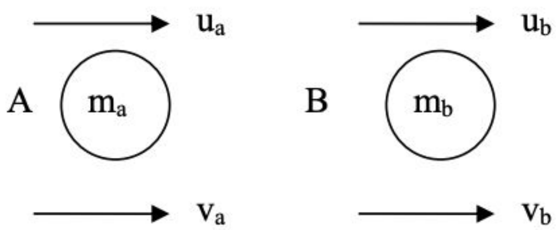
MOMENTUM, IMPULSE & MOMENTS

Momentum

- If an object of mass **m** has velocity **v**, then the momentum of the object is its mass **x** velocity, i.e.
    - **Momentum = mv**
    - Momentum is a vector quantity because it has both a magnitude and direction associated with it. Its units are kg ms-1 or Newton seconds Ns.

Conservation law

- For a system of interacting particles, the total momentum of a system remains constant when there is no resultant external force acting. If two particles collide:



- Where:
    - $\mathbf{m_a}$ = mass of particle A
    - $\mathbf{m_b}$ = mass of particle B
    - $\mathbf{u_a}$ = velocity of particle A before collision
    - $\mathbf{u_b}$ = velocity of particle B before collision
    - $\mathbf{v_a}$ = velocity of particle A after collision
    - $\mathbf{v_b}$ = velocity of particle B after collision
- Then, the principle of conservation of momentum is:

$$\mathbf{m_a u_a + m_b u_b = m_a v_a + m_b v_b}$$

    - i.e. Total momentum before collision = Total momentum after collision
- Direction of the velocities matter and must be considered in equation

Impulse

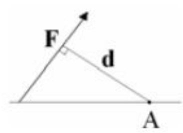
- The impulse of a force **F**, acting for a short time, **t**, on a body is the quantity **Ft**, i.e.
    - **Impulse = Ft**
- The impulse of F can be given by:

$$\int \mathbf{F}dt.$$

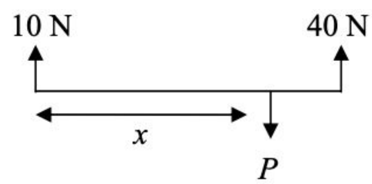
- For motion in one dimension it can be shown that:
    - The impulse of F = Change in momentum = m**v** - m**u**
    - Where **u** is the initial velocity and **v** is the final velocity
- Impulse is often found indirectly by reviewing the change in momentum.
    - This is very useful when the force and time are unknown but both the momentum before and after are know (or can be easily found).

Moments (moment of a force)

- The moment of the force F, about an axis through A, perpendicular to the plane containing A and F, is Fd



- Moments taken about a hinge or a fulcrum, as the reaction has no moment about that point
- **Example:**

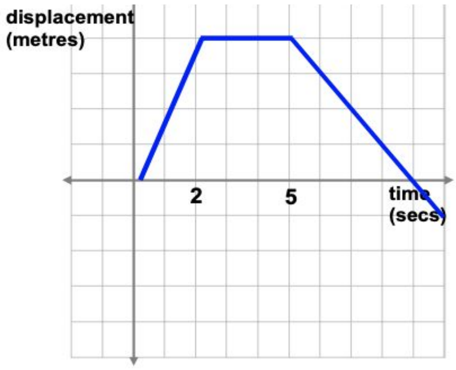


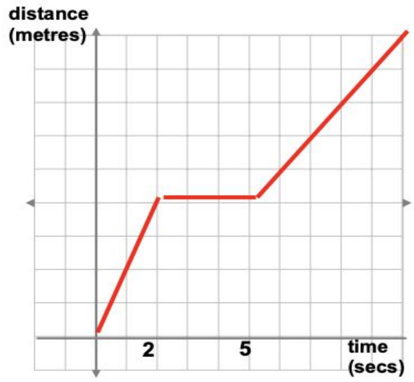
Resolving vertically: $10 + 40 - P = 0, \Rightarrow P = 50$ N
Taking moments about left-hand end:

$$40 \times 1.5 - Px = 0$$
$$50x = 60$$
$$x = \frac{6}{5} = 1.2 \text{ m}$$

MOTION GRAPHS

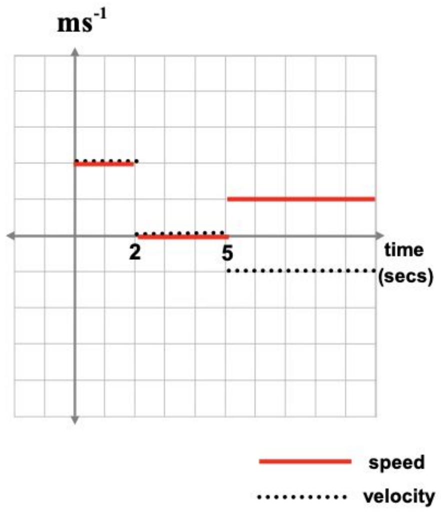
Displacement-time graphs and distance-time graphs





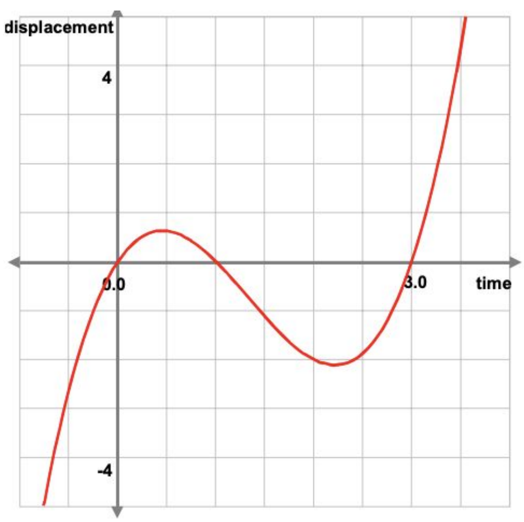
- **Displacement time** graph plots (metres) the displacement of the object from some fixed origin against time.
- For example, the journey described in the graph to the right can be described as follows,
    - **Phase 1:** The object moves away from the origin (at a constant speed) for 2 seconds
    - **Phase 2:** It remains motionless for 3 seconds.
    - **Phase 3:** It starts moving back towards the origin at a constant speed returning to the origin after about 4 seconds and then continues in the same direction another second.
- Gradient of the distance-time graph is the modulus of the gradient of the displacement-time graph

Velocity-time graphs and speed-time graphs



- **Velocity time graph** plots the velocity of the object against time
- **Speed-time graph** plots speed against time.
- For example for the journey we looked at in the displacement-time graph over the page the velocity-time graphs is drawn to the right as a dotted line and the speed time graph is drawn in as a solid line.
    - **Phase 1:** Object moving in direction (which has been designated the "positive direction" at constant speed of 2 ms^{-1}.
    - **Phase 2:** Stationary, zero velocity
    - **Phase 3:** Moving in the opposite direction to phase 1(the negative direction) at a constant speed of 1 , and therefore with a velocity of –1 .

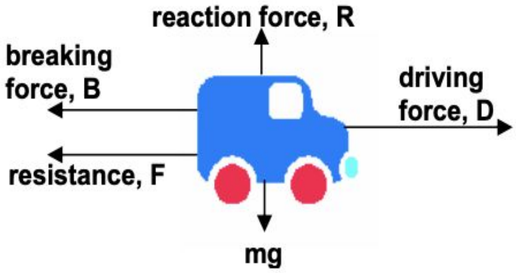
Interpreting features of a graph



- Gradient of velocity time graph = **acceleration**
- The area underneath (taking area below the x-axis to have a negative sign) a velocity time graph = **displacement.**
- The gradient of a displacement-time graph = **velocity**
- The gradient of a distance-time graph = **speed.**
- **In cases of non-constant acceleration:**
    - Gradients and areas calculated via differentiation and integration

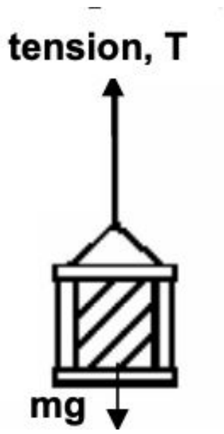
NEWTON'S LAWS: ALONG A LINE

Moving in the horizontal plane



- When the vehicle is moving along a horizontal road, the reaction force and the weight force (labeled *mg* in the diagram) are equal and opposite i.e. **R = mg** and as they are perpendicular to the horizontal motion they do not contribute (unless Friction is involved, which is related to R).
- So the resultant force acts in the horizontal direction and equals (taking right as positive): **D – B – F**

Moving in the vertical plane

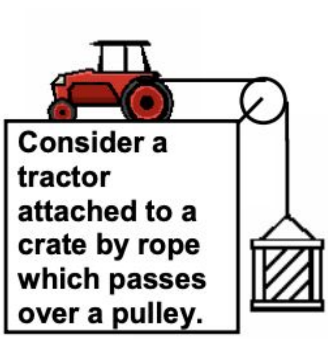


- The only forces acting on the crate are in the vertical direction. Hence the resultant force acts in the vertical direction.
- If the tension is greater than mg the crate will accelerate upwards (or decelerate if the crate is already moving down).
- If the tension equals mg the crate will maintain its current velocity (or remain stationary).
- If the tension is less than mg the crate will accelerate downwards (or decelerate if the crate is already moving upwards).

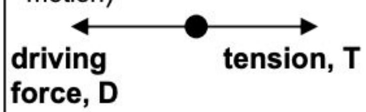
Pulleys

- The tension in a rope passing over a pulley is constant throughout the length of the rope
  - *(assumption: no friction in the pulley)*

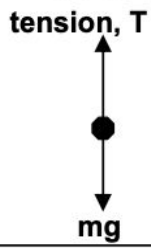
- The value of T is the same in all three of the below force diagrams.



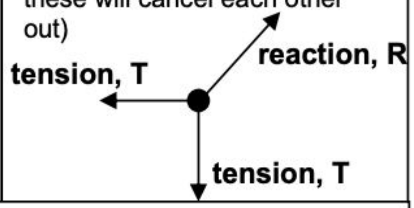
Forces acting on tractor
(ignoring vertical forces which are perpendicular to the motion and hence do not contribute to the horizontal motion)



Forces acting on crate



Forces acting on pulley (of negligible mass)
(assuming the pulley is fixed, these will cancel each other out)



CONSTANT ACCELERATION & “SUVAT” EQUATIONS

Constant acceleration

- Problems involving bodies or systems acted upon by constant forces often begin by calculating the acceleration using **Newton’s second law;**
  - once the acceleration has been found, they become suvat problems.

- Variables of **SUVAT** equations
  - **u** = initial velocity
  - **v** = velocity after t seconds
  - **a** = acceleration
  - **t** = time
  - **s** = displacement (from the initial displacement) at time t.

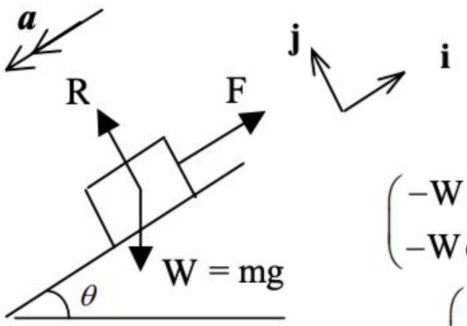
- SUVAT equations only used for objects moving under constant acceleration.

- If over the course of a journey, the acceleration changes from one constant rate to another constant rate.
  - the SUVAT equations must be applied to each leg of the journey separately.
  - The final velocity for the first leg will be the initial velocity for the second leg.

VECTORS AND NEWTON’S LAW IN 2 DIMENSIONS

Resolving forces into components (horizontally and vertically)

- Forces must be resolved horizontally and vertically, or parallel and perpendicular to a slope.
- Newton’s second law applied: **F** = m**a**



$$\begin{pmatrix} -W\sin\theta \\ -W\cos\theta \end{pmatrix} + \begin{pmatrix} F \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ R \end{pmatrix}$$

$$= m\begin{pmatrix} -a \\ 0 \end{pmatrix}$$