

C1: Algebra and Functions

1a) $4x - 3 > 7 - x$

$$5x - 3 > 7$$

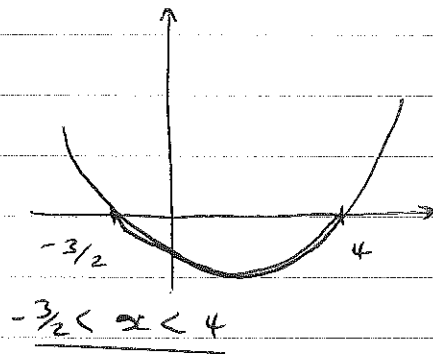
$$5x > 10$$

$$x > 2$$

b) $2x^2 - 5x - 12 < 0$

$$(2x + 3)(x - 4) < 0$$

$$x = -3/2 \quad x = 4$$



c) $2 < x < 4$

2) $x^2 + 3px + p = 0$

$$a=1 \quad b=3p \quad c=p$$

equal roots so $b^2 - 4ac = 0$

$$(3p)^2 - 4(1)(p) = 0$$

$$9p^2 - 4p = 0$$

$$p(9p - 4) = 0$$

$$p=0 \quad p=4/9$$

$$\therefore p=4/9$$

3) $x^3 - 9x$

$$x(x^2 - 9)$$

$$x(x+3)(x-3)$$

4a)

$$y = x - 4$$

$$2x^2 - xy = 8$$

$$2x^2 - x(x-4) = 8$$

$$2x^2 - x^2 + 4x = 8$$

$$x^2 + 4x - 8 = 0$$

$$(x+2)^2 - 12 = 0$$

$$(x+2)^2 = 12$$

$$x+2 = \pm\sqrt{12}$$

$$x = -2 \pm 2\sqrt{3}$$

$$y = -6 \pm 2\sqrt{3}$$

5a)

$$x^2 - 8x - 29$$

$$(x-4)^2 - 45$$

$$a = -4 \quad b = -45$$

b/

$$x^2 - 8x - 29 = 0$$

$$(x-4)^2 - 45 = 0$$

$$(x-4)^2 = 45$$

$$x-4 = \pm\sqrt{45}$$

$$x = 4 \pm \sqrt{45}$$

$$= \underline{4 \pm 3\sqrt{5}}$$

6/

$$y = x - 2$$

$$y^2 + x^2 = 10$$

$$(x-2)^2 + x^2 = 10$$

$$x^2 - 4x + 4 + x^2 = 10$$

$$2x^2 - 4x - 6 = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

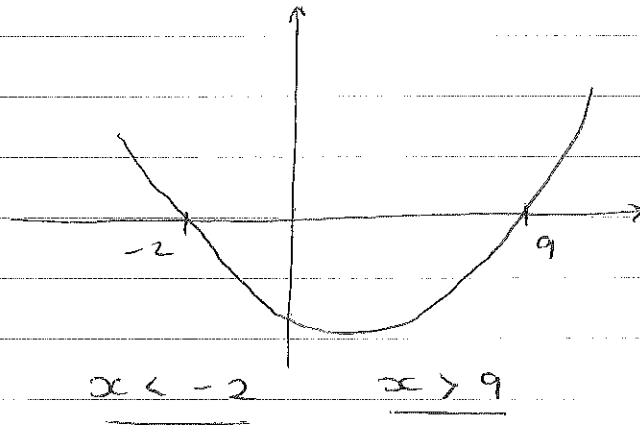
$$x = 3 \quad x = -1$$

$$y = 1 \quad y = -3$$

$$7/ \quad x^2 - 7x - 18 > 0$$

$$(x - 9)(x + 2) > 0$$

$$x = 9 \quad x = -2$$



$$8/ \quad x^3 - 4x^2 + 3x$$

$$x(x^2 - 4x + 3)$$

$$x(x - 3)(x - 1)$$

$$9/ \quad kx^2 + 4x + (5 - k) = 0$$

$$a = k \quad b = 4 \quad c = (5 - k)$$

$$2 \text{ Solutions} \quad \therefore b^2 - 4ac > 0$$

$$(4)^2 - 4(k)(5 - k) > 0$$

$$16 - 4k(5 - k) > 0$$

$$16 - 20k + 4k^2 > 0$$

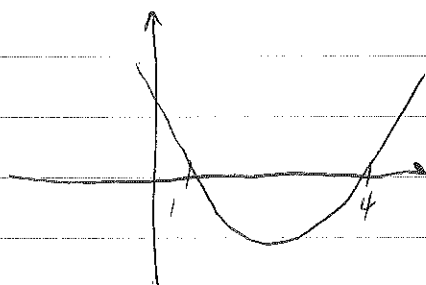
$$4k^2 - 20k + 16 > 0$$

$$k^2 - 5k + 4 > 0$$

b/

$$(k - 4)(k - 1) > 0$$

$$k = 4 \quad k = 1$$



$$k < 1 \text{ or } k > 4$$

10/

$$x - 2y = 1$$

$$x = 1 + 2y$$

$$x^2 + y^2 = 29$$

$$(1 + 2y)^2 + y^2 = 29$$

$$1 + 4y + 4y^2 + y^2 = 29$$

$$5y^2 + 4y - 28 = 0$$

$$(5y + 14)(y - 2) = 0$$

$$y = -\frac{14}{5}$$

$$y = 2$$

$$x = -\frac{23}{5}$$

$$x = 5$$

11/

$$2qx^2 + qx - 1 = 0$$

$$a = 2q \quad b = q \quad c = -1$$

$$\text{no roots} \therefore b^2 - 4ac < 0$$

$$(q)^2 - 4(2q)(-1) < 0$$

$$q^2 + 4(2q) < 0$$

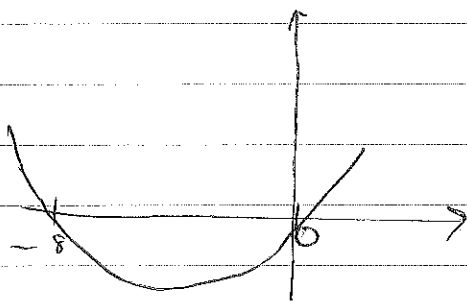
$$q^2 + 8q < 0$$

b/

$$q(q + 8) < 0$$

$$q = 0$$

$$q = -8$$



$$\underline{-8 < q < 0}$$

13a)

$$x^2 + kx + (k+3) = 0$$

$$\text{different roots} \therefore b^2 - 4ac > 0$$

$$a = 1 \quad b = k \quad c = (k+3) \quad (k)^2 - 4(1)(k+3) > 0$$

$$k^2 - 4k - 12 > 0$$

12/

$$x^2 + kx + 8 = k$$
$$x^2 + kx + (8 - k) = 0$$

No real solutions $\therefore b^2 - 4ac < 0$

$$k^2 - 4(1)(8 - k) < 0$$

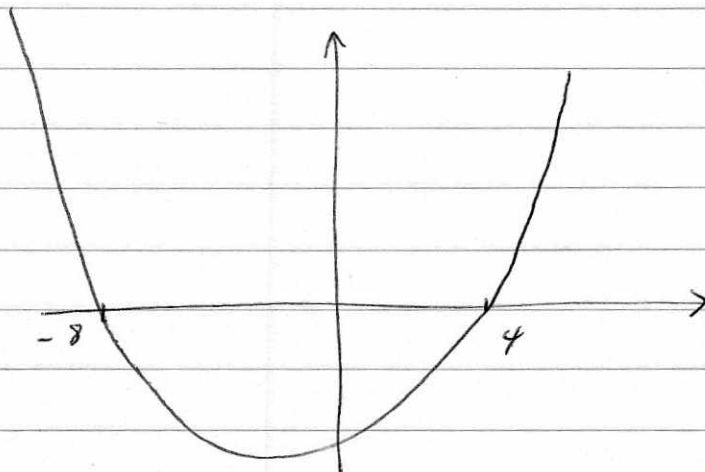
$$k^2 - 4(8 - k) < 0$$

$$k^2 - 32 + 4k < 0$$

$$k^2 + 4k - 32 < 0$$

b/ $(k + 8)(k - 4) < 0$

$$k = -8 \quad k = 4$$



$$\underline{\underline{-8 < k < 4}}$$

$$13a) \quad x^2 + kx + (k+3) = 0$$

different roots $\therefore b^2 - 4ac > 0$

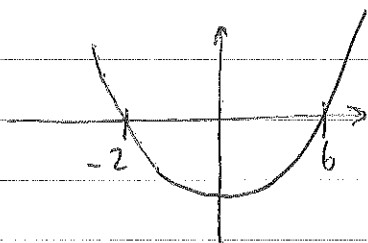
$$a=1 \quad b=k \quad c=(k+3) \quad (k)^2 - 4(1)(k+3) > 0$$

$$k^2 - 4k - 12 > 0$$

$$b) \quad k^2 - 4k - 12 > 0$$

$$(k-6)(k+2) > 0$$

$$k=6 \quad k=-2$$



$$k < -2 \text{ or } k > 6$$

14/

$$x + y = 2$$

$$x = 2 - y$$

$$x^2 + 2y = 12$$

$$(2-y)^2 + 2y = 12$$

$$4 - 4y + y^2 + 2y = 12$$

$$y^2 - 2y - 8 = 0$$

$$(y-4)(y+2) = 0$$

$$y=4 \quad y=-2$$

$$x=-2 \quad x=4$$

15/

$$2x^2 - 3x - (k+1) = 0$$

$$a=2 \quad b=-3 \quad c=-(k+1)$$

no real roots $\therefore b^2 - 4ac < 0$

$$(-3)^2 - 4(2)(-k-1) < 0$$

$$9 - 8(-k-1) < 0$$

$$9 + 8k + 8 < 0$$

$$8k < -17$$

$$k < -17/8$$

16/

$$x^2 + 2px + (3p+4) = 0$$

$$a=1 \quad b=2p \quad c=3p+4 \quad b^2 - 4ac = 0$$

$$(2p)^2 - 4(1)(3p+4) = 0$$

$$4p^2 - 12p - 16 = 0$$

$$p^2 - 3p - 4 = 0$$

$$(p-4)(p+1) = 0$$

$$\underline{\underline{p=4}}$$

b/ $p=4$

$$x^2 + 2(4)x + (3(4) + 4) = 0$$

$$x^2 + 8x + 16 = 0$$

$$(x+4)(x+4) = 0$$

$$\underline{x = -4}$$

17a/ $3(2x+1) > 5 - 2x$

$$6x + 3 > 5 - 2x$$

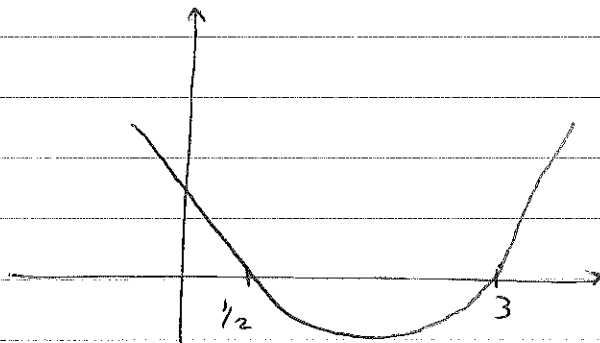
$$8x > 2$$

$$x > \frac{1}{4}$$

b/ $2x^2 - 7x + 3 > 0$

$$(2x-1)(x-3) > 0$$

$$x = \frac{1}{2} \quad x = 3$$



$$\underline{x < \frac{1}{2} \text{ or } x > 3}$$

c/ $\frac{1}{4} < x < \frac{1}{2} \text{ or } x > 3$

18/ $kx^2 + 12x + k = 0$

$$a=k \quad b=12 \quad c=k$$

$$\text{equal roots } \therefore b^2 - 4ac = 0$$

$$(12)^2 - 4(k)(k) = 0$$

$$144 - 4k^2 = 0$$

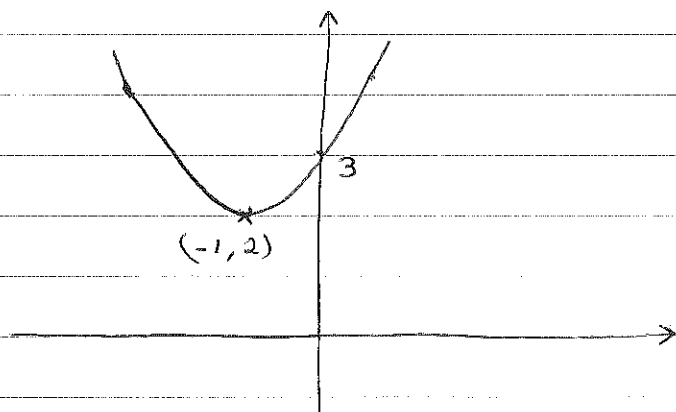
$$144 = 4k^2$$

$$36 = k^2$$

$$\underline{k = 6}$$

19a/ $x^2 + 2x + 3$
 $(x+1)^2 + 2$
 $a=1 \quad b=2$

b/ $y = x^2 + 2x + 3$



c/ $b^2 - 4ac$
 $(2)^2 - 4(1)(3)$
 $4 - 12$
 -8

as $b^2 - 4ac < 0$ there are no solutions to $x^2 + 2x + 3 = 0$
it does not cross the x axis

d/ $x^2 + kx + 3 = 0$

$a=1 \quad b=k \quad c=3$

$b^2 - 4ac < 0$

$k^2 - 4(1)(3) < 0$

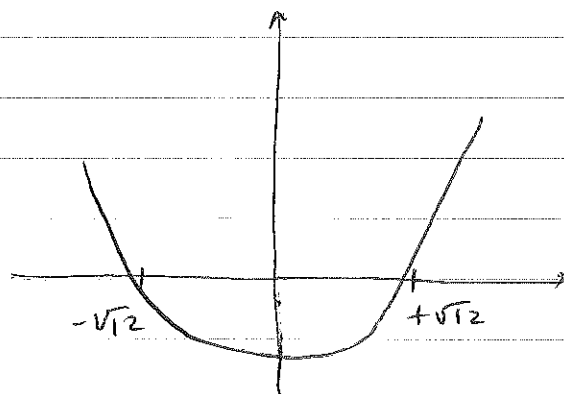
$k^2 - 12 < 0$

$k^2 = 12 \quad k^2 < 12$

$k = \pm\sqrt{12}$

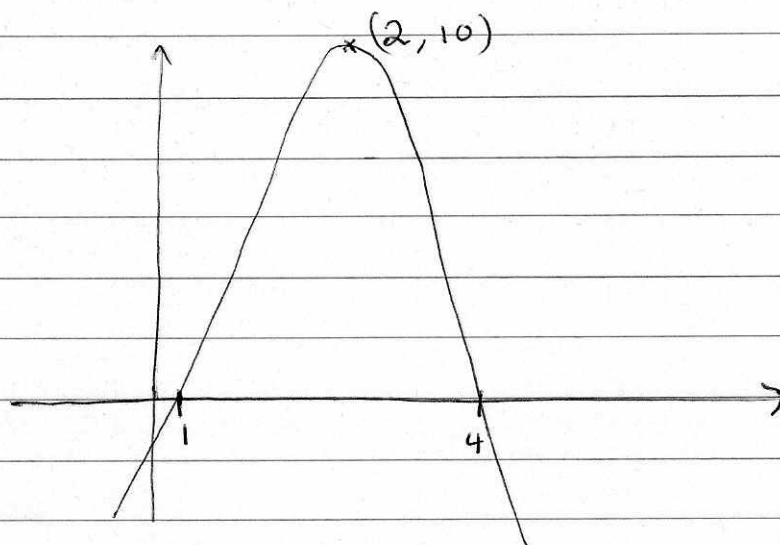
$-\sqrt{12} < k < \sqrt{12}$

$-2\sqrt{3} < k < 2\sqrt{3}$

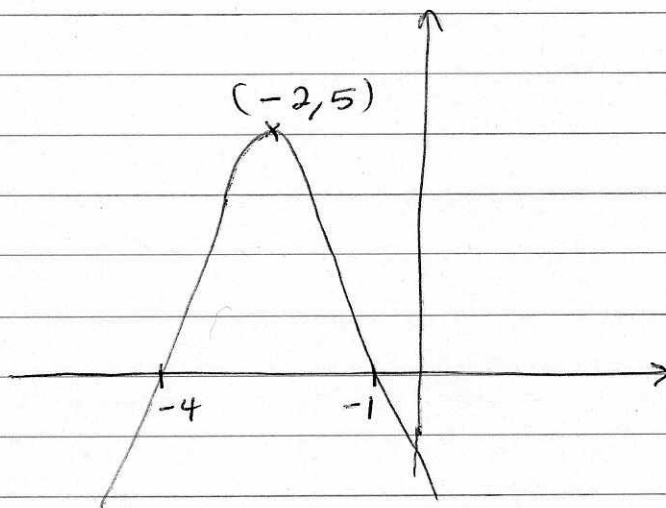


C1 - ALGEBRA & FUNCTIONS (GRAPHS)

1a)

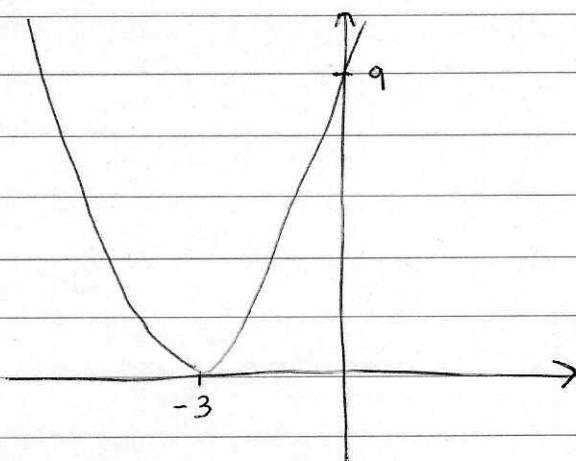


b/

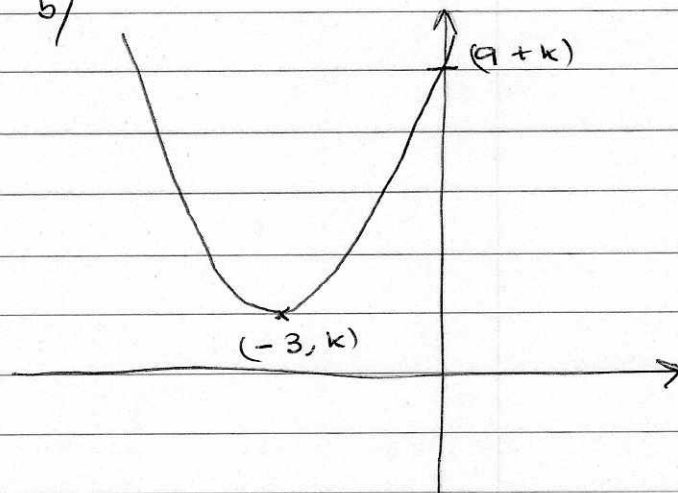


c/ $a = 2$

2a/



b/



3a)

$$y = (x+1)^2(2-x)$$

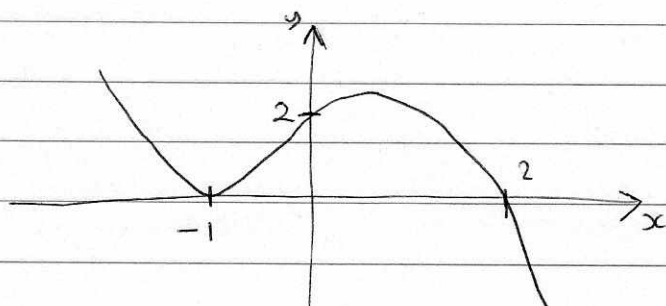
$$(1, a) \quad a = (1+1)^2(2-1)$$

$$= (2)^2(1)$$

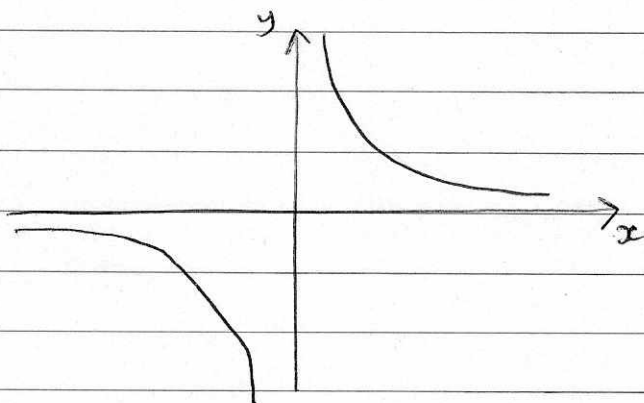
$$= \underline{\underline{4}}$$

b/

i/



ii/



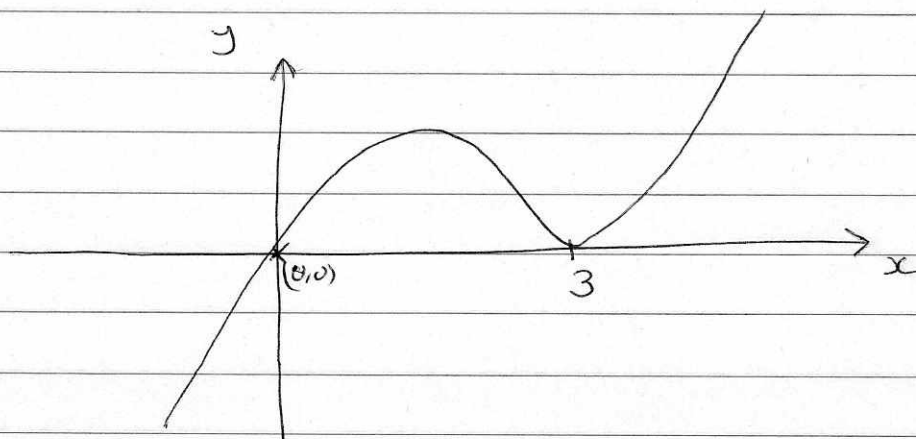
c/ 2 solutions. The graphs would intersect twice.

4a)

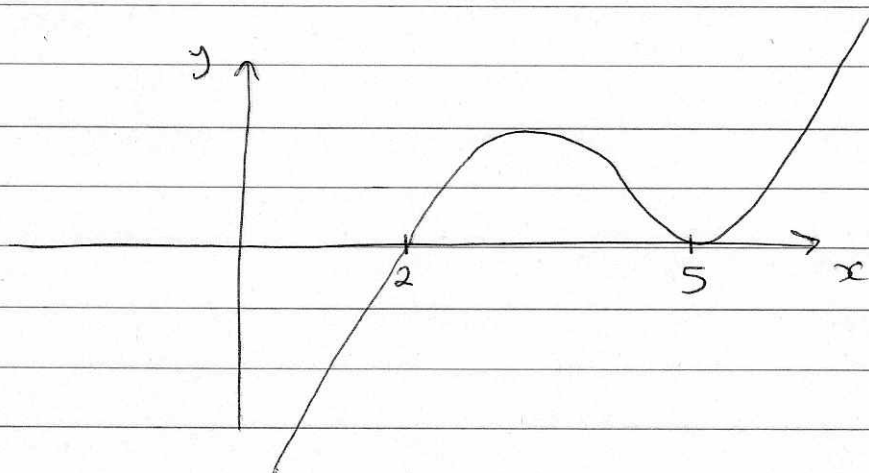
$$x(x^2 - 6x + 9)$$

$$x(x-3)(x-3)$$

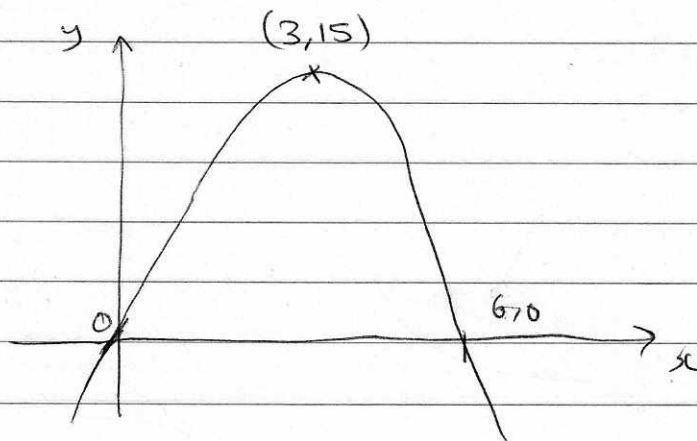
b/



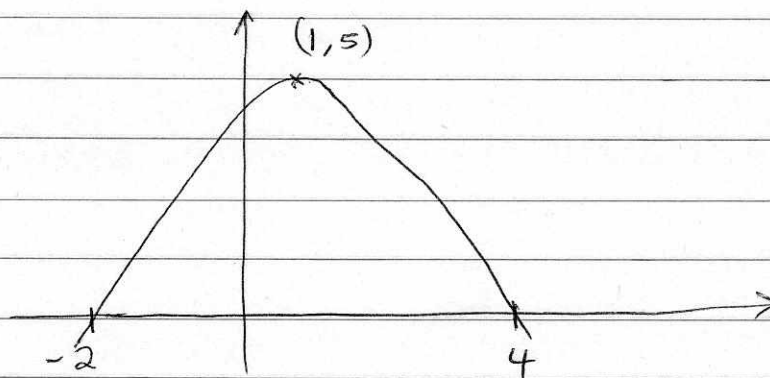
c/



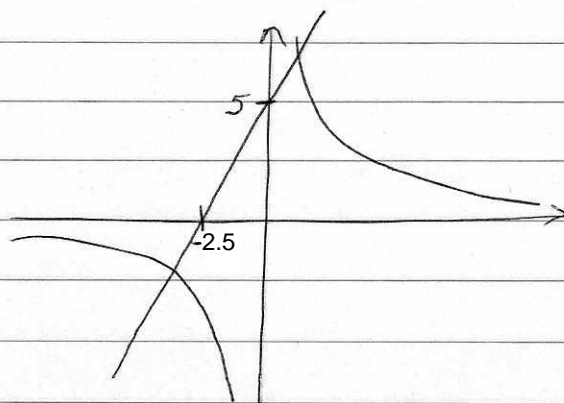
5a)



b/

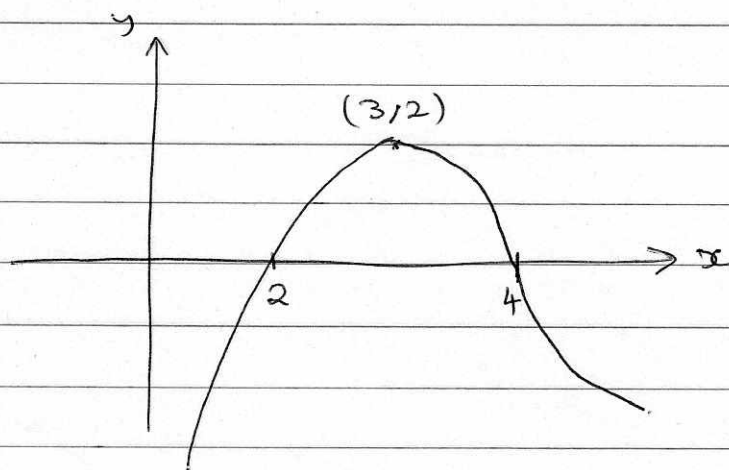


6a/

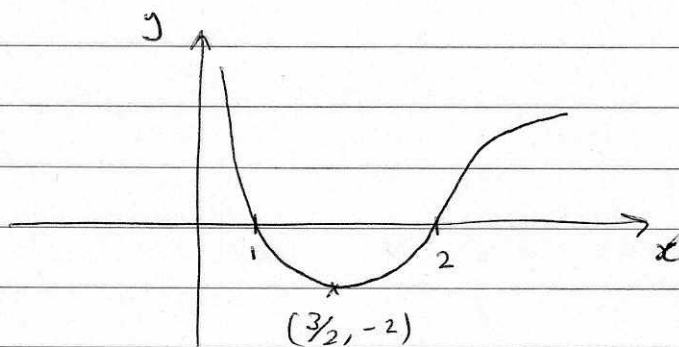


$$\begin{aligned}
 \text{b/} \quad \frac{3}{x} &= 2x + 5 \\
 3 &= 2x^2 + 5x \\
 0 &= 2x^2 + 5x - 3 \\
 0 &= (2x - 1)(x + 3) \\
 x &= \frac{1}{2} \quad x = -3 \\
 y &= 6 \quad y = -1 \\
 \underline{\underline{(\frac{1}{2}, 6)}} \quad & \underline{\underline{(-3, -1)}}
 \end{aligned}$$

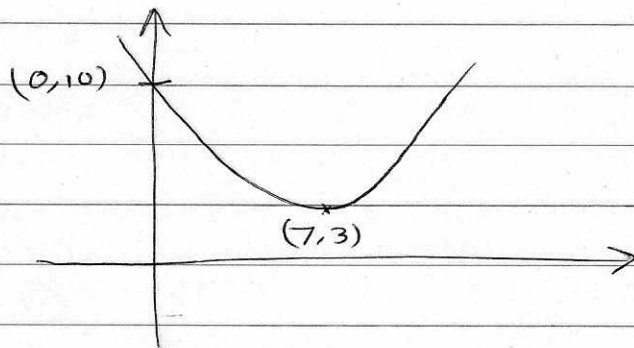
7a)



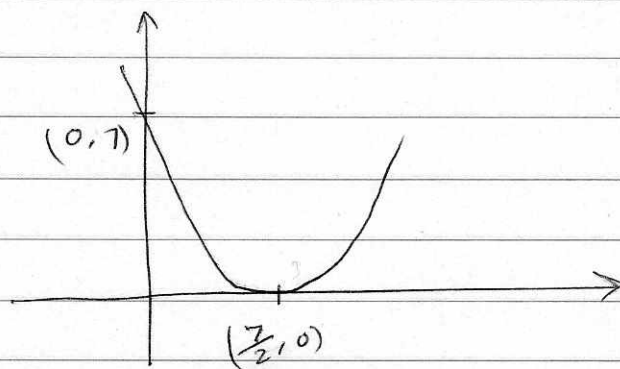
b)



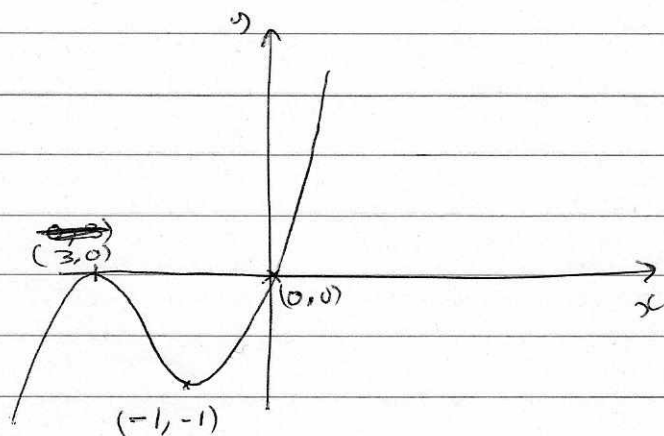
8a)



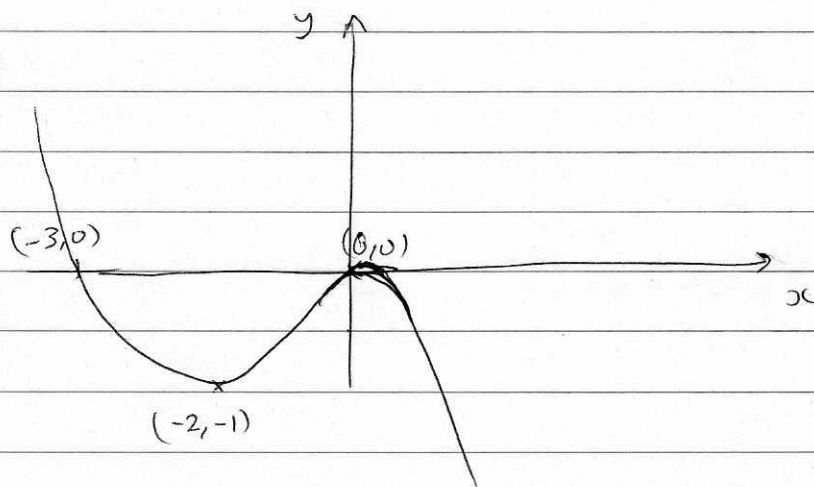
b)



9a)



b)



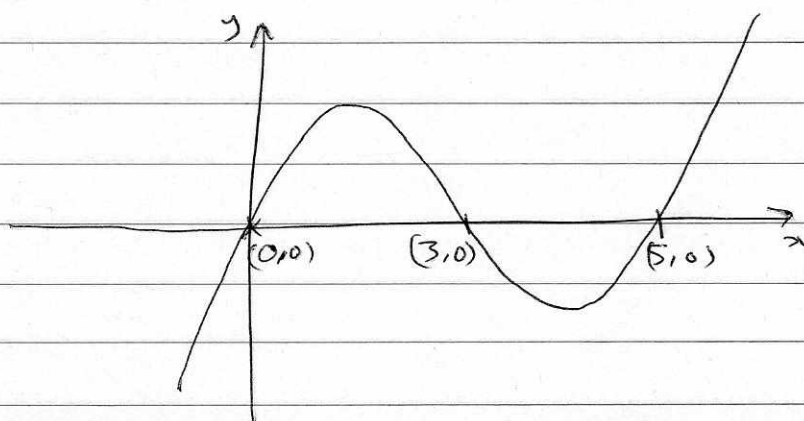
10a)

$$\begin{aligned} f(x) &= (x^2 - 6x)(x - 2) + 3x \\ &= x^3 - 2x^2 - 6x^2 + 12x + 3x \\ &= x^3 - 8x^2 + 15x \\ &= x(x^2 - 8x + 15) \end{aligned}$$

b/

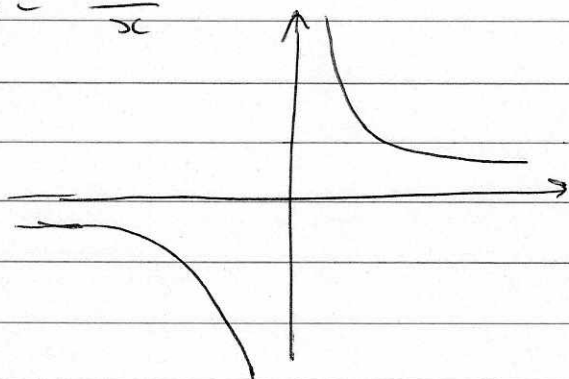
$$x(x - 3)(x - 5)$$

c/

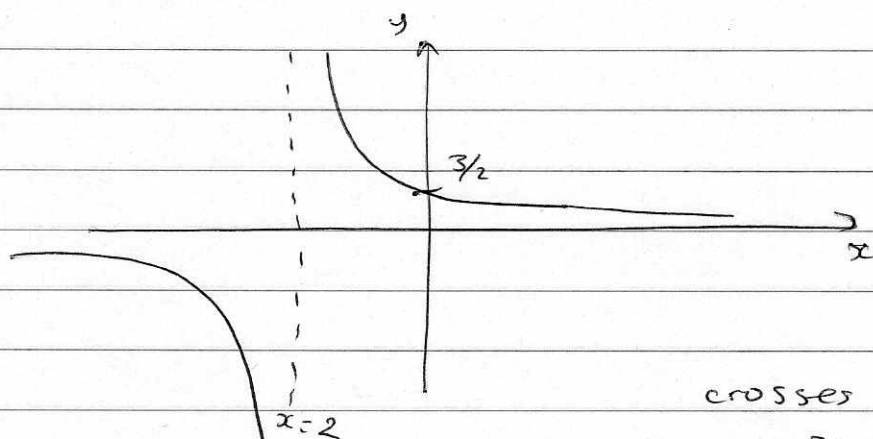


11.

$$y = \frac{3}{x}$$



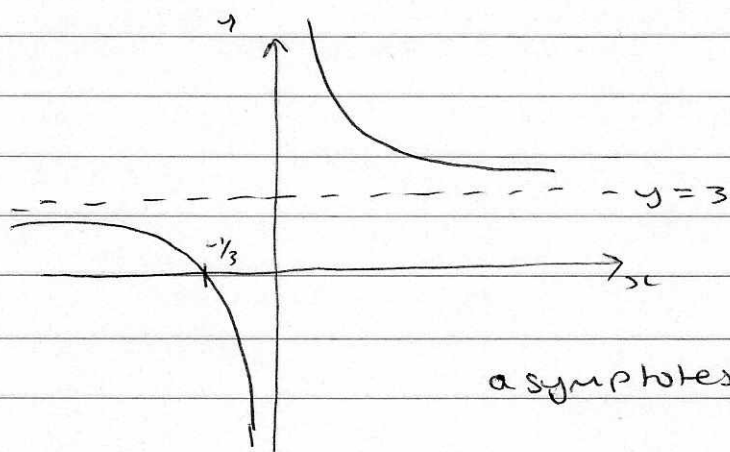
a/



crosses y when $x=0$
 $y = 3/2$

b/ asymptotes at $y=0$ and $x=2$

12 a/



asymptotes at $x=0$ and $y=3$

b) crosses x when $y=0$

$$y = \frac{1}{x} + 3$$

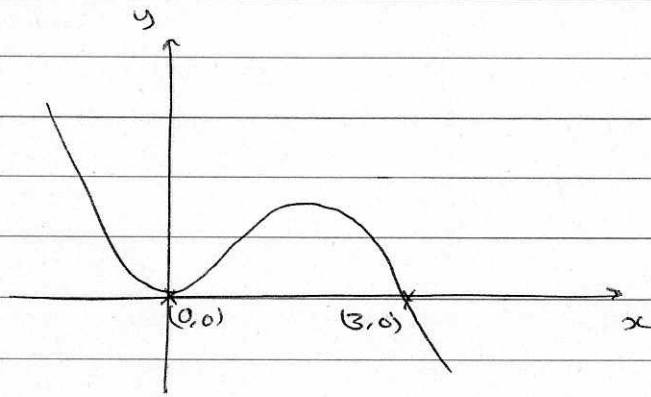
$$0 = \frac{1}{x} + 3$$

$$-3 = \frac{1}{x}$$

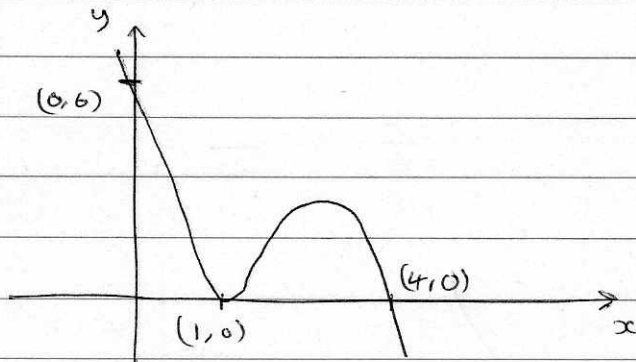
$$x = -\frac{1}{3}$$

$$\underline{\underline{(-1/3, 0)}}$$

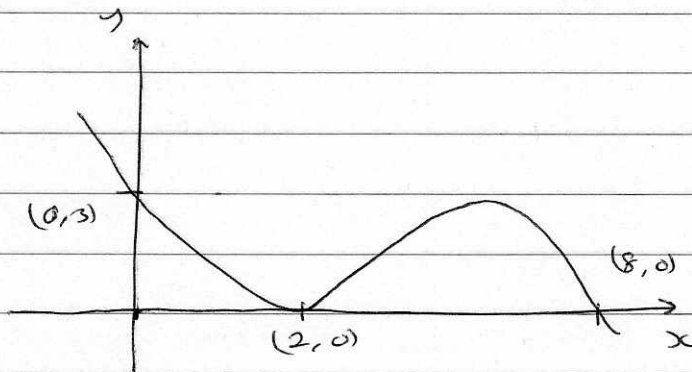
13a1



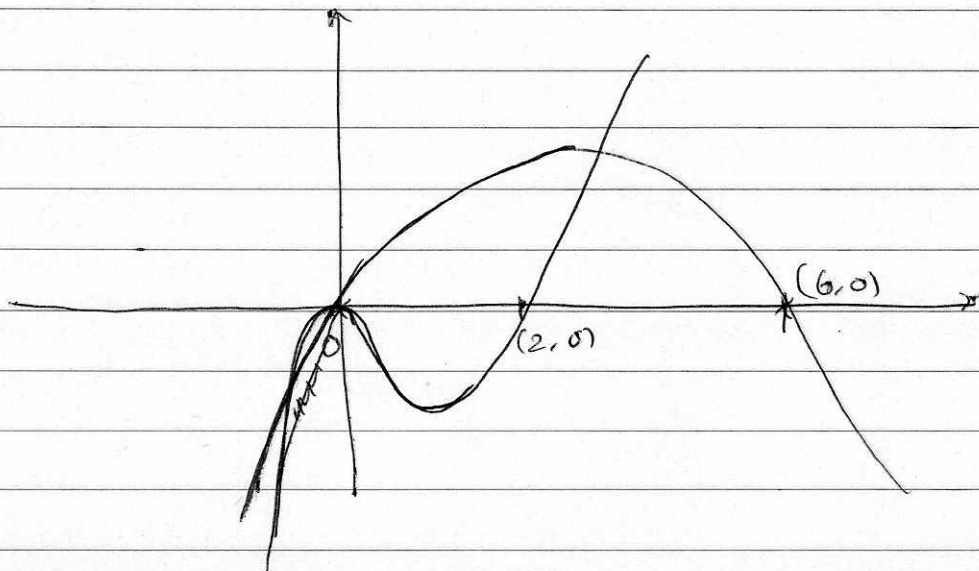
b1



c1



14a1



b/

$$x^2(x-2) = x(6-x)$$

$$x^3 - 2x^2 = 6x - x^2$$

$$x^3 - x^2 - 6x = 0$$

$$x(x^2 - x - 6) = 0$$

$$x(x-3)(x+2) = 0$$

$$x=0 \quad x=3 \quad x=-2$$

$$\text{when } x=0 \quad y=0$$

$$\text{when } x=3 \quad y=3(6-3) \\ = 9$$

$$\text{when } x=-2 \quad y=-2(6-(-2)) \\ = -16$$

$$\underline{(0,0)}, \underline{(3,9)}, \underline{(-2,-16)}$$

~~Ha.~~