Practice Set: 2

3.3.1. Consider the pushdown automaton $M = (K, \Sigma, \Gamma, \Delta, s, F)$, where

$$\begin{aligned}
K &= \{s, f\},\\
F &= \{f\},\\
\Sigma &= \{a, b\},\\
\Gamma &= \{a\},\\
\Delta &= \{((s, a, e), (s, a)), ((s, b, e), (s, a)), ((s, a, e), (f, e)),\\
&\quad ((f, a, a), (f, e)), ((f, b, a), (f, e))\}.
\end{aligned}$$

(a) Trace all possible sequences of transitions of M on input aba.

(b) Show that $aba, aa, abb \notin L(M)$, but $baa, bab, baaaa \in L(M)$.

(c) Describe $L(M)$ in English.

3.3.2. Construct pushdown automata that accept each of the following.

(a) The language generated by the grammar $G = (V, \Sigma, R, S)$, where

$$
\begin{aligned}
V &= \{S, (,), [,]\},\\
\Sigma &= \{(,), [,]\},\\
R &= \{S \to e,\\
&\qquad S \to SS,\\
&\qquad S \to [S],\\
&\qquad S \to (S)\}.
\end{aligned}
$$

(b) The language $\{a^m b^n : m \leq n \leq 2m\}$.

(c) The language $\{w \in \{a, b\}^* : w = w^R\}$.

(d) The language $\{w \in \{a, b\}^* :$ w has twice as many b's as a's$\}$.

3.4.1. Carry out the construction of Lemma 3.4.1 for the grammar of Example 3.1.4. Trace the operation of the automaton you have constructed on the input string (()()).

Lemma 3.4.1: *Each context-free language is accepted by some pushdown automaton.*

Example 3.1.4: The following grammar generates all strings of properly balanced left and right parentheses: every left parenthesis can be paired with a unique subsequent right parenthesis, and every right parenthesis can be paired with a unique preceding left parenthesis. Moreover, the string between any such pair has the same property. We let $G = (V, \Sigma, R, S)$, where

$$\begin{aligned} V &= \{S, (,)\}, \\ \Sigma &= \{(,)\}, \\ R &= \{S \to e, \\ &\qquad S \to SS, \\ &\qquad S \to (S)\}. \end{aligned}$$