ALGEBRA AND FUNCTIONS

Indices

i) $a^0 = 1$

ii) $a^m \times a^n = a^{m+n}$

iii) $\dfrac{a^m}{a^n} = a^{m-n}$

iv) $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

v) $(a^m)^n = a^{mn}$

vi) $a^{\frac{1}{n}} = \sqrt[n]{a}$

vii) $a^{-n} = \dfrac{1}{a^n}$

In general,

$\sqrt{x \pm y} \neq \sqrt{x} \pm \sqrt{y}$

Surds

i) $\sqrt{xy} = \sqrt{x} \times \sqrt{y}$

ii) $\sqrt{\dfrac{x}{y}} = \dfrac{\sqrt{x}}{\sqrt{y}}$

iii) $a\sqrt{x} \pm b\sqrt{x} = (a \pm b)\sqrt{x}$

Rationalising Surds

Given that $\dfrac{1}{\sqrt{a}}$ multiply by $\dfrac{\sqrt{a}}{\sqrt{a}}$.

Given that $\dfrac{1}{a \pm \sqrt{b}}$ multiply by $\dfrac{a \mp \sqrt{b}}{a \mp \sqrt{b}}$

Quadratic functions

If f(x) = $ax^2 + bx + c$, the discriminant is:

$b^2 - 4ac$

For f(x) = 0 ...

$b^2 - 4ac < 0 \Rightarrow$ **two unreal roots**

$b^2 - 4ac > 0 \Rightarrow$ **two real, distinct roots**

$b^2 - 4ac = 0 \Rightarrow$ **two real, equal roots,**

Factorisation and Complete the square

If f(x) = 0, then:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sketching the quadratic functions

- Finding the point of intersection with the y-axis:
  - Put x = 0 in y = f(x)
- Finding the point of intersection with x-axis:
  - Solve f(x) = 0
- Finding the maximum/minimum point:
  - Use completing the square,
  - Symmetry OR
  - Solve f'(x) = 0

Transformations

Transformation (for a > 0)	Solution
y = f(x) + a	Translate y=f(x) through $\begin{pmatrix} 0 \\ a \end{pmatrix}$
y = f(x + a)	Translate y = f(x) through $\begin{pmatrix} -a \\ 0 \end{pmatrix}$
y = af(x)	Stretch y = f(x) parallel to y-axis with scale factor **a**
y = f(ax)	Stretch y = f(x) parallel to x-axis with scale factor $\dfrac{1}{a}$

COORDINATE GEOMETRY

Given points $P(x_1, y_1)$ and $Q(x_2, y_2)$

- Gradient of PQ = $\dfrac{y_2 - y_1}{x_2 - x_1}$
- Distance PQ =

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of PQ: $M\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

Given gradient of l_1 is m_1, gradient of line l_2 is m_2:

- If l_1 parallel to l_2:
  - $m_1 = m_2$
- If l1 is perpendicular to l_2:
  - $m_1 \times m_2 = -1$

Equation of straight line

- Given gradient, m, and vertical intercept (0, c):
  - **y = mx + c**
- Given a point P (x_1, y_1) and the gradient, m:
  - $\mathbf{y - y_1 = m(x - x_1)}$
- Given two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ on the line:
  - $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$

SEQUENCES AND SERIES

Sigma notation e.g. $\sum_{r=1}^{4} (2r + 5) = 7 + 9 + 11 + 13$

$u_{n+1} = 3u_n + 5,\ n \geq 1,\ u_1 = -2$

- The first 5 terms of this sequence are:
  - −2, −1, 2, 11 and 38

Arithmetic series:

- each term is obtained from the previous term by adding a constant called the common difference, **d**

nth term: a + (n - 1)d

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{n}{2}[a + l] \text{ where last term } l = a + (n-1)d$$

Sum of first n natural numbers:

1 + 2 + 3 + ... + n:

$$S_n = \frac{n}{2}(n+1)$$

DIFFERENTIATION

Notation:

$$\text{If} \quad y = \mathrm{f}(x) \quad \text{then} \quad \frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}'(x) \text{ and } \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \mathrm{f}''(x)$$

y	dy/dx
ax^n	anx^{n-1} (a is constant)
$f(x) \pm g(x)$	$f'(x) \pm g'(x)$

Equation of tangents and normals:

Useful facts for these questions:

- Gradient of a tangent to a curve = **dy/dx**
- The normal to a curve at a particular point is perpendicular to the tangent at that point
- If two perpendicular lines have gradients m_1 and m_2 then $m_1 \times m_2 = -1$
- The equation of a line through (x_1, y_1) with gradient m is $y - y_1 = m(x - x_1)$

INTEGRATION

$$\int ax^n \, \mathrm{d}x = \frac{ax^{n+1}}{n+1} + c \quad \text{provided } n \neq -1$$

$$\int (\mathrm{f}'(x) + \mathrm{g}'(x)) \, \mathrm{d}x = \mathrm{f}(x) + \mathrm{g}(x) + c$$