ALGEBRA AND FUNCTIONS

- Simplifying rational expressions
  - Use factorisation and find common denominators
- Domain and range of functions
- Inverse function:
  - $f^{-1}(x)$ [$ff^{-1}(x) = f^{-1}f(x) = x$]
- Domain of f = range of f^{-1}
  - Range of f = domain of f^{-1}
- Composite functions e.g. fg(x)
- Modulus function

Transformations (expanded)

Transformation	Solution
y = f(x) + a *For a > 0*	Translate y=f(x) through $\begin{pmatrix} 0 \\ a \end{pmatrix}$
y = f(x + a) *For a > 0*	Translate y = f(x) through $\begin{pmatrix} -a \\ 0 \end{pmatrix}$
y = af(x) *For a > 0*	Stretch y = f(x) parallel to y-axis with scale factor **a**
y = f(ax) *For a > 0*	Stretch y = f(x) parallel to x-axis with scale factor $\frac{1}{a}$
y = |f(x)|	For y ≥ 0, sketch y = f(x) For y < 0, reflect y = f(x) in the x-axis
y = f(|x|)	For x ≥ 0, sketch y = f(x) For x < 0, reflect [y = f(x) for x > 0] in the y-axis
y = –f(x)	Reflect y = f(x) in the x-axis (line y = 0)
y = f(–x)	Reflect y = f(x) in the y-axis (line x = 0)

TRIGONOMETRY

Trig identities (expanded)

$$\sec x = \frac{1}{\cos x} \qquad \operatorname{cosec} x = \frac{1}{\sin x}$$

$$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

$$\sin^2 x + \cos^2 x = 1; \qquad 1 + \tan^2 x = \sec^2 x;$$

$$1 + \cot^2 x = \operatorname{cosec}^2 x$$

Trig identities: Double angle formulas

- sin2x = 2 sinx cosx
- $\cos 2x = \cos^2 x - \sin^2 x$
  - $2\cos^2 x - 1$
  - $1 - 2\sin^2 x$
- $\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

Graphs of inverse trig functions

$$-\pi/2 \le \arcsin x \le \pi/2 \qquad 0 \le \arccos x \le \pi \qquad -\pi/2 < \arctan x < \pi/2$$

Expressing acosθ + bsinθ in the form rcos(θ ± α) or rsin(θ ± α) and applications (e.g. solving equations, maxima, minima)

DIFFERENTIATION

$$\frac{d(e^x)}{dx} = e^x \qquad \frac{d(e^{kx})}{dx} = ke^{kx} \qquad \frac{d(\ln x)}{dx} = \frac{1}{x} \qquad \frac{d(\ln kx)}{dx} = \frac{1}{x}$$

$$\frac{d(\sin kx)}{dx} = k \cos kx \qquad \frac{d(\cos kx)}{dx} = -k \sin kx \qquad \frac{d(\tan kx)}{dx} = k \sec^2 kx$$

Differentiation rules

Chain rule $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$$

Product rule $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

Quotient rule $\frac{d(\frac{u}{v})}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

NUMERICAL METHODS

- **For a continuous function,** a change in sign of f(x) in the interval (a, b) ⇒ a root of f(x) = 0 in the interval (a, b)
- **Accuracy of roots** by choosing an interval (e.g. 1.47 to 2 d.pl. test f(1.465) and f(1.475) for change of sign)
- **Iterative methods:** rearranging equations in the form xn+1 = f(xn) and using repeated iterations