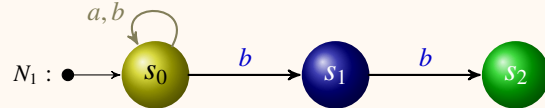
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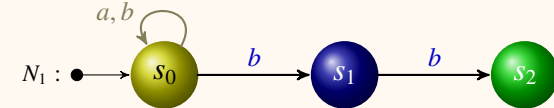
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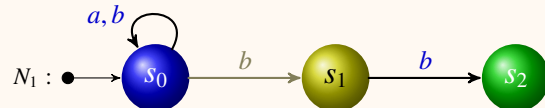
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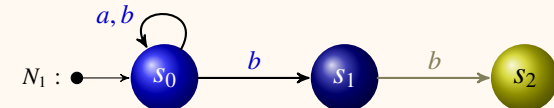
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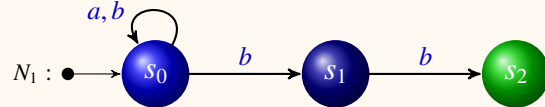
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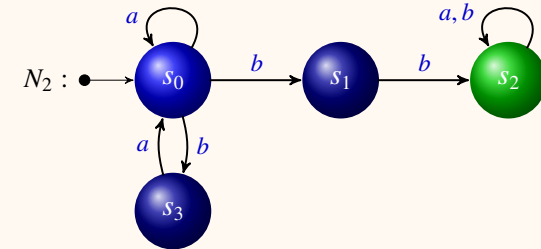
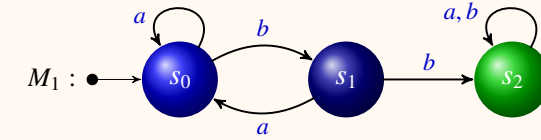
►  $[s_0, ababb] \mapsto [s_0, babbb] \mapsto [s_0, abb] \mapsto [s_0, bb] \mapsto [s_1, b] \mapsto [s_2, \epsilon] \mapsto [s_2]$



## Exemplos de NFA's

### Exemplo 1.49

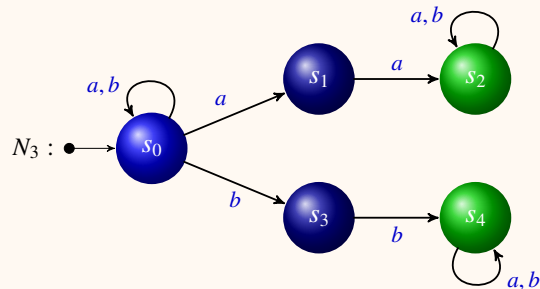
►  $\mathcal{L}(N_2) = (a \cup ba)^* bb(a \cup b)^*$ .



## Exemplos de NFA's

### Exemplo 1.50

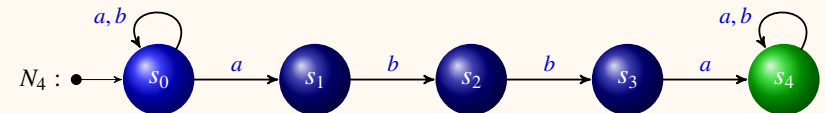
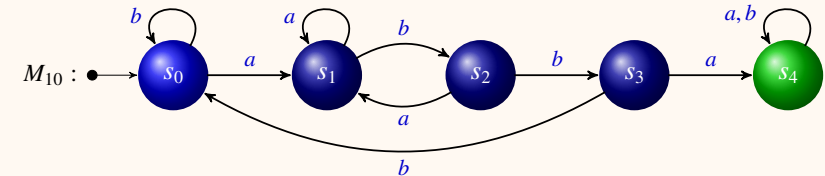
►  $\mathcal{L}(N_3) = \{w \in \{a, b\}^+ \mid w \text{ contém } aa \text{ ou } bb\}$ .



## Exemplos de NFA's

### Exemplo 1.51

►  $\mathcal{L}(N_4) = \{w \in \{a, b\}^* \mid w \text{ contém } abba\}$ .





## Transições vazias

### Definição 1.52

- Uma transição vazia não processa qualquer símbolo da cadeia de entrada.

### Definição 1.53

- NFA- $\varepsilon$  é a classe de NFA's que utilizam transições vazias.

### Definição 1.54

- Um NFA- $\varepsilon$  é uma quintupla  $N = \langle \Sigma, S, s_0, \delta, F \rangle$ , onde:  $\Sigma$ ,  $S$ ,  $s_0$  e  $F$  seguem a mesma definição de NFA. A função de transição é  $\delta : S \times (\Sigma \cup \{\varepsilon\}) \rightarrow \mathcal{P}(S)$ .

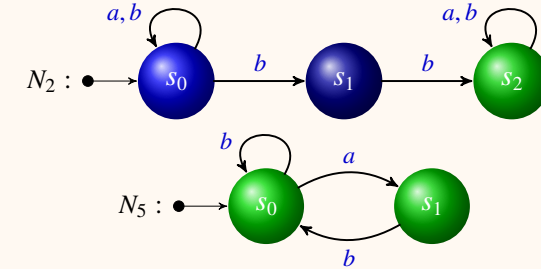


## Exemplos de NFA- $\varepsilon$ 's

### Exemplo 1.55

- Considere os NFA's  $N_2$  e  $N_5$ , tais que:

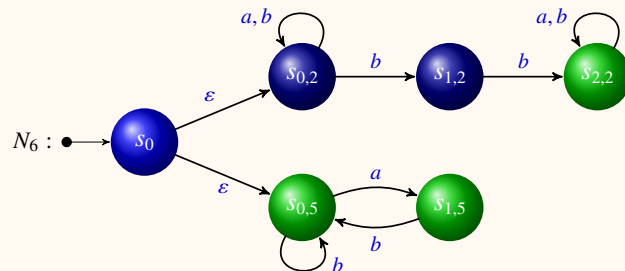
- $\mathcal{L}(N_2) = (a \cup b)^* bb(a \cup b)^*$ .
- $\mathcal{L}(N_5) = (b \cup ab)^*(a \cup \varepsilon)$ .



## Exemplos de NFA- $\varepsilon$ 's

### Exemplo 1.56

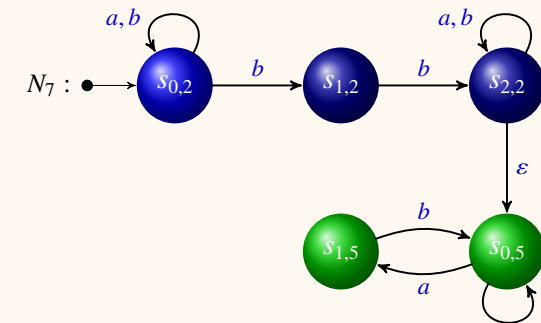
- NFA- $\varepsilon$   $N_6$  tal que  $\mathcal{L}(N_6) = \mathcal{L}(N_2) \cup \mathcal{L}(N_5)$ :



## Exemplos de NFA- $\varepsilon$ 's

### Exemplo 1.57

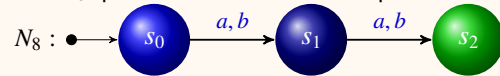
- NFA- $\varepsilon$   $N_7$  tal que  $\mathcal{L}(N_7) = \mathcal{L}(N_2)\mathcal{L}(N_5)$ :



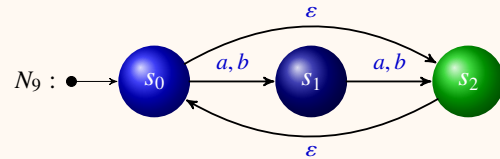
## Exemplos de NFA- $\varepsilon$ 's

### Exemplo 1.58

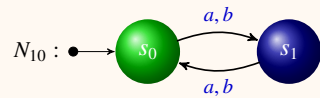
- NFA  $N_8$  que aceita cadeias de comprimento dois sobre o alfabeto  $\{a, b\}$ :



- NFA- $\varepsilon$   $N_9$  que aceita cadeias de comprimento par sobre o alfabeto  $\{a, b\}$ :



- Outro NFA que aceita cadeias de comprimento par sobre o alfabeto  $\{a, b\}$ :



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