Practice Set: 3

3.5.1. Use closure under union to show that the following languages are context-free.

(a) $\{a^m b^n : m \neq n\}$

(b) $\{a, b\}^* - \{a^n b^n : n \geq 0\}$

(c) $\{a^m b^n c^p d^q : n = q, \text{ or } m \leq p \text{ or } m + n = p + q\}$

(d) $\{a, b\}^* - L$, where L is the language

$$L = \{babaabaaab \ldots ba^{n-1}ba^n b : n \geq 1\}$$

(e) $\{w \in \{a, b\}^* : w = w^R\}$

3.5.2. Use Theorems 3.5.2 and 3.5.3 to show that the following languages are not context-free.

(a) $\{a^p : p \text{ is a prime}\}$

(b) $\{a^{n^2} : n \geq 0\}$

(c) $\{www : w \in \{a, b\}^*\}$

(d) $\{w \in \{a, b, c\}^* : w \text{ has equal numbers of } a\text{'s, } b\text{'s, and } c\text{'s}\}$

3.6.1. Convert the context-free grammar G given in Example 3.1.3 generating arithmetic expressions into an equivalent context-free grammar in Chomsky normal form. Apply the dynamic programming algorithm for deciding whether the string $x = (\mathsf{id} + \mathsf{id} + \mathsf{id}) * (\mathsf{id})$ is in $L(G)$.

3.6.2. How would you modify the dynamic programming algorithm in such a way that, when the input x is indeed in the language generated by G, then the algorithm produces an actual derivation of x in G?