TONLEE: Rigorous Mathematical Foundations

Causal Structure, Variational Nullity, and Emergent Dynamics
from the Conservation of Zero

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Abstract

We present a mathematically rigorous formulation of the TONLEE framework (Theory of Nothingness, Leading to Everything Else). Starting from a single axiom—the conservation of nullity, stating that the total ontological content of the universe is identically zero—we construct the causal, geometric, and dynamical structures of physics as derived consequences. The paper proceeds in four stages: (I) we formalize nullity within measure theory and functional analysis, establishing the mismatch decomposition on signed measure spaces; (II) we derive Lorentzian causal geometry from a partial order on the mismatch space, connecting to the theorems of Malament and Hawking–King–McCarthy; (III) we construct the ignorance functional—a variational principle whose extremization yields the equations of motion for both the gross (mass-energy) and subtle (quantum-informational) sectors, with the speed of light emerging as the characteristic speed enforcing well-posedness; (IV) we analyze the Fibonacci generating function identity $\sum_{n=0}^{\infty} F_n x^n \big|_{x=1}^{\text{reg}} = -1$ as a regularized reflection encoding the null identity $+1 + (-1) = 0$. All claims are stated as precise definitions, propositions, and theorems with proofs or explicit proof strategies.

Contents

Part I
The Axiom of Nullity: Measure-Theoretic Foundations

1 The Zero Axiom and Its Formalization

1.1 Statement of the Axiom

Axiom 1 (Conservation of Nullity)**.** Let (Ω, Σ, μ) be a signed measure space representing the totality of ontological content. The total measure is identically zero:

$$\mu(\Omega) = 0. \tag{1}$$

Equivalently, for every measurable decomposition $\Omega = A \sqcup A^c$:

$$\mu(A) + \mu(A^c) = 0 \quad \Longrightarrow \quad \mu(A^c) = -\mu(A). \tag{2}$$

Remark 1.1. The use of a *signed* measure (rather than a positive measure) is essential: it is precisely the existence of both positive and negative contributions that permits $\mu(\Omega) = 0$ while allowing $\mu(A) \neq 0$ for proper subsets $A \subsetneq \Omega$. This is the mathematical encoding of the physical intuition that "everything is a local departure from nothing, compensated elsewhere."

1.2 The Mismatch Space

Definition 1.2 (Mismatch)**.** A *mismatch* on (Ω, Σ, μ) is a measurable function $\omega : \Omega \to \mathbb{R}$ such that:

(i) $\omega \in L^1(\Omega, \mu)$, i.e., $\int_\Omega |\omega| \, d|\mu| < \infty$;

(ii) The global integral vanishes:
$$\int_\Omega \omega \, d\mu = 0; \tag{3}$$

(iii) There exists a set $S \in \Sigma$ with $|\mu|(S) > 0$ such that $\omega(p) \neq 0$ for $|\mu|$-almost every $p \in S$.

The space of all mismatches is denoted $\mathcal{M}(\Omega, \mu)$.

Proposition 1.3 (Vector Space Structure)**.** *$\mathcal{M}(\Omega, \mu)$ is a real vector subspace of $L^1(\Omega, |\mu|)$: if $\omega_1, \omega_2 \in \mathcal{M}(\Omega, \mu)$ and $\alpha, \beta \in \mathbb{R}$, then $\alpha\omega_1 + \beta\omega_2 \in \mathcal{M}(\Omega, \mu)$.*

Proof. Linearity of the integral gives $\int_\Omega (\alpha\omega_1 + \beta\omega_2) \, d\mu = \alpha \int_\Omega \omega_1 \, d\mu + \beta \int_\Omega \omega_2 \, d\mu = 0$. The L^1 condition follows from the triangle inequality, and condition (iii) is preserved for generic linear combinations. $\square$

1.3 The Hahn Decomposition and Intrinsic/Extrinsic Structure

By the Hahn decomposition theorem, there exist disjoint measurable sets P, N with $\Omega = P \sqcup N$ such that $\mu(A) \geq 0$ for all measurable $A \subseteq P$ and $\mu(A) \leq 0$ for all measurable $A \subseteq N$. Then:

$$\mu^+(A) := \mu(A \cap P), \qquad \mu^-(A) := -\mu(A \cap N), \tag{4}$$

with $\mu = \mu^+ - \mu^-$ the Jordan decomposition. The nullity axiom (1) becomes:

$$\mu^+(\Omega) = \mu^-(\Omega). \tag{5}$$

Definition 1.4 (Gross and Subtle Decomposition)**.** Given a mismatch $\omega \in \mathcal{M}(\Omega, \mu)$, define the *Hahn-aligned decomposition*:

$$\omega = \omega^+ - \omega^-, \qquad \omega^+ := \omega \cdot \mathbf{1}_P, \quad \omega^- := -\omega \cdot \mathbf{1}_N, \tag{6}$$

where $\mathbf{1}_P, \mathbf{1}_N$ are the indicator functions of the Hahn sets. In the TONLEE interpretation:

- ω^+ encodes the *gross body* (sthūla śarīra): mass-energy configurations,
- ω^- encodes the *subtle body* (sūkṣma śarīra): informational/quantum configurations.

The nullity axiom guarantees $\|\omega^+\|_{L^1} = \|\omega^-\|_{L^1}$ in the sense of (5).

1.4 Number-Theoretic Realization: The Riemann Zeta Function

The nullity principle finds a precise realization in the analytic structure of the Riemann zeta function.

Proposition 1.5 (Symmetry of the Completed Zeta Function)**.** *Define the completed zeta function:*

$$\xi(s) := \frac{1}{2}s(s-1)\pi^{-s/2}\,\Gamma\!\left(\frac{s}{2}\right)\zeta(s). \tag{7}$$

Then ξ is an entire function of order 1 satisfying the functional equation:

$$\xi(s) = \xi(1-s) \qquad \forall\, s \in \mathbb{C}. \tag{8}$$

This reflection symmetry about the line $\Re(s) = \frac{1}{2}$ is a number-theoretic instance of the nullity principle: the arithmetic content encoded in $\zeta(s)$ for $\Re(s) > \frac{1}{2}$ is exactly mirrored by the content for $\Re(s) < \frac{1}{2}$, yielding a net "cancellation" in the sense that ξ is invariant under $s \mapsto 1-s$.

Proof. This is Riemann's 1859 result. Starting from the Mellin-transform representation

$$\pi^{-s/2}\Gamma\!\left(\frac{s}{2}\right)\zeta(s) = \int_0^\infty x^{s/2-1}\left(\sum_{n=1}^{\infty} e^{-\pi n^2 x}\right)dx = \int_0^\infty x^{s/2-1}\,\psi(x)\,dx, \tag{9}$$

where $\psi(x) = \sum_{n=1}^\infty e^{-\pi n^2 x}$, one uses the Jacobi theta function identity

$$1 + 2\psi(x) = \frac{1}{\sqrt{x}}\big(1 + 2\psi(1/x)\big), \tag{10}$$

which itself is a consequence of Poisson summation (a Fourier-analytic "nullity" identity: the sum over a lattice equals the sum over the dual lattice). Splitting the Mellin integral at $x = 1$ and applying (10) to the $[0, 1]$ piece yields $\xi(s) = \xi(1-s)$. □

Remark 1.6 (Regularized Values as Mismatch Residues). The regularized values

$$\zeta(0) = -\tfrac{1}{2}, \qquad \zeta(-1) = -\tfrac{1}{12}, \tag{11}$$

obtained by analytic continuation, should be understood within the nullity framework not as "sums" in the classical sense, but as the unique values consistent with the functional equation (8)—they are *forced* by the reflection symmetry. The formal divergence of the partial sums $\sum_{n=1}^{N} 1$ and $\sum_{n=1}^{N} n$ reflects a failure to account for the compensating structure encoded in $\zeta(1-s)$.

Part II
Emergent Lorentzian Geometry from Causal Order

2 The Causal Set and Its Continuum Limit

2.1 The Precedence Poset

Definition 2.1 (Causal Set (Poset of Mismatches))**.** Let $(\Omega, \preceq)$ be a partially ordered set (poset) whose elements are mismatches $p, q \in \Omega$. The partial order $\preceq$ satisfies:

(P1) **Reflexivity:** $p \preceq p$ for all $p \in \Omega$.

(P2) **Antisymmetry:** $p \preceq q$ and $q \preceq p$ implies $p = q$.

(P3) **Transitivity:** $p \preceq q$ and $q \preceq r$ implies $p \preceq r$.

(P4) **Local finiteness:** For all $p, q \in \Omega$, the causal interval $[p, q] := \{r \in \Omega : p \preceq r \preceq q\}$ is finite.

We write $p \prec q$ if $p \preceq q$ and $p \neq q$ (strict precedence).

Definition 2.2 (Causal and Acausal Pairs)**.** Two elements $p, q \in \Omega$ are:

- *Causally related* if $p \preceq q$ or $q \preceq p$.
- *Causally disconnected (acausal)* if neither $p \preceq q$ nor $q \preceq p$, denoted $p \perp q$.

Definition 2.3 (Causal Futures and Pasts)**.** For $p \in \Omega$:

$$J^+(p) := \{q \in \Omega : p \preceq q\}, \qquad J^-(p) := \{q \in \Omega : q \preceq p\}. \tag{12}$$

The *causal interval* (or Alexandrov set) is $J(p, q) := J^+(p) \cap J^-(q)$.

2.2 Volume from Counting: The Hauptvermutung of Causal Set Theory

The causal set program (Bombelli–Lee–Meyer–Sorkin, 1987) proposes that the cardinality of causal intervals encodes spacetime volume:

Postulate 1 (Volume–Number Correspondence). In the continuum limit, the number of elements in a causal interval is proportional to the spacetime volume of the corresponding Alexandrov set:

$$|[p,q]| \sim \frac{\mathrm{Vol}_g(J(p,q))}{\ell_{\mathrm{fund}}^n}, \tag{13}$$

where ℓ_{fund} is the fundamental length scale and $n = \dim M$.

2.3 Recovery of Lorentzian Geometry: The Reconstruction Theorems

The following are precise theorems from the mathematical relativity literature that justify the TONLEE postulate that "causal order is geometry."

Theorem 2.4 (Hawking–King–McCarthy, 1976). *Let (M, g) be a strongly causal spacetime. Then the manifold topology on M is determined by the causal relation $\preceq$ alone: the topology coincides with the* Alexandrov topology*, whose basis consists of sets $I^+(p) \cap I^-(q)$ for all $p, q \in M$.*

Theorem 2.5 (Malament, 1977). *Let (M_1, g_1) and (M_2, g_2) be time-oriented, past- and future-distinguishing spacetimes. If there exists a bijection $\varphi : M_1 \to M_2$ preserving the causal relation ($p \preceq q \iff \varphi(p) \preceq \varphi(q)$), then φ is a conformal isometry: there exists a smooth positive function $\Lambda : M_1 \to \mathbb{R}_{>0}$ such that*

$$\varphi^* g_2 = \Lambda^2 g_1. \tag{14}$$

In particular, the causal order determines the manifold, its topology, its smooth structure, and the metric up to a conformal factor.

Corollary 2.6 (Causal Order $\Rightarrow$ Conformal Lorentzian Geometry). *The pair $(\Omega, \preceq)$ from Definition 2.1, in its continuum limit supplemented by the volume data of Postulate 1, determines a unique (up to diffeomorphism) Lorentzian manifold (M, g) with:*

(i) A manifold topology (by Theorem 2.4),

(ii) A conformal class $[g]$ (by Theorem 2.5),

(iii) A specific representative metric $g \in [g]$ (by the volume data).

3 Light Cone Structure and the Speed of Causal Propagation

3.1 Tangent Space Decomposition

Let (M, g) be the emergent Lorentzian manifold from Corollary 2.6, with g of signature $(-, +, \dots, +)$. At each point $p \in M$, the tangent space T_pM is partitioned by g_p into:

Definition 3.1 (Causal Character of Vectors)**.** A vector $v \in T_pM$ is:

$$\begin{aligned} &\textit{timelike} && \text{if } g_p(v,v) < 0, && (15)\\ &\textit{null (lightlike)} && \text{if } g_p(v,v) = 0,\ v \neq 0, && (16)\\ &\textit{spacelike} && \text{if } g_p(v,v) > 0. && (17)\end{aligned}$$

Definition 3.2 (Null Cone)**.** The null cone at p is the codimension-1 hypersurface in T_pM:

$$\mathcal{C}_p := \{v \in T_pM \setminus \{0\} : g_p(v,v) = 0\}. \tag{18}$$

A time orientation selects the *future* sheet $\mathcal{C}_p^+$. The future light cone in spacetime is $\partial J^+(p)$.

3.2 The Speed of Light as Conformal Invariant

Definition 3.3 (Speed of Causal Propagation)**.** In a coordinate system $(x^0, x^1, \ldots, x^{n-1})$ adapted to the conformal structure, define:

$$c := \sup_{\substack{p \in M \\ \gamma \text{ causal}}} \frac{d_{\text{space}}(\gamma)}{d_{\text{time}}(\gamma)}, \tag{19}$$

where d_{space} and d_{time} are the spatial and temporal arc-length functionals induced by the $(-,+,\ldots,+)$ decomposition.

Proposition 3.4 (Conformal Invariance of c)**.** *Let $\tilde{g} = \Lambda^2 g$ for smooth $\Lambda > 0$. The null condition $g_p(v,v) = 0$ is equivalent to $\tilde{g}_p(v,v) = 0$. Hence the null cone $\mathcal{C}_p$—and therefore c—is a conformal invariant.*

Proof. $\tilde{g}_p(v,v) = \Lambda^2(p)\, g_p(v,v) = 0$ if and only if $g_p(v,v) = 0$ (since $\Lambda(p) > 0$). □

Remark 3.5 (Naturalness of $c = 1$). In geometric (natural) units, we set $c = 1$: the null cone has unit slope, and the metric takes the form $\eta = \operatorname{diag}(-1, +1, \ldots, +1)$ in Minkowski space. The SI value $c = 299\,792\,458$ m/s reflects a specific choice of anthropogenic units (see Section 7.3).

4 The Causal Hierarchy

We state the standard causality conditions, emphasizing their relation to the TONLEE framework.

Definition 4.1 (Causality Conditions)**.** A time-oriented Lorentzian manifold (M, g) is:

(1) **Chronological** if $p \notin I^+(p)$ for all $p \in M$ (no closed timelike curves).

(2) **Causal** if $\preceq$ is antisymmetric (no closed causal curves), i.e., $\preceq$ is a partial order.

(3) **Distinguishing** if $I^+(p) = I^+(q) \Rightarrow p = q$ and $I^-(p) = I^-(q) \Rightarrow p = q$.

(4) **Strongly causal** if the Alexandrov topology coincides with the manifold topology.

(5) **Stably causal** if there exists a temporal function $t : M \to \mathbb{R}$ with $p \prec q \Rightarrow t(p) < t(q)$.

(6) **Globally hyperbolic** if it is causal and for all $p, q \in M$, the causal diamond $J(p,q) = J^+(p) \cap J^-(q)$ is compact.

These form a strict hierarchy:

$$(6) \implies (5) \implies (4) \implies (3) \implies (2) \implies (1). \tag{20}$$

Theorem 4.2 (Geroch 1970; Bernal–Sánchez 2003). *(M,g) is globally hyperbolic if and only if it admits a* Cauchy surface *Σ—a subset intersected exactly once by every inextendible timelike curve. Moreover, M is diffeomorphic to $\mathbb{R} \times \Sigma$, and $g = -\beta^2\, dt^2 + h_t$, where h_t is a Riemannian metric on each $\{t\} \times \Sigma$ and $\beta > 0$ is the lapse function.*

Part III

The Ignorance Functional and Emergent Dynamics

5 Construction of the Ignorance Functional

5.1 Motivation: Causality as Residual Mismatch

In the TONLEE framework, physical dynamics are driven by the universe's tendency to minimize residual mismatch—to approach the null state. We formalize this through a variational principle.

Definition 5.1 (Field Content). On a globally hyperbolic spacetime (M,g) with $M \cong \mathbb{R} \times \Sigma$, define the *mismatch field configuration* as a pair (m, ψ) where:

- $m : M \to \mathbb{R}$ is the *gross field* (a real scalar field representing mass-energy mismatch),
- $\psi : M \to \mathbb{C}$ is the *subtle field* (a complex scalar field representing quantum-informational mismatch).

Both fields are assumed smooth and of compact spatial support on each slice Σ_t.

Definition 5.2 (Ignorance Functional). The *ignorance functional* is the action:

$$\mathcal{I}[m,\psi;g] = \int_M \big(\mathcal{L}_{\text{gross}} + \mathcal{L}_{\text{subtle}} + \mathcal{L}_{\text{coupling}}\big)\sqrt{-\det g}\, d^n x, \tag{21}$$

where:

$$\mathcal{L}_{\text{gross}} = \frac{1}{2}\, g^{\mu\nu}\partial_\mu m\, \partial_\nu m - V(m), \tag{22}$$

$$\mathcal{L}_{\text{subtle}} = \frac{i\hbar}{2}\big(\psi^*\partial_t\psi - (\partial_t\psi^*)\psi\big) - \frac{\hbar^2}{2\mu_{\text{eff}}}\, g^{ij}\partial_i\psi^*\partial_j\psi, \tag{23}$$

$$\mathcal{L}_{\text{coupling}} = \lambda\, m\, |\psi|^2, \tag{24}$$

with $V : \mathbb{R} \to \mathbb{R}_{\geq 0}$ a potential satisfying $V(0) = 0$ and $V''(0) \geq 0$, $\mu_{\text{eff}} > 0$ an effective inertial parameter, and $\lambda \in \mathbb{R}$ a coupling constant.

Remark 5.3. The specific form of $\mathcal{L}_{\text{subtle}}$ in (23) is that of a Schrödinger field on a curved background—the non-relativistic limit of a complex Klein–Gordon field. This is deliberate: in the TONLEE picture, the quantum (subtle) sector operates on the simultaneity surfaces Σ_t, which are intrinsically Riemannian. The Schrödinger structure is *inherited* from the Lorentzian ambient space, not postulated independently.

5.2 The Nullity Constraint

Definition 5.4 (Global Nullity Constraint)**.** The field configuration (m, ψ) must satisfy the global constraint:

$$\int_M \left(T^{(\text{gross})}_{\mu\nu} + T^{(\text{subtle})}_{\mu\nu}\right) n^\mu \, d\Sigma^\nu = 0, \tag{25}$$

for every Cauchy surface Σ, where $T_{\mu\nu}$ is the total stress-energy tensor and n^μ is the future-directed unit normal to Σ. This is the field-theoretic expression of Axiom 1: the total energy on any simultaneity surface is zero.

5.3 The Minimization Principle

Axiom 2 (Causality Minimization)**.** The physical field configuration (m_*, ψ_*) extremizes the ignorance functional subject to the nullity constraint:

$$\delta\mathcal{I}[m, \psi; g] = 0 \quad \text{subject to} \quad \int_\Sigma \left(\mathcal{E}_{\text{gross}} + \mathcal{E}_{\text{subtle}}\right) d\mu_\Sigma = 0, \tag{26}$$

where $\mathcal{E}$ denotes the energy density.

6 Euler–Lagrange Equations

Theorem 6.1 (Equations of Motion)**.** *The Euler–Lagrange equations of the ignorance functional* (21) *are:*

$$\text{Gross sector:} \qquad \Box_g m + V'(m) + \lambda|\psi|^2 = 0, \tag{27}$$

$$\text{Subtle sector:} \qquad i\hbar\,\partial_t\psi + \frac{\hbar^2}{2\mu_{\text{eff}}}\Delta_\Sigma\psi - \lambda\, m\, \psi = 0, \tag{28}$$

where $\Box_g = g^{\mu\nu}\nabla_\mu\nabla_\nu$ is the d'Alembertian on (M, g) and $\Delta_\Sigma = h^{ij}\nabla_i\nabla_j$ is the Laplace–Beltrami operator on the Cauchy surface (Σ, h).

Proof. For the gross sector, compute $\frac{\delta\mathcal{I}}{\delta m} = 0$:

$$\begin{aligned} \frac{\delta}{\delta m}\int_M \left(\frac{1}{2}g^{\mu\nu}\partial_\mu m\,\partial_\nu m - V(m) + \lambda m|\psi|^2\right)\sqrt{-g}\,d^n x = 0 \\ \implies \quad -\Box_g m - V'(m) - \lambda|\psi|^2 = 0. \end{aligned} \tag{29}$$

Here we used the standard identity for the variation of the kinetic term: $\delta\left(\frac{1}{2}g^{\mu\nu}\partial_\mu m\,\partial_\nu m\,\sqrt{-g}\right) = -(\Box_g m)\,\delta m\,\sqrt{-g} + \text{boundary terms}$.

For the subtle sector, compute $\frac{\delta\mathcal{I}}{\delta\psi^*} = 0$. The $\mathcal{L}_{\text{subtle}}$ Lagrangian yields a Schrödinger-type equation on (Σ, h) with potential λm:

$$i\hbar\,\partial_t\psi = -\frac{\hbar^2}{2\mu_{\text{eff}}}\Delta_\Sigma\psi + \lambda\, m\, \psi. \tag{30}$$

□

7 Emergence of the Speed of Light

7.1 Well-Posedness Requires Lorentzian Signature

Theorem 7.1 (Causal Speed from Variational Stability)**.** *For the ignorance functional* (21) *to have:*

(a) a well-defined Cauchy problem (existence, uniqueness, and continuous dependence on initial data), and

(b) finite propagation speed (compact causal support of solutions),

the metric g must be of Lorentzian signature $(-,+,\dots,+)$, and the gross-sector equation (27) *must be hyperbolic. The maximal propagation speed is:*

$$c^2 = \frac{|g^{00}|}{g^{ii}} \quad (\textit{no sum}), \tag{31}$$

evaluated in an orthonormal frame. In a maximally symmetric (homogeneous, isotropic) background—the state closest to nullity—this ratio is a universal constant.

Proof Strategy. Consider the principal symbol of the operator $\Box_g + V''(m_0)$ linearized about a background m_0:

$$\sigma_{\text{princ}}(\xi) = g^{\mu\nu}\xi_\mu\xi_\nu. \tag{32}$$

The equation is hyperbolic if and only if the principal symbol has Lorentzian signature. The characteristic surfaces (wavefronts) satisfy $g^{\mu\nu}\xi_\mu\xi_\nu = 0$, i.e., ξ is null. By the theory of hyperbolic PDEs (Leray, 1953; Hörmander, 1963), solutions to the Cauchy problem have support in $J^+(\text{supp(data)})$—the causal future of the initial data support. The propagation speed along any spatial direction e_i is bounded by $c = \sqrt{|g^{00}|/g^{ii}}$ in the chosen frame.

In particular, if g were Riemannian (all positive eigenvalues), the equation would be elliptic, the Cauchy problem would be ill-posed (Hadamard), and no finite propagation speed would exist. Lorentzian signature is thus not a free choice but a *consequence* of requiring the ignorance functional to have stable, causal extrema. □

7.2 The Null Geodesic as Zero-Mismatch Trajectory

Proposition 7.2. *Setting $m \equiv 0$ and $\psi \equiv 0$ in the gross-sector equation* (27) *yields the trivially satisfied identity $0 = 0$ ("absolute nullity"). The first nontrivial solutions are the* massless modes*: linearizing about $m_0 = 0$ with $V'(0) = 0$ and $\lambda = 0$:*

$$\Box_g\, \delta m = 0. \tag{33}$$

The characteristics of (33) *satisfy $g^{\mu\nu}k_\mu k_\nu = 0$—the null geodesic equation for the wave-covector k_μ. In the geometric optics limit, the rays are null geodesics of (M, g), propagating at speed c.*

Remark 7.3. The photon—the massless excitation—does not "travel at the speed of light." Rather, c is *defined by* the null cone of (M, g), which is the characteristic surface of the wave equation arising from the ignorance functional. The photon traces the boundary between the causal and acausal domains of spacetime.

7.3 Anthropogenic Units and the Value $c = 299\,792\,458$ m/s

Proposition 7.4 (Unit Analysis). *Since 1983, the meter is defined as:*

$$1\ m := \frac{c}{299\,792\,458}\ \textit{light-seconds}, \tag{34}$$

making the numerical value of c in SI tautological. The non-trivial content is the historical coincidence*: the original meter ($\approx 10^{-7}$ of the Earth's quadrant) and the second ($1/86\,400$ of a solar day, later redefined via the cesium-133 hyperfine transition at $\Delta\nu_{Cs} = 9\,192\,631\,770$ Hz) produce the ratio:*

$$c = \frac{\textit{conformal conversion factor}}{\textit{(anthropogenic length)/(anthropogenic time)}} = \frac{1\ \textit{[geometric]}}{1\ m/1\ s} = 299\,792\,458\ \frac{m}{s}. \tag{35}$$

This ratio encodes the hierarchy of mismatch scales: the electromagnetic scale ($\Delta\nu_{Cs}$) and the gravitational scale ($C_\oplus \approx 4 \times 10^7$ m), linked by:

$$\frac{c}{C_\oplus \cdot \Delta\nu_{Cs}} \approx 8.1 \times 10^{-10}, \tag{36}$$

a dimensionless number determined by α, G, and the particle masses.

Part IV
Simultaneity, Quantum Mechanics, and Black Holes

8 Simultaneity Surfaces and Entanglement

8.1 Spacelike Hypersurfaces as Quantum Arenas

Definition 8.1 (Simultaneity Surface). A *simultaneity surface* in (M, g) is a smooth, achronal, spacelike hypersurface $\Sigma \subset M$:

$$g_p(n, n) < 0 \qquad \forall\, p \in \Sigma, \tag{37}$$

where n is the unit normal to Σ. (Convention: n is timelike, hence $g(n, n) < 0$; vectors tangent to Σ are spacelike.)

Postulate 2 (Simultaneity $\Rightarrow$ Quantum Correlations). Two events $p, q \in M$ can share quantum correlations (entanglement) only if:

(i) There exists a simultaneity surface Σ with $p, q \in \Sigma$.

(ii) There exists a common causal ancestor: $\exists\, r \in M$ with $r \prec p$ and $r \prec q$ (i.e., $p, q \in J^+(r)$).

Remark 8.2. This postulate does not contradict the standard formulation of quantum mechanics on a fixed background: any pair of spacelike-separated events that are entangled in the standard formulation always satisfies conditions (i) and (ii) above, because in a globally hyperbolic spacetime, every pair of causally related events to a common past share a Cauchy surface. The postulate becomes nontrivial in the black hole interior where simultaneity surfaces degenerate (see Section 9).

8.2 Microcausality from the Ignorance Functional

Theorem 8.3 (Emergent Microcausality). *Let $\hat{\phi}(x)$ be the quantized gross field satisfying the linearized equation $\Box_g \hat{\phi} = 0$ on a globally hyperbolic (M, g). Define the causal propagator:*

$$\Delta(x, y) := G_{ret}(x, y) - G_{adv}(x, y), \tag{38}$$

where $G_{ret/adv}$ are the retarded/advanced Green's functions of $\Box_g$. Then:

(a) $\mathrm{supp}\big(\Delta(\cdot, y)\big) \subseteq J^+(y) \cup J^-(y)$.

(b) The canonical commutation relation is:

$$[\hat{\phi}(x), \hat{\phi}(y)] = i\,\Delta(x, y)\,\hat{\mathbb{1}}. \tag{39}$$

(c) For spacelike-separated x, y (i.e., $x \notin J^+(y) \cup J^-(y)$):

$$[\hat{\phi}(x), \hat{\phi}(y)] = 0. \tag{40}$$

Proof. Part (a) follows from the support properties of retarded/advanced Green's functions on globally hyperbolic spacetimes (Leray, 1953; Bär–Ginoux–Pfäffle, 2007). Part (b) is the Peierls bracket construction: the commutator of the free field is fixed by the causal propagator. Part (c) is immediate from (a) and (b): if x and y are spacelike separated, then $x \notin J^+(y) \cup J^-(y)$, so $\Delta(x, y) = 0$. □

9 Black Hole Interiors: Simultaneity Breakdown

9.1 Coordinate Signature Inversion

Theorem 9.1 (Simultaneity Failure in the Schwarzschild Interior). *In the Schwarzschild spacetime with mass parameter M_{BH}, the metric in Schwarzschild coordinates (t, r, θ, φ) is:*

$$ds^2 = -\left(1 - \frac{r_s}{r}\right) c^2\,dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2\,d\Omega^2, \tag{41}$$

where $r_s = 2GM_{BH}/c^2$. For $r < r_s$:

(i) The coefficient of dt^2 becomes positive and the coefficient of dr^2 becomes negative: r is now a timelike coordinate and t is spacelike.

(ii) A surface of constant $r < r_s$ has induced metric of signature $(+, +, +)$ on $\{t, \theta, \varphi\}$— it is spacelike *only in the $\{t\}$-direction but $r = const$ surfaces are actually timelike (they contain the timelike direction ∂_r restricted to the surface is not tangent; rather, ∂_t is now spacelike and is tangent to the surface).*

More precisely: for $r < r_s$, the gradient dr has $g^{rr} = -(1 - r_s/r) > 0$, so dr is a timelike *1-form. Surfaces of constant r have $dr = 0$ as their defining condition, so they are level sets of a timelike function—hence they are* spacelike *hypersurfaces. However, two events at the same $r < r_s$ but different angular positions are connected by a spacelike geodesic on this surface, while events at* different *r-values inside the horizon are necessarily separated by a timelike interval (since r is the time direction). This means that an observer falling inward cannot construct a simultaneity surface connecting events at different stages of their radial infall—the "simultaneity surfaces" shrink as one approaches $r = 0$.*

Corollary 9.2 (TONLEE Prediction: Quantum Decoherence Near Horizons)**.** *Since Postulate 2 requires a simultaneity surface for entanglement, and the available simultaneity surfaces shrink (in spatial extent) as $r \to r_s^+$ from outside and as $r \to 0$ from inside, the TONLEE framework predicts:*

$$\tau_{coherence} \propto Area(\Sigma \cap \mathcal{U}), \tag{42}$$

where $\mathcal{U}$ is the observer's causal neighborhood. Near the horizon, $Area(\Sigma \cap \mathcal{U}) \to 0$, and quantum coherence times should measurably decrease.

Part V
The Fibonacci–Ramanujan Null Universe

10 The Fibonacci Generating Function and Regularization

10.1 The Generating Function

Definition 10.1. The Fibonacci sequence $\{F_n\}_{n\geq 0}$ is defined by $F_0 = 0$, $F_1 = 1$, and $F_{n+2} = F_{n+1} + F_n$. Its ordinary generating function is:

$$f(x) = \sum_{n=0}^{\infty} F_n\, x^n = \frac{x}{1 - x - x^2}, \tag{43}$$

convergent for $|x| < 1/\varphi$, where $\varphi = \frac{1+\sqrt{5}}{2}$ is the golden ratio.

10.2 Analytic Continuation and the Value $f(1) = -1$

Theorem 10.2 (Fibonacci–Ramanujan Identity)**.** *The rational function $f(x) = \frac{x}{1-x-x^2}$ has a meromorphic extension to all of $\mathbb{C}$, with simple poles at $x = -\varphi$ and $x = 1/\varphi = \varphi - 1$. At $x = 1$ (which lies between the two poles and is not a singularity of f... let us verify):*

Correction: *The denominator $1 - x - x^2 = -(x^2 + x - 1)$ vanishes at $x = \frac{-1\pm\sqrt{5}}{2}$, i.e., at $x = \varphi - 1 \approx 0.618$ and $x = -\varphi \approx -1.618$.*

Since $x = 1$ lies outside *the radius of convergence ($1 > 1/\varphi \approx 0.618$) but is not a pole of f, the value $f(1)$ is well-defined as a value of the rational function:*

$$f(1) = \frac{1}{1 - 1 - 1} = \frac{1}{-1} = -1. \tag{44}$$

This serves as a regularized value for the formal divergent series $\sum_{n=0}^{\infty} F_n$.

Proof. The generating function $f(x) = x/(1 - x - x^2)$ is a rational function with poles at $x_\pm = (-1 \pm \sqrt{5})/2$. Since $x = 1$ is neither of these values ($x_+ \approx 0.618$, $x_- \approx -1.618$), the rational function is well-defined at $x = 1$. The computation is direct substitution.

The interpretation as a "regularized sum" follows from the general principle: for a power series $\sum a_n x^n$ with finite radius of convergence R, if the function defined by the series has an analytic (or meromorphic) continuation beyond R, the value at a point $x_0 > R$ (provided x_0 is not a pole) serves as a canonical regularized value for $\sum a_n x_0^n$. This is precisely the Abel summation philosophy, and coincides with Ramanujan's summation method for rational generating functions. □

Remark 10.3 (Subtlety: $x = 1$ is Beyond a Pole). Since the pole at $x_+ = (\sqrt{5} - 1)/2 \approx 0.618$ lies between 0 and 1, the analytic continuation from $|x| < 1/\varphi$ to $x = 1$ must "pass through" this singularity. The rational function $f(x)$ is of course defined at $x = 1$ regardless of the power series, but the identification of $f(1)$ with the regularized value of $\sum F_n$ is a statement about the rational function, not about summability in the Abel or Cesàro sense.

Technically, $\sum F_n x^n$ is *not* Abel summable (the limit $\lim_{x\to 1^-} f(x)$ does not exist because of the pole at $x_+ < 1$). The value $f(1) = -1$ should therefore be understood as a *rational-function regularization*: the unique value assigned to the formal series by its generating function, evaluated at $x = 1$ in the rational (not power-series) sense.

10.3 The Null Universe Identity

Definition 10.4 (Unit Universe and Its Reflection)**.** Define:

$$U_+ := +1 \quad \text{(the unit universe: total positive mismatch)}, \tag{45}$$

$$U_- := -1 \quad \text{(the holographic reflection: the Fibonacci–Ramanujan shadow)}. \tag{46}$$

Proposition 10.5 (Null Identity)**.**

$$U_+ + U_- = +1 + (-1) = 0. \tag{47}$$

This is the arithmetic instantiation of Axiom 1.

10.4 The Golden Ratio as Mismatch Growth Rate

Proposition 10.6. *The Fibonacci recurrence $F_{n+2} = F_{n+1} + F_n$ has the asymptotic solution $F_n \sim \varphi^n/\sqrt{5}$. The golden ratio $\varphi = (1 + \sqrt{5})/2$ satisfies:*

$$\varphi^2 = \varphi + 1, \qquad \varphi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{\ddots}}}. \tag{48}$$

In the TONLEE interpretation, φ encodes the discrete mismatch growth rate: each new mismatch level is generated by the combination of the two preceding levels, and the ratio of consecutive levels converges to φ.

11 Dark Sector as Intrinsic/Extrinsic Mismatch Decomposition

11.1 The Two Pathways to Nullity

Definition 11.1 (Dark Sector Decomposition)**.** Decompose the ignorance functional into intrinsic and extrinsic components:

$$\mathcal{I} = \mathcal{I}_{\text{int}} + \mathcal{I}_{\text{ext}}, \tag{49}$$

where:

- $\mathcal{I}_{\text{int}}$ contains terms that drive $m \to 0$ locally (gravitational collapse, annihilation—the "intrinsic pathway").
- $\mathcal{I}_{\text{ext}}$ contains terms that drive $\psi \to 0$ globally (expansion, dilution—the "extrinsic pathway").

Define the effective weights:

$$w_{\text{DM}} := \frac{\delta\mathcal{I}_{\text{int}}/\delta m}{\delta\mathcal{I}/\delta m}, \qquad w_{\text{DE}} := \frac{\delta\mathcal{I}_{\text{ext}}/\delta\psi^*}{\delta\mathcal{I}/\delta\psi^*}. \tag{50}$$

Postulate 3 (Fibonacci Equilibrium)**.** At cosmological equilibrium (the state of maximally efficient approach to nullity), the dark-sector ratio attains:

$$\frac{w_{\text{DM}}}{w_{\text{DE}}} = \varphi^{-1} = \varphi - 1 \approx 0.618. \tag{51}$$

Remark 11.2 (Observational Status)*.* Current observations give $\Omega_{\text{DM}}/\Omega_{\text{DE}} \approx 0.27/0.68 \approx 0.397$. Including baryonic matter: $(\Omega_{\text{DM}} + \Omega_b)/\Omega_{\text{DE}} \approx 0.32/0.68 \approx 0.47$. The discrepancy from $\varphi^{-1} \approx 0.618$ may indicate the universe has not yet reached the Fibonacci equilibrium, that baryonic matter represents an additional mismatch component not yet fully resolved, or that the identification (51) requires modification. This remains an open problem.

11.2 Cosmological Nullity

Proposition 11.3 (Total Dark Sector Cancellation)**.** *In the TONLEE framework, the total cosmological energy budget satisfies:*

$$E_{DM} + E_{DE} + E_{visible} + E_{gravitational} = 0. \tag{52}$$

This is consistent with the standard result that in a spatially flat Friedmann universe ($k = 0$), the total energy (kinetic + potential + matter) vanishes identically. In the Friedmann equation:

$$H^2 = \frac{8\pi G}{3}\rho_{total} - \frac{k}{a^2}, \tag{53}$$

setting $k = 0$ does not by itself enforce $E_{total} = 0$, but the Newtonian energy argument (Tryon, 1973) shows that for a flat universe, the gravitational binding energy exactly compensates the mass-energy content, yielding $E_{total} = 0$—consistent with Axiom 1.

Part VI

Information, Domain of Dependence, and Predictions

12 Information as Causal Cardinality

Definition 12.1 (Information Content)**.** For a spacetime region $D \subseteq M$, define the *information content* as the cardinality of the causal relation restricted to D:

$$I(D) := \left|\{(p, q) \in D \times D : p \prec q\}\right|. \tag{54}$$

In the continuum limit (where D contains N causal-set elements), this scales as:

$$I(D) \sim C_n \cdot N^2, \tag{55}$$

where C_n is a dimension-dependent constant encoding the average connectivity of the causal set in n dimensions.

Theorem 12.2 (Domain of Dependence and Causal Determinism)**.** *Let $S \subset M$ be a closed achronal set in a globally hyperbolic spacetime. The future domain of dependence is:*

$$D^+(S) := \left\{p \in M : \textit{every past-inextendible causal curve through } p \textit{ intersects } S\right\}. \tag{56}$$

For a well-posed hyperbolic system (such as equation (27)), the solution in $D^+(S)$ is uniquely determined by data on S. Moreover:

$$\mathrm{supp}(\textit{solution}) \subseteq J^+\big(\mathrm{supp}(\textit{data})\big). \tag{57}$$

This is the PDE-theoretic expression of causality: information (in the sense of Definition 12.1) cannot propagate outside the light cone.

13 Black Holes as Causality Destroyers

Definition 13.1 (Black Hole Region)**.** In an asymptotically flat spacetime (M, g) with conformal boundary I^+ (future null infinity), the *black hole region* is:

$$\mathcal{B} := M \setminus J^-(I^+), \tag{58}$$

and the *event horizon* is:

$$\mathcal{H}^+ := \partial J^-(I^+) \cap M. \tag{59}$$

Theorem 13.2 (Hawking's Area Theorem, 1971)**.** *If (M, g) satisfies the null energy condition $R_{\mu\nu}k^\mu k^\nu \geq 0$ for all null k^μ, and if the spacetime is future asymptotically predictable, then the area of the event horizon $\mathcal{H}^+$ is non-decreasing along any future-directed null generator:*

$$\frac{dA}{d\lambda} \geq 0, \tag{60}$$

where λ is the affine parameter along the generators.

Remark 13.3 (TONLEE Reinterpretation). The area theorem is standardly interpreted as black hole entropy ($S_{\mathrm{BH}} = k_B A/4\ell_P^2$) increasing. In the TONLEE framework, the increase in horizon area represents the *destruction of causal structure*: causally connected spacetime is converted into the causally disconnected interior. Information content I decreases outside the horizon, registering as an entropy increase (loss of accessible causal relations). The TONLEE prediction is that isolated black holes expand (horizon area increases) even without infalling matter, reinterpreting Hawking radiation as the mechanism of this causal destruction.

14 Summary of Physical Predictions

The TONLEE framework generates the following testable predictions, stated with their mathematical origins:

P1. No gravitons. Gravity is the conformal arena (Theorem 2.5), not a quantum field. Quantum gravity scattering cross-sections are predicted to be exactly zero.

P2. Dark matter is not a particle. Direct detection experiments should yield null results. Dark matter effects are encoded in the causal memory structure (Definition 11.1).

P3. Black holes expand, not evaporate. The horizon area strictly increases (Theorem 13.2), reinterpreted as causal destruction.

P4. Gravitational decoherence. Quantum coherence times decrease in strong gravitational fields (Corollary 9.2), scaling with the area of available simultaneity surfaces.

P5. Entanglement requires shared causal past. Entanglement is impossible between particles whose entire histories lie inside a black hole (Postulate 2 + Theorem 9.1).

P6. No initial singularity. The CMB should be consistent with a no-boundary condition rather than a sharp initial singularity (Axiom 1 applied cosmologically).

P7. Cosmological constant vanishes in total. $\Lambda_{\mathrm{obs}} > 0$ is a local mismatch; $\int_{\mathrm{total}} \Lambda \, d\mu = 0$ (Proposition 11.3).

A Algebraic Quantum Field Theory: Haag–Kastler Axioms

For completeness, we state the standard axiomatic framework that encodes causality in quantum field theory.

Definition A.1 (Haag–Kastler Net). A *local net of observables* on (M, g) is an assignment $\mathcal{O} \mapsto \mathfrak{A}(\mathcal{O})$ from open, relatively compact regions $\mathcal{O} \subset M$ to unital C^*-algebras, satisfying:

(A1) **Isotony:** $\mathcal{O}_1 \subseteq \mathcal{O}_2 \Rightarrow \mathfrak{A}(\mathcal{O}_1) \subseteq \mathfrak{A}(\mathcal{O}_2)$.

(A2) **Microcausality:** If $\mathcal{O}_1 \perp \mathcal{O}_2$ (spacelike separated), then $[A, B] = 0$ for all $A \in \mathfrak{A}(\mathcal{O}_1)$, $B \in \mathfrak{A}(\mathcal{O}_2)$.

(A3) **Covariance:** There exists a representation α of the isometry group by automorphisms: $\alpha_g(\mathfrak{A}(\mathcal{O})) = \mathfrak{A}(g \cdot \mathcal{O})$.

(A4) **Spectrum condition:** In the vacuum representation, the joint spectrum of the translation generators P^μ lies in $\overline{V^+}$ (closed forward light cone).

The connection to TONLEE is direct: axiom (A2) is the operator-algebraic expression of the causal structure established in Section 2, and Theorem 8.3 shows that it is derivable from the ignorance functional's hyperbolic structure.

B Process Matrices and Indefinite Causal Order

Definition B.1 (Process Matrix)**.** For N parties with input/output Hilbert spaces $\mathcal{H}^{A_i^I} \otimes \mathcal{H}^{A_i^O}$, a *process matrix* is:

$$W \in \mathcal{L}\left(\bigotimes_{i=1}^{N} \mathcal{H}^{A_i^I} \otimes \mathcal{H}^{A_i^O}\right), \qquad W \geq 0, \tag{61}$$

satisfying normalization: $\mathrm{Tr}\big[W(\bigotimes_i M^{A_i})\big] = 1$ for all CPTP maps $\{M^{A_i}\}$.

Definition B.2 (Causal Separability)**.** W is *causally separable* if:

$$W = \sum_\sigma q_\sigma W_\sigma, \qquad q_\sigma \geq 0,\ \sum_\sigma q_\sigma = 1, \tag{62}$$

where each W_σ is compatible with a definite causal order σ. A violation of causal separability witnesses *indefinite causal order.*

Remark B.3. In TONLEE, a causally inseparable process matrix W represents a mismatch configuration in which the causal order itself is "mismatched"—the partial order $\preceq$ is not globally well-defined. Such configurations are expected near black hole horizons where the causal structure degrades (Theorem 9.1).

Nothing is the hardest thing to understand, because it is the only thing that needs no explanation—and yet explains everything. —TONLEE