

图算法篇：有向图的深度优先搜索

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深度优先搜索回顾：算法思想

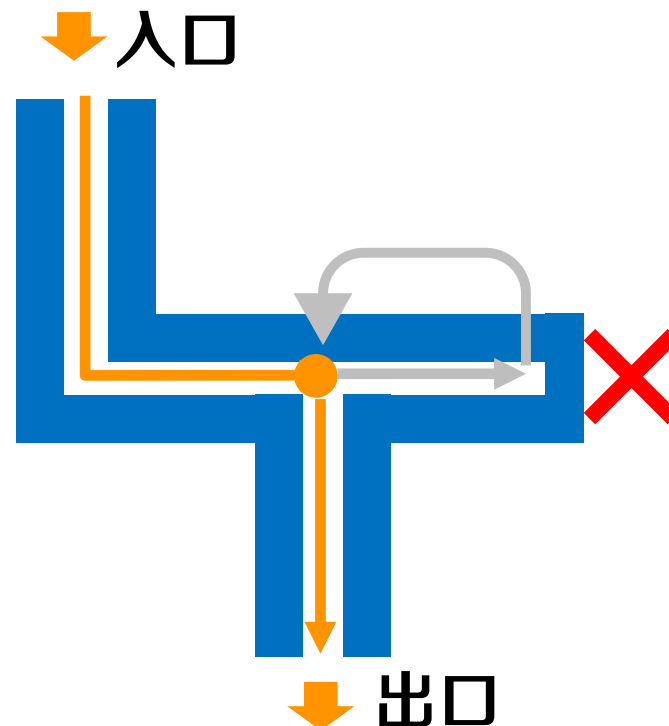


- 算法步骤

- 分叉时，任选一条边深入
- 无边时，后退一步找新边
- 找到边，从新边继续深入

- 辅助数组

- *color*: 表示顶点状态
 - *White*: 白色顶点尚未被发现
 - *Black*: 黑色顶点已被处理
 - *Gray*: 正在处理，尚未完成
- *pred*: 顶点 u 由 $pred[u]$ 发现
- *d*: 顶点发现时刻（变成灰色的时刻）
- *f*: 顶点完成时刻（变成黑色的时刻）



- DFS(G)

输入: 图 G

输出: 祖先数组 $pred$, 发现时刻 d , 结束时刻 f

新建数组 $color[1..V], pred[1..V], d[1..V], f[1..V]$

新建数组

//初始化

for $v \in V$ do

$pred[v] \leftarrow NULL$

$color[v] \leftarrow WHITE$

end

$time \leftarrow 0$

for $v \in V$ do

 if $color[v] = WHITE$ then

 DFS-Visit(G, v)

 end

end

return $pred, d, f$

$d[i], f[i]$ 分别记录顶点 i 的发现时刻与结束时刻

- DFS(G)

输入: 图 G

输出: 祖先数组 $pred$, 发现时刻 d , 结束时刻 f

新建数组 $color[1..V], pred[1..V], d[1..V], f[1..V]$

//初始化

for $v \in V$ do

$pred[v] \leftarrow NULL$

$color[v] \leftarrow WHITE$

end

$time \leftarrow 0$

for $v \in V$ do

 if $color[v] = WHITE$ then

 DFS-Visit(G, v)

 end

end

return $pred, d, f$

初始化

- DFS(G)

输入: 图 G

输出: 祖先数组 $pred$, 发现时刻 d , 结束时刻 f

新建数组 $color[1..V], pred[1..V], d[1..V], f[1..V]$

//初始化

for $v \in V$ do

$pred[v] \leftarrow NULL$

$color[v] \leftarrow WHITE$

end

$time \leftarrow 0$

for $v \in V$ do

 if $color[v] = WHITE$ then

 DFS-Visit(G, v)

 end

end

return $pred, d, f$

保证搜索完全

- DFS-Visit(G, v)

输入: 图 G , 顶点 v

$color[v] \leftarrow GRAY$

$time \leftarrow time + 1$

$d[v] \leftarrow time$

for $w \in G.Adj[v]$ do

 if $color[w] = WHITE$ then

$pred[w] \leftarrow v$

 DFS-Visit(G, w)

 end

end

$color[v] \leftarrow BLACK$

$time \leftarrow time + 1$

$f[v] \leftarrow time$

修改当前顶点颜色、发现时刻

- DFS-Visit(G, v)

输入: 图 G , 顶点 v

$color[v] \leftarrow GRAY$

$time \leftarrow time + 1$

$d[v] \leftarrow time$

```
(for  $w \in G.Adj[v]$  do
  | if  $color[w] = WHITE$  then
  |   |  $pred[w] \leftarrow v$ 
  |   | DFS-Visit( $G, w$ )
  | end
end)
```

$color[v] \leftarrow BLACK$

$time \leftarrow time + 1$

$f[v] \leftarrow time$

搜索相邻顶点

- DFS-Visit(G, v)

输入: 图 G , 顶点 v

$color[v] \leftarrow GRAY$

$time \leftarrow time + 1$

$d[v] \leftarrow time$

for $w \in G.Adj[v]$ **do**

if $color[w] = WHITE$ **then**

$pred[w] \leftarrow v$

 DFS-Visit(G, w)

end

end

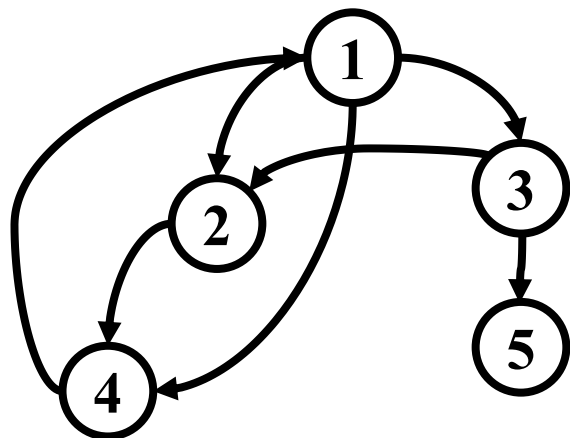
$color[v] \leftarrow BLACK$

$time \leftarrow time + 1$

$f[v] \leftarrow time$

结束搜索

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 0

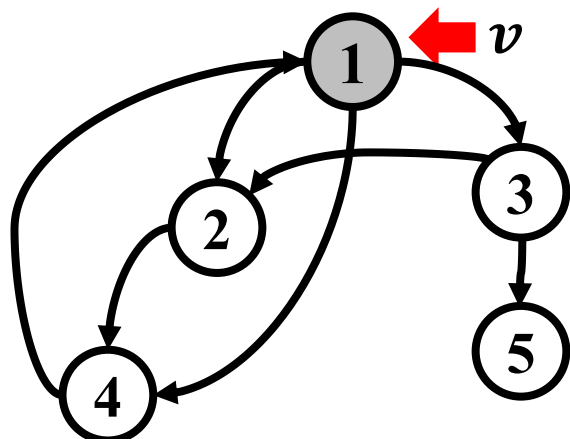
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	N	N	N	N

<i>v</i>	1	2	3	4	5
<i>color</i>	W	W	W	W	W

<i>v</i>	1	2	3	4	5
<i>d</i>					

<i>v</i>	1	2	3	4	5
<i>f</i>					

深度优先搜索回顾：有向图算法实例



$color[v] \leftarrow GRAY$

$time \leftarrow time + 1$

$d[v] \leftarrow time$

for $w \in Adj[v]$ **do**

if $color[w] = WHITE$ **then**

$pred[w] \leftarrow v$

 DFS-Visit(w)

end

end

$color[v] \leftarrow BLACK$

$time \leftarrow time + 1$

$f[v] \leftarrow time$

$time = 0$

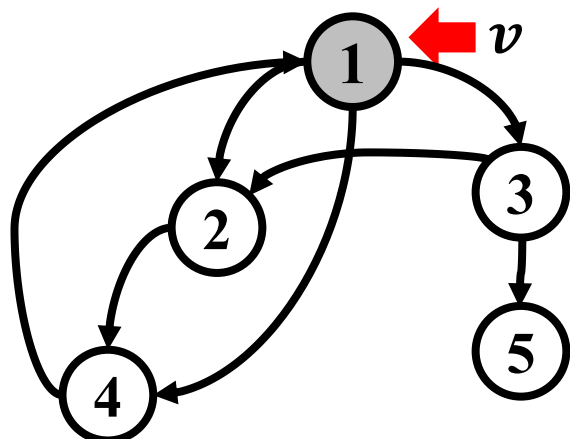
v	1	2	3	4	5
$pred$	N	N	N	N	N

v	1	2	3	4	5
$color$	G	W	W	W	W

v	1	2	3	4	5
d					

v	1	2	3	4	5
f					

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 1

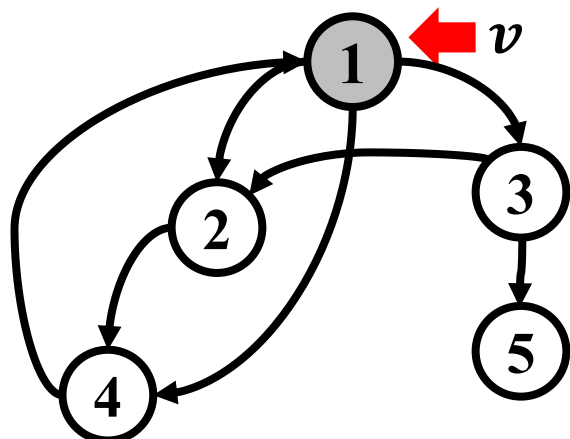
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	N	N	N	N

<i>v</i>	1	2	3	4	5
<i>color</i>	G	W	W	W	W

<i>v</i>	1	2	3	4	5
<i>d</i>					

<i>v</i>	1	2	3	4	5
<i>f</i>					

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 1

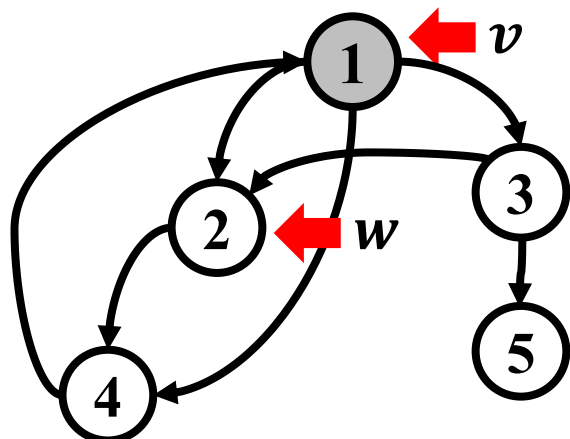
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	N	N	N	N

<i>v</i>	1	2	3	4	5
<i>color</i>	G	W	W	W	W

<i>v</i>	1	2	3	4	5
<i>d</i>	1				

<i>v</i>	1	2	3	4	5
<i>f</i>					

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for  $w \in Adj[v]$  do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

$time = 1$

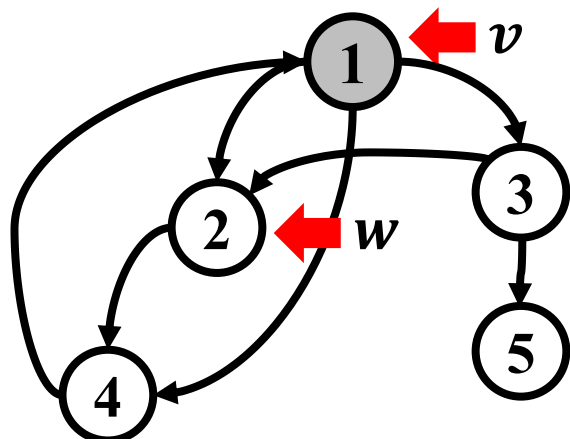
v	1	2	3	4	5
$pred$	N	N	N	N	N

v	1	2	3	4	5
$color$	G	W	W	W	W

v	1	2	3	4	5
d	1				

v	1	2	3	4	5
f					

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 1

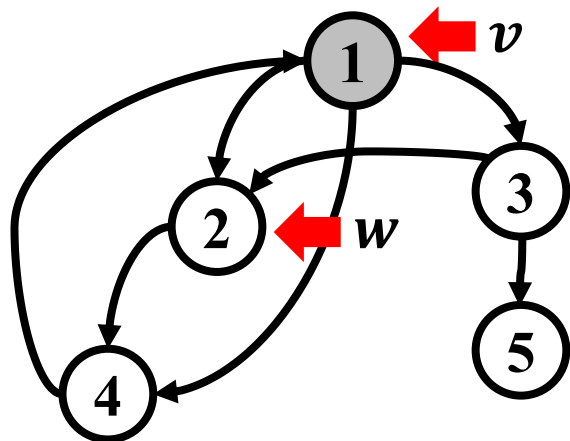
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	N	N	N	N

<i>v</i>	1	2	3	4	5
<i>color</i>	G	W	W	W	W

<i>v</i>	1	2	3	4	5
<i>d</i>	1				

<i>v</i>	1	2	3	4	5
<i>f</i>					

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 1

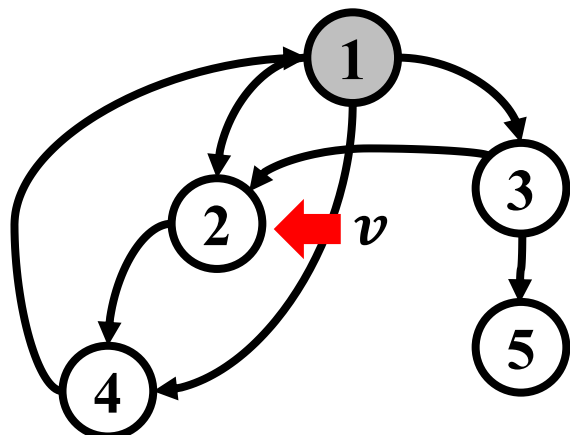
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	N	N	N

<i>v</i>	1	2	3	4	5
<i>color</i>	G	W	W	W	W

<i>v</i>	1	2	3	4	5
<i>d</i>	1				

<i>v</i>	1	2	3	4	5
<i>f</i>					

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for  $w \in \text{Adj}[v]$  do
    if  $\text{color}[w] = \text{WHITE}$  then
         $\text{pred}[w] \leftarrow v$ 
        DFS-Visit( $w$ )
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

$\text{time} = 1$

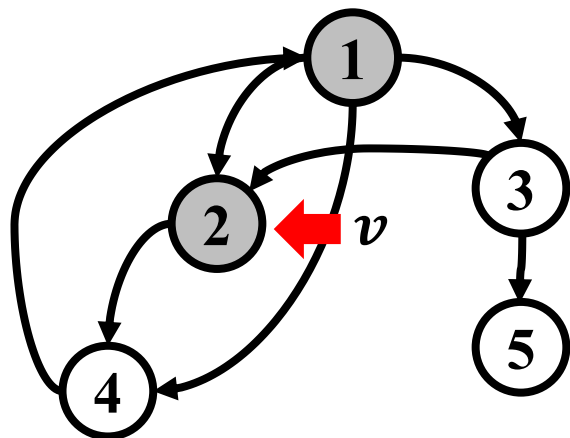
v	1	2	3	4	5
pred	N	1	N	N	N

v	1	2	3	4	5
color	G	W	W	W	W

v	1	2	3	4	5
d	1				

v	1	2	3	4	5
f					

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 1

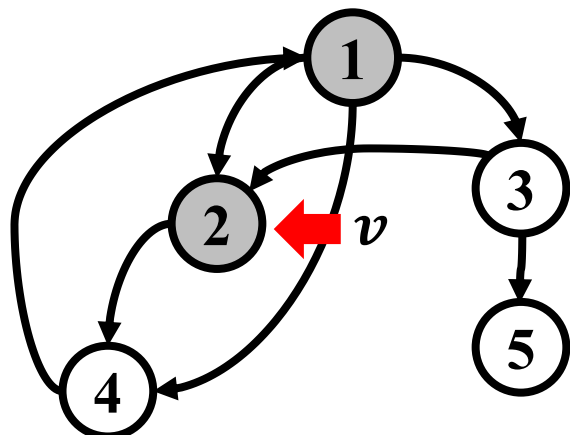
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	N	N	N

<i>v</i>	1	2	3	4	5
<i>color</i>	G	G	W	W	W

<i>v</i>	1	2	3	4	5
<i>d</i>	1				

<i>v</i>	1	2	3	4	5
<i>f</i>					

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 2

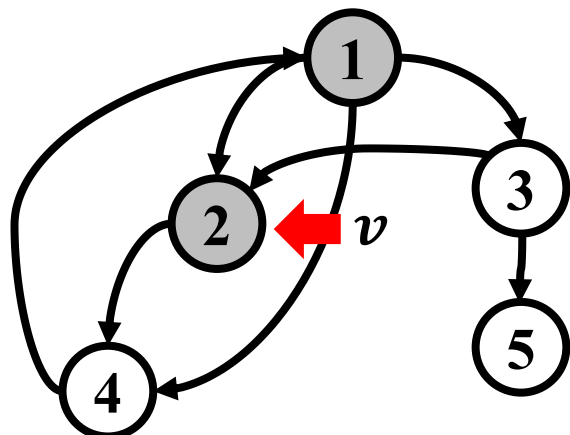
v	1	2	3	4	5
$pred$	N	1	N	N	N

v	1	2	3	4	5
$color$	G	G	W	W	W

v	1	2	3	4	5
d	1				

v	1	2	3	4	5
f					

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

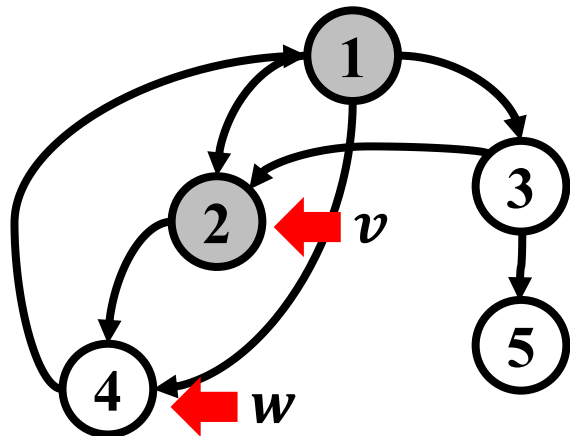
$time = 2$

v	1	2	3	4	5
$pred$	N	1	N	N	N

v	1	2	3	4	5
$color$	G	G	W	W	W

v	1	2	3	4	5
d	1	2			

v	1	2	3	4	5
f					



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

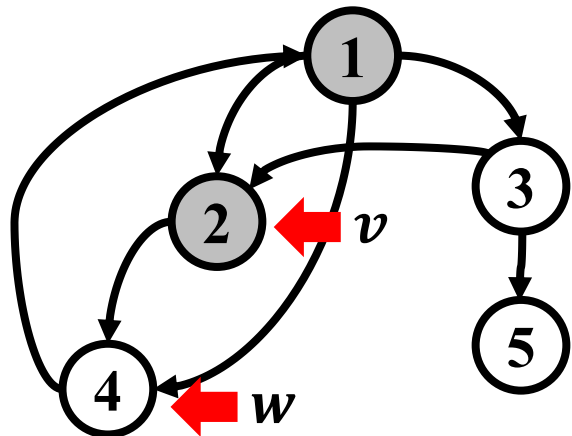
time = 2

<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	N	N	N

<i>v</i>	1	2	3	4	5
<i>color</i>	G	G	W	W	W

<i>v</i>	1	2	3	4	5
<i>d</i>	1	2			

<i>v</i>	1	2	3	4	5
<i>f</i>					



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 2

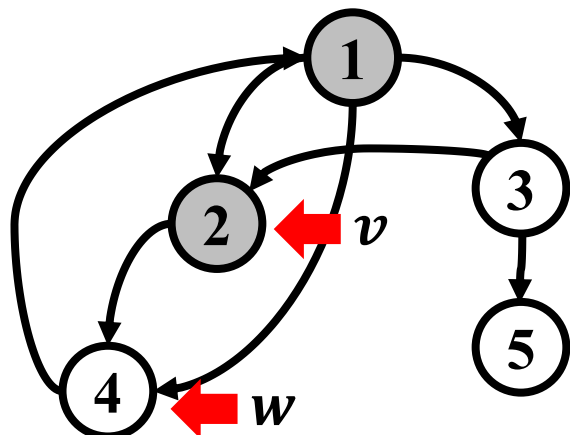
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	N	N	N

<i>v</i>	1	2	3	4	5
<i>color</i>	G	G	W	W	W

<i>v</i>	1	2	3	4	5
<i>d</i>	1	2			

<i>v</i>	1	2	3	4	5
<i>f</i>					

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

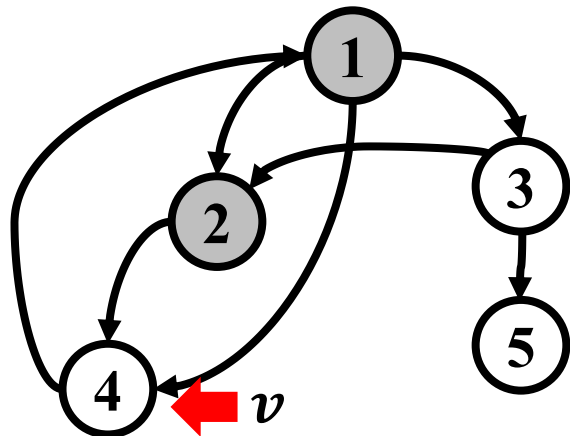
time = 2

<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	N	2	N

<i>v</i>	1	2	3	4	5
<i>color</i>	G	G	W	W	W

<i>v</i>	1	2	3	4	5
<i>d</i>	1	2			

<i>v</i>	1	2	3	4	5
<i>f</i>					



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 2

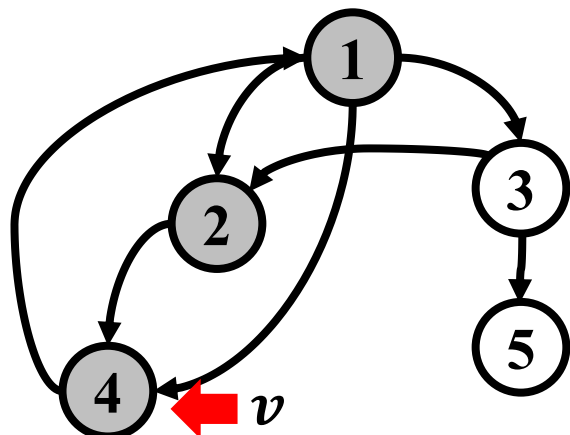
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	N	2	N

<i>v</i>	1	2	3	4	5
<i>color</i>	G	G	W	W	W

<i>v</i>	1	2	3	4	5
<i>d</i>	1	2			

<i>v</i>	1	2	3	4	5
<i>f</i>					

深度优先搜索回顾：有向图算法实例



$color[v] \leftarrow GRAY$

$time \leftarrow time + 1$

$d[v] \leftarrow time$

for $w \in Adj[v]$ **do**

if $color[w] = WHITE$ **then**

$pred[w] \leftarrow v$

 DFS-Visit(w)

end

end

$color[v] \leftarrow BLACK$

$time \leftarrow time + 1$

$f[v] \leftarrow time$

$time = 2$

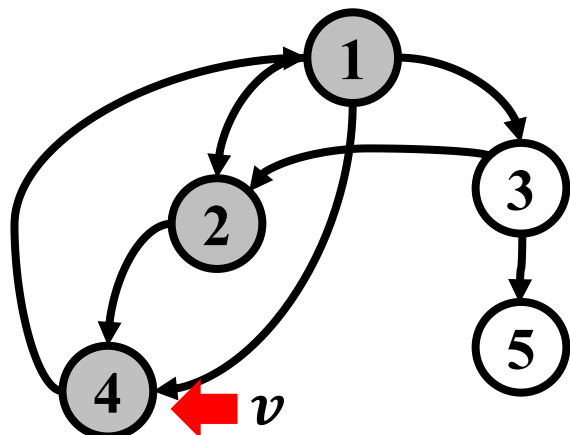
v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	G	W	G	W

v	1	2	3	4	5
d	1	2			

v	1	2	3	4	5
f					

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 3

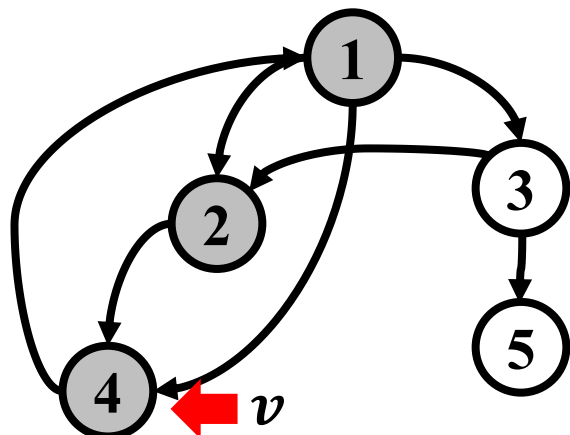
v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	G	W	G	W

v	1	2	3	4	5
d	1	2			

v	1	2	3	4	5
f					

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

$time = 3$

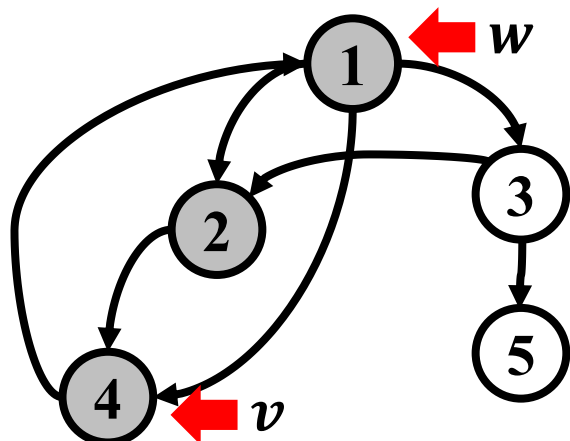
v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	G	W	G	W

v	1	2	3	4	5
d	1	2		3	

v	1	2	3	4	5
f					

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 3

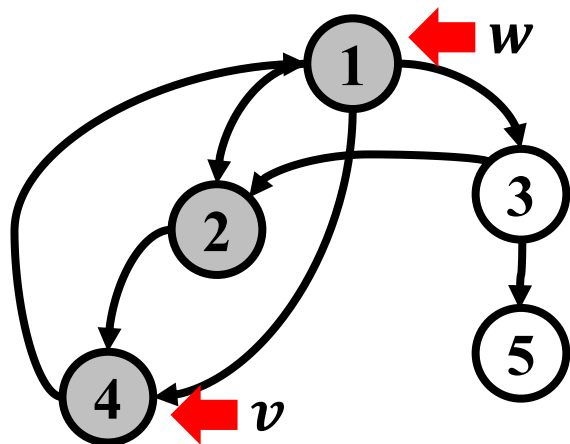
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	N	2	N

<i>v</i>	1	2	3	4	5
<i>color</i>	G	G	W	G	W

<i>v</i>	1	2	3	4	5
<i>d</i>	1	2		3	

<i>v</i>	1	2	3	4	5
<i>f</i>					

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

$time = 3$

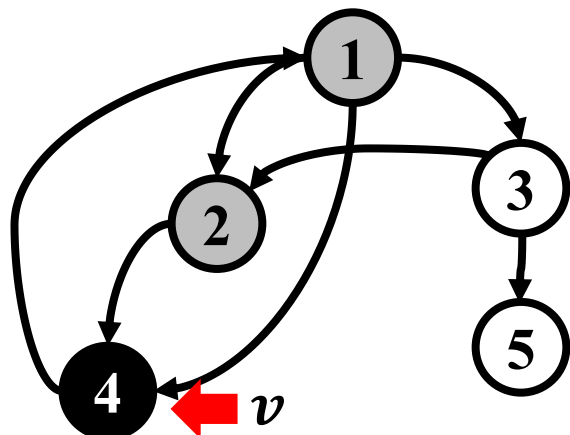
v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	G	W	G	W

v	1	2	3	4	5
d	1	2		3	

v	1	2	3	4	5
f					

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

$time = 3$

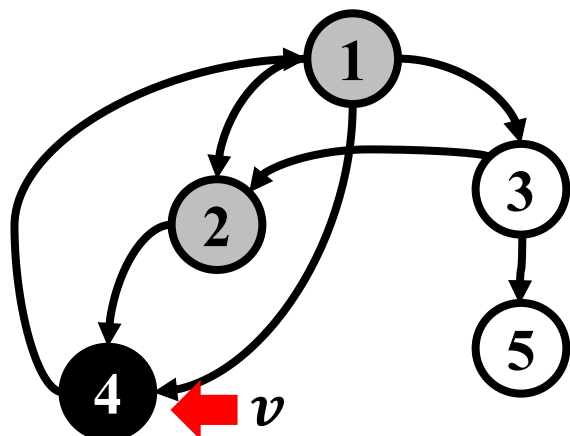
v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	G	W	B	W

v	1	2	3	4	5
d	1	2		3	

v	1	2	3	4	5
f					

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 4

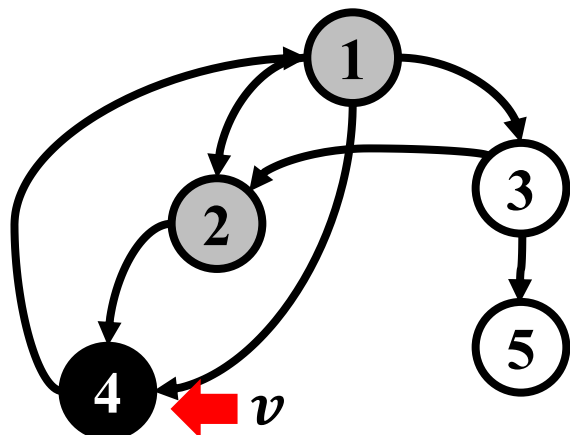
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	N	2	N

<i>v</i>	1	2	3	4	5
<i>color</i>	G	G	W	B	W

<i>v</i>	1	2	3	4	5
<i>d</i>	1	2		3	

<i>v</i>	1	2	3	4	5
<i>f</i>					

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

$time = 4$

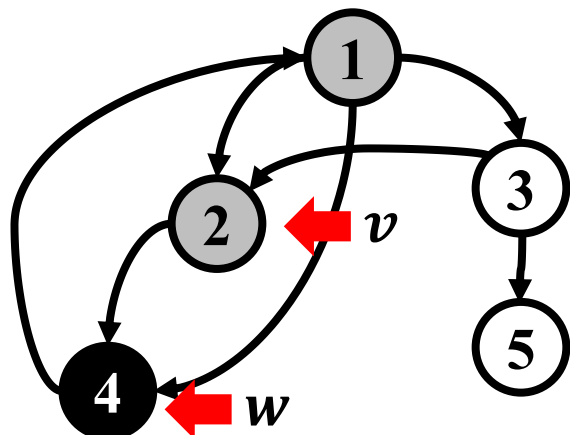
v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	G	W	B	W

v	1	2	3	4	5
d	1	2		3	

v	1	2	3	4	5
f				4	

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 4

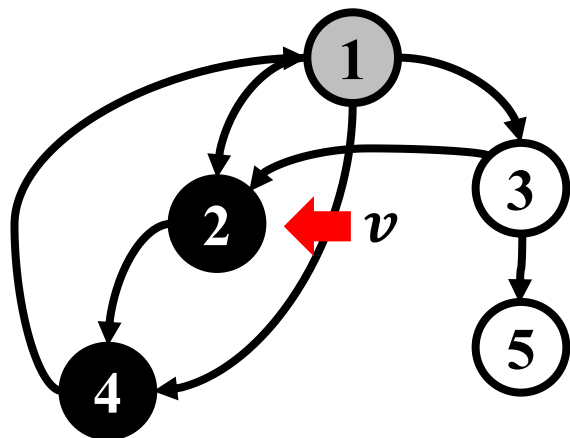
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	N	2	N

<i>v</i>	1	2	3	4	5
<i>color</i>	G	G	W	B	W

<i>v</i>	1	2	3	4	5
<i>d</i>	1	2		3	

<i>v</i>	1	2	3	4	5
<i>f</i>				4	

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

$time = 4$

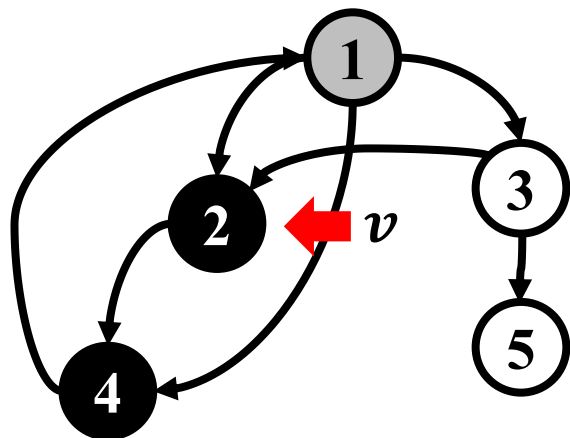
v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	B	W	B	W

v	1	2	3	4	5
d	1	2		3	

v	1	2	3	4	5
f				4	

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 5

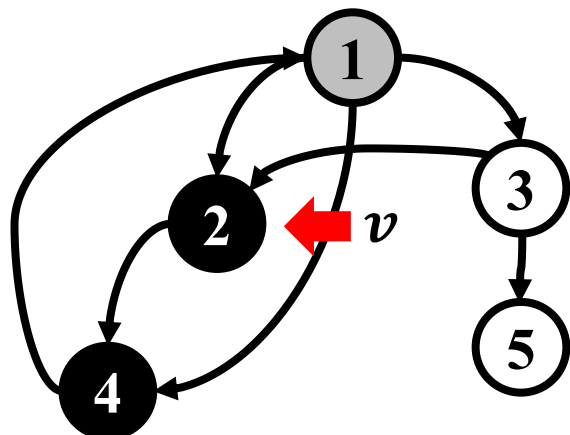
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	N	2	N

<i>v</i>	1	2	3	4	5
<i>color</i>	G	B	W	B	W

<i>v</i>	1	2	3	4	5
<i>d</i>	1	2		3	

<i>v</i>	1	2	3	4	5
<i>f</i>				4	

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

$time = 5$

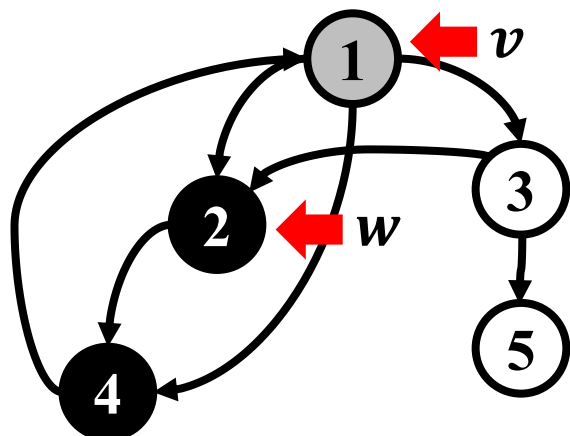
v	1	2	3	4	5
$pred$	N	1	N	2	N

v	1	2	3	4	5
$color$	G	B	W	B	W

v	1	2	3	4	5
d	1	2		3	

v	1	2	3	4	5
f		5		4	

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 5

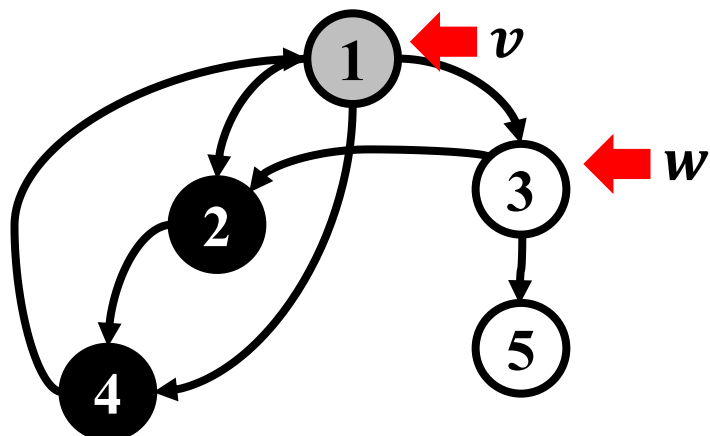
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	N	2	N

<i>v</i>	1	2	3	4	5
<i>color</i>	G	B	W	B	W

<i>v</i>	1	2	3	4	5
<i>d</i>	1	2		3	

<i>v</i>	1	2	3	4	5
<i>f</i>		5		4	

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 5

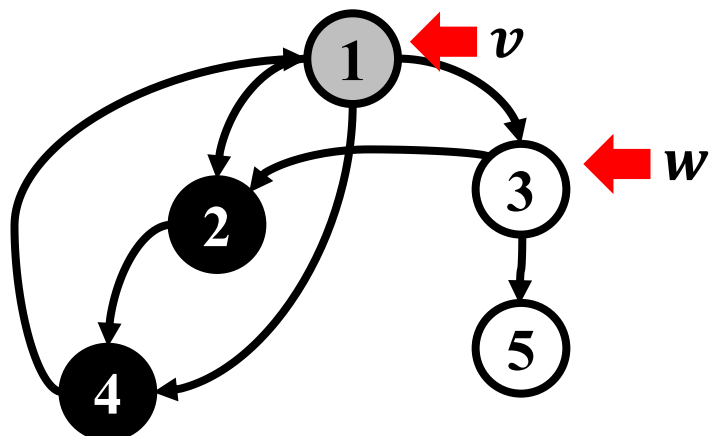
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	N	2	N

<i>v</i>	1	2	3	4	5
<i>color</i>	G	B	W	B	W

<i>v</i>	1	2	3	4	5
<i>d</i>	1	2		3	

<i>v</i>	1	2	3	4	5
<i>f</i>		5		4	

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 5

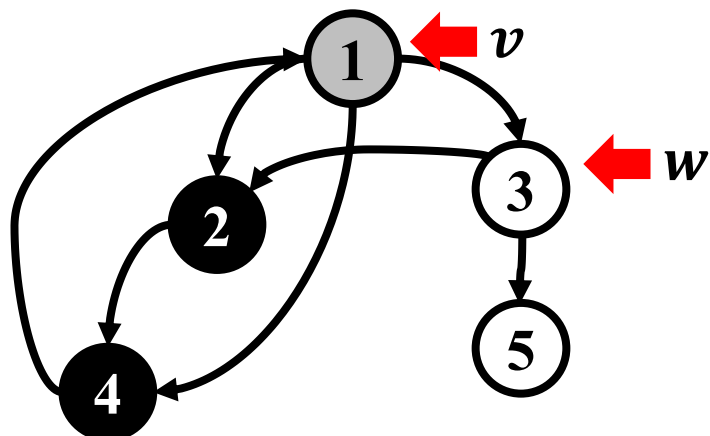
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	N	2	N

<i>v</i>	1	2	3	4	5
<i>color</i>	G	B	W	B	W

<i>v</i>	1	2	3	4	5
<i>d</i>	1	2		3	

<i>v</i>	1	2	3	4	5
<i>f</i>		5		4	

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 5

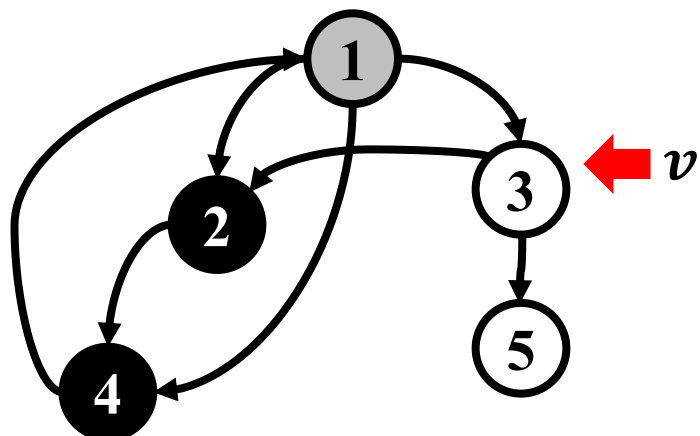
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	1	2	N

<i>v</i>	1	2	3	4	5
<i>color</i>	G	B	W	B	W

<i>v</i>	1	2	3	4	5
<i>d</i>	1	2		3	

<i>v</i>	1	2	3	4	5
<i>f</i>		5		4	

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 5

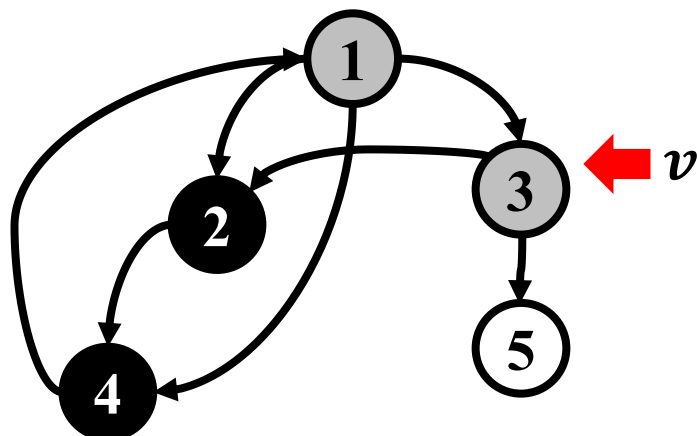
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	1	2	N

<i>v</i>	1	2	3	4	5
<i>color</i>	G	B	W	B	W

<i>v</i>	1	2	3	4	5
<i>d</i>	1	2		3	

<i>v</i>	1	2	3	4	5
<i>f</i>		5		4	

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

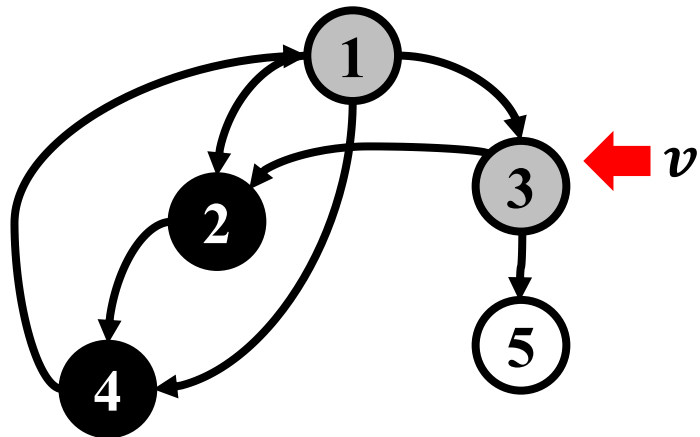
time = 5

<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	1	2	N

<i>v</i>	1	2	3	4	5
<i>color</i>	G	B	G	B	W

<i>v</i>	1	2	3	4	5
<i>d</i>	1	2		3	

<i>v</i>	1	2	3	4	5
<i>f</i>		5		4	



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 6

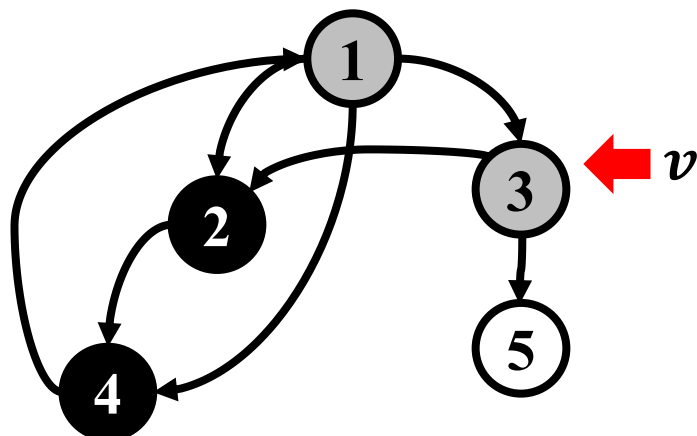
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	1	2	N

<i>v</i>	1	2	3	4	5
<i>color</i>	G	B	G	B	W

<i>v</i>	1	2	3	4	5
<i>d</i>	1	2		3	

<i>v</i>	1	2	3	4	5
<i>f</i>		5		4	

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 6

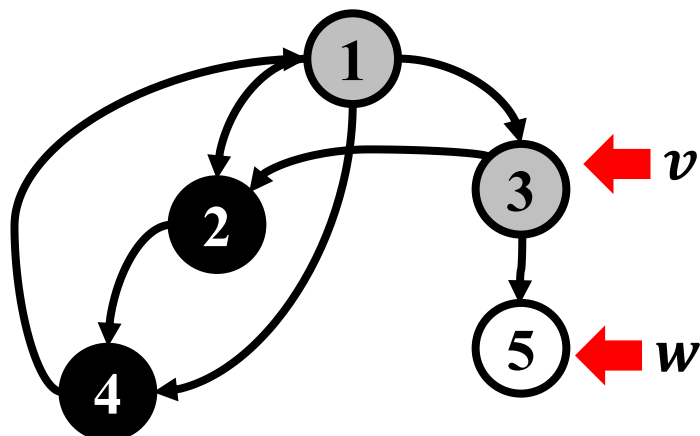
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	1	2	N

<i>v</i>	1	2	3	4	5
<i>color</i>	G	B	G	B	W

<i>v</i>	1	2	3	4	5
<i>d</i>	1	2	6	3	

<i>v</i>	1	2	3	4	5
<i>f</i>		5		4	

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 6

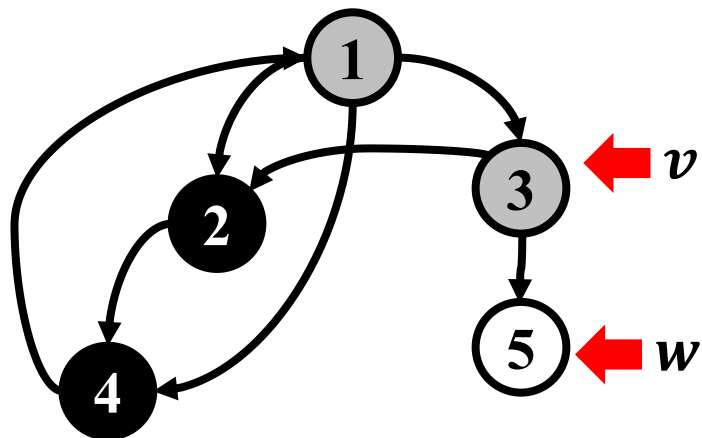
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	1	2	N

<i>v</i>	1	2	3	4	5
<i>color</i>	G	B	G	B	W

<i>v</i>	1	2	3	4	5
<i>d</i>	1	2	6	3	

<i>v</i>	1	2	3	4	5
<i>f</i>		5		4	

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 6

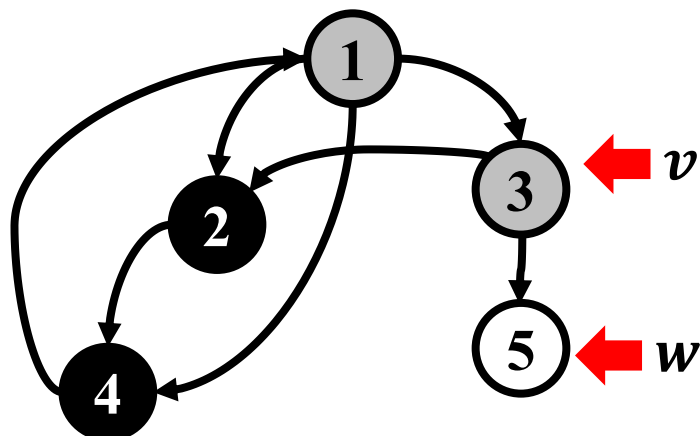
v	1	2	3	4	5
$pred$	N	1	1	2	N

v	1	2	3	4	5
$color$	G	B	G	B	W

v	1	2	3	4	5
d	1	2	6	3	

v	1	2	3	4	5
f		5		4	

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

$time = 6$

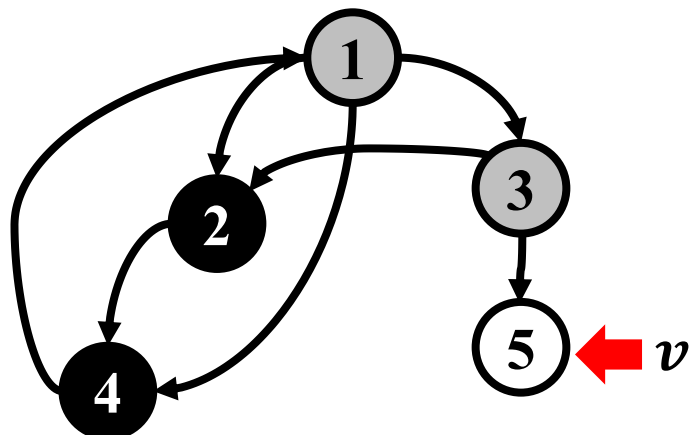
v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	G	B	G	B	W

v	1	2	3	4	5
d	1	2	6	3	

v	1	2	3	4	5
f		5		4	

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

$time = 6$

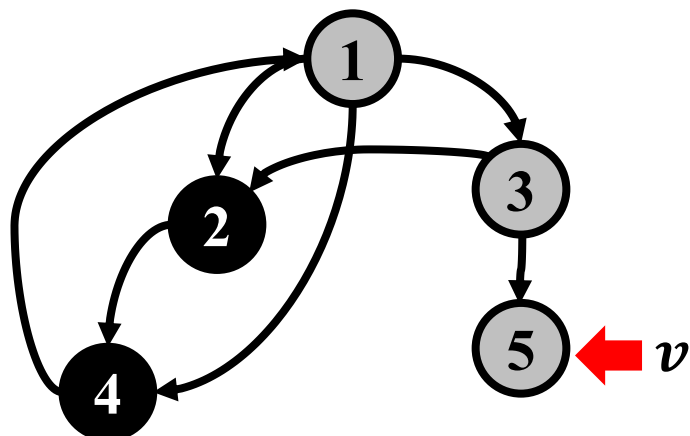
v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	G	B	G	B	W

v	1	2	3	4	5
d	1	2	6	3	

v	1	2	3	4	5
f		5		4	

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 6

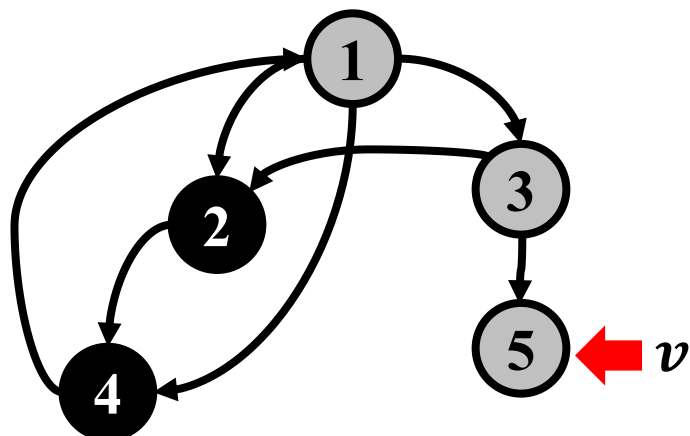
v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	G	B	G	B	G

v	1	2	3	4	5
d	1	2	6	3	

v	1	2	3	4	5
f		5		4	

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 7

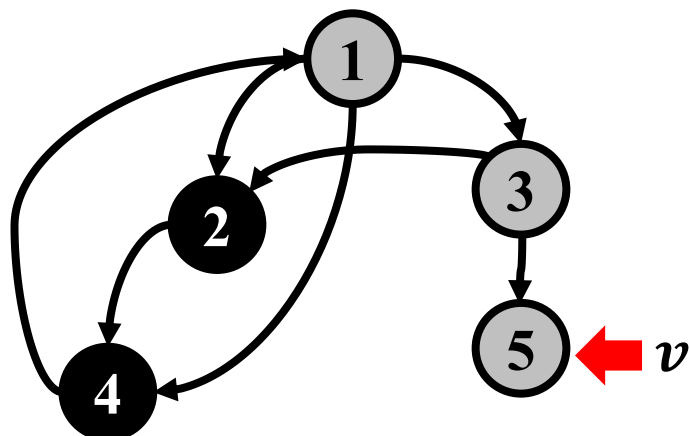
v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	G	B	G	B	G

v	1	2	3	4	5
d	1	2	6	3	

v	1	2	3	4	5
f		5		4	

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

$time = 7$

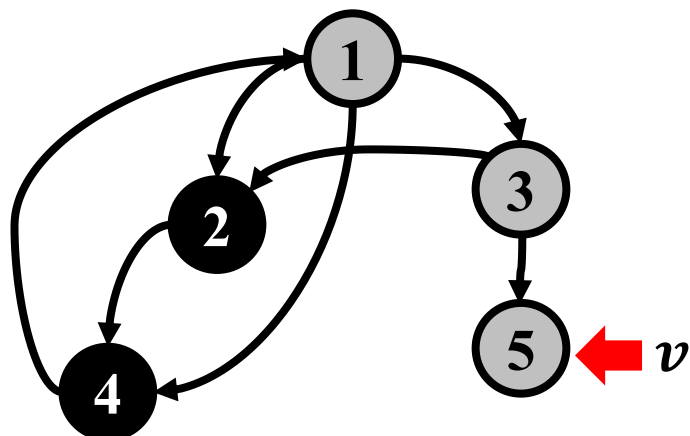
v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	G	B	G	B	G

v	1	2	3	4	5
d	1	2	6	3	7

v	1	2	3	4	5
f		5		4	

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for  $w \in Adj[v]$  do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

$time = 7$

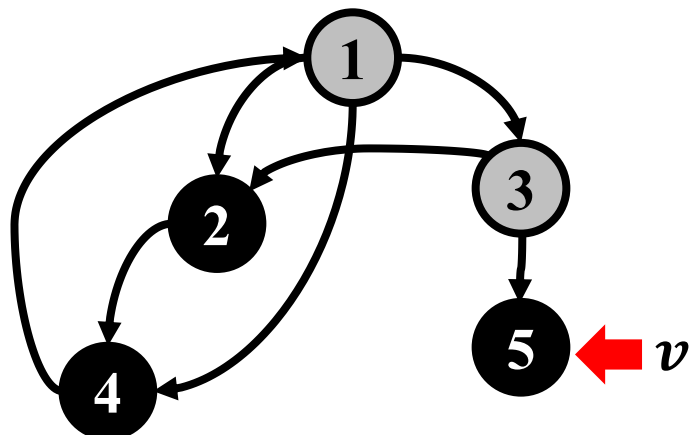
v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	G	B	G	B	G

v	1	2	3	4	5
d	1	2	6	3	7

v	1	2	3	4	5
f		5		4	

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

$time = 7$

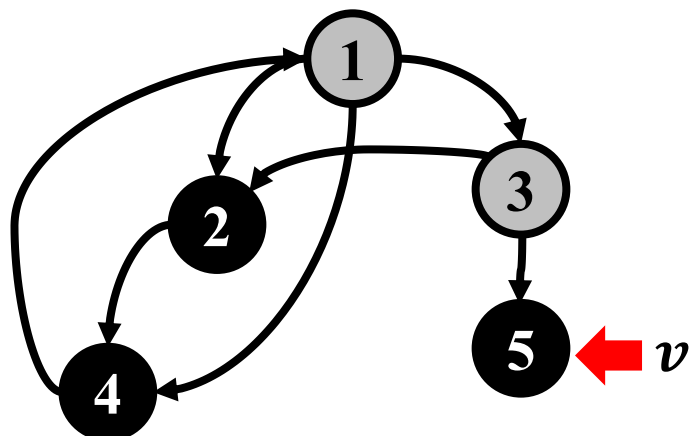
v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	G	B	G	B	B

v	1	2	3	4	5
d	1	2	6	3	7

v	1	2	3	4	5
f		5		4	

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 8

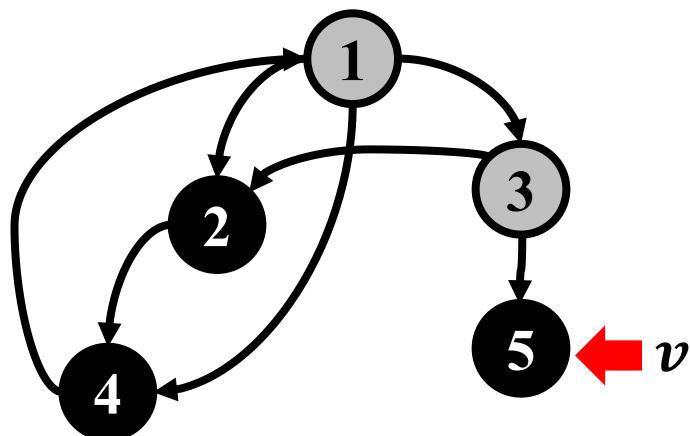
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	1	2	3

<i>v</i>	1	2	3	4	5
<i>color</i>	G	B	G	B	B

<i>v</i>	1	2	3	4	5
<i>d</i>	1	2	6	3	7

<i>v</i>	1	2	3	4	5
<i>f</i>		5		4	

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 8

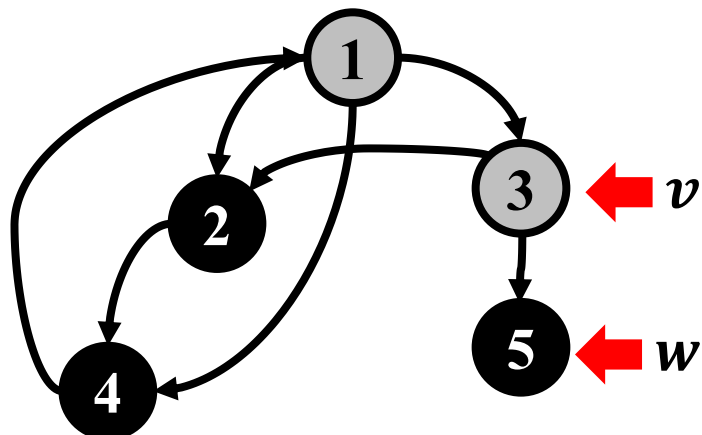
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	1	2	3

<i>v</i>	1	2	3	4	5
<i>color</i>	G	B	G	B	B

<i>v</i>	1	2	3	4	5
<i>d</i>	1	2	6	3	7

<i>v</i>	1	2	3	4	5
<i>f</i>		5		4	8

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

$time = 8$

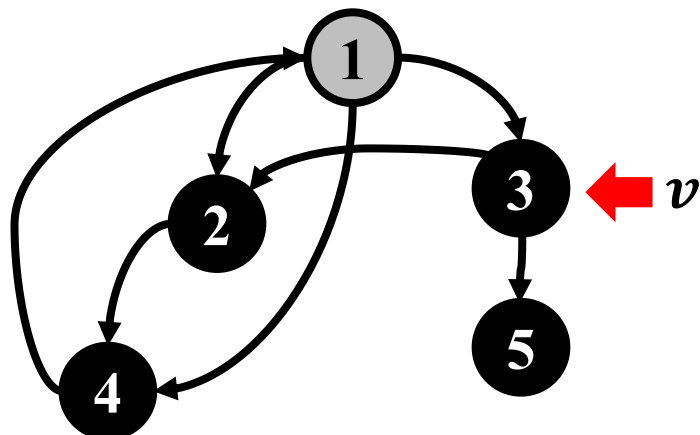
v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	G	B	G	B	B

v	1	2	3	4	5
d	1	2	6	3	7

v	1	2	3	4	5
f		5		4	8

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

$time = 8$

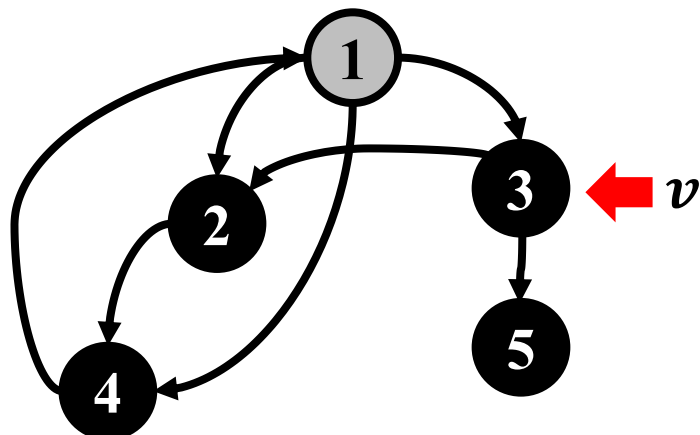
v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	G	B	B	B	B

v	1	2	3	4	5
d	1	2	6	3	7

v	1	2	3	4	5
f		5		4	8

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 9

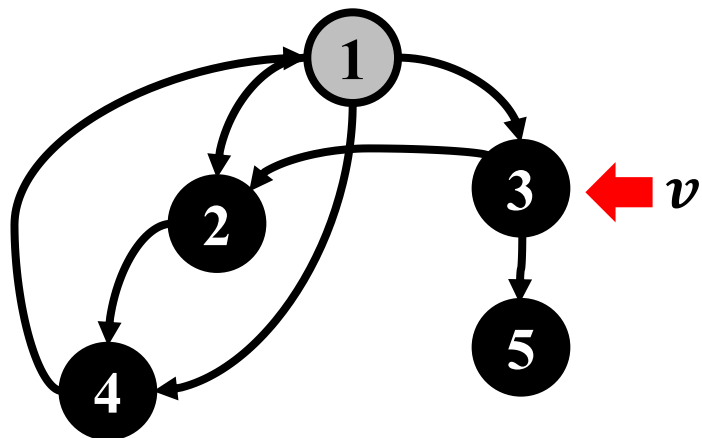
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	1	2	3

<i>v</i>	1	2	3	4	5
<i>color</i>	G	B	B	B	B

<i>v</i>	1	2	3	4	5
<i>d</i>	1	2	6	3	7

<i>v</i>	1	2	3	4	5
<i>f</i>		5		4	8

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

$time = 9$

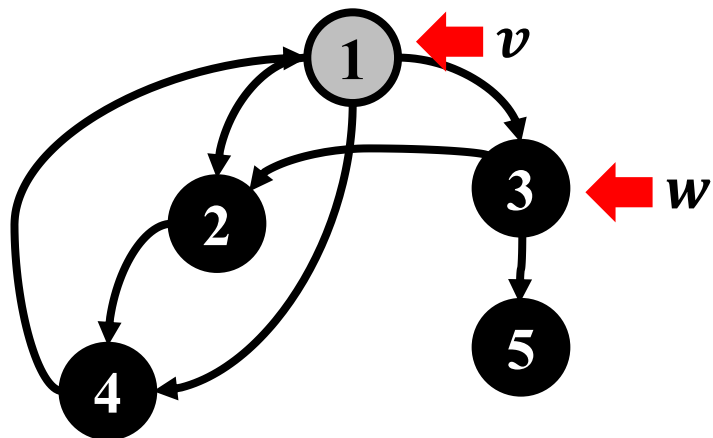
v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	G	B	B	B	B

v	1	2	3	4	5
d	1	2	6	3	7

v	1	2	3	4	5
f		5	9	4	8

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 9

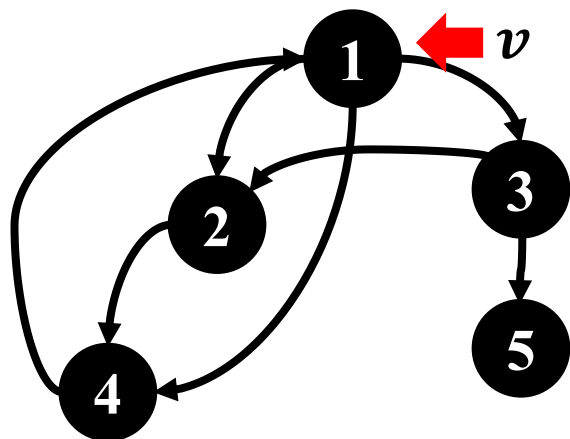
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	1	2	3

<i>v</i>	1	2	3	4	5
<i>color</i>	G	B	B	B	B

<i>v</i>	1	2	3	4	5
<i>d</i>	1	2	6	3	7

<i>v</i>	1	2	3	4	5
<i>f</i>		5	9	4	8

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 9

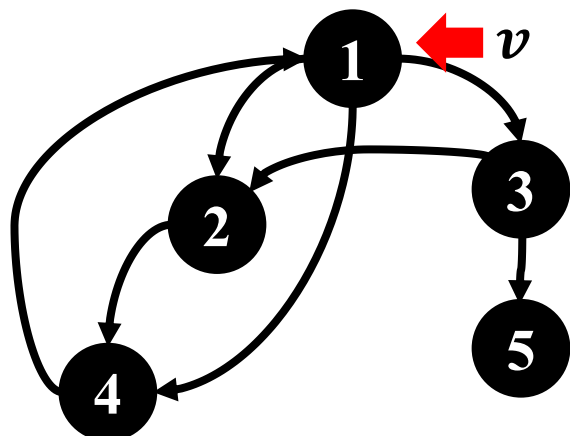
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	1	2	3

<i>v</i>	1	2	3	4	5
<i>color</i>	B	B	B	B	B

<i>v</i>	1	2	3	4	5
<i>d</i>	1	2	6	3	7

<i>v</i>	1	2	3	4	5
<i>f</i>		5	9	4	8

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

time = 10

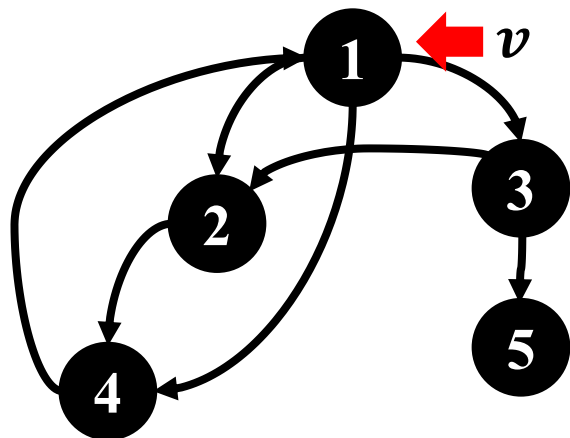
<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	1	2	3

<i>v</i>	1	2	3	4	5
<i>color</i>	B	B	B	B	B

<i>v</i>	1	2	3	4	5
<i>d</i>	1	2	6	3	7

<i>v</i>	1	2	3	4	5
<i>f</i>		5	9	4	8

深度优先搜索回顾：有向图算法实例



```
color[v] ← GRAY
time ← time + 1
d[v] ← time
for w ∈ Adj[v] do
    if color[w] = WHITE then
        pred[w] ← v
        DFS-Visit(w)
    end
end
color[v] ← BLACK
time ← time + 1
f[v] ← time
```

$time = 10$

v	1	2	3	4	5
$pred$	N	1	1	2	3

v	1	2	3	4	5
$color$	B	B	B	B	B

v	1	2	3	4	5
d	1	2	6	3	7

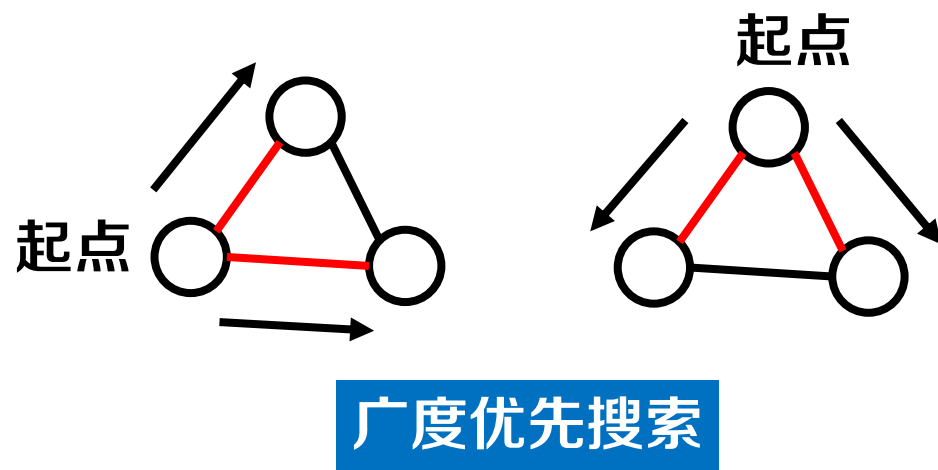
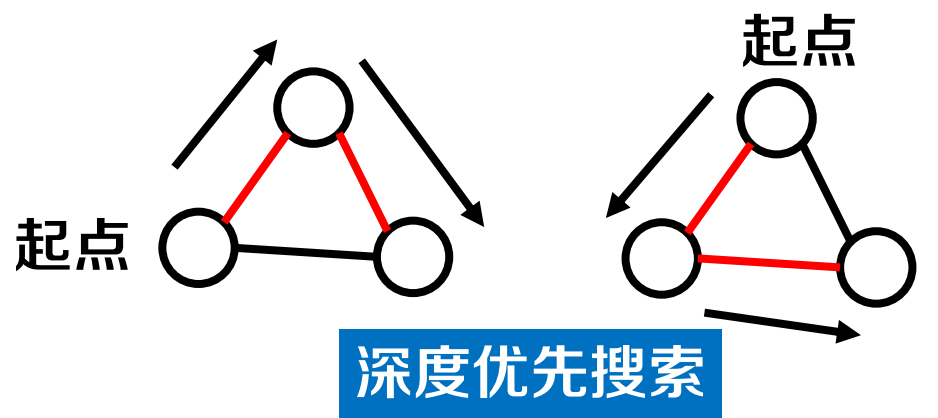
v	1	2	3	4	5
f	10	5	9	4	8

连通无向图的优先树与连通有向图的优先森林



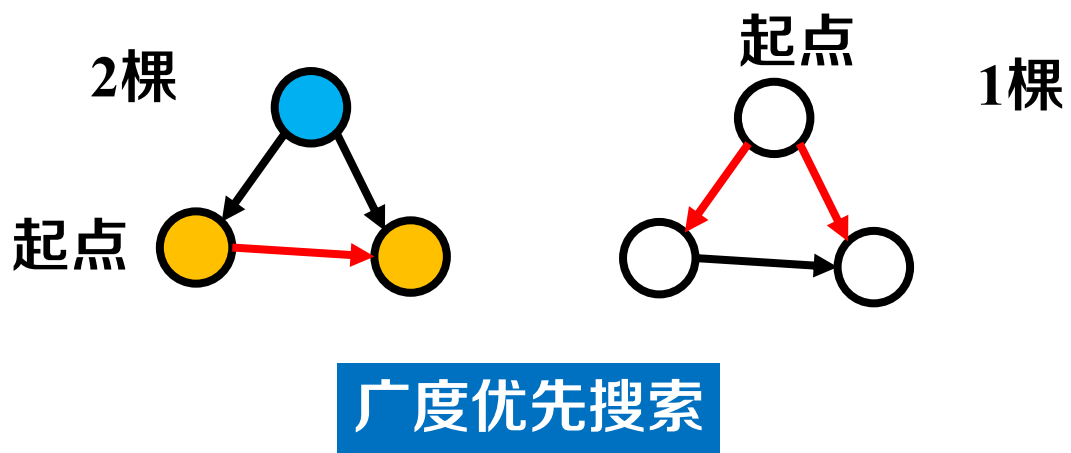
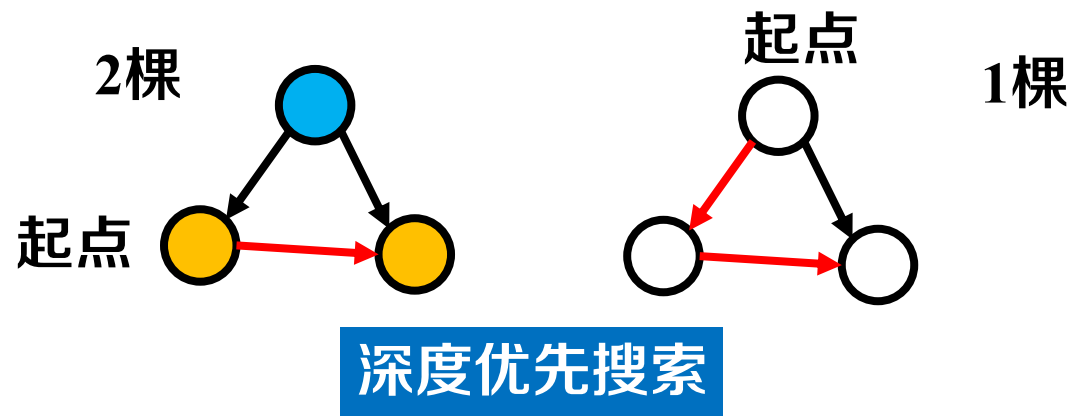
• 无向图

- 树的形状：取决于搜索顺序
- 树的数量：确定1棵优先树

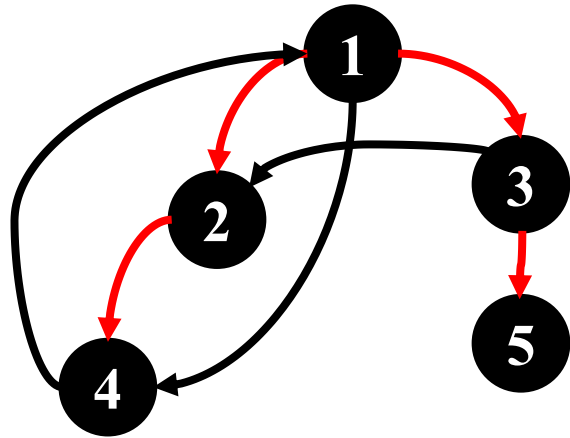


• 有向图

- 树的形状：取决于搜索顺序
- 树的数量：取决于搜索顺序

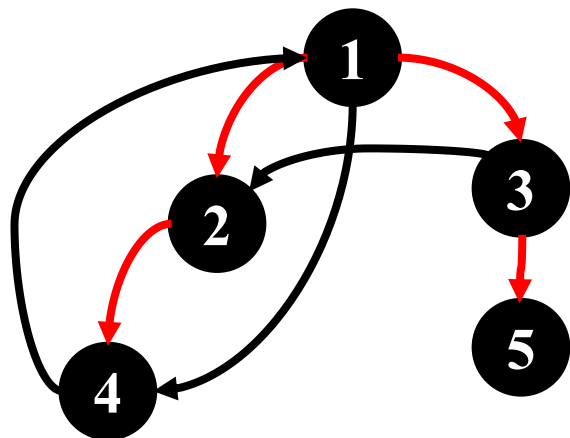


有向图深度优先森林



time = 10

<i>v</i>	1	2	3	4	5
<i>pred</i>	N	1	1	2	3

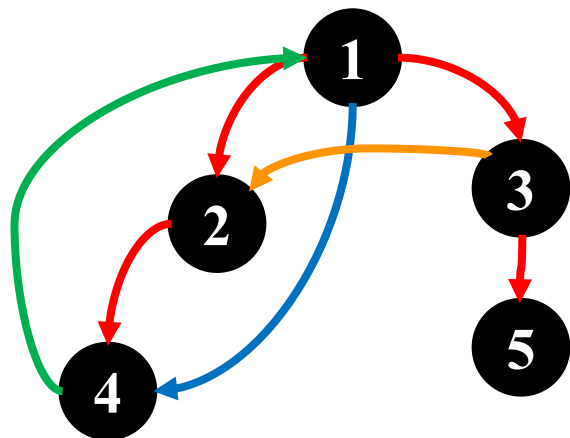


$time = 10$

v	1	2	3	4	5
$pred$	N	1	1	2	3

- 回顾深度优先搜索边的性质

- 后向边：不是树边，但两顶点在深度优先树中是祖先后代关系
- 对于**无向图**，非树边一定是后向边



$time = 10$

v	1	2	3	4	5
$pred$	N	1	1	2	3

区别1: 祖先指向后代? 还是相反?

- 回顾深度优先搜索边的性质

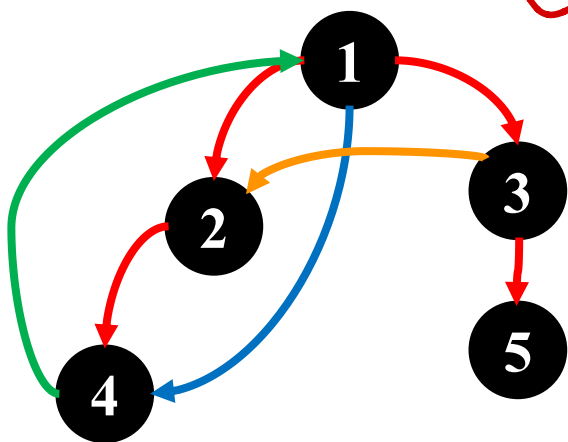
- 后向边: 不是树边, 但两顶点在深度优先树中是**祖先后代关系**
- 对于**无向图**, 非树边一定是后向边

区别2: 非树边出现在兄弟顶点之间

深度优先搜索边的分类

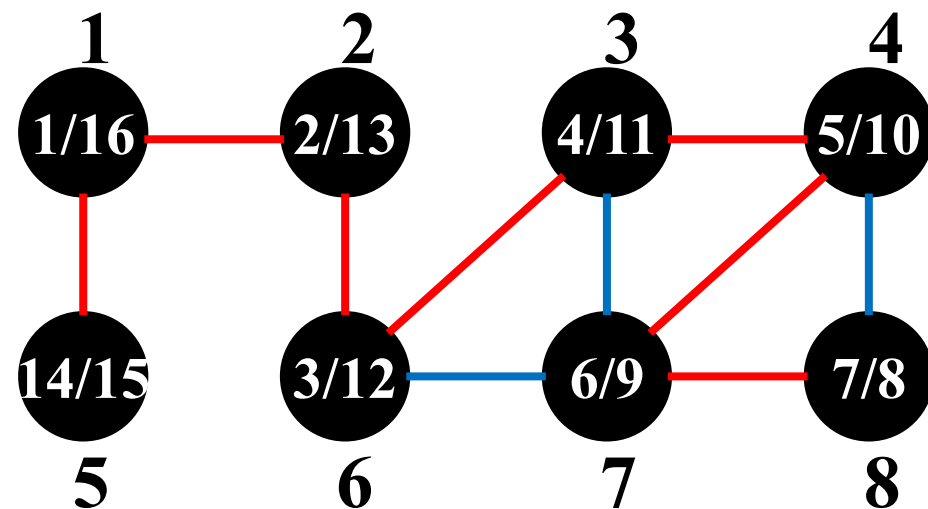
- 有向图，深度优先搜索有4类边

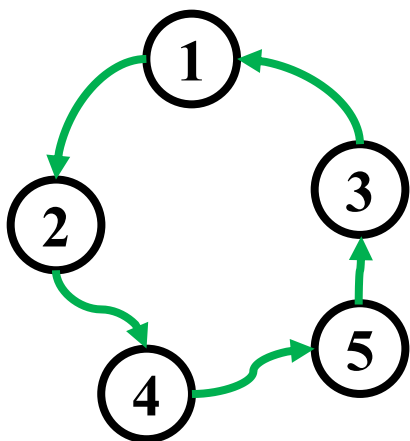
- 树边：在深度优先树中的边
- 前向边：不在深度优先树中，从祖先指向后代的边
- 后向边：从后代指向祖先的边
- 横向边：顶点不具有祖先后代关系的边



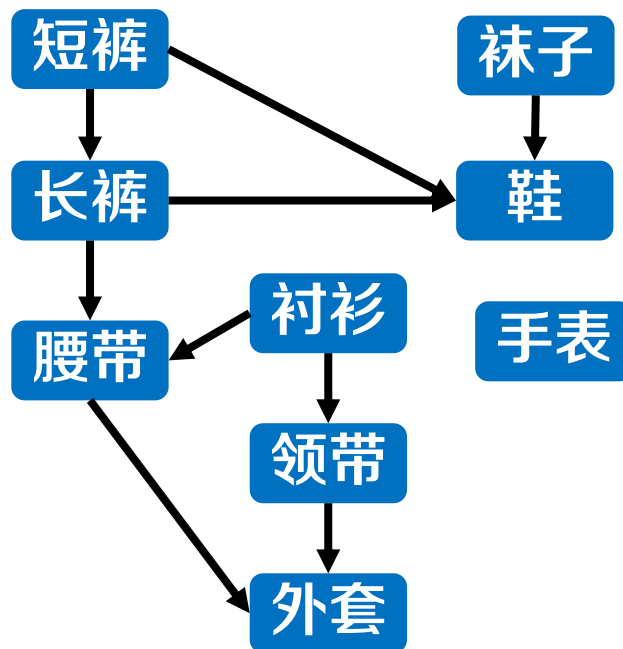
- 无向图，深度优先搜索有2类边

- 树边：在深度优先树中的边
- 后向边：两顶点有祖先后代关系的非树边

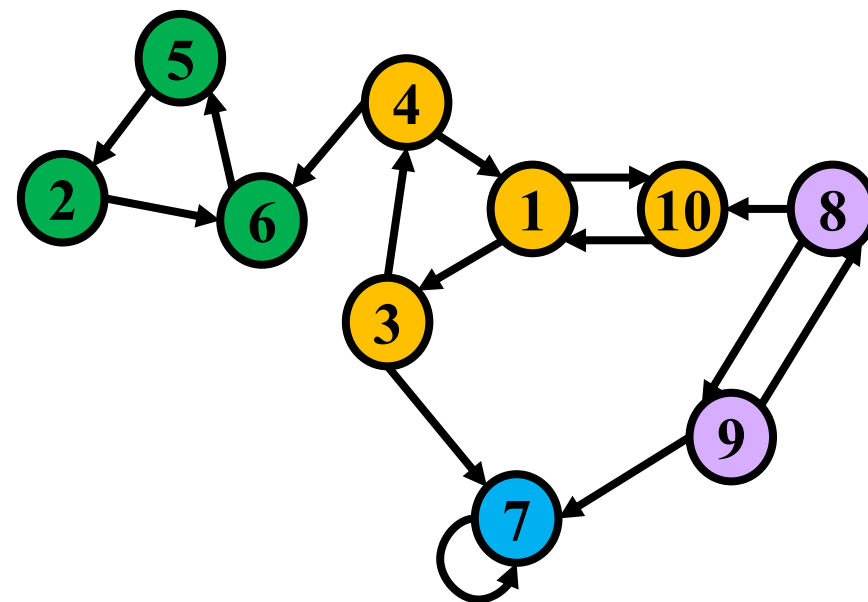




环路的存在性判断



拓扑排序



强连通分量