

Practice set: 4 TM

**4.1.1.** Let  $M = (K, \Sigma, \delta, s, \{h\})$ , where

$$K = \{q_0, q_1, h\},$$

$$\Sigma = \{a, b, \sqcup, \triangleright\},$$

$$s = q_0,$$

and  $\delta$  is given by the following table.

$q,$	$\sigma$	$\delta(q, \sigma)$
$q_0$	$a$	$(q_1, b)$
$q_0$	$b$	$(q_1, a)$
$q_0$	$\sqcup$	$(h, \sqcup)$
$q_0$	$\triangleright$	$(q_0, \rightarrow)$
$q_1$	$a$	$(q_0, \rightarrow)$
$q_1$	$b$	$(q_0, \rightarrow)$
$q_1$	$\sqcup$	$(q_0, \rightarrow)$
$q_1$	$\triangleright$	$(q_1, \rightarrow)$

- (a) Trace the computation of  $M$  starting from the configuration  $(q_0, \triangleright \underline{a} abbbba)$ .
- (b) Describe informally what  $M$  does when started in  $q_0$  on any square of a tape.

**4.1.2.** Repeat Problem 4.1.1 for the machine  $M = (K, \Sigma, \delta, s, \{h\})$ , where

$$K = \{q_0, q_1, q_2, h\},$$

$$\Sigma = \{a, b, \sqcup, \triangleright\},$$

$$s = q_0,$$

and  $\delta$  is given by the following table (the transitions on  $\triangleright$  are  $\delta(q, \triangleright) = (q, \triangleright)$ , and are omitted).

$q, \sigma$	$\delta(q, \sigma)$
$q_0, a$	$(q_1, \leftarrow)$
$q_0, b$	$(q_0, \rightarrow)$
$q_0, \sqcup$	$(q_0, \rightarrow)$
$q_1, a$	$(q_1, \leftarrow)$
$q_1, b$	$(q_2, \rightarrow)$
$q_1, \sqcup$	$(q_1, \leftarrow)$
$q_2, a$	$(q_2, \rightarrow)$
$q_2, b$	$(q_2, \rightarrow)$
$q_2, \sqcup$	$(h, \sqcup)$

Start from the configuration  $(q_0, \triangleright a \underline{b} b \sqcup b b \sqcup \sqcup \sqcup a b a)$ .

**4.1.4.** Let  $M$  be the Turing machine  $(K, \Sigma, \delta, s, \{h\})$ , where

$$K = \{q_0, q_1, q_2, h\},$$

$$\Sigma = \{a, \sqcup, \triangleright\},$$

$$s = q_0,$$

and  $\delta$  is given by the following table.

Let  $n \geq 0$ . Describe carefully what  $M$  does when started in the configuration  $(q_0, \triangleright \sqcup a^n \underline{a})$ .

- 4.1.7.** Design and write out in full a Turing machine that scans to the right until it finds two consecutive  $a$ 's and then halts. The alphabet of the Turing machine should be  $\{a, b, \sqcup, \triangleright\}$ .

**4.1.8.** Give the full details of the Turing machines illustrated.

$>LL.$        $>R$  

$>L \xrightarrow{\sqcup} R$

**4.1.10.** Explain what this machine does.

$$>R \xrightarrow{a \neq \sqcup} R \xrightarrow{b \neq \sqcup} R_{\sqcup}aR_{\sqcup}b$$

**4.1.11.** Trace the operation of the Turing machine of Example 4.1.8 when started on  $\triangleright \underline{1} aabb$ .