

To prove that the balance factors of all nodes in an AVL tree remain unchanged except for those on the path through which a node is inserted, we first need to understand the characteristics of AVL trees and the process of insertion.

Properties of AVL Trees

Definition: An AVL tree is a type of self-balancing binary search tree where the height difference between the left and right subtrees (the balance factor) of any node is at most

1. Balance Factor: For any node N , the balance factor $BF(N)$ is defined as the height of the left subtree minus the height of the right subtree.

Insertion Process in an AVL Tree

When inserting a node into an AVL tree, the following steps are carried out:

Standard BST Insertion: The node is inserted according to the rules of a binary search tree (BST).

Height Updates: The heights of all ancestors of the newly inserted node are updated.

Balancing: The tree is checked for balance violations, and rotations are performed if necessary to restore the balance.

Key Observation

The key point to note is that only the ancestors of the newly inserted node, which lie on the path from the root to the insertion point, have their heights and balance factors potentially altered. Nodes that are not on this path, and their subtrees, remain unaffected in terms of their heights and balance factors.

Proof

Denote the path from the root to the node where the insertion occurs as PP . Nodes on PP are directly affected by the insertion, but nodes not on PP are not affected in terms of their balance factors.

Nodes on Path PP :

After the node is inserted, the heights and balance factors of nodes on PP are updated.

If any node becomes unbalanced, rotations are performed to restore balance, which may change the balance factors of nodes on PP and possibly their immediate neighbors.

Nodes not on Path PP :

These nodes reside in subtrees that are not affected by the insertion path.

Since no nodes are added or removed from these subtrees, their heights remain the same before and after the insertion.

Consequently, the balance factors of these nodes remain unchanged.

Conclusion

Given that the insertion only impacts the nodes on the path from the root to the inserted node, and the heights and balance factors of nodes not on this path remain unchanged, we conclude that the balance factors of all nodes except those on the insertion path remain the same.

This demonstrates that in an AVL tree, the balance factors are only altered for nodes along the path through which a new node is inserted, while nodes not on this path retain their original balance factors.