ALGEBRA AND FUNCTIONS

Partial fractions:
Methods for dealing with degree of numerator ≥ degree of denominator

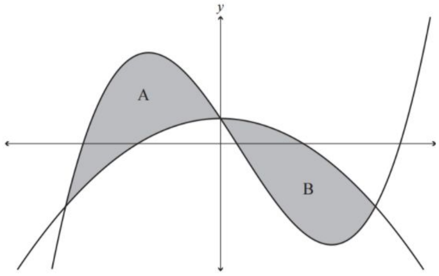
E.g. Partial fractions of the form

$$\frac{2x+3}{x(x-1)(2x+1)} \equiv \frac{A}{x} + \frac{B}{x-1} + \frac{C}{2x+1} \text{ and } \frac{5-x}{(x+2)(x-3)^2} \equiv \frac{A}{x+2} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$$

COORDINATE GEOMETRY

Changing equations of curves between Cartesian and parametric form

Finding area under curve:



$$\int y \frac{dx}{dt}\, dt$$

SEQUENCES AND SERIES

Expansion of (ax + b) n for any rational n and for |x| < $\frac{b}{a}$, using:

$$(1+x)^n = 1 + nx + {}^nC_2x^2 + \ldots + {}^nC_rx^r + \ldots + x^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!} + \ldots + x^n$$

$$\text{where } {}^nC_r = \frac{n!}{r!(n-r)!}$$

DIFFERENTIATION

- Implicit and parametric differentiation including applications to tangents and normals
- Exponential growth and decay

$$\frac{d(a^x)}{dx} = a^x \ln a$$

- Formation of differential equations

INTEGRATION

- Integration by substitution
- Integration by parts
- Integration of other trig functions can be found in ***formula book***
- Volume: Use $\int \pi y^2$ when rotating about x-axis

$$\int e^x\, dx = e^x + c \qquad \int e^{kx}\, dx = \frac{1}{k}e^{kx} + c$$

$$\int \frac{1}{x}\, dx = \ln|x| + c \qquad \int \frac{1}{ax}\, dx = \int \frac{1}{a}.\frac{1}{x}\, dx = \frac{1}{a}\ln|x| + c_1 \quad \text{or} \quad \int \frac{1}{ax}\, dx = \frac{1}{a}\ln|ax| + c_2$$

$$\int \cos kx\, dx = \frac{1}{k}\sin kx + c \qquad \int \sin kx\, dx = -\frac{1}{k}\cos kx + c \qquad \int \sec^2 kx\, dx = \frac{1}{k}\tan kx + c$$

$$\text{Use of } \int \frac{f'(x)}{f(x)}\, dx = \ln|f(x)| + c \quad \text{and} \quad \int f'(x)[f(x)]^n\, dx = \frac{[f(x)]^{n+1}}{n+1} + c$$

- Use of partial fractions in integration

Differential equations: first order separate variables

e.g. $\mathrm{f}(x)\mathrm{g}(y)\dfrac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{h}(x)\mathrm{k}(y) \quad \Rightarrow \quad \displaystyle\int \frac{\mathrm{g}(y)}{\mathrm{k}(y)}\,\mathrm{d}y = \int \frac{\mathrm{h}(x)}{\mathrm{f}(x)}\,\mathrm{d}x$

Trapezium rule (expanded)

$$\int_a^b \mathrm{f}(x)\,\mathrm{d}x \approx \tfrac{1}{2}h[y_0 + y_n + 2(y_1 + \ldots + y_{n-1})] \quad \text{where } y_i = \mathrm{f}(a + ih) \quad \text{and} \quad h = \frac{b-a}{n}$$

VECTORS

If $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, $\quad |\mathbf{a}| = \sqrt{(x^2 + y^2 + z^2)}$

If $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, the unit vector in the direction of $\mathbf{a}$ is $[(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \div \sqrt{(x^2 + y^2 + z^2)}]$

Scalar product

If $\mathbf{OP} = \mathbf{p} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $\mathbf{OQ} = \mathbf{q} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ and $\angle POQ = \theta$, then

$\mathbf{p}.\mathbf{q} = |\mathbf{p}||\mathbf{q}|\cos\theta$ and $\mathbf{p}.\mathbf{q} = (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \,.\, (a\mathbf{i} + b\mathbf{j} + c\mathbf{k}) = xa + by + cz$

If OP and OQ are perpendicular, **p.q** = 0

Vector equation of line where **a** is the position vector of a point on the line and **m** is a vector parallel to the line:

r = a + λm

Vector equation of line where **a** and **b** are the position vectors of points on the line:

r = a + λ(b – a)