Centennial sea level projections

Semi-empirical modelling with Markov-chain Monte Carlo and Forward Euler

Anton, Steffen & Johann

October 25th 2024

Introduction

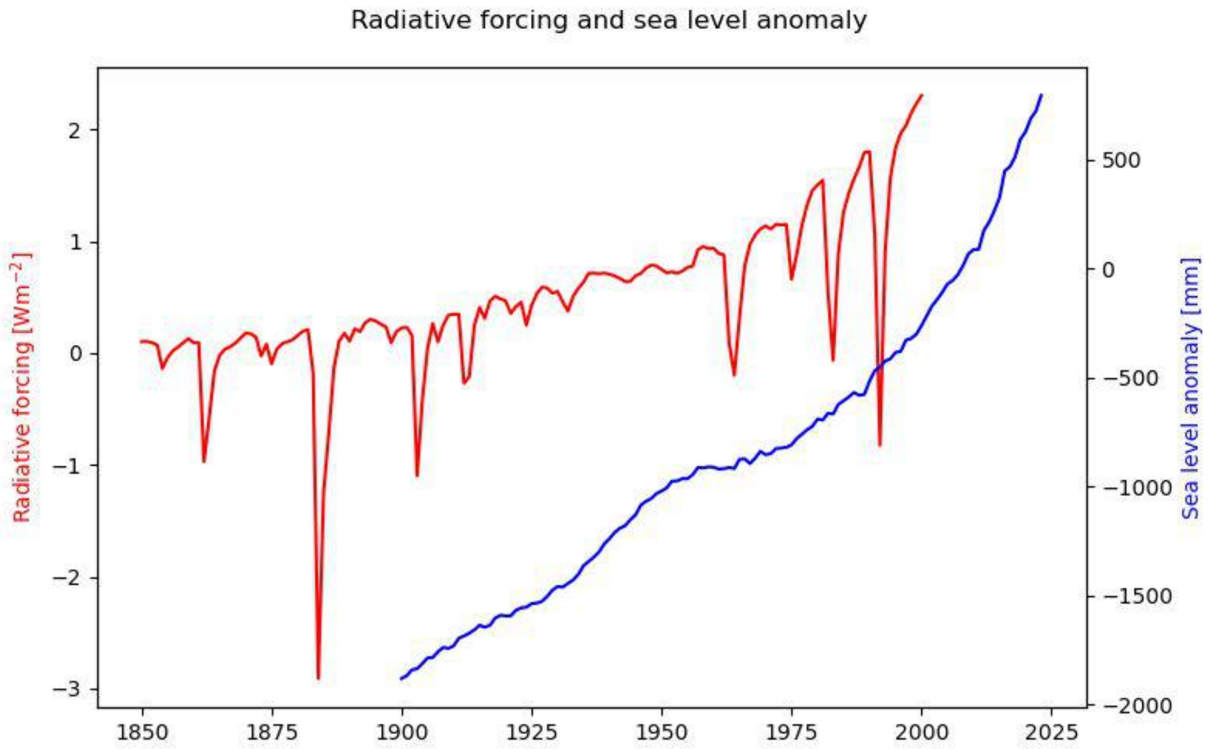
Sea-level and radiative forcing

Why (narrative):

- Climate change
- Vulnerable to sea level change
- Observed + extended sea level change

Why do we use radiative forcing?

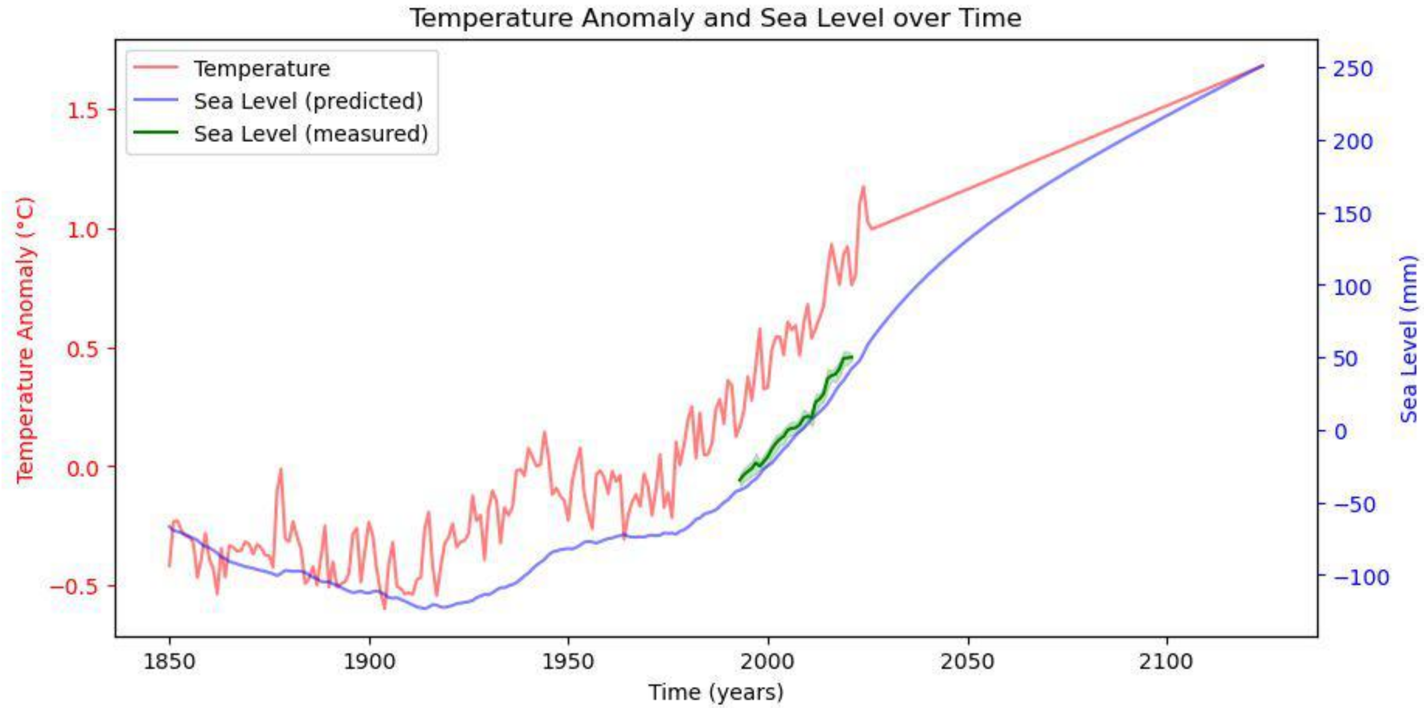
- Take different factors into account
- Specific contribution for each factor



Introduction

Sea-level and radiative forcing

Purpose of our project:



Inspiration & Idea

Inspirational papers

Reconstructing sea level from paleo and projected temperatures 200 to 2100 AD

Aslak Grinsted · J. C. Moore · S. Jevrejeva

Received: 7 August 2008 / Accepted: 5 December 2008 / Published online: 6 January 2009

Abstract We use a physically plausible four parameter linear response equation to relate 2,000 years of global temperatures and sea level. We estimate likelihood distributions of equation parameters using Monte Carlo inversion, which then allows visualization of past and future sea level scenarios. The model has good predictive power when calibrated on the pre-1990 period and validated against the high rates of sea level rise from the satellite altimetry. Future sea level is projected from intergovernmental panel on climate change (IPCC) temperature scenarios and past sea level from established multi-proxy reconstructions assuming that the established relationship between temperature and sea level holds from 200 to 2100 AD. Over the last 2,000 years minimum sea level (−19 to −26 cm) occurred around 1730 AD, maximum sea level (12–21 cm) around 1150 AD. Sea level 2090–2099 is projected to be 0.9 to 1.3 m for the A1B scenario, with low probability of the rise being within IPCC confidence limits.

1 Introduction

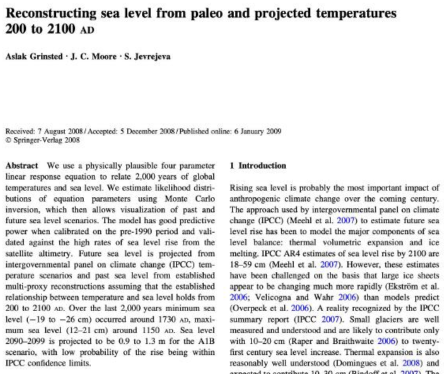
Rising sea level is probably the most important impact of anthropogenic climate change over the coming century. The approach used by intergovernmental panel on climate change (IPCC) (Meehl et al. 2007) to estimate future sea level rise has been to model the major components of sea level balance: thermal volumetric expansion and ice melting. IPCC AR4 estimates of sea level rise by 2100 are 18–59 cm (Meehl et al. 2007). However, these estimates have been challenged on the basis that large ice sheets appear to be changing much more rapidly (Ekström et al. 2006; Velicogna and Wahr 2006) than models predict (Overpeck et al. 2006). A reality recognized by the IPCC summary report (IPCC 2007). Small glaciers are well measured and understood and are likely to contribute only with 10–20 cm (Raper and Braithwaite 2006) to twenty-first century sea level increase. Thermal expansion is also reasonably well understood (Domingues et al. 2008) and

$$S_{eq} = aT + b$$

$$\frac{\partial S}{\partial t} = \frac{S_{eq} - S}{\tau}.$$

Inspiration & Idea

Contributing factors to sea level change



Reconstructing sea level from paleo and projected temperatures 200 to 2100 AD

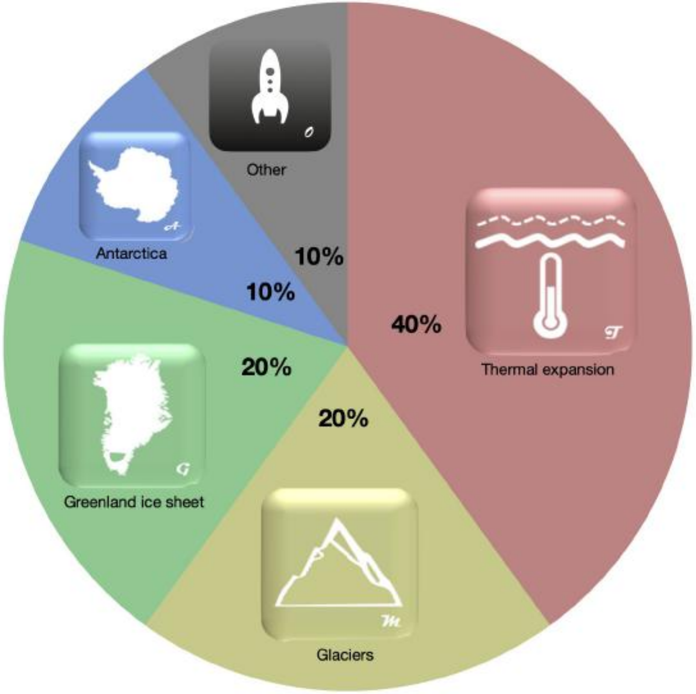
Aslak Grinsted · J. C. Moore · S. Jevrejeva

Received: 7 August 2008 / Accepted: 5 December 2008 / Published online: 6 January 2009

Abstract We use a physically plausible four parameter linear response equation to relate 2,000 years of global temperatures and sea level. We estimate likelihood distributions of equation parameters using Monte Carlo inversion, which then allows visualization of past and future sea level scenarios. The model has good predictive power when calibrated on the pre-1990 period and validated against the high rates of sea level rise from the satellite altimetry. Future sea level is projected from intergovernmental panel on climate change (IPCC) temperature scenarios and past sea level from established multi-proxy reconstructions assuming that the established relationship between temperature and sea level holds from 200 to 2100 AD. Over the last 2,000 years minimum sea level (−19 to −26 cm) occurred around 1730 AD, maximum sea level (12–21 cm) around 1150 AD. Sea level 2090–2099 is projected to be 0.9 to 1.3 m for the A1B scenario, with low probability of the rise being within IPCC confidence limits.

1 Introduction

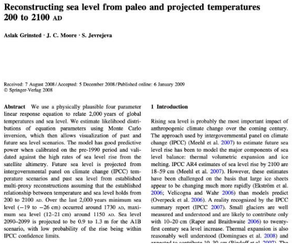
Rising sea level is probably the most important impact of anthropogenic climate change over the coming century. The approach used by intergovernmental panel on climate change (IPCC) (Meehl et al. 2007) to estimate future sea level rise has been to model the major components of sea level balance: thermal volumetric expansion and ice melting. IPCC AR4 estimates of sea level rise by 2100 are 18–59 cm (Meehl et al. 2007). However, these estimates have been challenged on the basis that large ice sheets appear to be changing much more rapidly (Ekström et al. 2006; Velicogna and Wahr 2006) than models predict (Overpeck et al. 2006). A reality recognized by the IPCC summary report (IPCC 2007). Small glaciers are well measured and understood and are likely to contribute only with 10–20 cm (Raper and Braithwaite 2006) to twenty-first century sea level increase. Thermal expansion is also reasonably well understood (Domingues et al. 2008) and



Based on IPCC AR6 table 9.5

Inspiration & Idea

Modelling contributing factors individually



Reconstructing sea level from paleo and projected temperatures 200 to 2100 AD

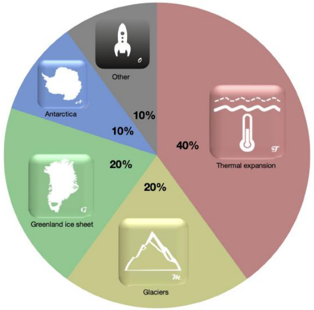
Aslak Grinsted · J. C. Moore · S. Jevrejeva

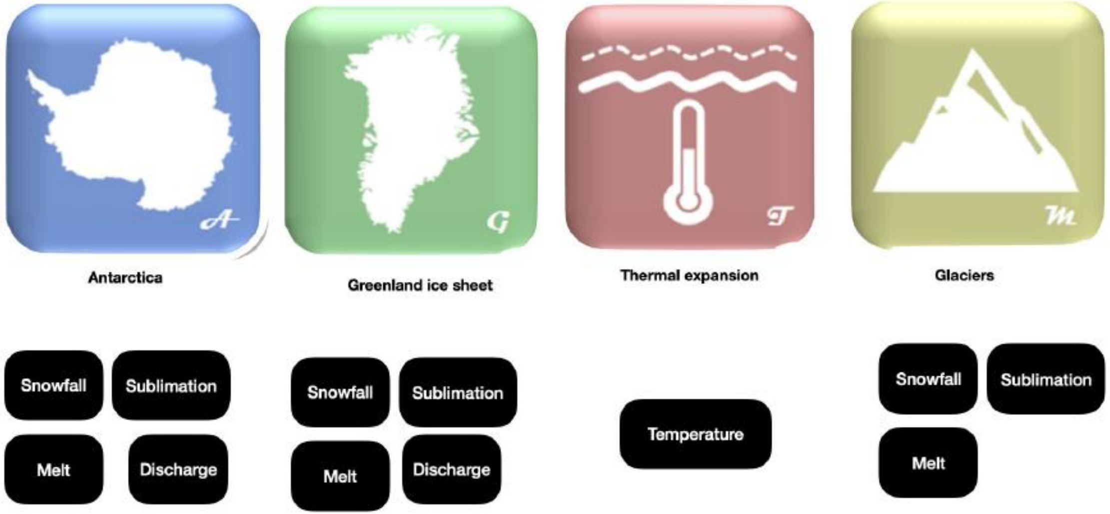
Received: 7 August 2008 / Accepted: 5 December 2008 / Published online: 6 January 2009

Abstract We use a physically plausible four parameter linear response equation to relate 2,000 years of global temperatures and sea level. We estimate likelihood distributions of equation parameters using Monte Carlo inversion, which then allows visualization of past and future sea level scenarios. The model has good predictive power when calibrated on the pre-1990 period and validated against the high rates of sea level rise from the satellite altimetry. Future sea level is projected from intergovernmental panel on climate change (IPCC) temperature scenarios and past sea level from established multi-proxy reconstructions assuming that the established relationship between temperature and sea level holds from 200 to 2100 AD. Over the last 2,000 years minimum sea level (−19 to −26 cm) occurred around 1730 AD, maximum sea level (12–21 cm) around 1150 AD. Sea level 2090–2099 is projected to be 0.9 to 1.3 m for the A1B scenario, with low probability of the rise being within IPCC confidence limits.

1 Introduction

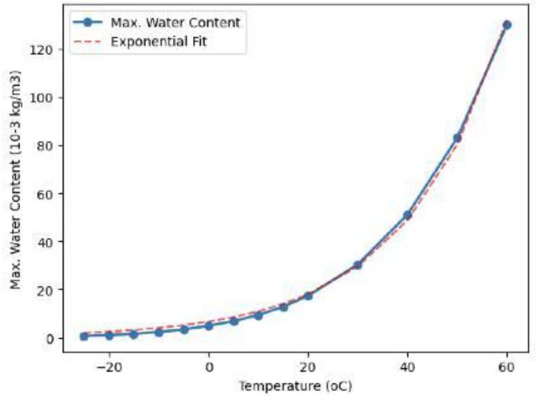
Rising sea level is probably the most important impact of anthropogenic climate change over the coming century. The approach used by intergovernmental panel on climate change (IPCC) (Meehl et al. 2007) to estimate future sea level rise has been to model the major components of sea level balance: thermal volumetric expansion and ice melting. IPCC AR4 estimates of sea level rise by 2100 are 18–59 cm (Meehl et al. 2007). However, these estimates have been challenged on the basis that large ice sheets appear to be changing much more rapidly (Ekström et al. 2006; Velicogna and Wahr 2006) than models predict (Overpeck et al. 2006). A reality recognized by the IPCC summary report (IPCC 2007). Small glaciers are well measured and understood and are likely to contribute only with 10–20 cm (Raper and Braithwaite 2006) to twenty-first century sea level increase. Thermal expansion is also reasonably well understood (Domingues et al. 2008) and

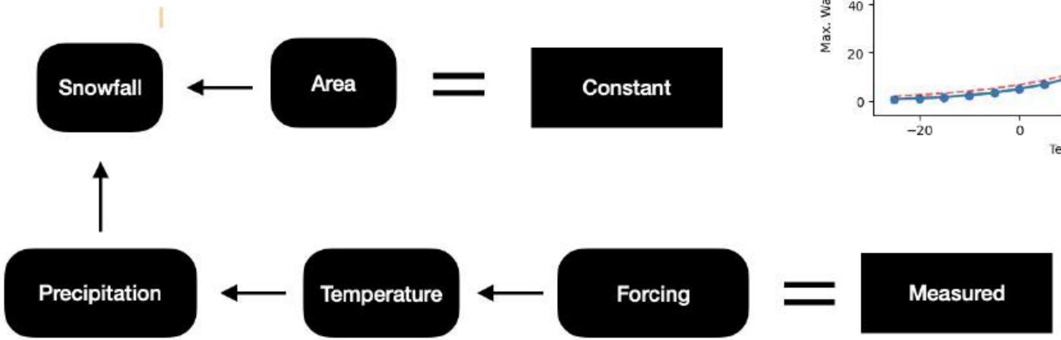




Model

Ice sheets - Snowfall





$$P(T) = A_{\text{ice sheet}} \cdot C(T_{\text{ice sheet}}) \cdot \zeta_1, \tag{1}$$

in which, $P(T)$ is the snowfall, and ζ_1 is a fitting constant. $T_{\text{ice sheet}}$ will tend toward an equilibrium value, which is set by the radiative forcing

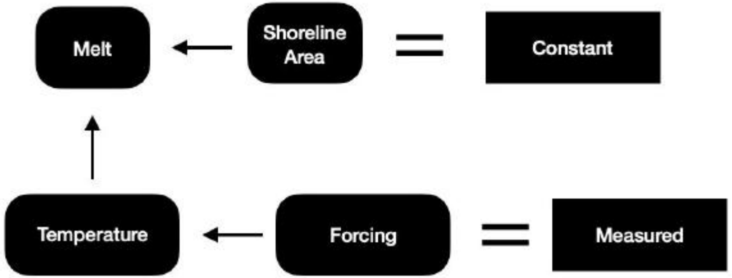
$$T_{\text{eq, ice sheet}} = \zeta_2 F + \zeta_3 \tag{2}$$

$$\frac{\partial T_{\text{ice sheet}}}{\partial t} = \frac{T_{\text{eq, ice sheet}} - T_{\text{ice sheet}}}{\tau_1} \tag{3}$$

Model

Ice sheets - Melt

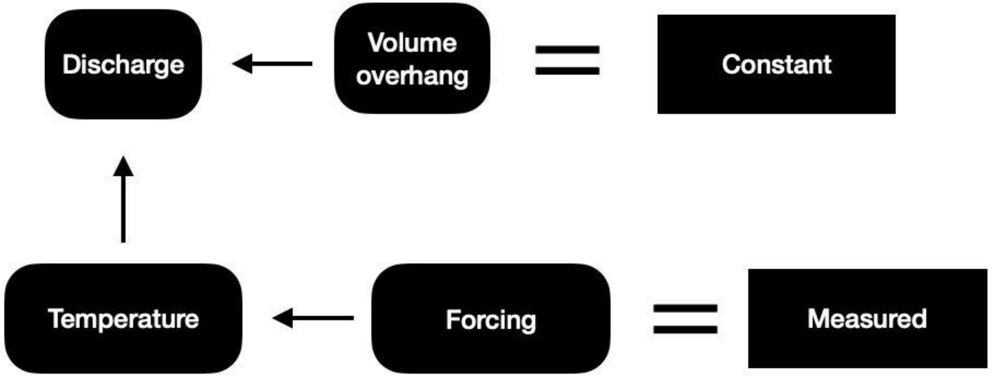
Ice sheets melt along the perimeter, in accordance with temperature



$$L(T) = A_{\text{edge area}} \cdot \zeta_4 \cdot \exp(T - T_C)$$

Model

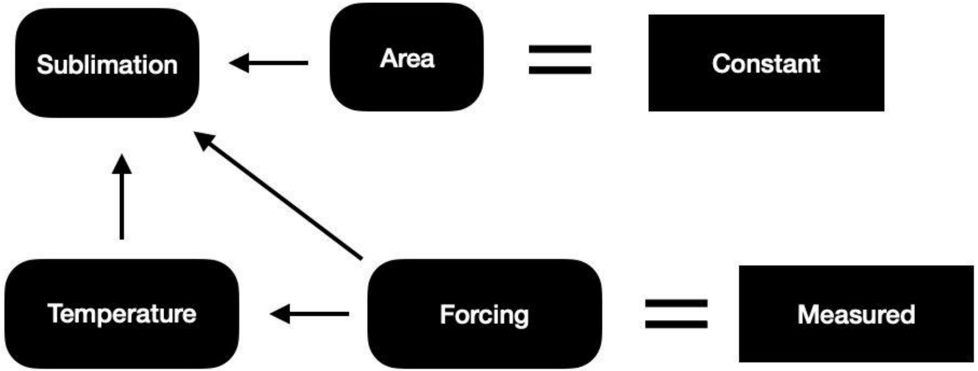
Ice sheets - Discharge



$$D(T) = V_{\text{overhang}} \cdot \exp(T_{\text{ocean}} - T_C) \cdot \zeta_5$$

Model

Ice sheets - Sublimation



$$B(F,T) = \zeta_6 AFT$$

Model

Ice sheets

2.1.5 Combined ice sheet model

If we add the different contributing factors to the differential equation for the ice sheet mass balance, we get the following equation:

$$\frac{\partial M}{\partial t} = P - (L + D + B).$$

Model

Glaciers

- Approximation that all the glaciers are one mass and one shape
- Use of the same model as the ice sheet without discharge but other possibilities are offered

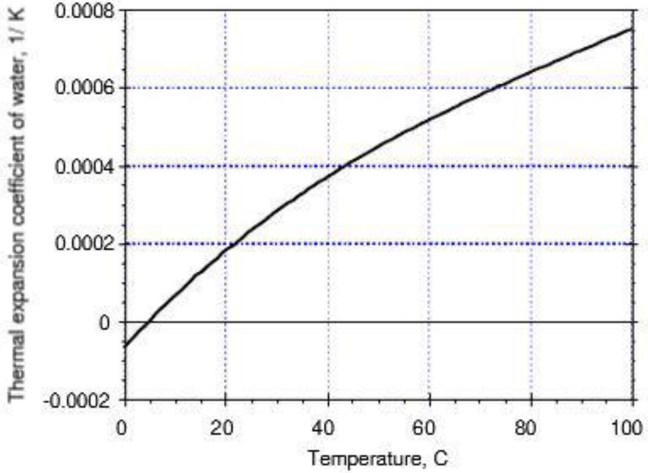
Model

<u>Thermal expansion</u>

- Thermal coefficient for water :α
- Sea water 25% higher in mass that tap water
- Expansion in 3D= factor 3

=>DV/DT=3αV1.25

=>DS/DT=(3.α.V1.25)/(A ocean)

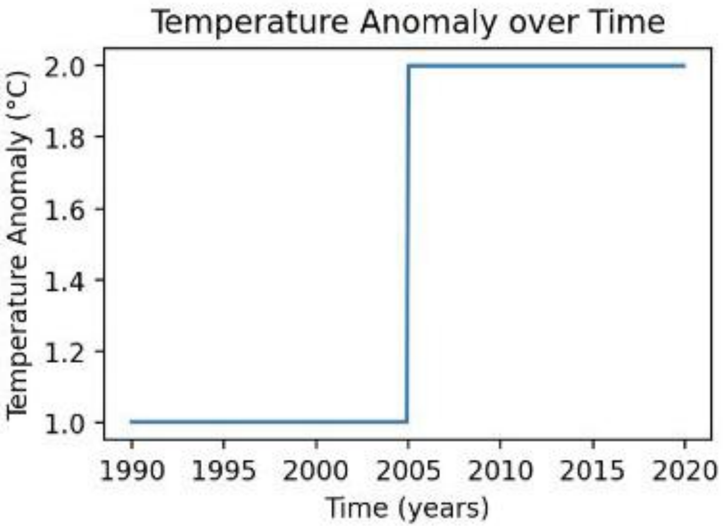


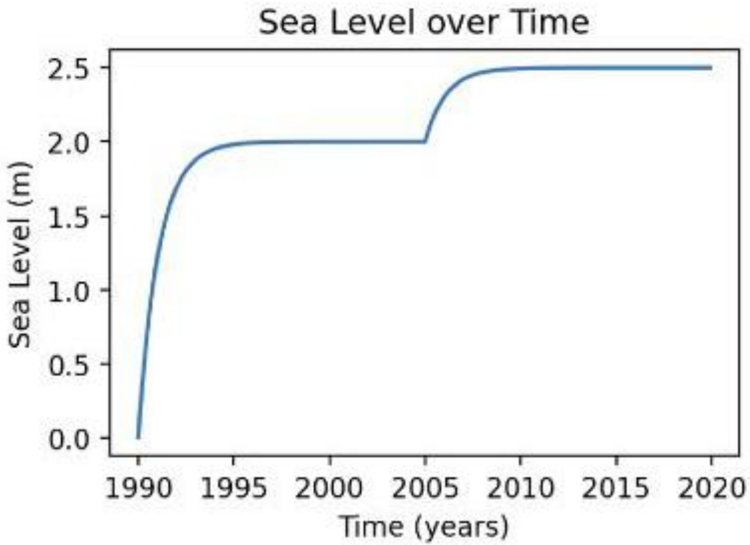
Source: Oraevsky et al 2001

Method

Box-model

$$S_{eq} = aT + b$$
$$\frac{\partial S}{\partial t} = \frac{S_{eq} - S}{\tau}.$$





Method

Forward Euler method

$$S_{eq} = aT + b$$

$$\frac{\partial S}{\partial t} = \frac{S_{eq} - S}{\tau}.$$

```
# Parameters
a = ...      # constant a
b = ...      # constant b
tau = ...    # time constant

dt = ...     # time step
time_steps = ... # number of time steps

T = ...      # temperature measured
S0 = ...      # initial S value

# Forward Euler Method
S = [0, 0, ..., 0]
for t in range(1, time_steps + 1):
    S_eq = a * T + b # calculate equilibrium value
    dS_dt = (S_eq - S[t-1]) / tau  # step toward equil.
    S[t] = S[t-1] + dS_dt * dt # Forward Euler step
```

Listing 1: Forward Euler Method for temperature forced sea level change

Method

Markov Chain Monte Carlo (MCMC)

- Metropolis-Hastings algorithm

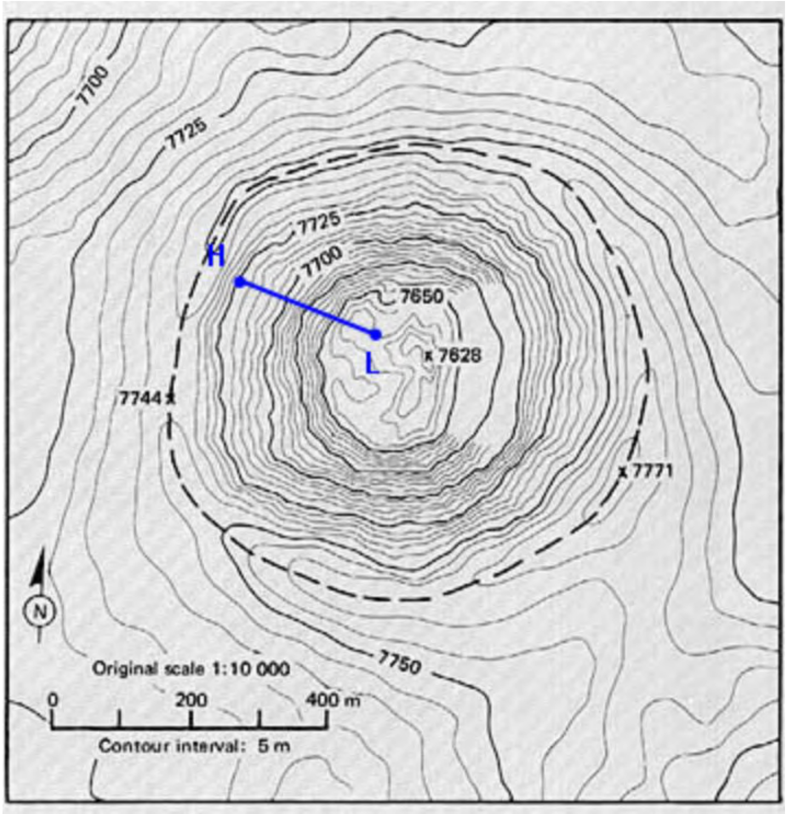
$$P(\theta) = \begin{cases} 1, & \text{if } \mathcal{L}(\theta) < \mathcal{L}(\theta_{\text{previous}}) \\ \frac{\mathcal{L}(\theta_{\text{previous}})}{\mathcal{L}(\theta)}, & \text{otherwise.} \end{cases}$$

- Negative log likelihood (Mosegaard & Tarantola)

$$\mathcal{L}(\theta) = \exp\left(-\frac{||y-\hat{y}||^2}{2\sigma_d^2}\right)$$

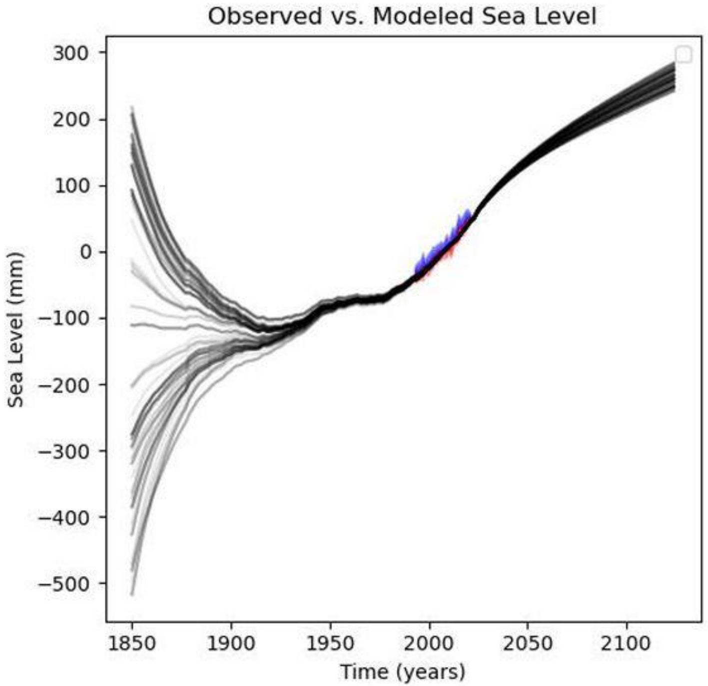
- Optimization

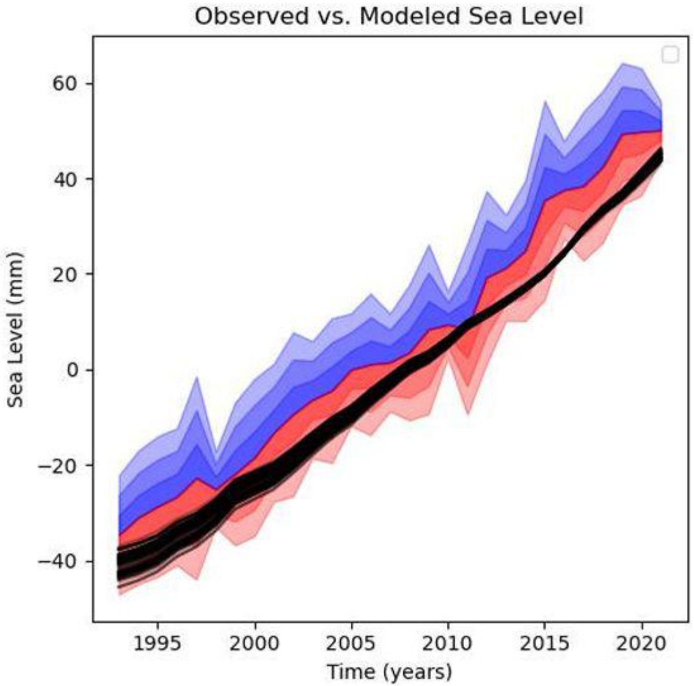
$$\tilde{\theta} = \arg\min_{\theta} \left(||d - f(\theta)||^2 + \epsilon^2||\theta||^2\right),$$



Results

One-box Model: temperature driven





Conclusions

Model comparisons

$$\mathrm{AIC} = 2k - 2\ln(\hat{L})$$

- We need still complete results for larger models

Model	**min Deviation**	**no. params**	**AIC?**
One-box	3.4	3	3.55
Four-box	?	7	?
Ours	?	16	?

Discussion/considerations

Model shortcomings

Possible shortcomings of our model

- Lacking data
- Large uncertainties in data used for each box in the four-box model
- Negative feedbacks
- Any bias and errors inherited by our forecasting from NASA

Discussion/considerations

<u>Approximations/Impacts</u>

Uncertainties in priors

Simple models

Approximations

Local impacts on glaciers could be quite different (generally shrinking but some gains mass)

Complex phenomenon and interaction not fully understood

Different models for each part possible and need to add interaction between them

More complex models with more interactions and high resolution needs huge calculation power

Discussion

<u>Numerical</u>

- It was a bad idea to predict intermediate temperature.
- Many params is maybe bad?
- We could additionally consider error based on measured temperature.

Thank you!