## ALGEBRA AND FUNCTIONS

- Algebraic division by (x ± a)
- **Remainder theorem:** When f(x) is divided by (x – a), f(x) = (x – a)Q(x) + R where Q(x) is the quotient and R is the remainder
- **Factor theorem:** If f(a) = 0 then (x – a) is a factor of f(x)

## SEQUENCES AND SERIES

### Geometric series:

- each term is obtained from the previous term by multiplying by a constant called the common ratio, **r**

**nth term:** $ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r},$$

**Sum to infinity:**

$$S_\infty = \frac{a}{1-r} \text{ where } |r| < 1$$

## COORDINATE GEOMETRY

- Circle, centre (0, 0) radius r: $x^2 + y^2 = r^2$
- Circle centre (a, b) radius r: $(x - a)^2 + (y - b)^2 = r^2$

### Useful circle facts:

- The angle between the tangent and the radius is 90°
- Tangents drawn from a common point to a circle are equal in length
- The centre of a circle is on the perpendicular bisector of any chord
- The angle subtended by a diameter at the circumference is 90°

### Binomial expansion

The following expansions are valid for all n ∈ N:

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-1}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n$$

$$(1+x)^n = 1 + nx + {}^nC_2 x^2 + \dots + {}^nC_r x^r + \dots + x^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!} + \dots + x^n$$

$$\text{where } {}^nC_r = \frac{n!}{r!(n-r)!}$$

## TRIGONOMETRY

### Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

### Cosine rule

$$a^2 = b^2 + c^2 - 2bc\cos A \quad \text{or} \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

**Area of triangle ABC = ½absinC**

$$\sin x° = \cos(90 - x)°, \ \cos x° = \sin(90 - x)°, \ \tan x° = \frac{1}{\tan(90° - x)}$$

### Graphs of trigonometric functions

$$\sin(-x) = -\sin x, \ \cos(-x) = \cos x, \ \tan(-x) = \tan x$$

$$\sin 30° = \cos 60° = ½, \ \cos 30° = \sin 60° = \tfrac{\sqrt{3}}{2}, \ \tan 30° = \tfrac{1}{\sqrt{3}}, \ \tan 60° = \sqrt{3}$$

$$\cos 45° = \sin 45° = \tfrac{1}{\sqrt{2}}, \ \tan 45° = 1$$

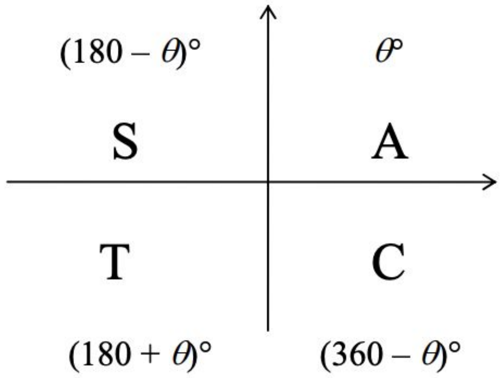
### Degrees / Radians conversion

| Degrees | 360° | 180° | 90° | 45° | 60° | 30° | 270° | 120° | 135° | etc |
|---|---|---|---|---|---|---|---|---|---|---|
| Radians | $2\pi$ | $\pi$ | $\frac{\pi}{2}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{6}$ | $\frac{3\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | |

Arc length = **r$\theta$** , Area of sector = **½r²$\theta$**

### Trig identities

$$\cos^2\theta + \sin^2\theta = 1, \quad \tan\theta = \frac{\sin\theta}{\cos\theta}$$




## EXPONENTIALS AND LOGARITHMS

If $y = a^x$ then $\log_a y = x$

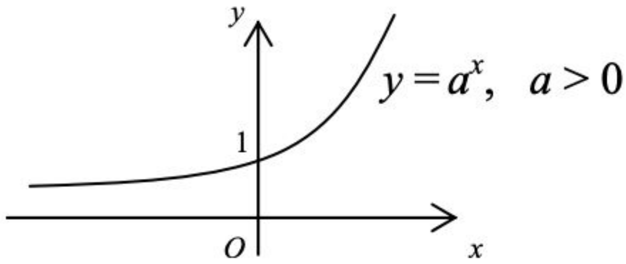
### Laws of logarithms:

$$\log_a pq = \log_a p + \log_a q, \quad \log_a \frac{p}{q} = \log_a p - \log_a q, \quad \log_a x^n = n.\log_a x$$

$$\log_a x = \frac{\log_b x}{\log_b a}, \quad \log_a 1 = 0, \quad \log_a a = 1$$

$f : x \to a^x$ $x \in \mathbb{R}$ $a > 0$ (a is constant) y is an exponential function, e.g. $6^{3x+4}$

Solving equations of the form $a^x = b$




## DIFFERENTIATION

The stationary point is a **minimum** turning point if:

$$\frac{dy}{dx} = 0 \text{ and } \frac{d^2y}{dx^2} > 0$$

The stationary point is a **maximum** turning point if:

$$\frac{dy}{dx} = 0 \text{ and } \frac{d^2y}{dx^2} < 0$$

**Increasing function:**

$$\frac{dy}{dx} > 0,$$

**Decreasing function:**

$$\frac{dy}{dx} < 0$$

### Maxima and minima problems:

- Find the point at which f'(x) = 0
- Find the nature of the turning point to confirm if value is a minimum or a maximum.
  - As well as value of x at which it occurs

## INTEGRATION

$$\text{If } \int f(x)\,dx = F(x) + c \text{ then } \int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$$

$$\text{If } y > 0 \text{ for } a \le x \le b\text{, then area is given by } A = \int_a^b y\,dx$$

### Trapezium rule

$$\int_a^b f(x)\,dx \approx \tfrac{1}{2}h[y_0 + y_n + 2(y_1 + \ldots + y_{n-1})] \quad \text{where } y_i = f(a + ih) \text{ and } h = \frac{b-a}{n}$$