Tower of Hanoi

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1 Introduction

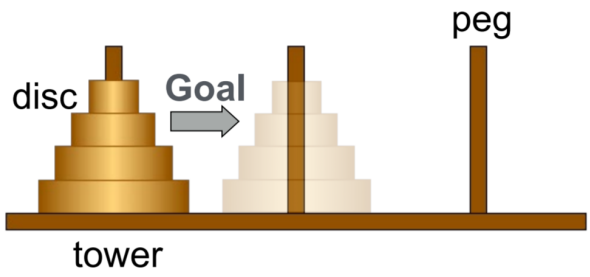

Figure 1: The purpose of Tower of Hanoi

Tower of Hanoi is a puzzle consisting of three pegs and many different-sized disks. The purpose of this puzzle is to move all disks to another peg obeying three rules:

1. Only one disk can be moved at one time.
2. Only the top disk of each peg can be moved.
3. No larger disk is placed above a smaller disk.

It is proved that the puzzle can be played with any number of disks, and the minimum times to solve is $2^n - 1$ [1].

Therefore, this project aims to solve the Tower of Hanoi Puzzle by receiving the number of disks and showing the solution to move all disks from the first peg to the second peg step-by-step. Moreover, this project illustrates the solution as a model of the puzzle in order that the solution would be easier to understand.

2 Template

The template of this project is a list with three lists inside, and inside each list there are numbers. The three inside lists represent the three pegs of Tower of

Hanoi, and the numbers inside each list represents the disks on each peg. The bigger number, the larger disk. Hence, the bigger numbers must be on the left side of the smaller numbers according to the rule. As in the Figure 2, this step is represented by the list [[3],[2,1],[]]. The variable used to keep the template is "pegs".

Figure 2: Tower of Hanoi in the template [[3],[2,1],[]]

3 Used Function

To solve Tower of Hanoi, this project uses 4 functions: print_pegs, move_disks, move_towers and hanoi.

3.1 Function print_pegs(pegs)

print_pegs is a function used to illustrate the model of each step from the template. The input of this function is the template pegs. For example, with the input [[3],[2,1],[]], this function prints out the model as in Figure 3. Then, this function can be applied to the move_disk function. (See Section 3.2)

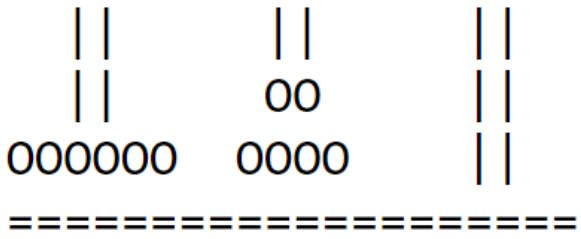

Figure 3: Tower of Hanoi model from the function print_pegs

3.2 Function move_disk(pegs, source, dest)

move_disk function receive the input pegs, source and dest representing the template, the beginning peg and the destination peg respectively. This function

moves the top disk of the beginning peg to the destination peg obeying to the rules. (If it violates the rules, a warning will appear.)

To simply describe this function, in the template, this function will pop out the last member of the source list and add it to the dest list using two list functions, pop and append.

3.3 Function move_tower(pegs, nb_disk, source, dest)

move_tower is a recursive function. The main point of this function is to move all disks except the base (nb_disk-1 disks) to spare peg (another peg that is not source or dest), move the base disk to dest peg and then move all disks at the spare peg (that are just moved) to the dest peg.

However, there is a rule that only one disk can be moved at one time, so the recursive function is applied. Before, the function moves nb_disk-1 disks, the functions moved nb_disk-2 disks to the dest peg, and, to do that, more functions were doing the same thing with nb_disk-3, nb_disk-4,... disks before. Therefore, this function checks if nb_disk equals 0 or not. If so, the function will pass to let the the base disk move to the dest peg. If not, the function will recursively do until the nb_disk is 0. Then, the recursive function can move nb_disk disks to another peg.

This function uses the function move_disk to move the base disk to the desk peg.

3.4 Function hanoi(n)

hanoi includes print_pegs function and move_tower function. This function receives the number of starting disks and prints each step of the solution. This function is used in the main program to get the solution.

References

[1] Miodrag Petkovic. *Famous puzzles of great mathematicians.* American Mathematical Society, 2013.