C4  Partial Fractions & Binomial.

1)  $f(x) = (4 + x)^{-\frac{1}{2}}$

$= 4^{-\frac{1}{2}}(1 + \frac{1}{4}x)^{-\frac{1}{2}}$

$= \frac{1}{2}(1 + \frac{1}{4}x)^{-\frac{1}{2}}$

$= \frac{1}{2}\left[1 + (-\frac{1}{2})(\frac{1}{4}x) + \dfrac{(-\frac{1}{2})(-\frac{3}{2})}{2}(\frac{1}{4}x)^2 + \dfrac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{6}(\frac{1}{4}x)^3\right]$

$= \frac{1}{2}\left[1 - \frac{1}{8}x + \frac{3}{128}x^2 - \frac{5}{1024}x^3\right]$

$= \frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 - \frac{5}{2048}x^3$

2)  $\dfrac{27x^2 + 32x + 16}{(3x+2)^2(1-x)} = \dfrac{A}{(3x+2)} + \dfrac{B}{(3x+2)^2} + \dfrac{C}{(1-x)}$

$27x^2 + 32x + 16 = A(3x+2)(1-x) + B(1-x) + C(3x+2)^2$

Let $x = 1$

$75 = 25C$

$\underline{\underline{C = 3}}$

Let $x = -\frac{2}{3}$

$\frac{20}{3} = \frac{5}{3}B$

$\underline{\underline{B = 4}}$

Let $x = 0$

$16 = 2A + B + 4C$

$16 = 2A + (4) + 4(3)$

$16 = 2A + 16$

$\underline{\underline{A = 0}}$

b)  $f(x) = \dfrac{4}{(3x+2)^2} + \dfrac{3}{1-x}$

$= 4(3x+2)^{-2} + 3(1-x)^{-1}$

$= 4(2^{-2}(1 + \frac{3}{2}x)^{-2}) + 3(1-x)^{-1}$

$= (1 + \frac{3}{2}x)^{-2} + 3(1-x)^{-1}$

$$= 1 + -2\left(\tfrac{3}{2}x\right) + \frac{(-2)(-3)}{2}\left(\tfrac{3}{2}x\right)^2 + 3\left(1 + (-1)(-x) + \frac{(-1)(-2)(-x)^2}{2}\right)$$

$$= 1 - 3x + \tfrac{27}{4}x^2 + 3\left(1 + x + x^2\right)$$

$$= 1 - 3x + \tfrac{27}{4}x^2 + 3 + 3x + 3x^2$$

$$= 4 + \tfrac{39}{4}x^2$$

c/ $\quad 4 + \tfrac{39}{4}(0.2)^2 = 4.39$

$$\frac{27(0.2)^2 + 32(0.2) + 16}{(3(0.2)+2)^2(1-0.2)} = 4.341715976$$

$$\frac{4.39 - ANS}{ANS} \times 100 = 1.1\% \quad (2sf)$$

3a) $\quad (4-3x)^{-1/2}$
$\qquad 4^{-1/2}\left(1 - \tfrac{3}{4}x\right)^{-1/2}$

$$\tfrac{1}{2}\left(1 - \tfrac{3}{4}x\right)^{-1/2}$$
$$\tfrac{1}{2}\left(1 + (-\tfrac{1}{2})(-\tfrac{3}{4}x) + \frac{(-\tfrac{1}{2})(-\tfrac{3}{2})}{2}\left(-\tfrac{3}{4}x\right)^2\right)$$

$$\tfrac{1}{2}\left(1 + \tfrac{3}{8}x + \tfrac{27}{128}x^2\right)$$

$$\tfrac{1}{2} + \tfrac{3}{16}x + \tfrac{27}{256}x^2$$

b) $\quad (x+8)\left(\tfrac{1}{2} + \tfrac{3}{16}x + \tfrac{27}{256}x^2\right)$
$\qquad \tfrac{1}{2}x + \tfrac{3}{16}x^2 + 4 + \tfrac{3}{2}x + \tfrac{27}{32}x^2$

$$\underline{4 + 2x + \tfrac{33}{32}x^2}$$

4a)  $(8-3x)^{\frac{1}{3}}$

$8^{\frac{1}{3}}\left(1-\frac{3}{8}x\right)^{\frac{1}{3}}$

$2\left(1-\frac{3}{8}x\right)^{\frac{1}{3}}$

$2\left(1+\frac{1}{3}\left(-\frac{3}{8}x\right)+\frac{\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)}{2}\left(-\frac{3}{8}x\right)^{2}+\frac{\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)\left(\frac{-5}{3}\right)}{6}\left(-\frac{3}{8}x\right)^{3}\right)$

$2\left(1-\frac{1}{8}x-\frac{1}{64}x^{2}-\frac{5}{1536}x^{3}\right)$

$2-\frac{1}{4}x-\frac{1}{32}x^{2}-\frac{5}{768}x^{3}$

b)  $8-3x=7.7$

$0.3=3x$

$x=0.1$

$2-\frac{1}{4}(0.1)-\frac{1}{32}(0.1)^{2}-\frac{5}{768}(0.1)^{3}$

$=1.9746810 \quad (7dp)$

5)  $(3+2x)^{-3}$

$3^{-3}\left(1+\frac{2}{3}x\right)^{-3}$

$\frac{1}{27}\left(1+\frac{2}{3}x\right)^{-3}$

$\frac{1}{27}\left(1+(-3)\left(\frac{2}{3}x\right)+\frac{(-3)(-4)}{2}\left(\frac{2}{3}x\right)^{2}+\frac{(-3)(-4)(-5)}{6}\left(\frac{2}{3}x\right)^{3}\right)$

$\frac{1}{27}\left(1-2x+\frac{8}{3}x^{2}-\frac{80}{27}x^{3}\right)$

$\frac{1}{27}-\frac{2}{27}x+\frac{8}{81}x^{2}-\frac{80}{729}x^{3}$

**6)** $(2-5x)^{-2}$

$2^{-2}(1-\frac{5}{2}x)^{-2}$

$\frac{1}{4}(1-\frac{5}{2}x)^{-2}$

$\frac{1}{4}\left(1 + (-2)(-\frac{5}{2}x) + \frac{(-2)(-3)}{2}(-\frac{5}{2}x)^2 + \frac{(-2)(-3)(-4)}{6}(-\frac{5}{2}x)^3\right)$

$\frac{1}{4}\left(1 + 5x + \frac{75}{4}x^2 + \frac{125}{2}x^3\right)$

$\frac{1}{4} + \frac{5}{4}x + \frac{75}{16}x^2 + \frac{125}{8}x^3$

**7)** $\dfrac{3x-1}{(1-2x)^2} = \dfrac{A}{(1-2x)} + \dfrac{B}{(1-2x)^2}$

$(3x-1) = A(1-2x) + B$

Let $x = \frac{1}{2}$

$\underline{\frac{1}{2} = B}$

Let $x = 0$

$-1 = A + \frac{1}{2}$

$\underline{A = -\frac{3}{2}}$

**b)** $\dfrac{-3}{2(1-2x)} + \dfrac{1}{2(1-2x)^2}$

$-\frac{3}{2}(1-2x)^{-1} + \frac{1}{2}(1-2x)^{-2}$

$-\frac{3}{2}\left(1 + (-1)(-2x) + \frac{(-1)(-2)}{2}(-2x)^2 + \frac{(-1)(-2)(-3)}{6}(-2x)^3\right) + \cdots$

$\cdots + \frac{1}{2}\left(1 + (-2)(-2x) + \frac{(-2)(-3)}{2}(-2x)^2 + \frac{(-2)(-3)(-4)}{6}(-2x)^3\right)$

$-\frac{3}{2}(1 + 2x + 4x^2 + 8x^3) + \frac{1}{2}(1 + 4x + 12x^2 + 32x^3)$

$-\frac{3}{2} - 3x - 6x^2 - 12x^3 + \frac{1}{2} + 2x + 6x^2 + 16x^3$

$\underline{-1 - x + 4x^3}$

8a) $\dfrac{3x^2+16}{(1-3x)(2+x)^2} = \dfrac{A}{1-3x} + \dfrac{B}{2+x} + \dfrac{C}{(2+x)^2}$

$3x^2 + 16 = A(2+x)^2 + B(1-3x)(2+x) + C(1-3x)$

Let $x = -2$

$\qquad 28 = 7C$

$\qquad \underline{C = 4}$

Let $x = \dfrac{1}{3}$

$\qquad \dfrac{49}{3} = \dfrac{49}{9}A$

$\qquad \underline{A = 3}$

Let $x = 0$

$\qquad 16 = 4A + 2B + C$

$\qquad 16 = 4(3) + 2(B) + 4$

$\qquad 16 = 12 + 2B + 4$

$\qquad 16 = 16 + 2B$

$\qquad \underline{B = 0}$

b) $\qquad 3(1-3x)^{-1} + 4(2+x)^{-2}$

$3(1 + (-1) \quad 3(1-3x)^{-1} + 4\left(\dfrac{1}{8}2^{-2}\left(2\ 1 + \dfrac{1}{2}x\right)\right)^{-2}$

$\qquad 3(1-3x)^{-1} + \left(1 + \dfrac{1}{2}x\right)^{-2}$

$3\left(1 + (-1)(-3x) + \dfrac{(-1)(-2)}{2}(-3x)^2 + \dfrac{(-1)(-2)(-3)}{6}(-3x)^3\right) + \ldots$

$\qquad 1 + (-2)\left(\dfrac{+1}{2}x\right) + \dfrac{(-2)(-3)}{2}\left(\dfrac{+1}{2}x\right)^2 + \dfrac{(-2)(-3)(-4)}{6}\left(\dfrac{+1}{2}x\right)^3$

$3(1 + 3x + 9x^2 + 27x^3) + 1 - x + \dfrac{3}{4}x^2 - \dfrac{1}{2}x^3$

$3 + 9x + 27x^2 + 81x^3 + 1 - x + \dfrac{3}{4}x^2 - \dfrac{1}{2}x^3$

$\cancel{4} + 10x + \dfrac{111}{4}x^2 + \dfrac{163}{2}x^3$

$\underline{4 + 8x + \dfrac{111}{4}x^2 + \dfrac{161}{2}x^3}$