Practice 6

4.3.1. Formally define:

(a) M semidecides L, where M is a two-way infinite tape Turing machine;

(b) M computes f, where M is a k-tape Turing machine and f is a function from strings to strings.

4.3.2. Formally define:

(a) a k-head Turing machine (with a single one-way infinite tape);

(b) a configuration of such a machine;

(c) the yields in one step relation between configurations of such a machine. (There is more than one correct set of definitions.)

4.3.3. Describe (in an extension of our notation for k-tape Turing machines) a 2-head Turing machine that compute the function $f(w) = ww$.

4.3.4. The stack of a pushdown automaton can be considered as a tape that can be written and erased only at the right end; in this sense a Turing machine is a generalization of the deterministic pushdown automaton. In this problem we consider a generalization in another direction, namely the *deterministic pushdown automaton with two stacks*.

(a) Define informally but carefully the operation of such a machine. Define what it means for such a machine to decide a language.

(b) Show that the class of languages decided by such machines is precisely the class of recursive languages.

4.3.5. Give a three-tape Turing machine which, when started with two binary integers separated by a ';' on its first tape, computes their product. (*Hint:* Use the adding machine of Example 4.3.2 as a "subroutine."

4.3.6. Formally define a Turing machine with a 2-dimensional tape, its configurations, and its computation. Define what it means for such a machine to decide a language L. Show that t steps of this machine, starting on an input of length n, can be simulated by a standard Turing machine in time that is *polynomial* in t and n.

5.1. Give (in abbreviated notation) nondeterministic Turing machines that accept these languages.

(a) $a^*abb^*baa^*$
(b) $\{ww^Ruu^R : w, u \in \{a, b\}^*\}$

4.5.2. Let $M = (K, \Sigma, \delta, s, \{h\})$ be the following nondeterministic Turing machine:

$$\begin{aligned} K =& \{q_0, q_1, h\}, \\ \Sigma =& \{a, \triangleright, \sqcup\}, \\ s =& q_0, \\ \Delta =& \{(q_0, \sqcup, q_1, a), (q_0, \sqcup, q_1, \sqcup), (q_1, \sqcup, q_1, \sqcup), (q_1, a, q_0, \rightarrow), (q_1, a, h, \rightarrow)\} \end{aligned}$$

Describe all possible computations of five steps or less by M starting from the configuration $(q_0, \triangleright\underline{\sqcup})$. Explain in words what M does when started from this configuration. What is the number r (in the proof of Theorem 4.5.1) for this machine?

4.5.3. Although nondeterministic Turing machines are not helpful in showing closure under complement of the recursive languages, they are very convenient for showing other closure properties. Use nondeterministic Turing machines to show that the class of recursive languages is closed under union, concatenation, and Kleene star. Repeat for the class of recursively enumerable languages.