图算法篇：广度优先搜索

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中国大学MOOC北航《算法设计与分析》

提纲

图的搜索

算法思想

算法实例

算法分析

算法应用

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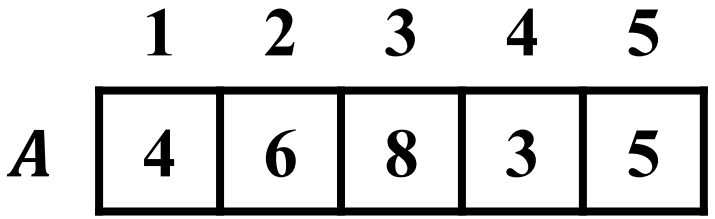
- 数组结构
  - 查询最大值：简单循环搜索所有元素，记录最大值



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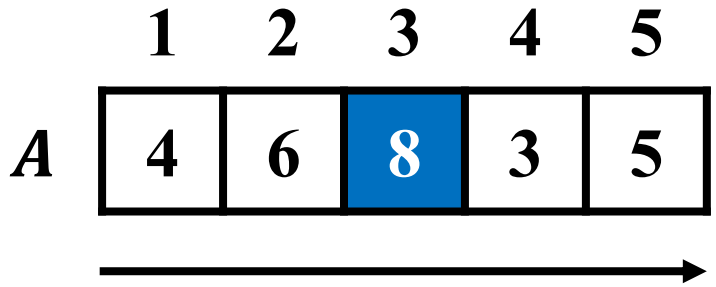

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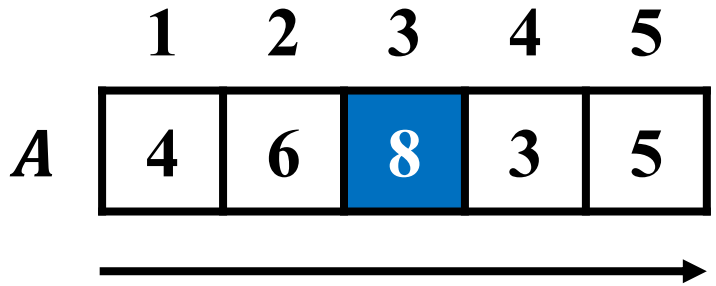

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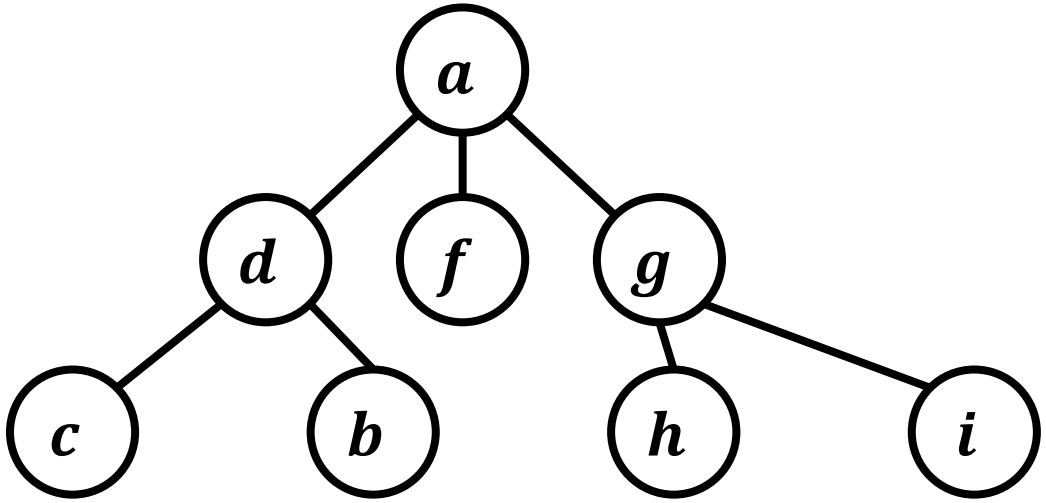

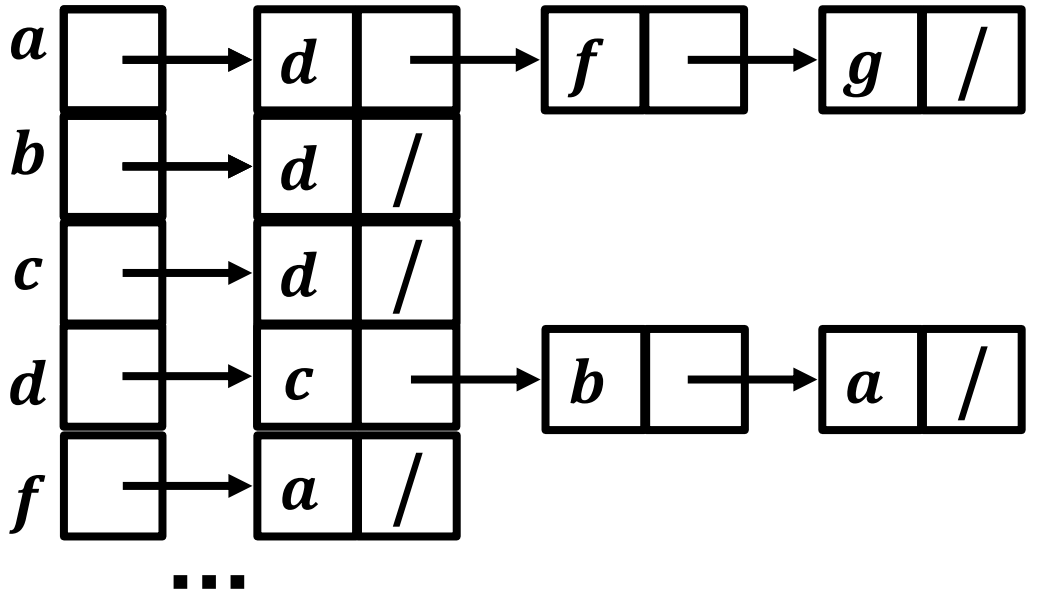

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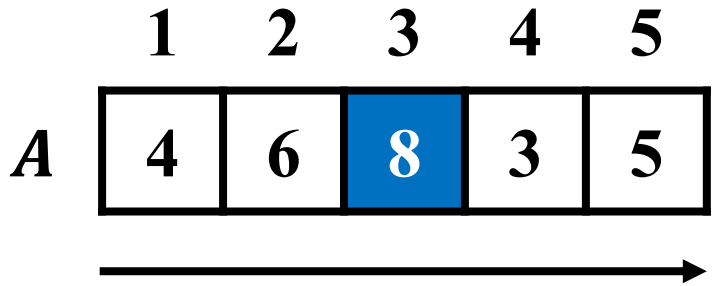

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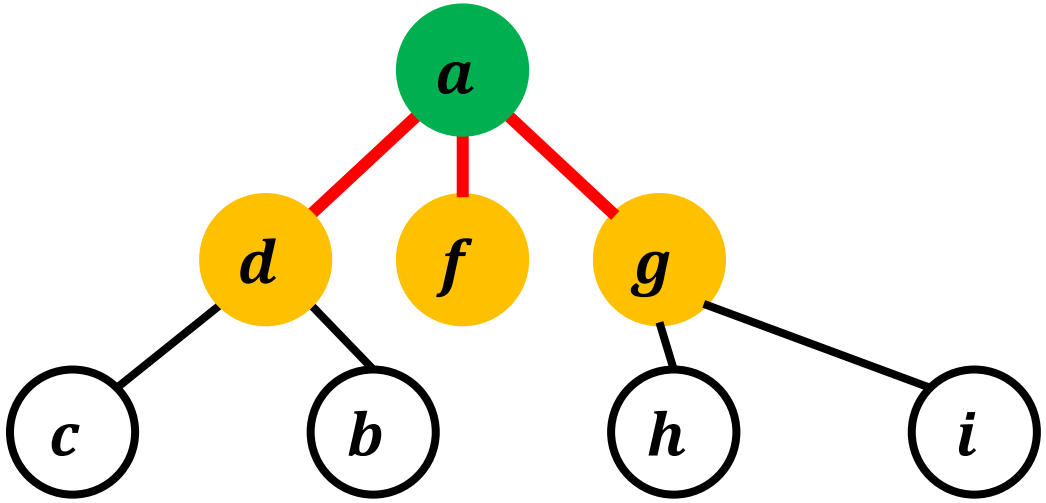
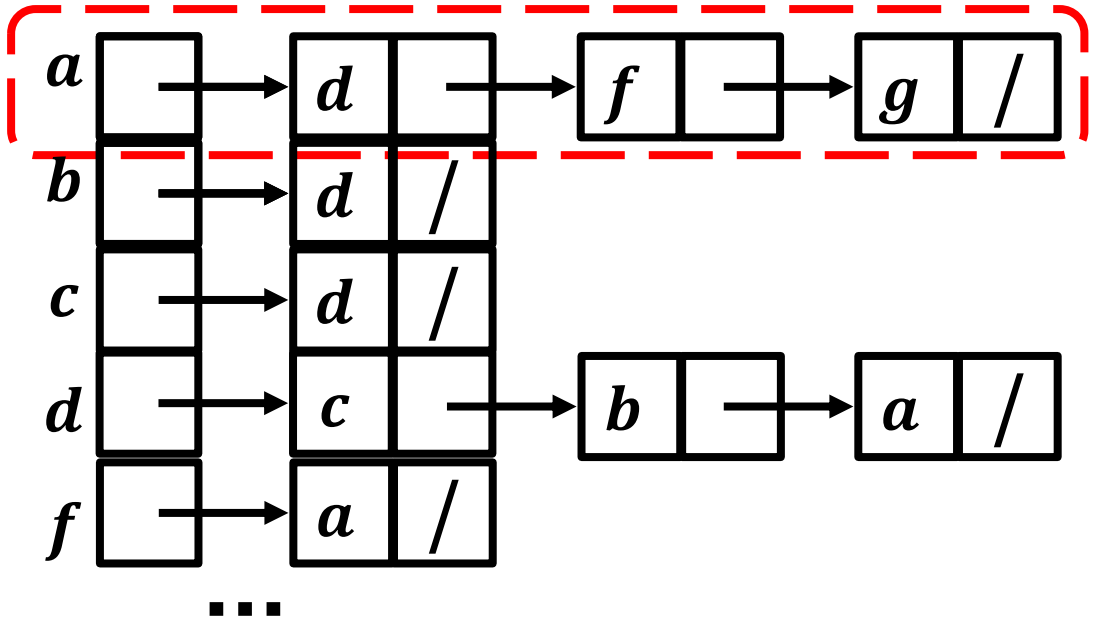
顶点a与$\{d, f, g\}$相邻





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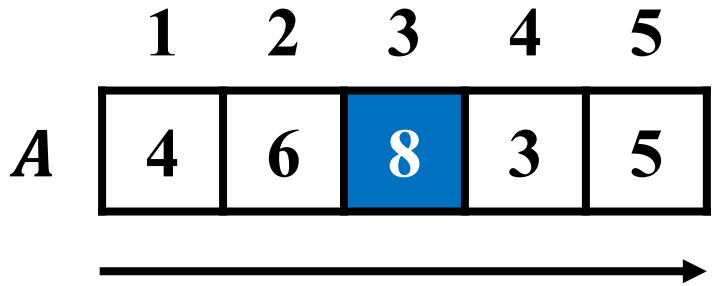

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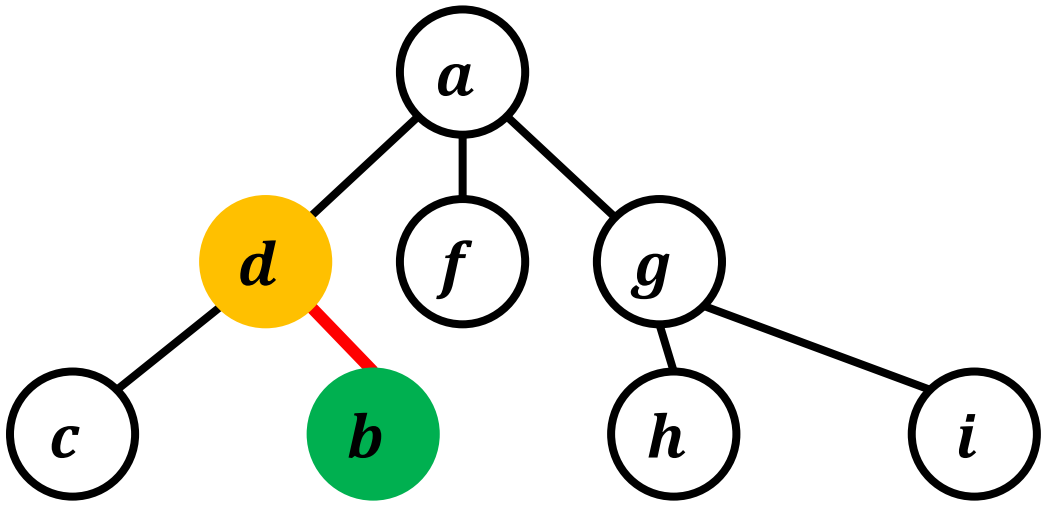
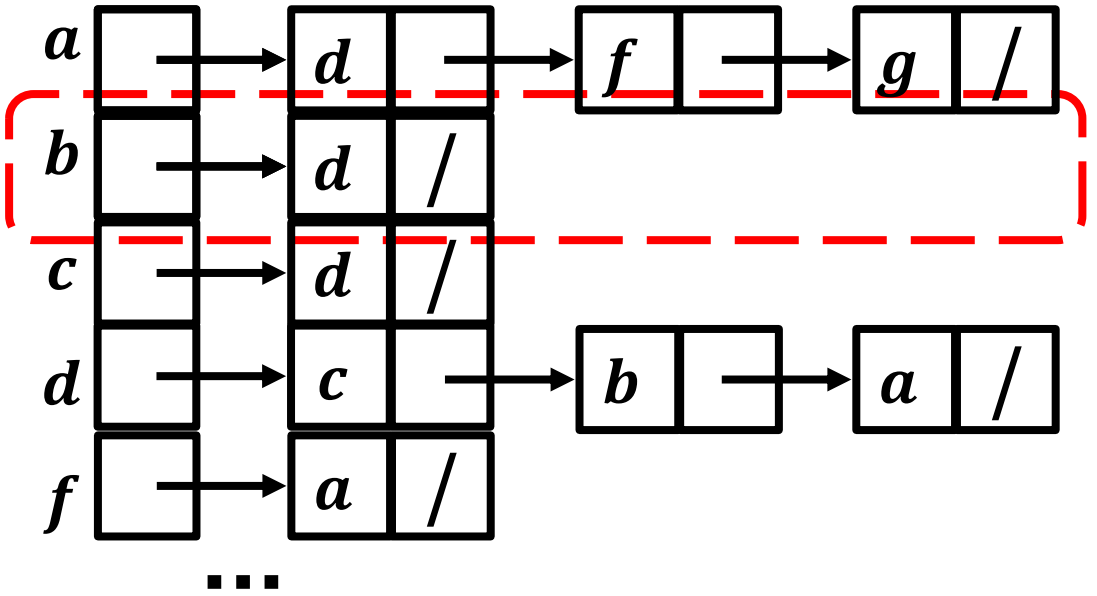
顶点b与$\{d\}$相邻





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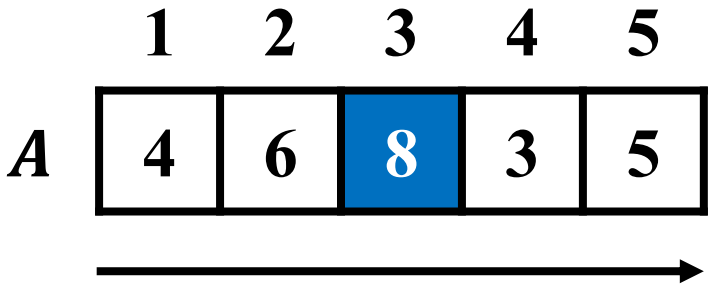

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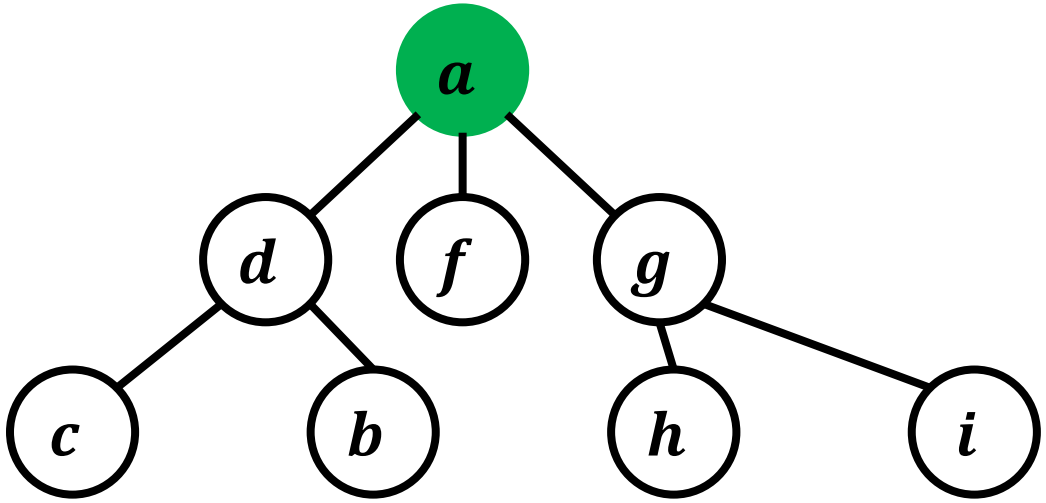

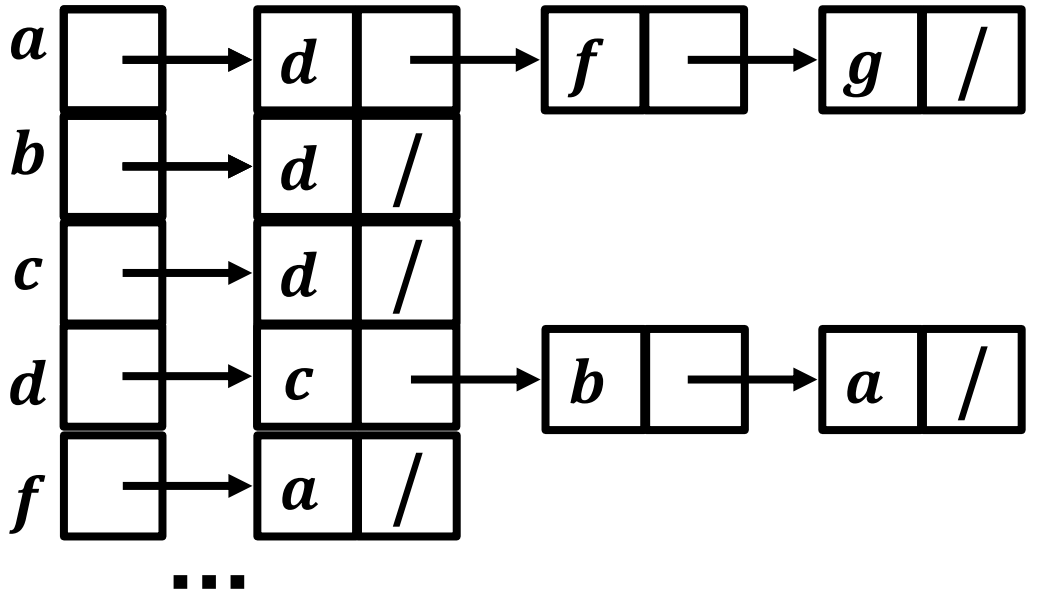

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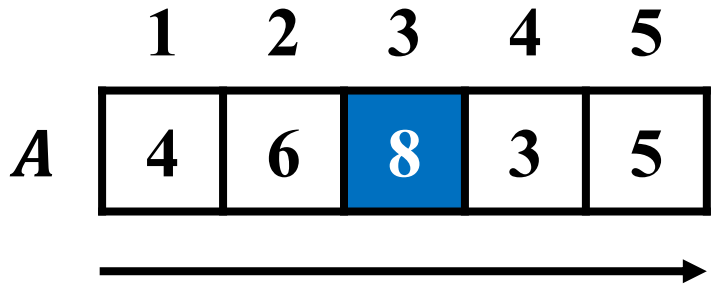

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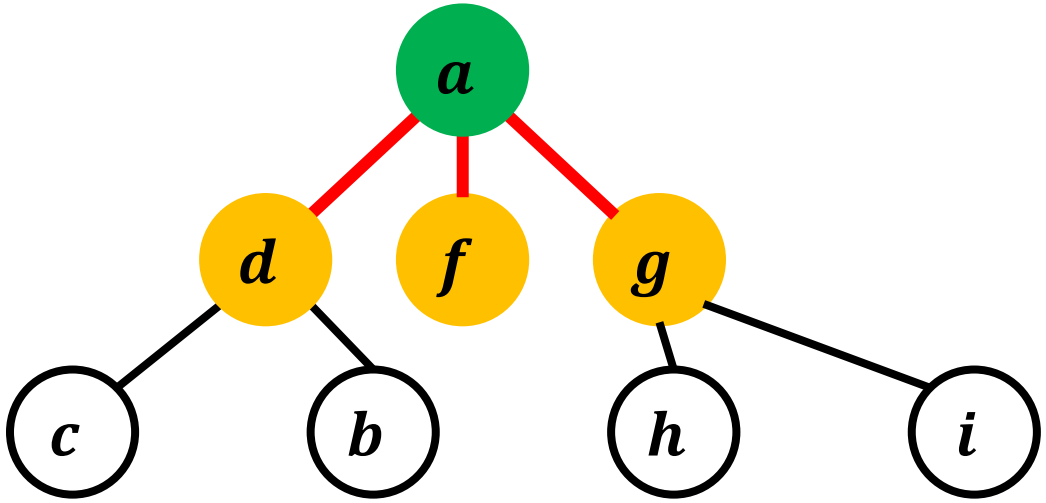

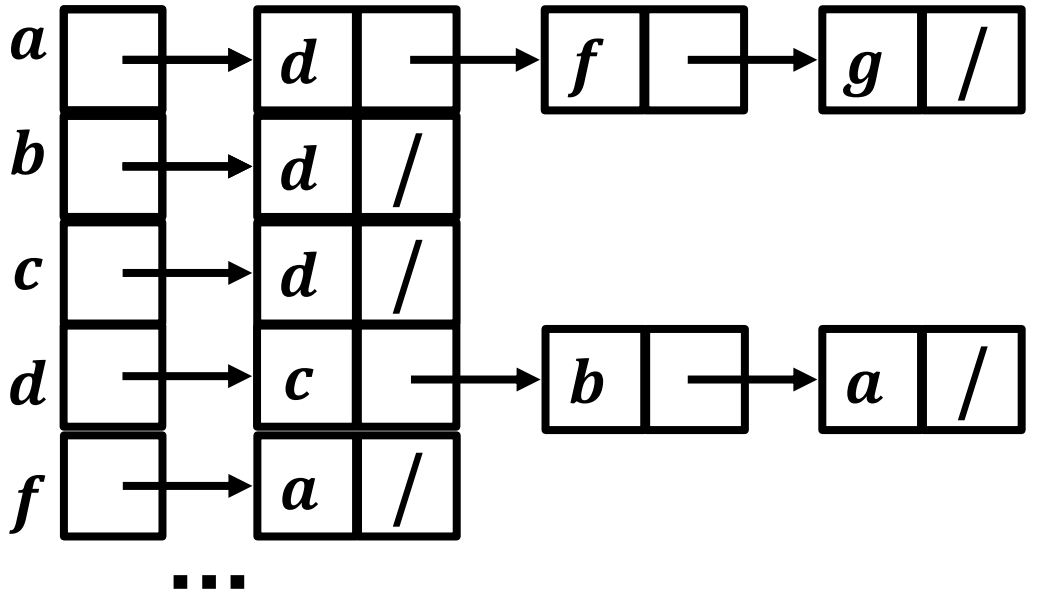

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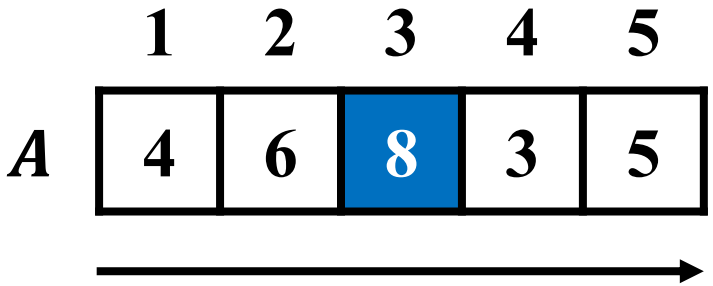

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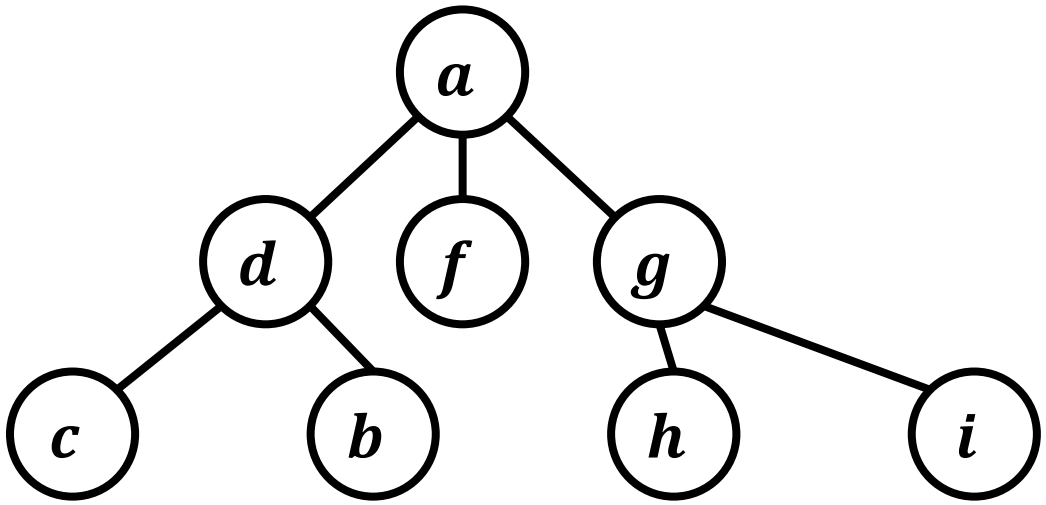

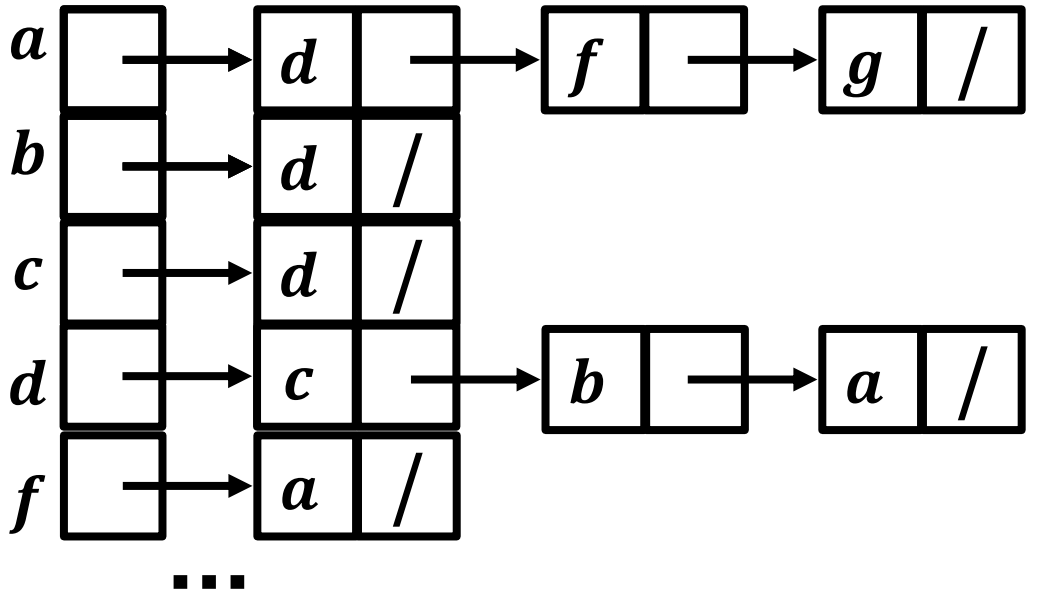

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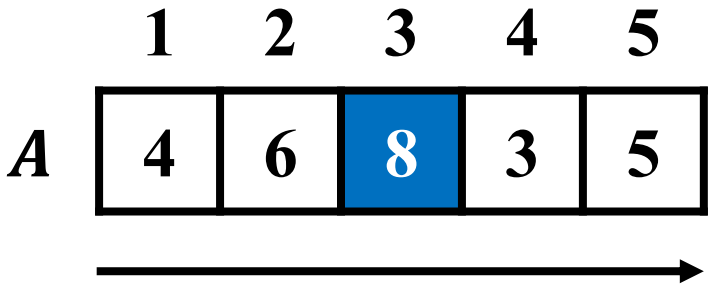

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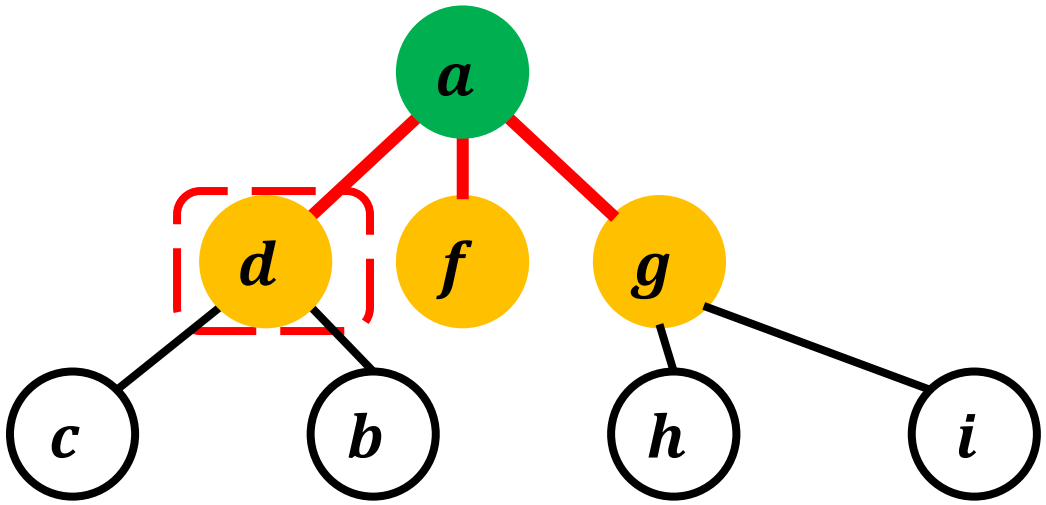

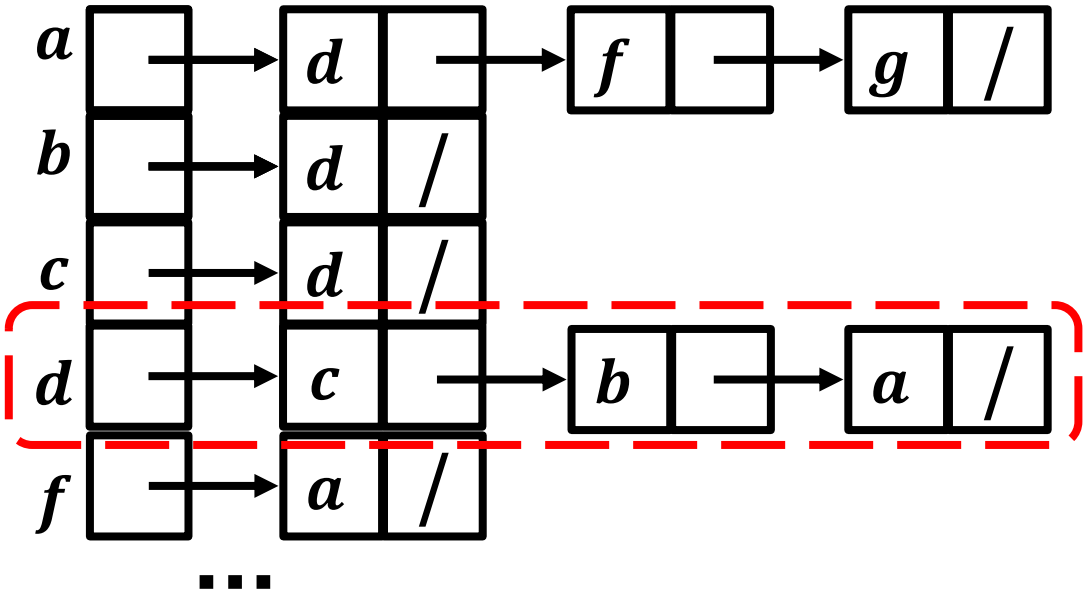

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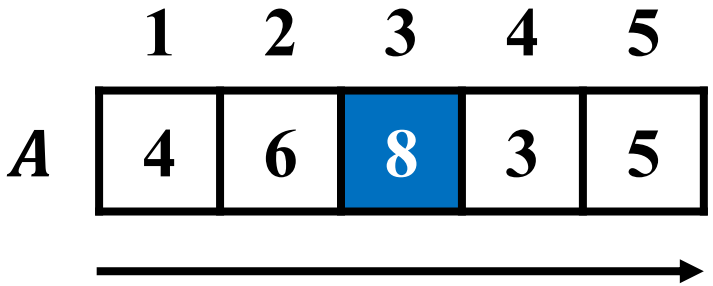

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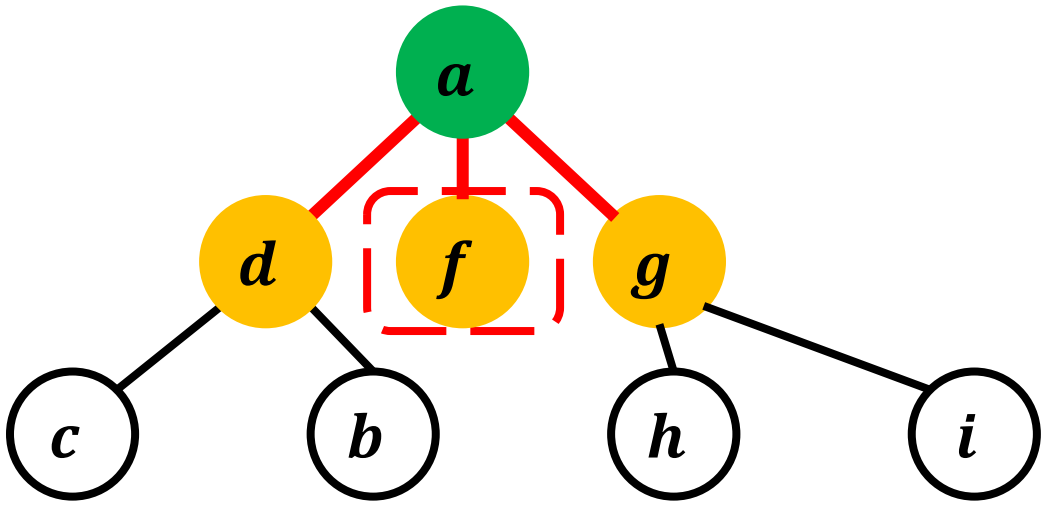

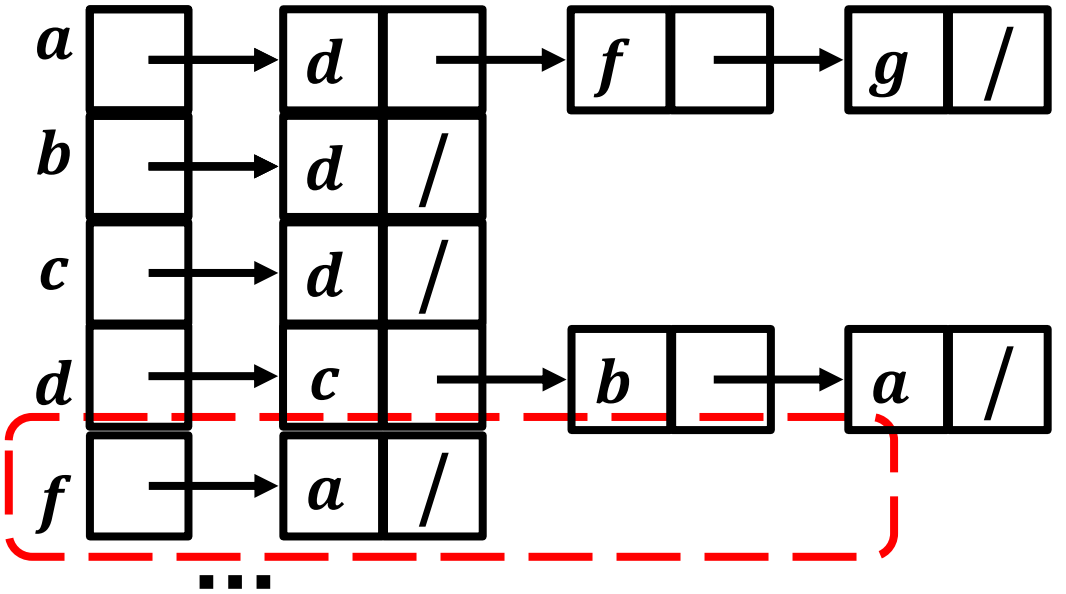

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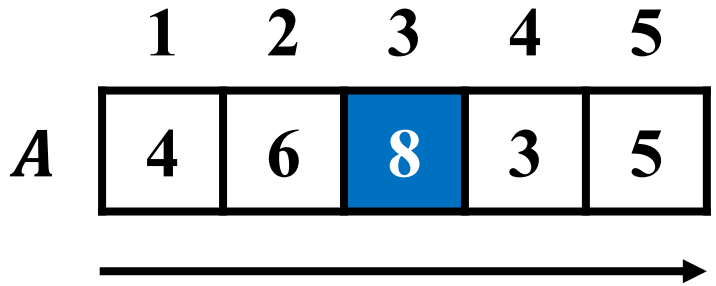

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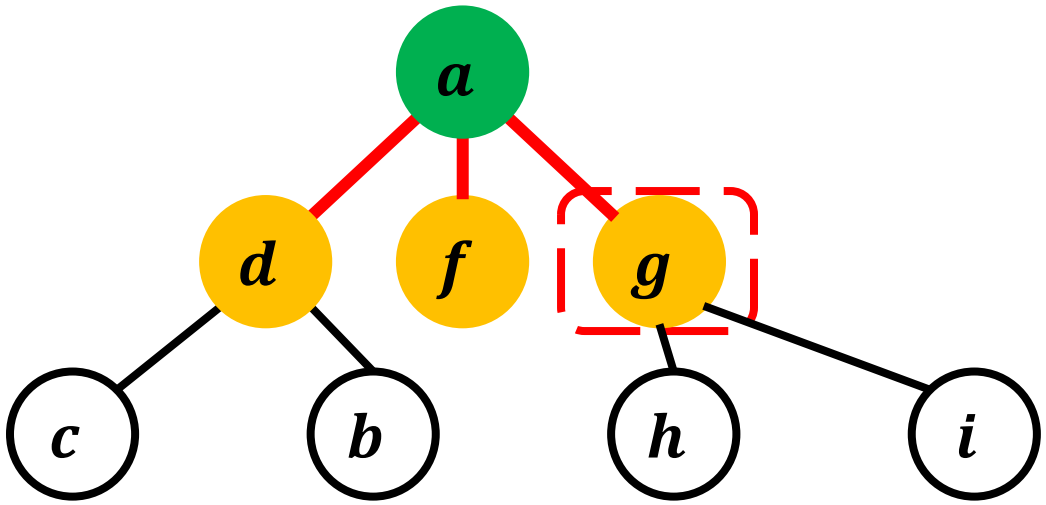

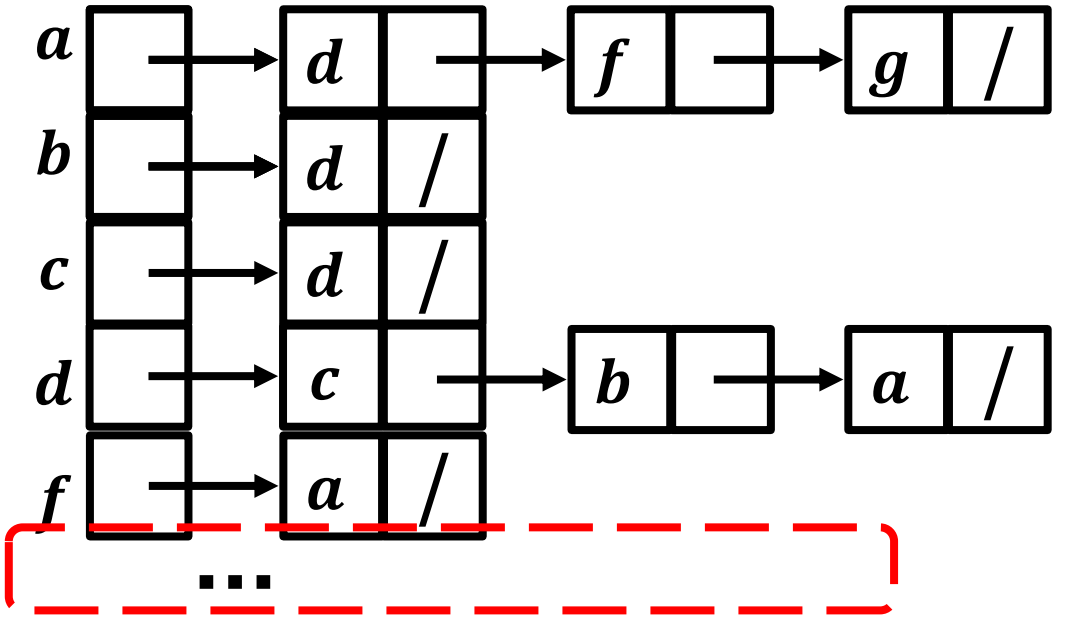

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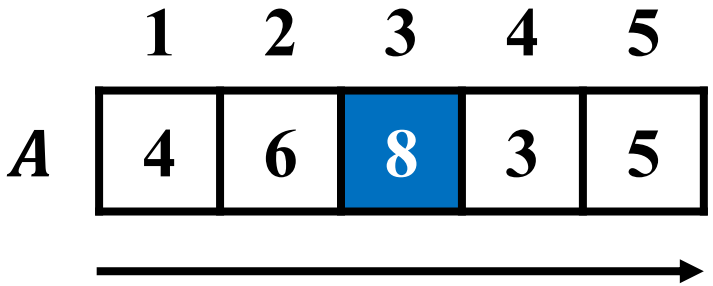

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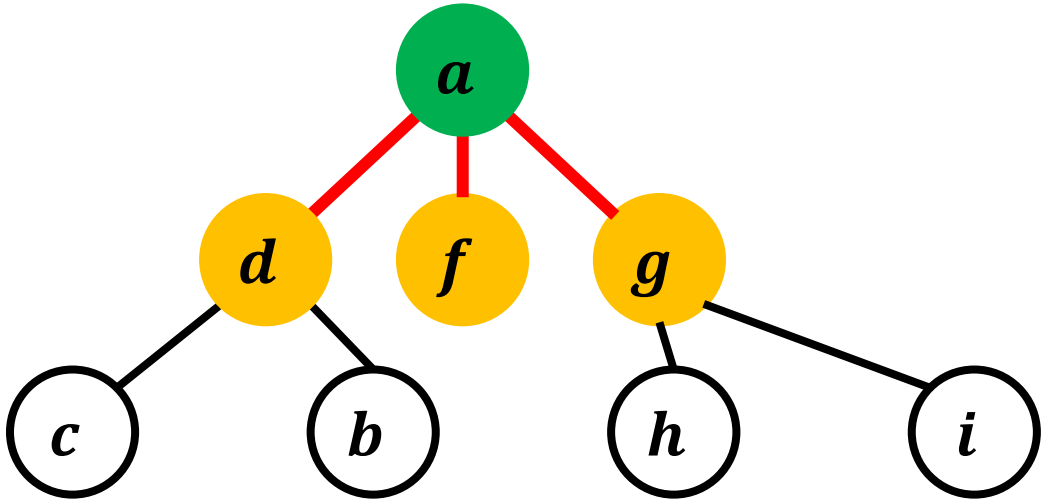

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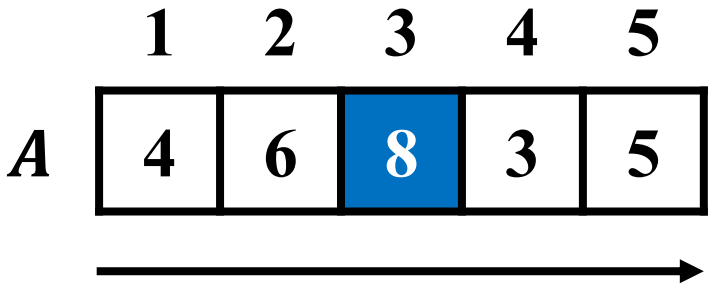

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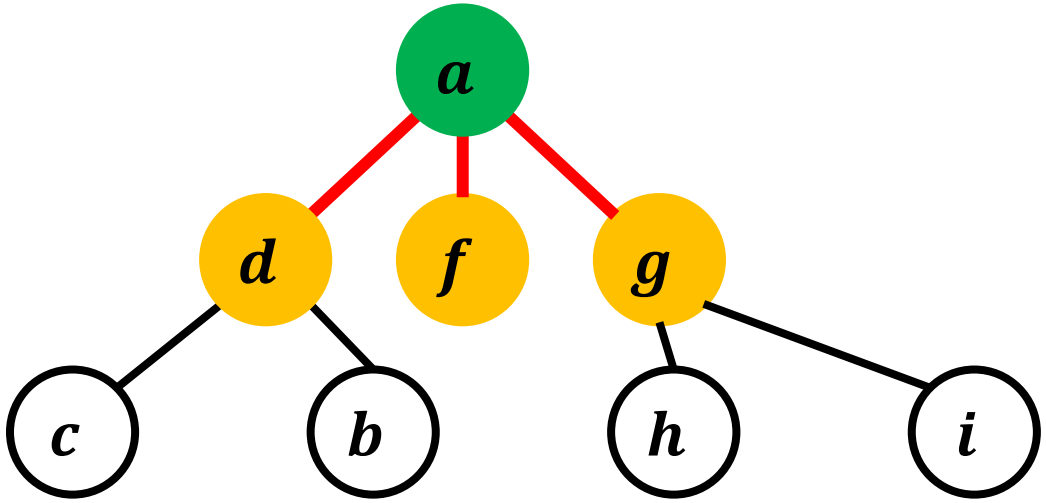

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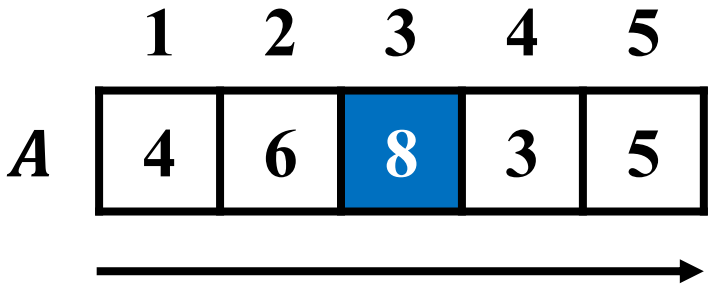

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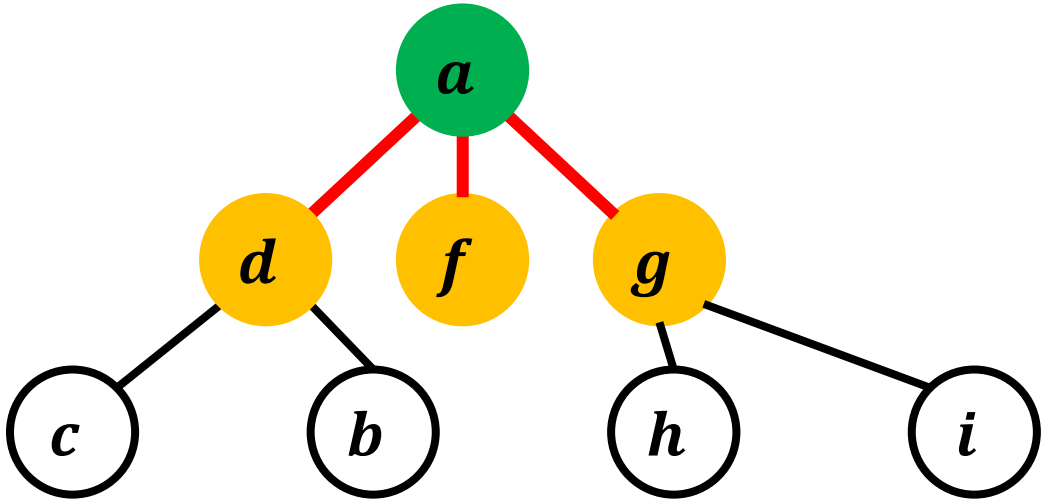

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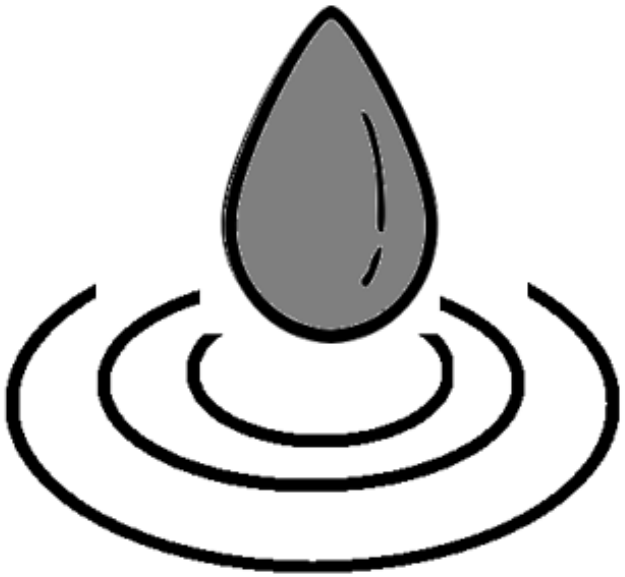
- 水滴落入水面所激起的涟漪会向相邻区域逐渐扩散



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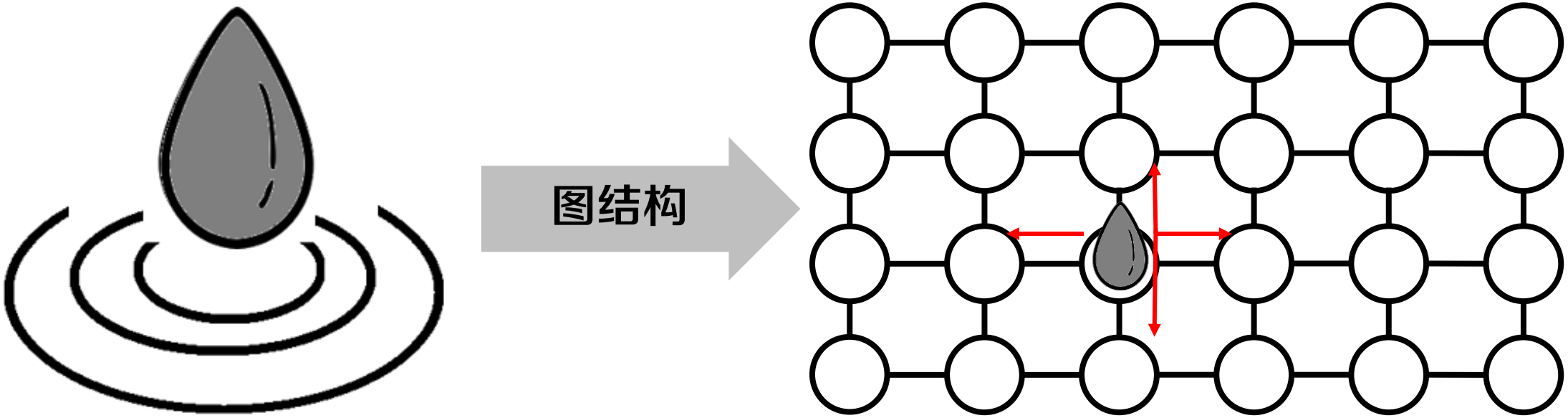

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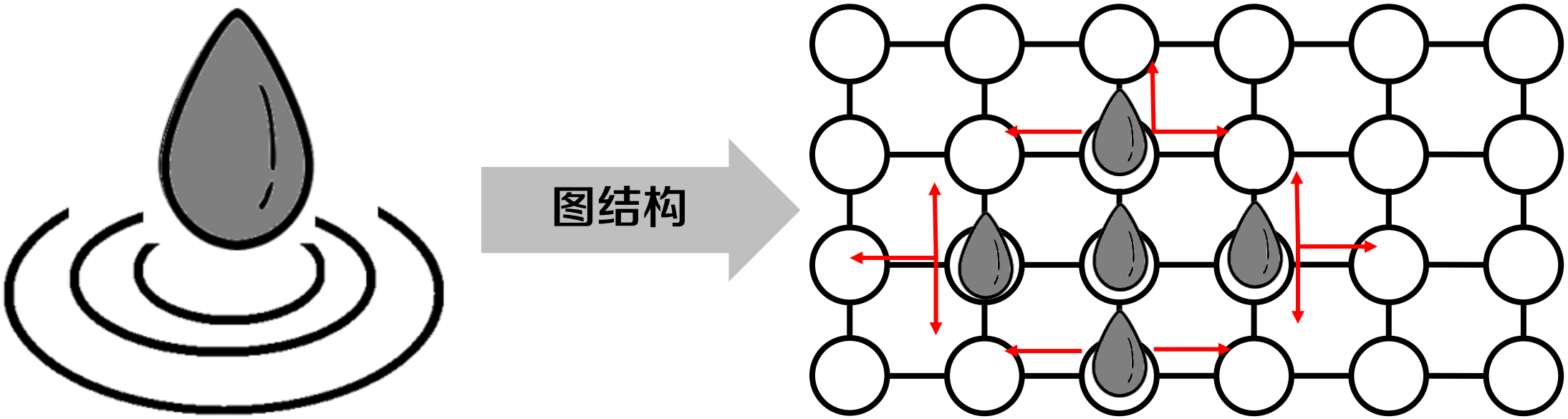

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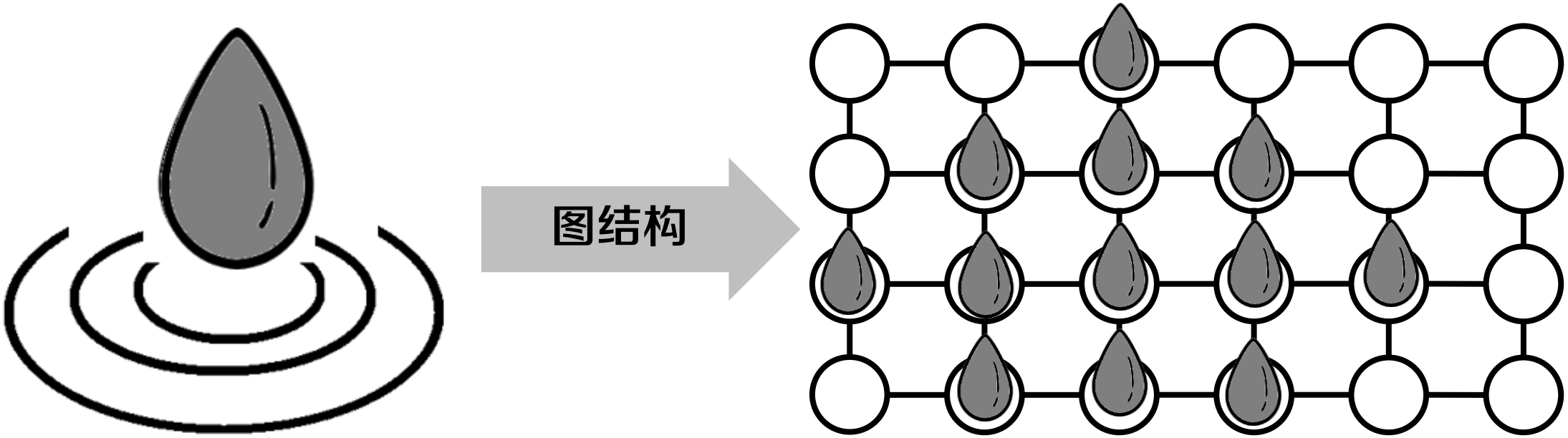
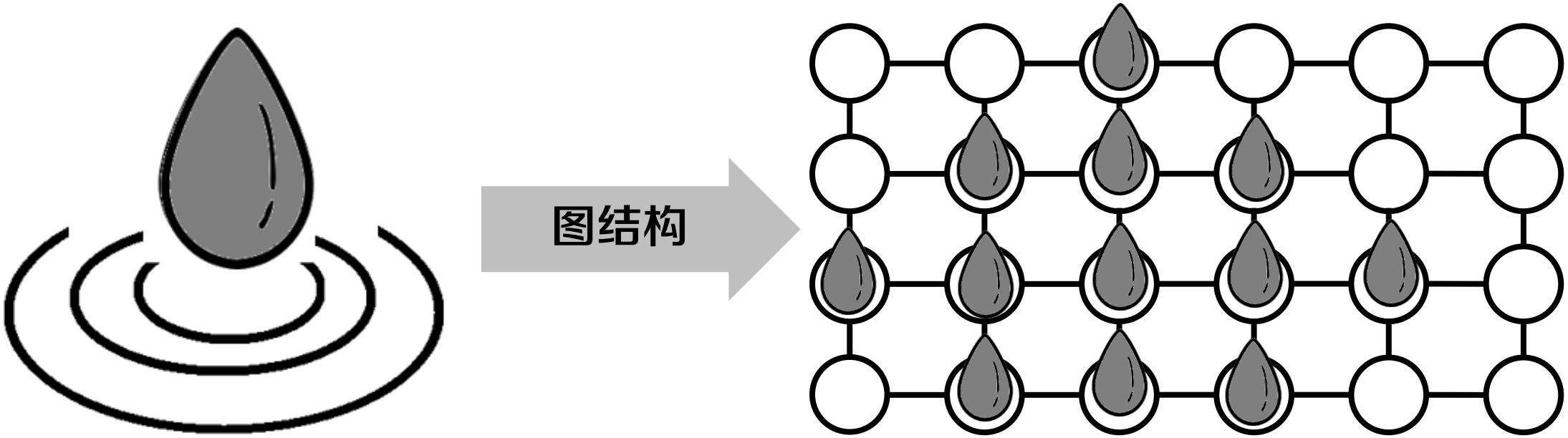


这种依次扩散的现象，蕴含了搜索图结构的一种顺序

算法思想：现象启发

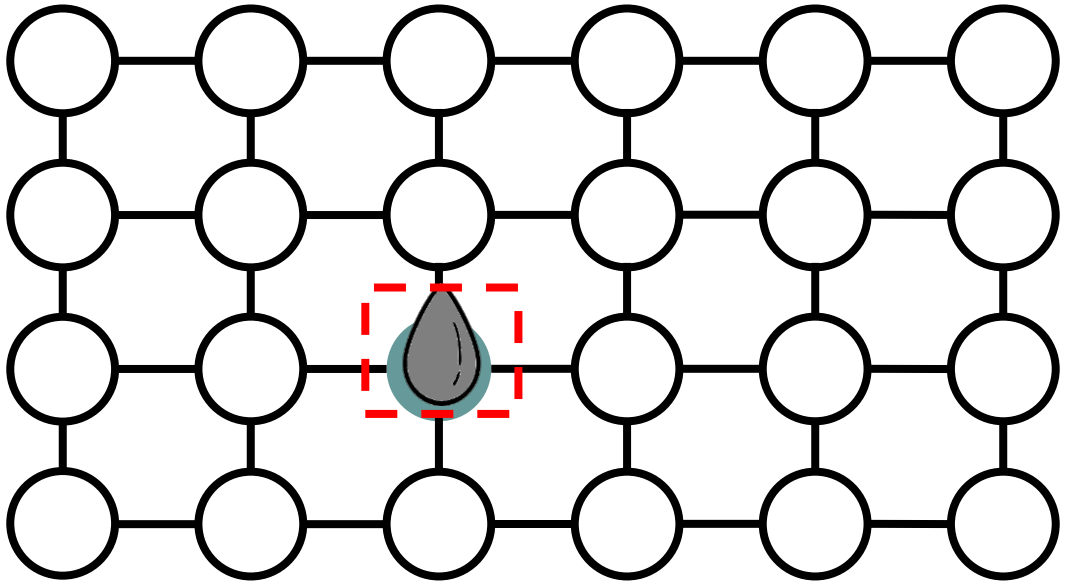
- 水滴落入水面所激起的涟漪会向相邻区域逐渐扩散



处理某顶点时，一次性发现所有其相邻顶点

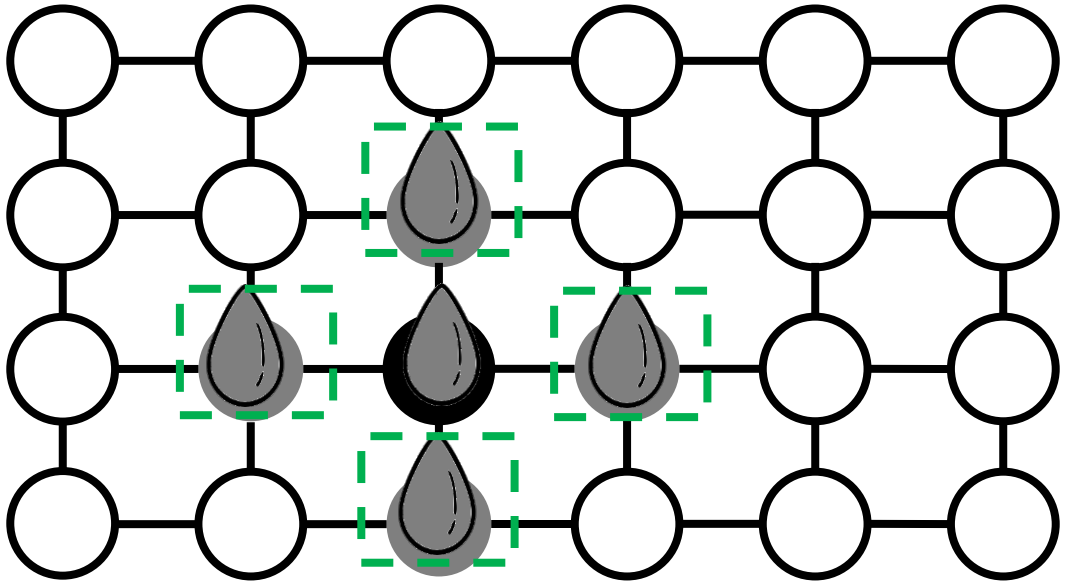
算法思想：广度优先搜索

- 核心思想
  - 处理某顶点时，一次性发现其所有相邻顶点



算法思想：广度优先搜索

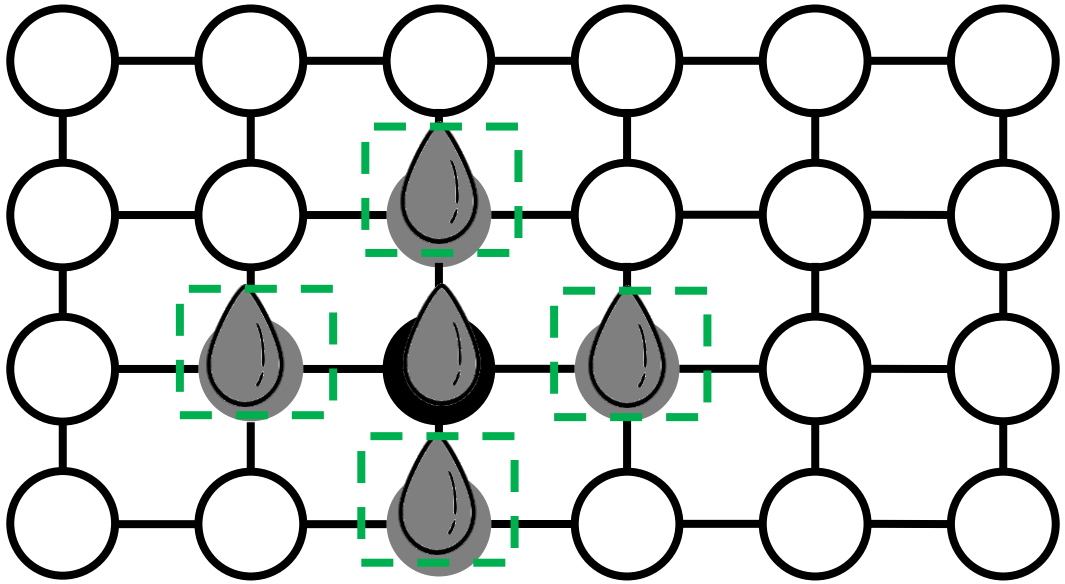
- 核心思想
  - 处理某顶点时，一次性发现其所有相邻顶点



算法思想：广度优先搜索

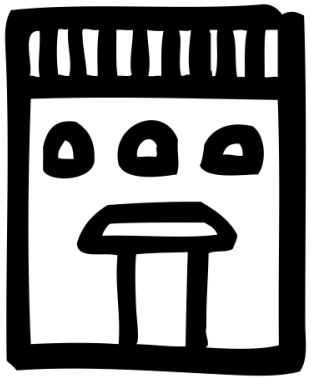
- 核心思想
  - 处理某顶点时，一次性发现其所有相邻顶点

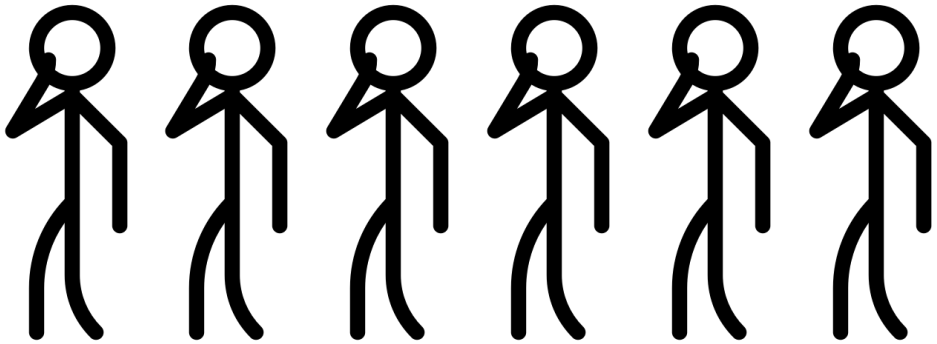
扩散中多处同时进行，一次只能处理单个顶点怎么办？



算法思想：队列

- 队列
  - 先来先服务：队尾加入，队首离开
    - 加入队列，$Q.Enqueue()$
    - 离开队列，$Q.Dequeue()$

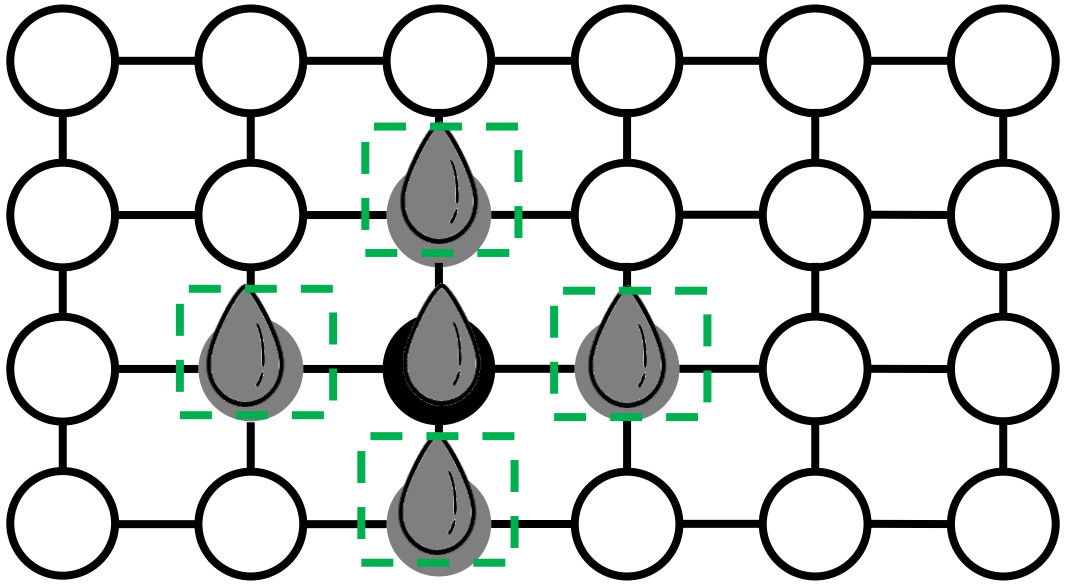




队列Q

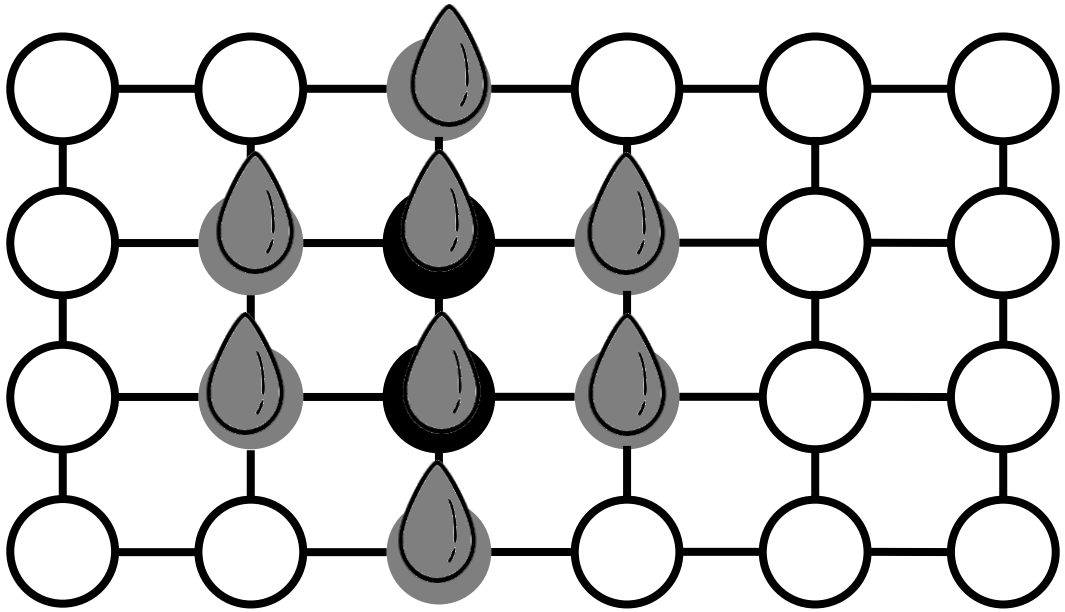
算法思想：广度优先搜索

- 核心思想
  - 处理某顶点时，一次性发现其所有相邻顶点，未处理顶点加入等待队列



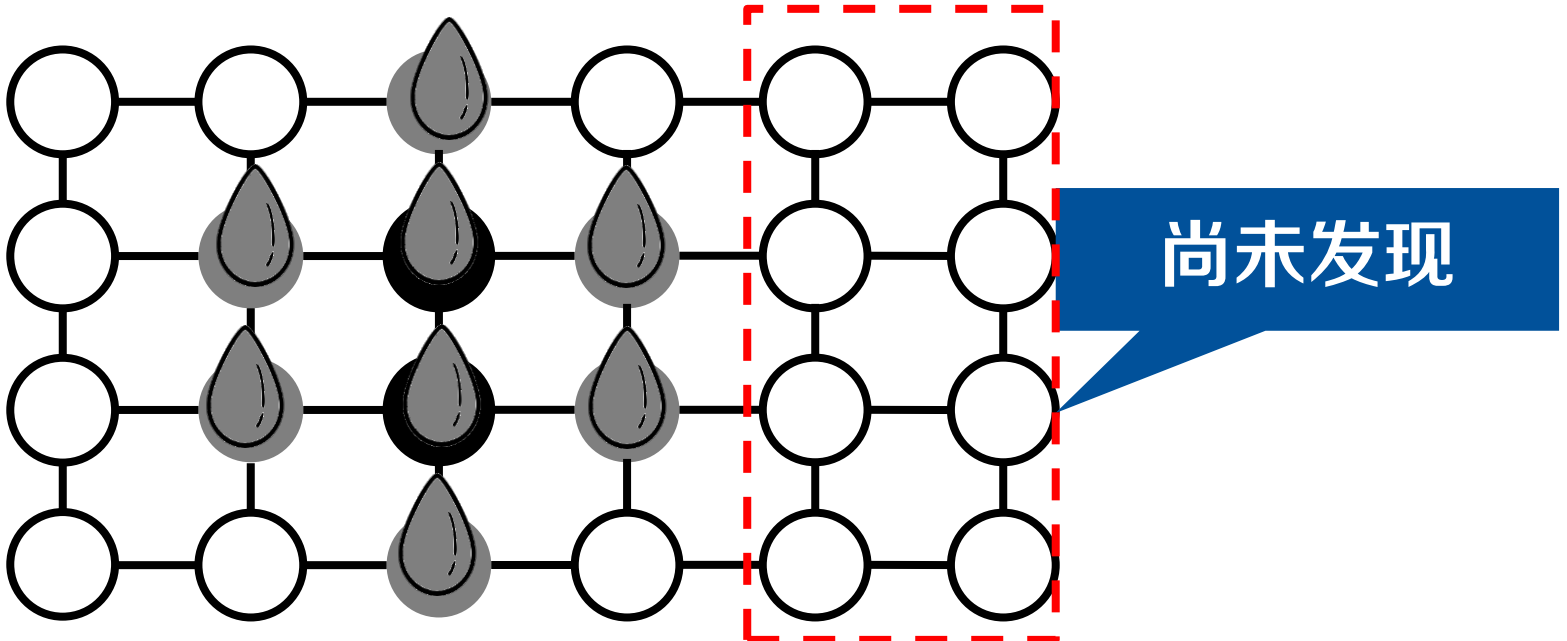
算法思想：广度优先搜索

- 辅助数组
  - $color$表示顶点状态



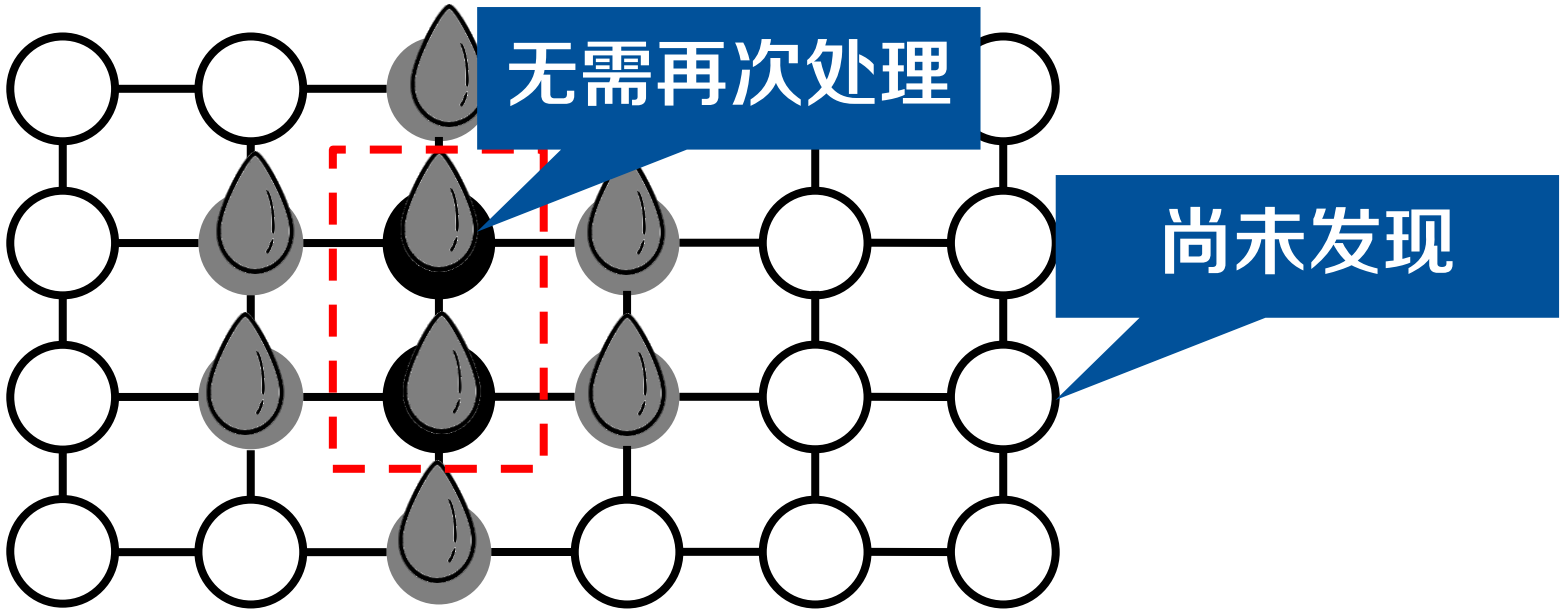
算法思想：广度优先搜索

- 辅助数组
  - $color$表示顶点状态
    - $White$：白色顶点u尚未被发现，发现后直接入队



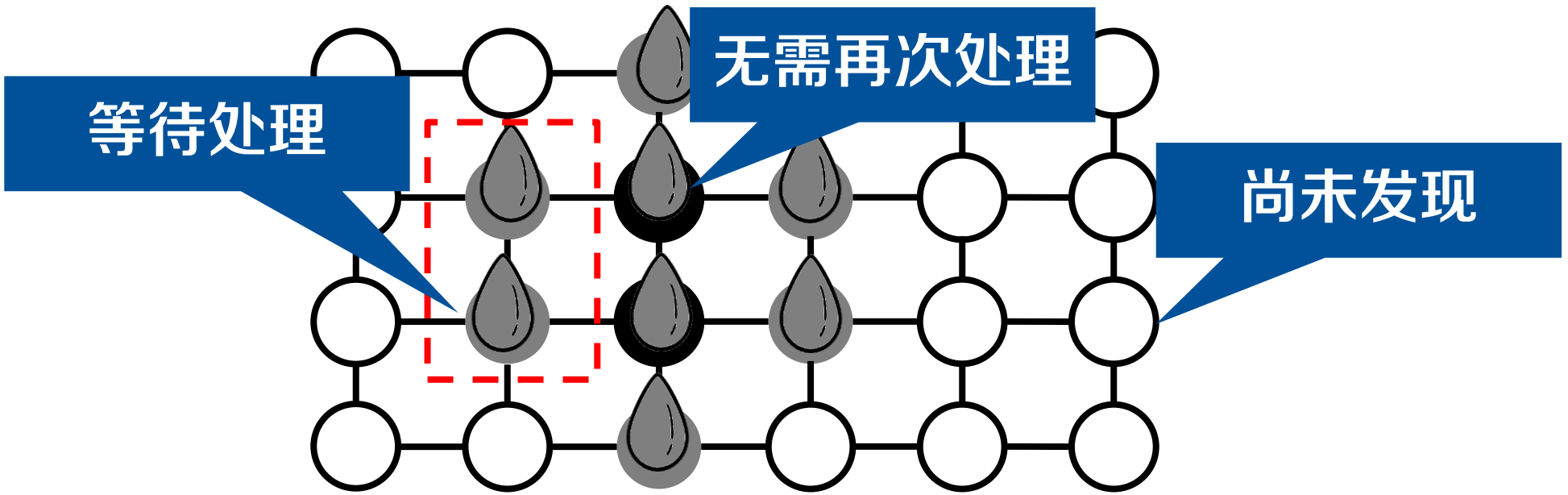
算法思想：广度优先搜索

- 辅助数组
  - $color$表示顶点状态
    - $White$：白色顶点u尚未被发现，发现后直接入队
    - $Black$：黑色顶点u已被处理，无需再次入队



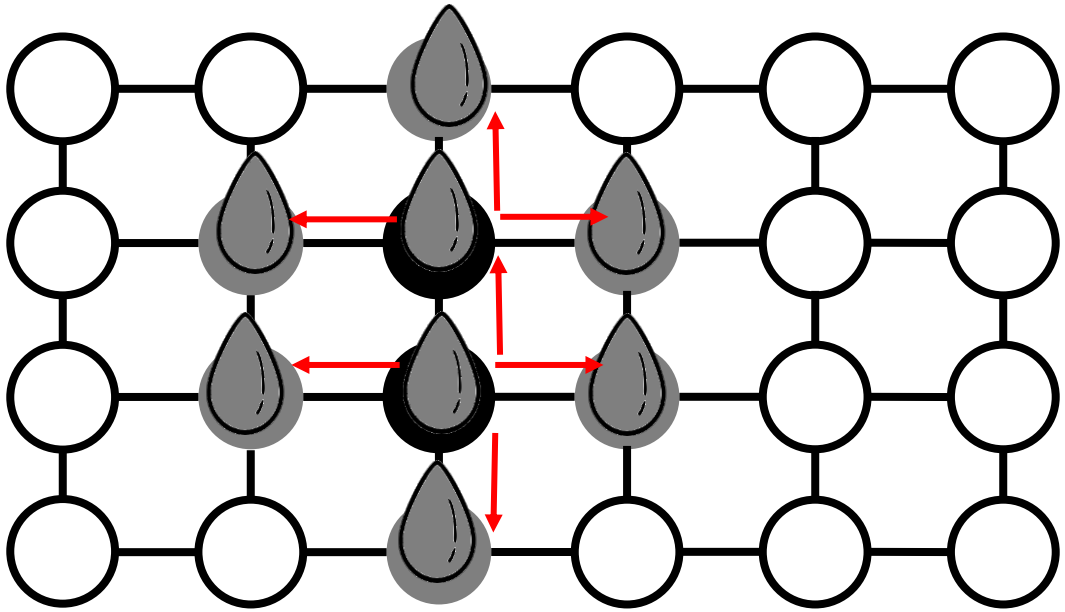
算法思想：广度优先搜索

- 辅助数组
  - $color$表示顶点状态
    - $White$：白色顶点u尚未被发现，发现后直接入队
    - $Black$：黑色顶点u已被处理，无需再次入队
    - $Gray$：灰色顶点u已加入队列，无需再次入队



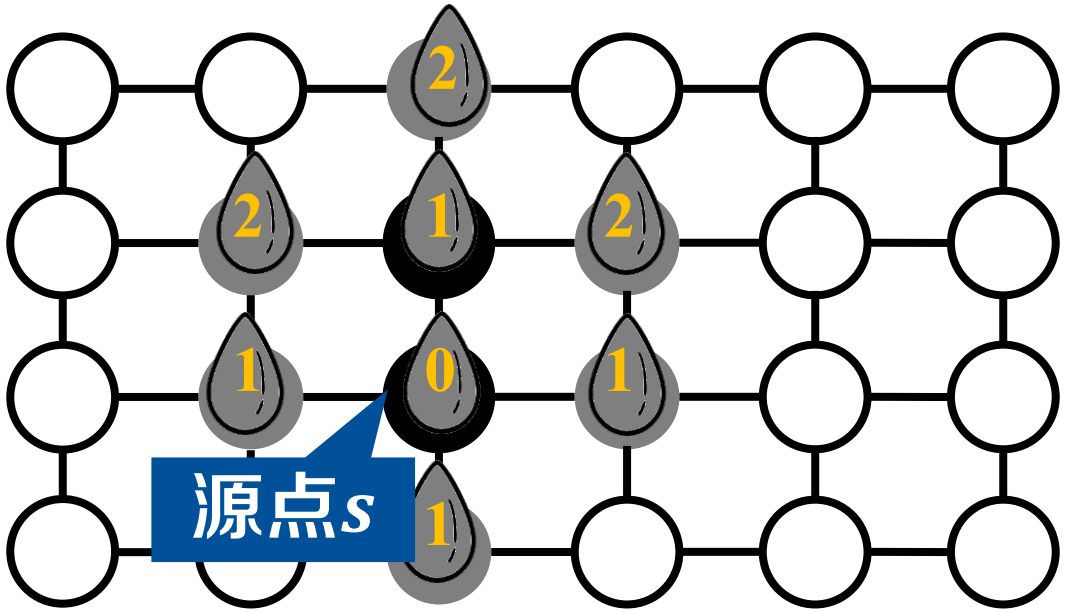
算法思想：广度优先搜索

- 辅助数组
  - $color$表示顶点状态
    - $White$：白色顶点u尚未被发现，发现后直接入队
    - $Black$：黑色顶点u已被处理，无需再次入队
    - $Gray$：灰色顶点u已加入队列，无需再次入队
  - $pred$：顶点u由$pred[u]$发现



算法思想：广度优先搜索

- 辅助数组
  - $color$表示顶点状态
    - $White$：白色顶点u尚未被发现，发现后直接入队
    - $Black$：黑色顶点u已被处理，无需再次入队
    - $Gray$：灰色顶点u已加入队列，无需再次入队
  - $pred$：顶点u由$pred[u]$发现
  - $dist$：顶点u距离源点s的距离



提纲

图的搜索

算法思想

算法实例

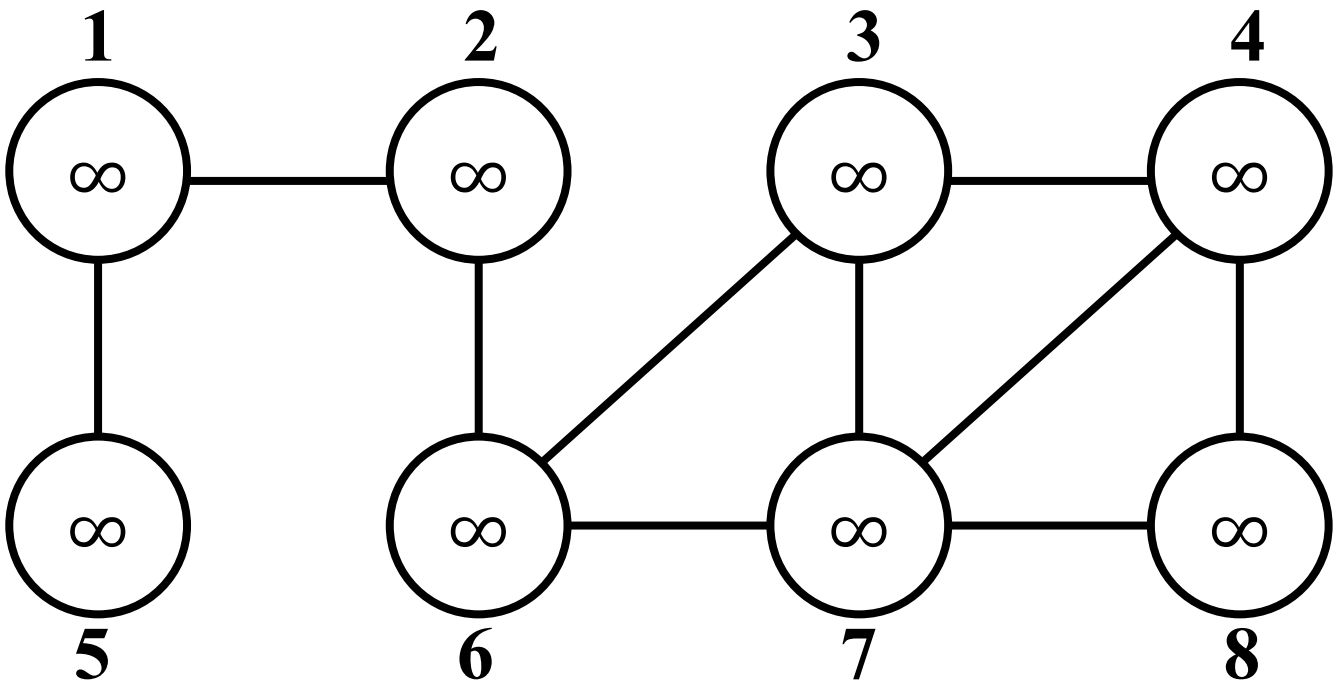
算法分析

算法应用

算法实例

V	1	2	3	4	5	6	7	8
$color$	W	W	W	W	W	W	W	W
$pred$	N	N	N	N	N	N	N	N
$dist$	∞	∞	∞	∞	∞	∞	∞	∞

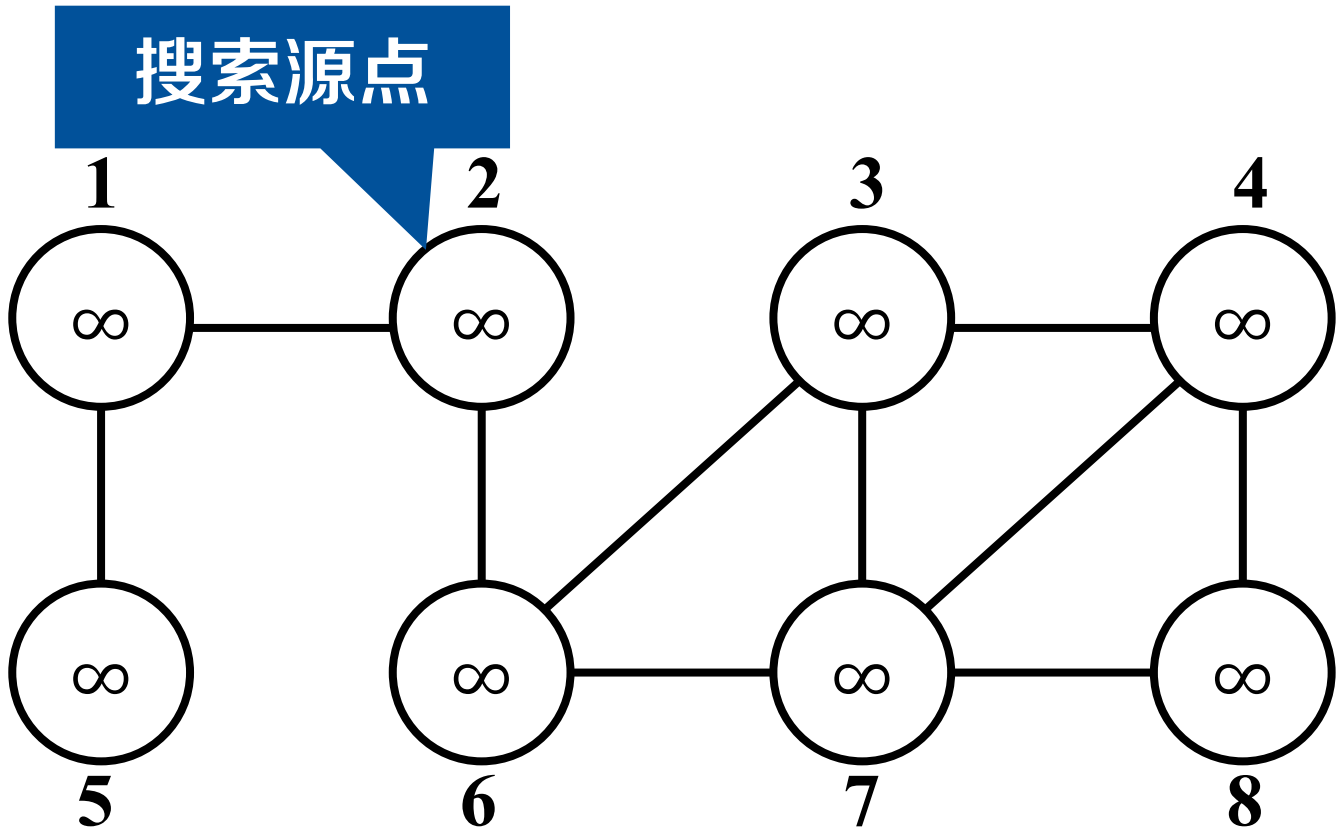
待处理队列



算法实例

V	1	2	3	4	5	6	7	8
$color$	W	W	W	W	W	W	W	W
$pred$	N	N	N	N	N	N	N	N
$dist$	∞	∞	∞	∞	∞	∞	∞	∞

待处理队列

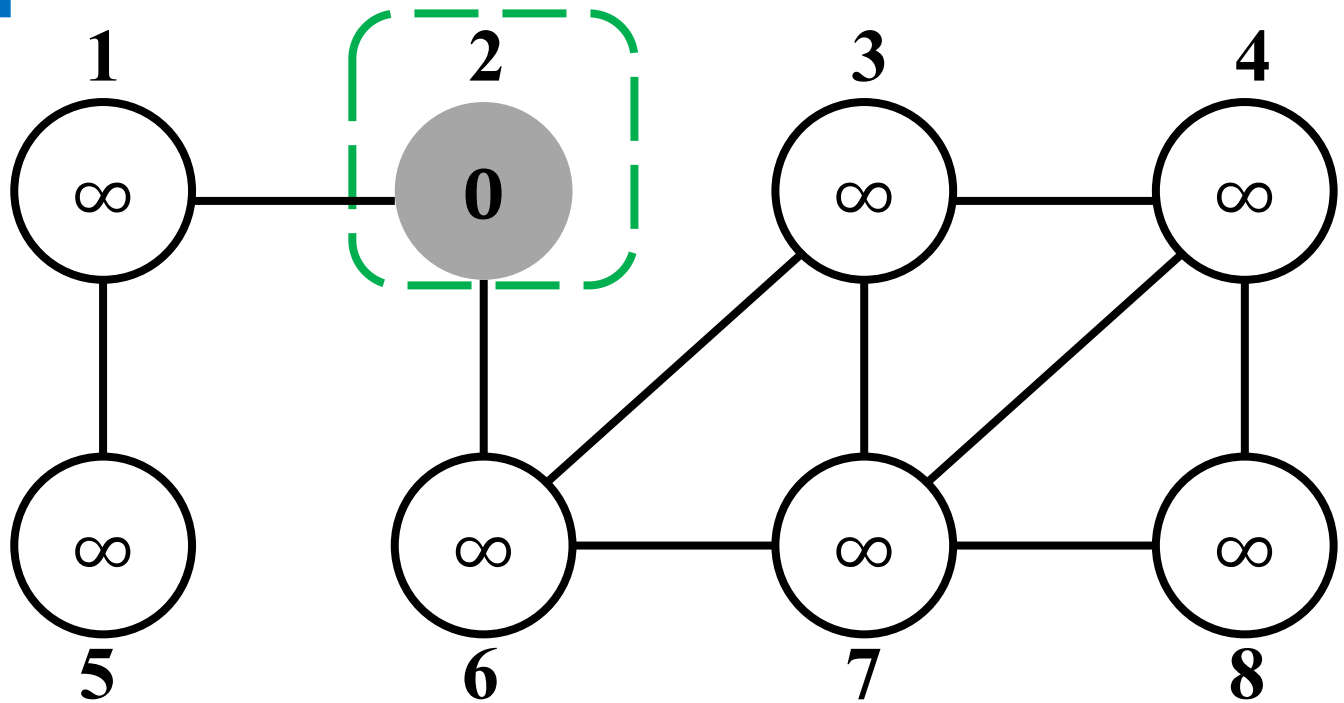


算法实例

V	1	2	3	4	5	6	7	8
$color$	W	G	W	W	W	W	W	W
$pred$	N	N	N	N	N	N	N	N
$dist$	∞	0	∞	∞	∞	∞	∞	∞

待处理队列	2

源点2

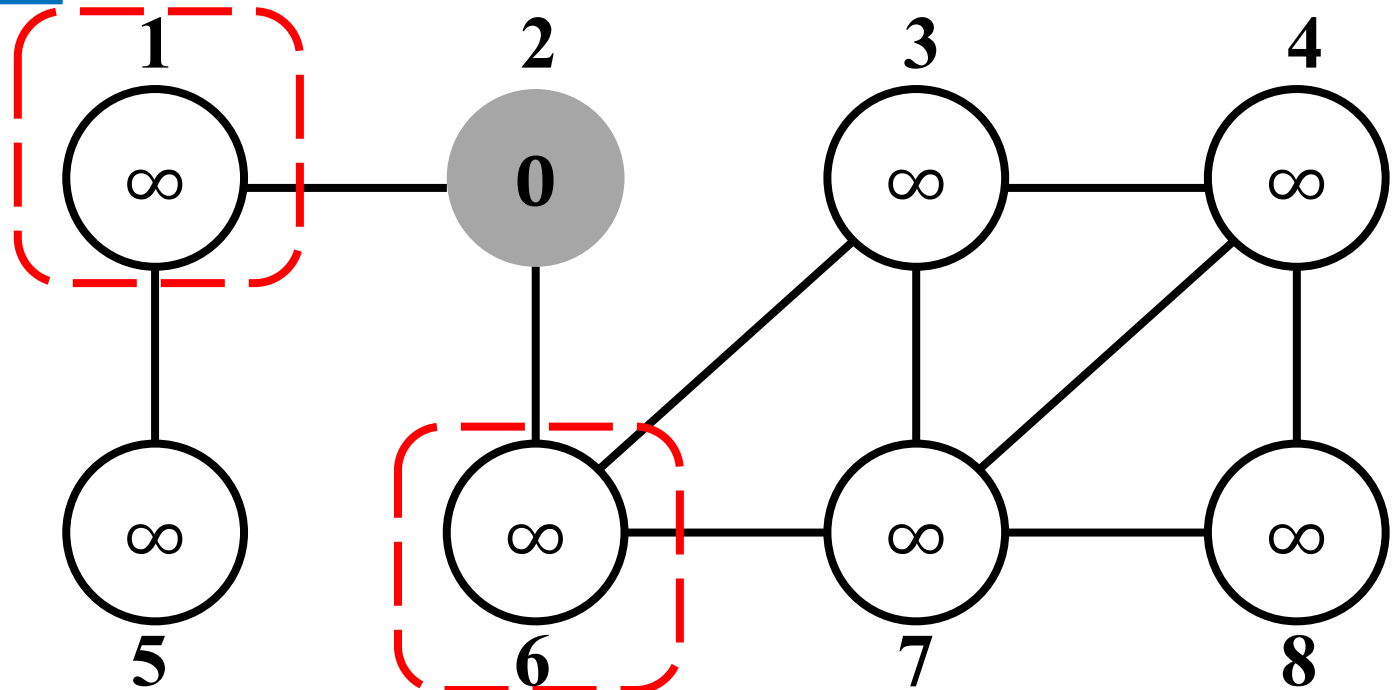


算法实例

V	1	2	3	4	5	6	7	8
$color$	W	G	W	W	W	W	W	W
$pred$	N	N	N	N	N	N	N	N
$dist$	∞	0	∞	∞	∞	∞	∞	∞

待处理队列	2

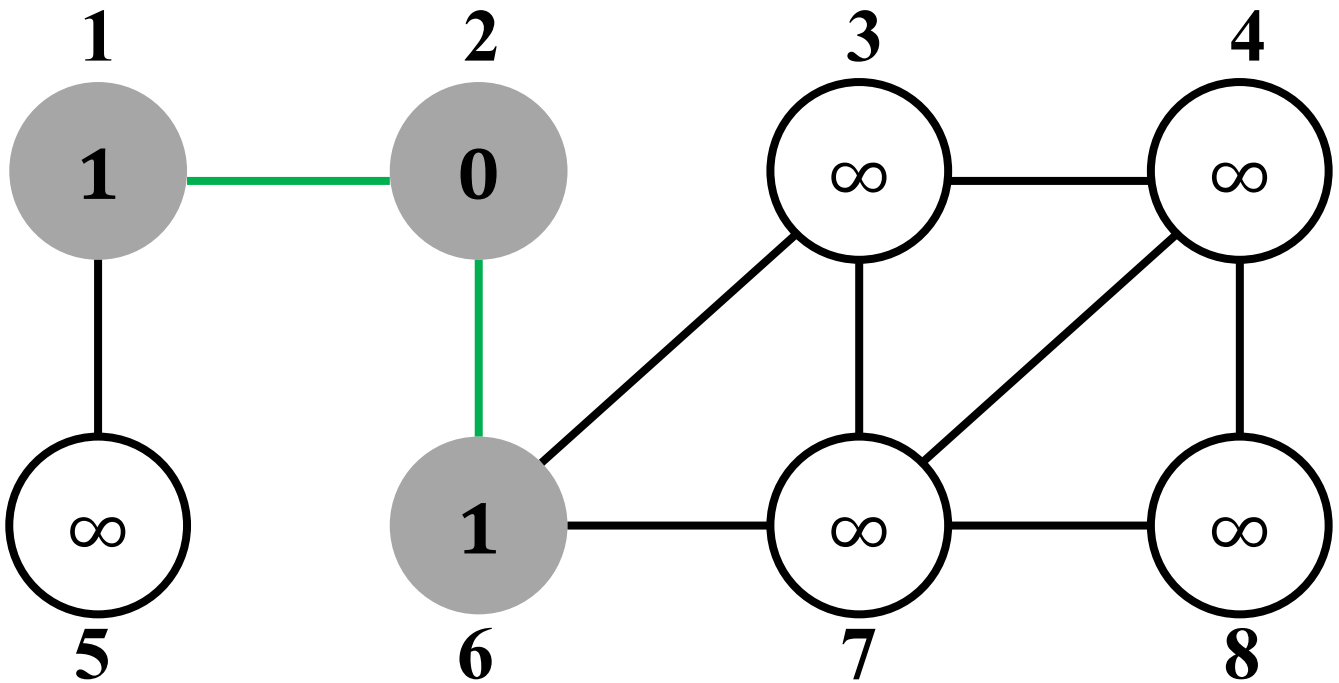
源点2



算法实例

V	1	2	3	4	5	6	7	8
$color$	G	G	W	W	W	G	W	W
$pred$	2	N	N	N	N	2	N	N
$dist$	1	0	∞	∞	∞	1	∞	∞
待处理队列	2	1	6					

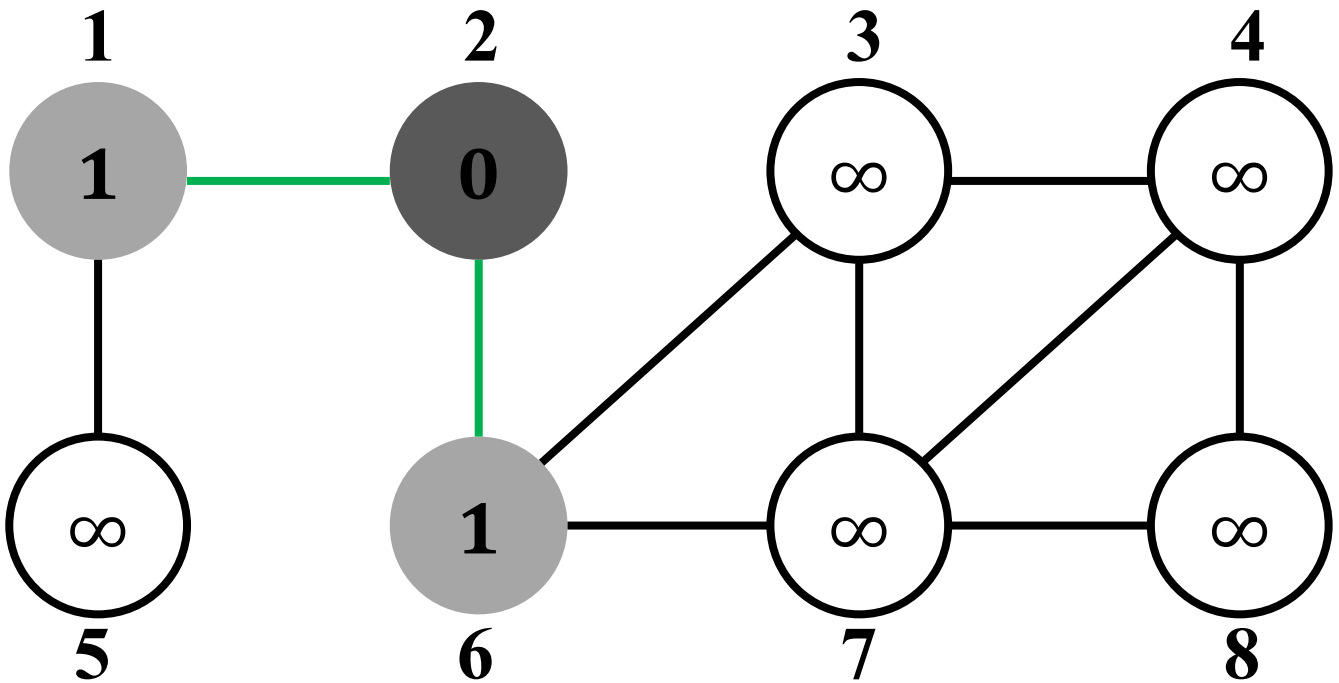
源点2



算法实例

V	1	2	3	4	5	6	7	8
$color$	G	B	W	W	W	G	W	W
$pred$	2	N	N	N	N	2	N	N
$dist$	1	0	∞	∞	∞	1	∞	∞
待处理队列	2	1	6					

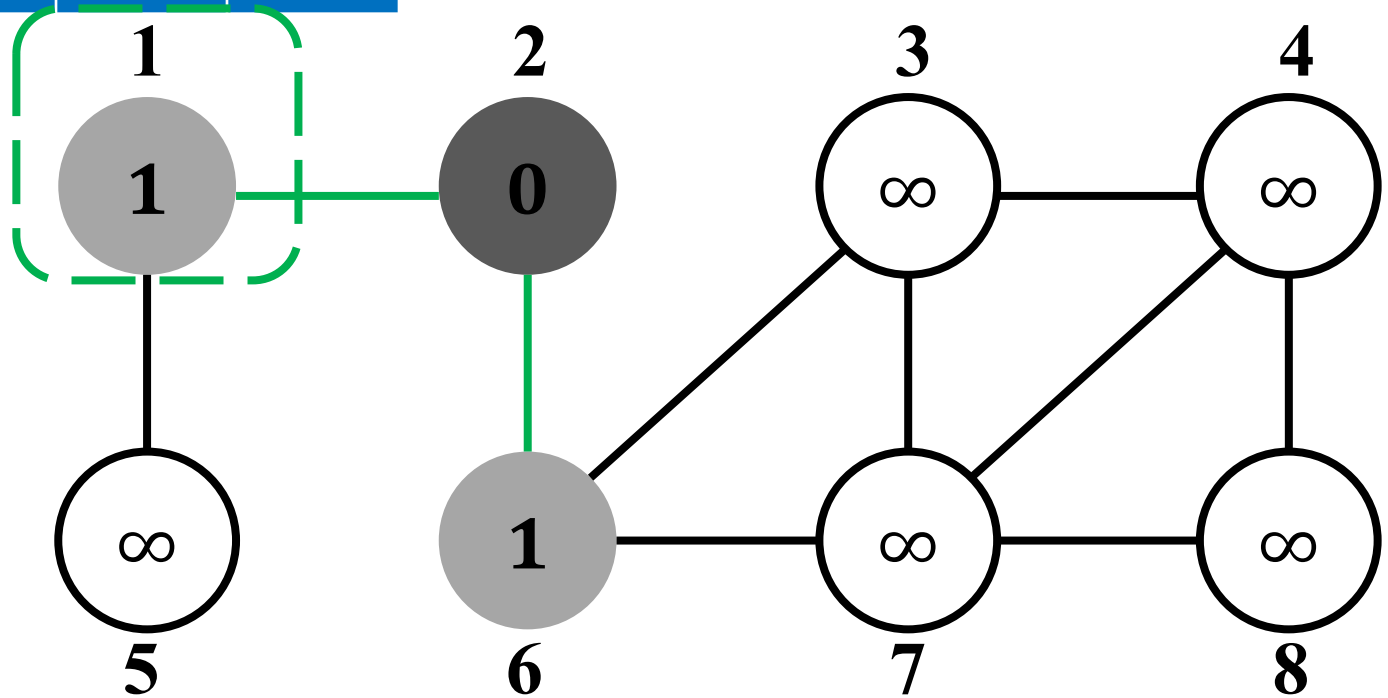
源点2



算法实例

V	1	2	3	4	5	6	7	8
$color$	G	B	W	W	W	G	W	W
$pred$	2	N	N	N	N	2	N	N
$dist$	1	0	∞	∞	∞	1	∞	∞
待处理队列	2	1	6					

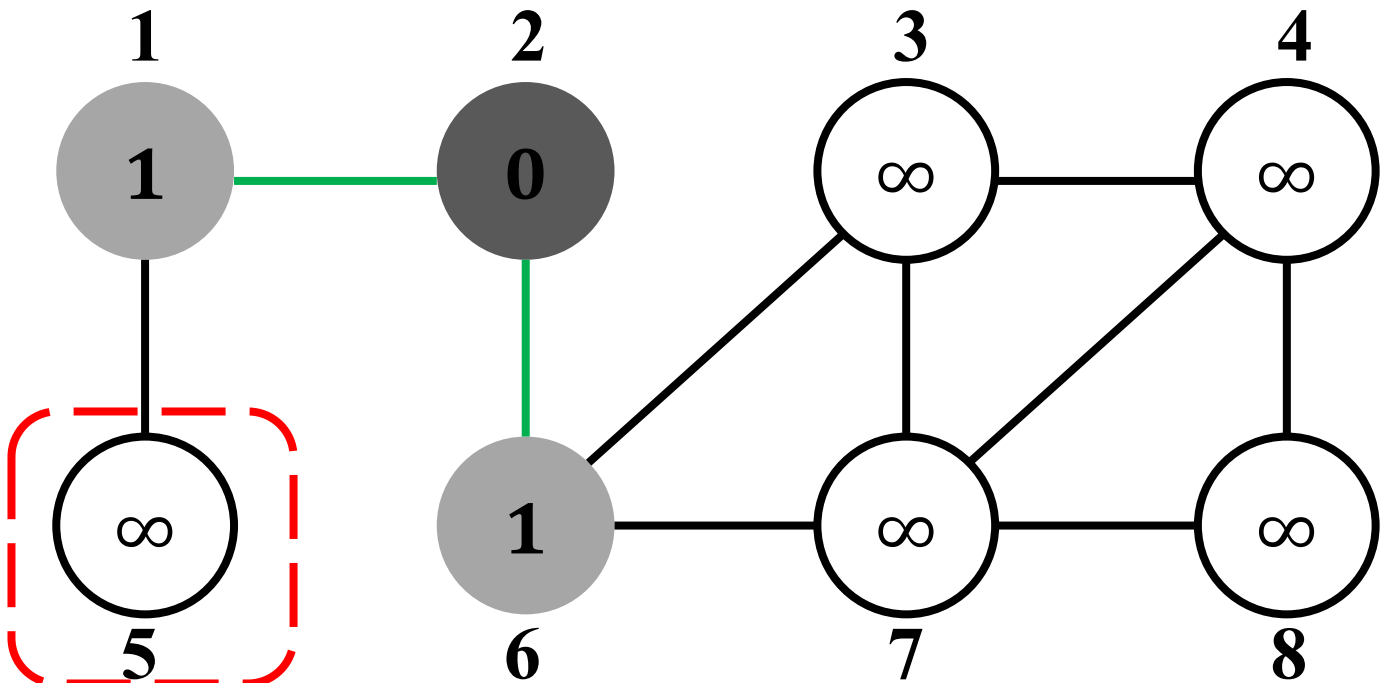
源点2



算法实例

V	1	2	3	4	5	6	7	8
color	G	B	W	W	W	G	W	W
pred	2	N	N	N	N	2	N	N
dist	1	0	∞	∞	∞	1	∞	∞
待处理队列	2	1	6					

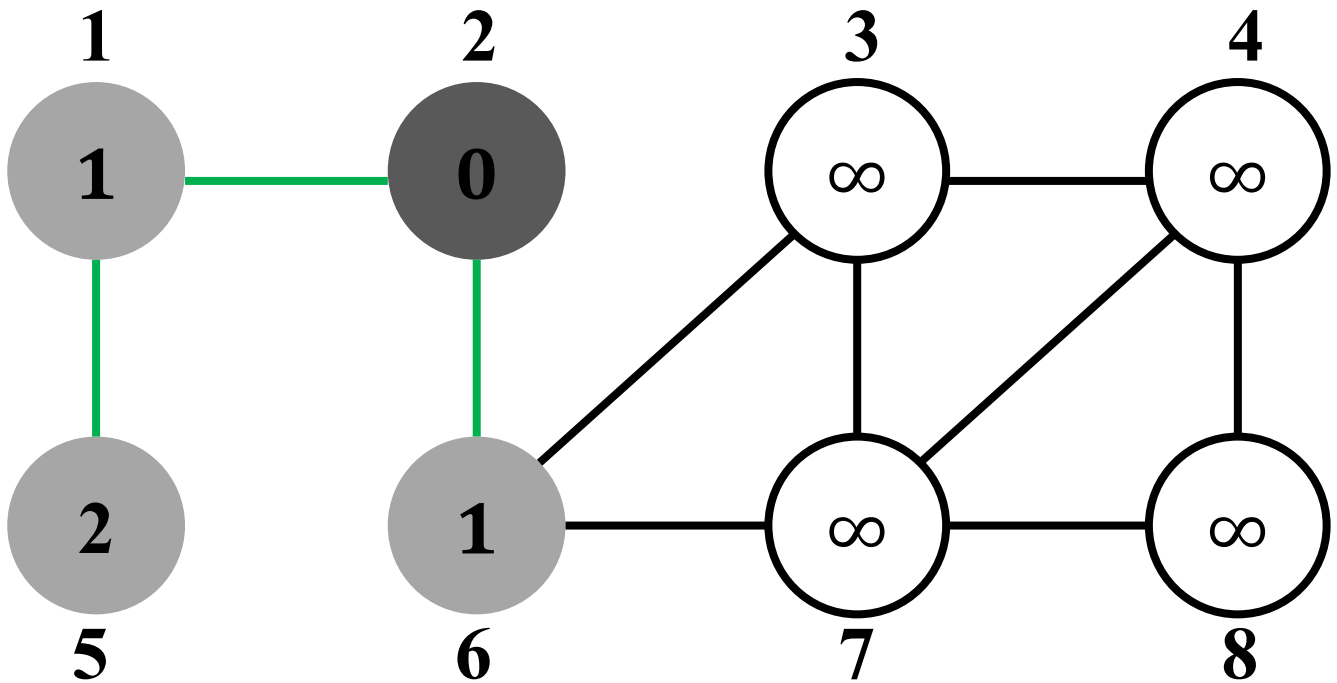
源点2



算法实例

V	1	2	3	4	5	6	7	8
$color$	G	B	W	W	G	G	W	W
$pred$	2	N	N	N	1	2	N	N
$dist$	1	0	∞	∞	2	1	∞	∞
待处理队列	2	1	6	5				

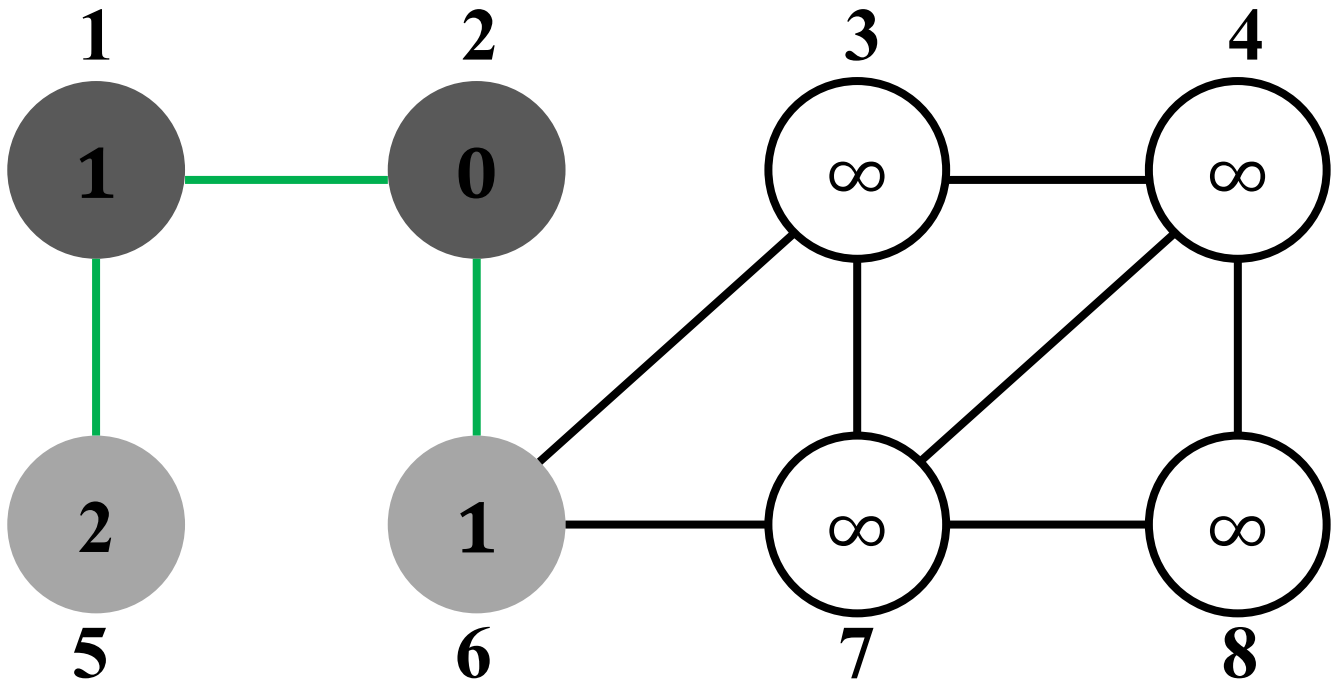
源点2



算法实例

V	1	2	3	4	5	6	7	8
$color$	B	B	W	W	G	G	W	W
$pred$	2	N	N	N	1	2	N	N
$dist$	1	0	∞	∞	2	1	∞	∞
待处理队列	2	1	6	5				

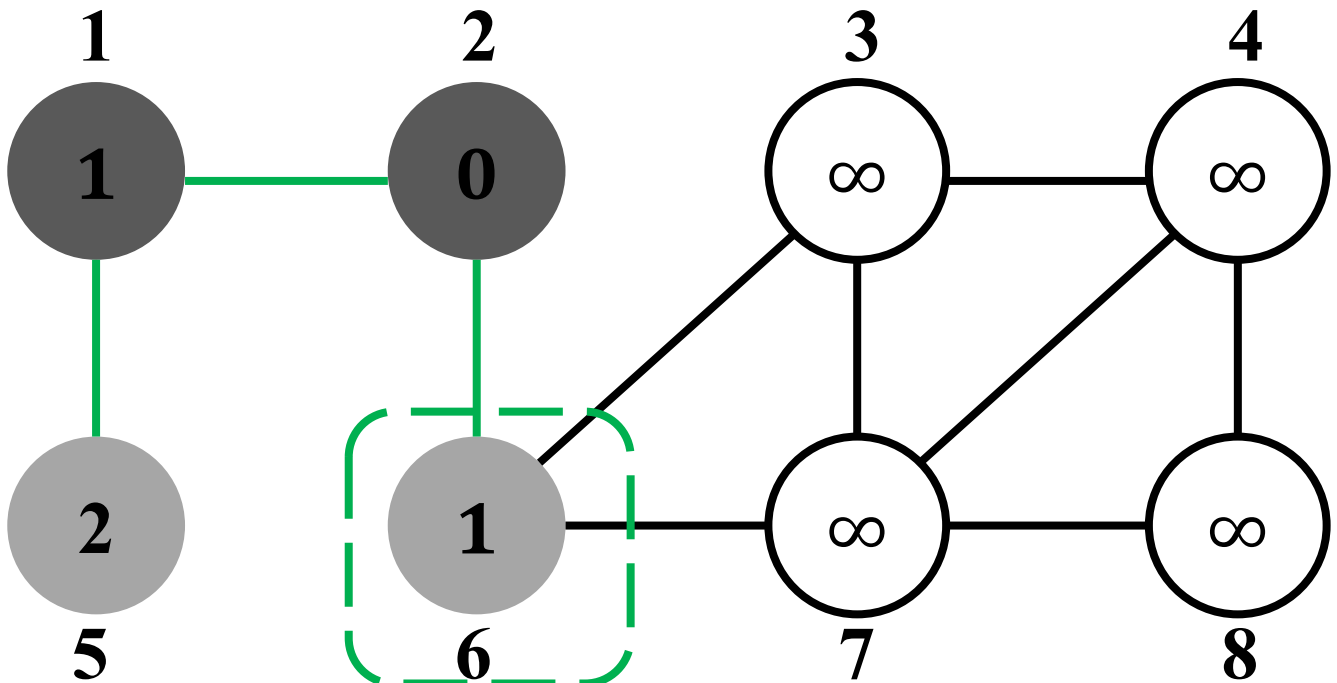
源点2



算法实例

V	1	2	3	4	5	6	7	8
$color$	B	B	W	W	G	G	W	W
$pred$	2	N	N	N	1	2	N	N
$dist$	1	0	∞	∞	2	1	∞	∞
待处理队列	2	1	6	5				

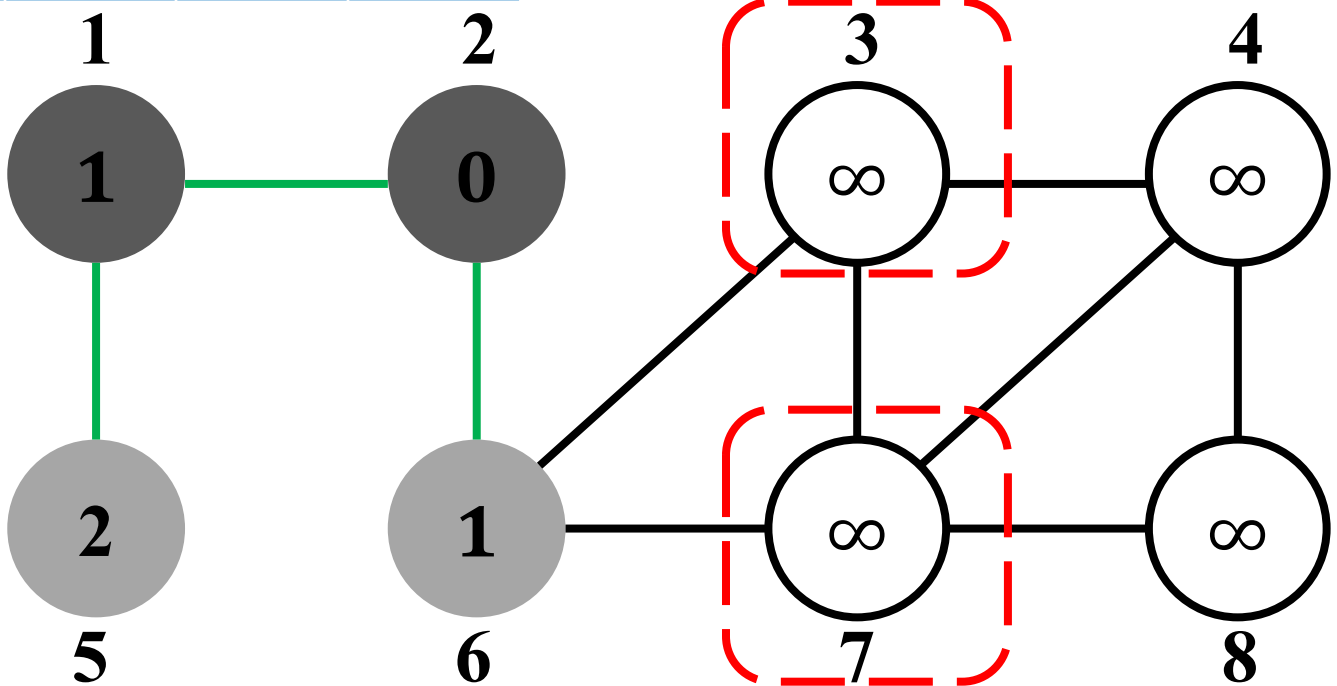
源点2



算法实例

V	1	2	3	4	5	6	7	8
$color$	B	B	W	W	G	G	W	W
$pred$	2	N	N	N	1	2	N	N
$dist$	1	0	∞	∞	2	1	∞	∞
待处理队列	2	1	6	5				

源点2

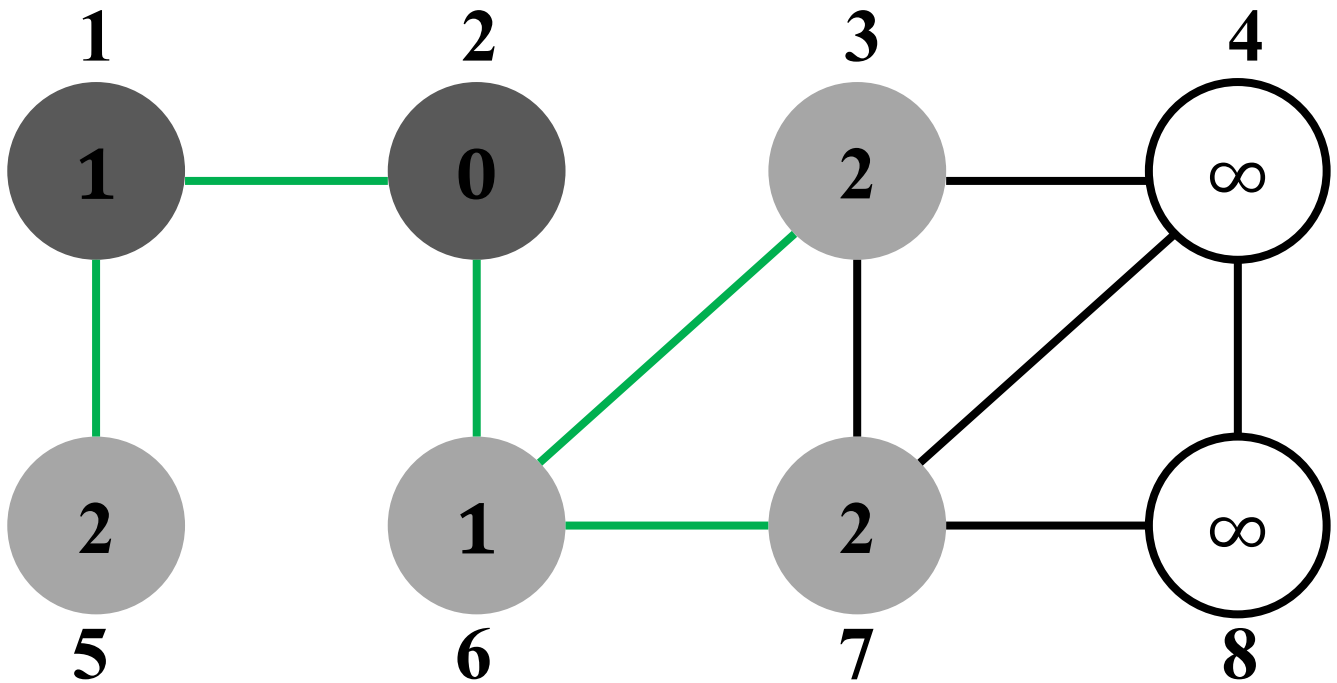


算法实例

V	1	2	3	4	5	6	7	8
$color$	B	B	G	W	G	G	G	W
$pred$	2	N	6	N	1	2	6	N
$dist$	1	0	2	∞	2	1	2	∞

待处理队列	2	1	6	5	3	7

源点2

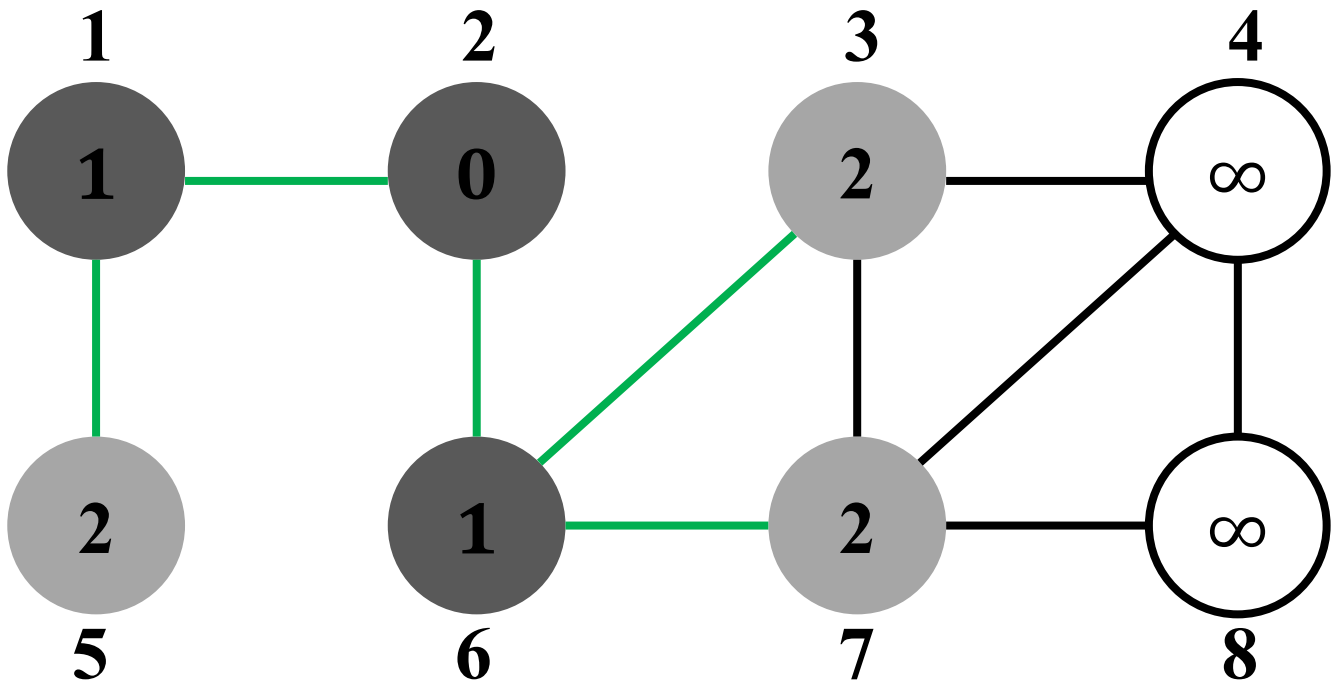


算法实例

V	1	2	3	4	5	6	7	8
$color$	B	B	G	W	G	B	G	W
$pred$	2	N	6	N	1	2	6	N
$dist$	1	0	2	∞	2	1	2	∞

待处理队列	2	1	6	5	3	7

源点2

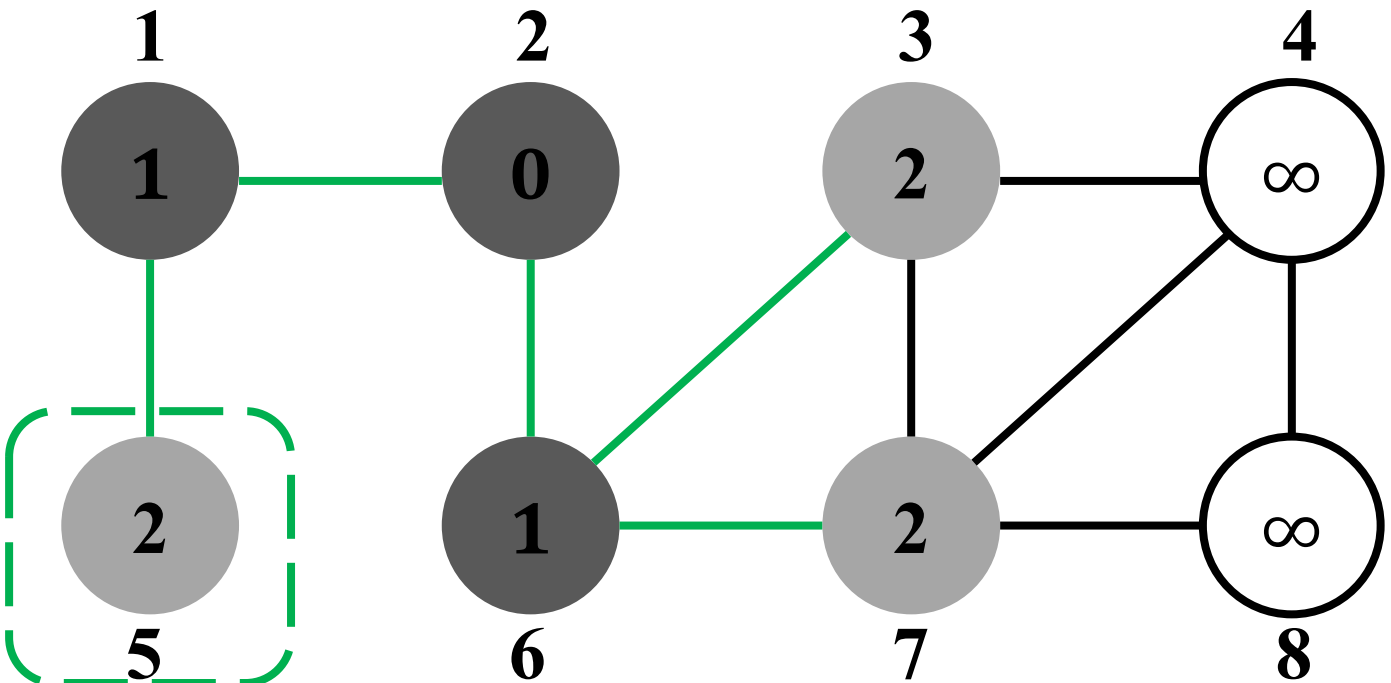


算法实例

V	1	2	3	4	5	6	7	8
$color$	B	B	G	W	G	B	G	W
$pred$	2	N	6	N	1	2	6	N
$dist$	1	0	2	∞	2	1	2	∞

待处理队列	2	1	6	5	3	7

源点2

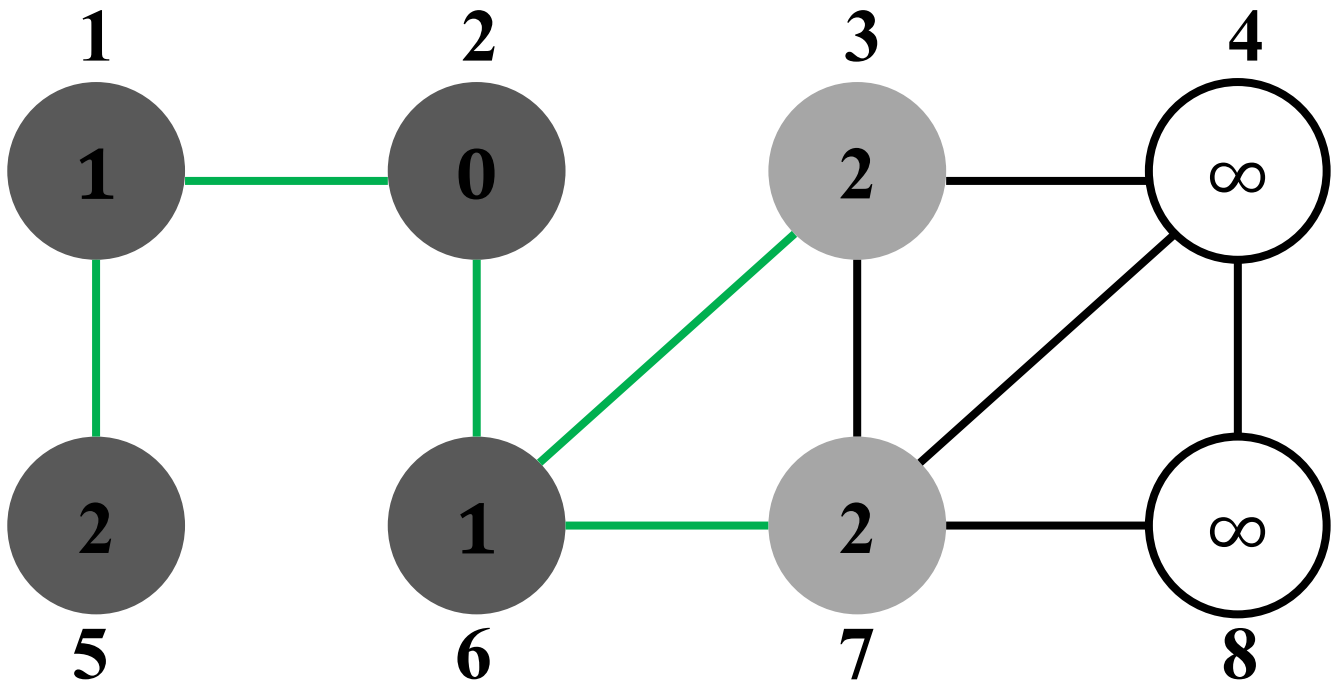


算法实例

V	1	2	3	4	5	6	7	8
$color$	B	B	G	W	B	B	G	W
$pred$	2	N	6	N	1	2	6	N
$dist$	1	0	2	∞	2	1	2	∞

待处理队列	2	1	6	5	3	7

源点2

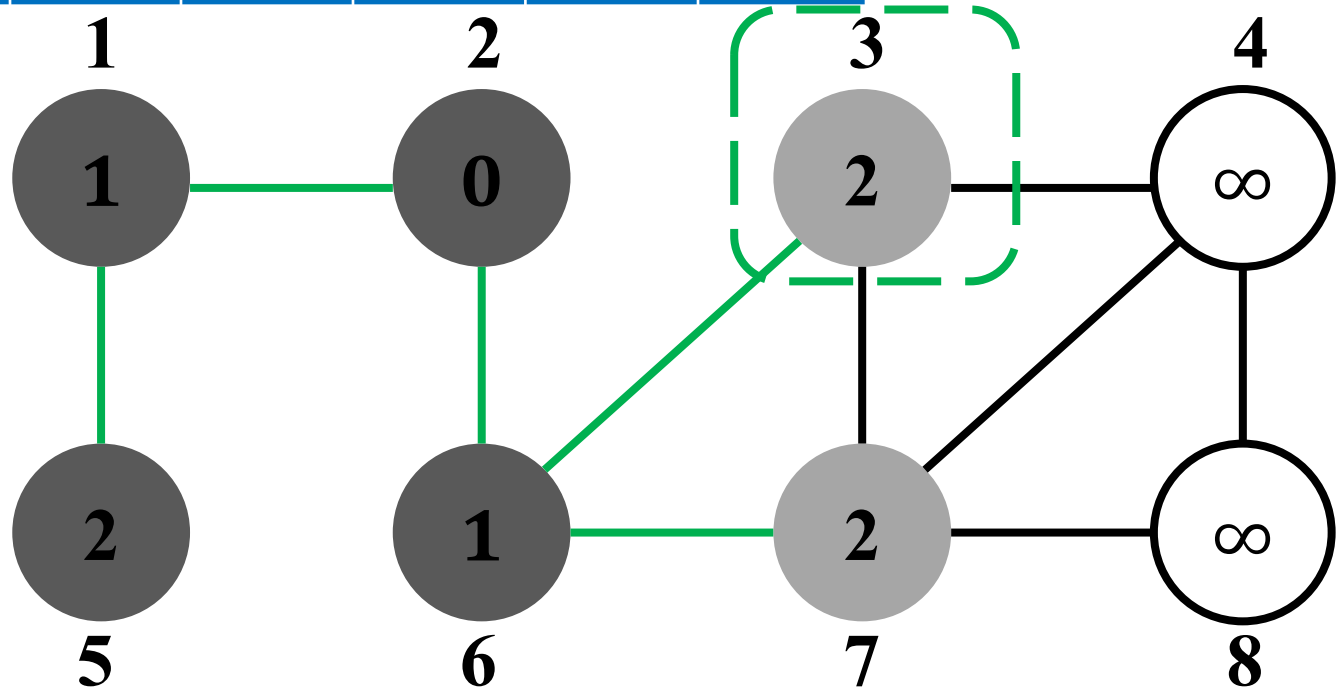


算法实例

V	1	2	3	4	5	6	7	8
$color$	B	B	G	W	B	B	G	W
$pred$	2	N	6	N	1	2	6	N
$dist$	1	0	2	∞	2	1	2	∞

待处理队列	2	1	6	5	3	7

源点2

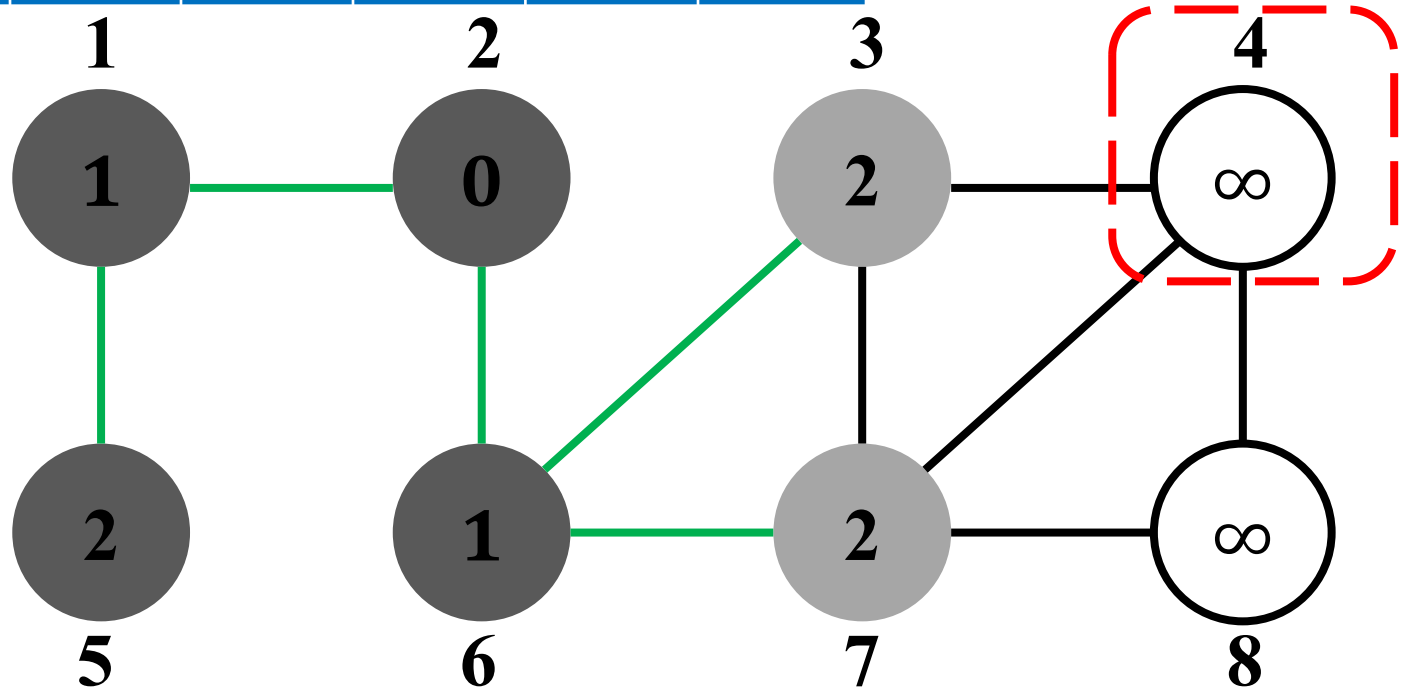


算法实例

V	1	2	3	4	5	6	7	8
$color$	B	B	G	W	B	B	G	W
$pred$	2	N	6	N	1	2	6	N
$dist$	1	0	2	∞	2	1	2	∞

待处理队列	2	1	6	5	3	7

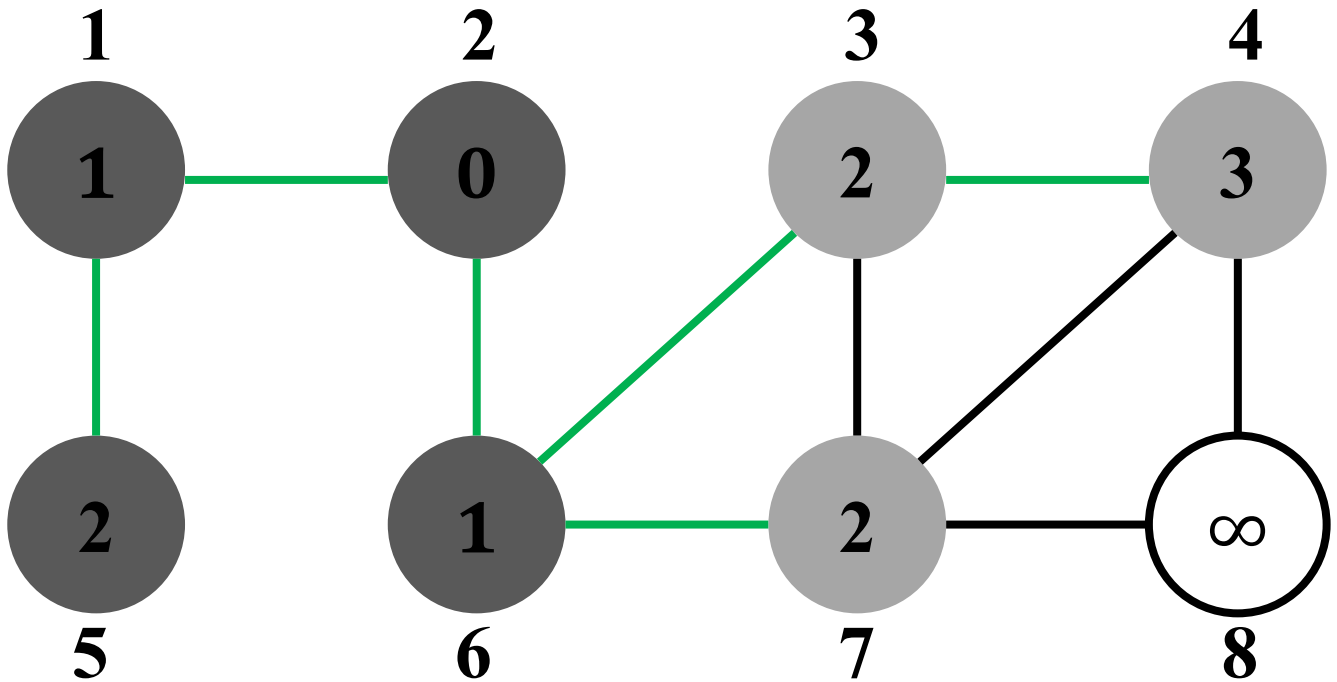
源点2



算法实例

V	1	2	3	4	5	6	7	8
$color$	B	B	G	G	B	B	G	W
$pred$	2	N	6	3	1	2	6	N
$dist$	1	0	2	3	2	1	2	∞
待处理队列	2	1	6	5	3	7	4	

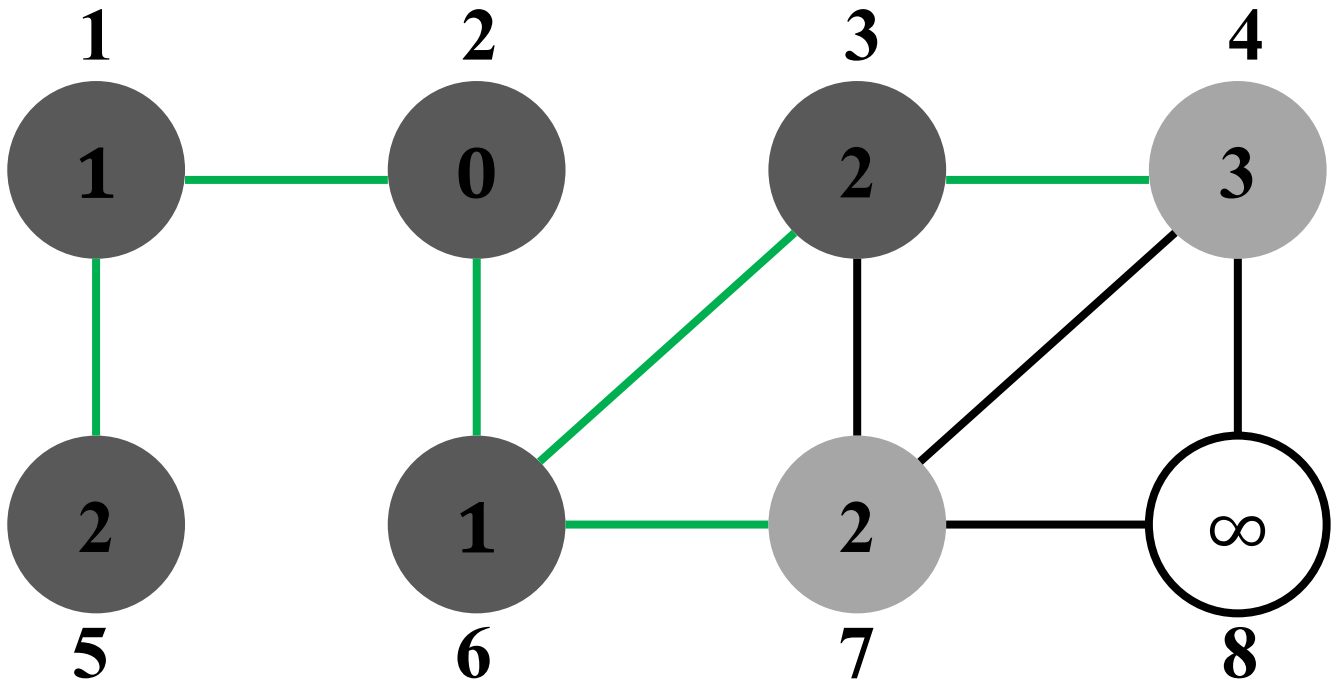
源点2



算法实例

V	1	2	3	4	5	6	7	8
$color$	B	B	B	G	B	B	G	W
$pred$	2	N	6	3	1	2	6	N
$dist$	1	0	2	3	2	1	2	∞
待处理队列	2	1	6	5	3	7	4	

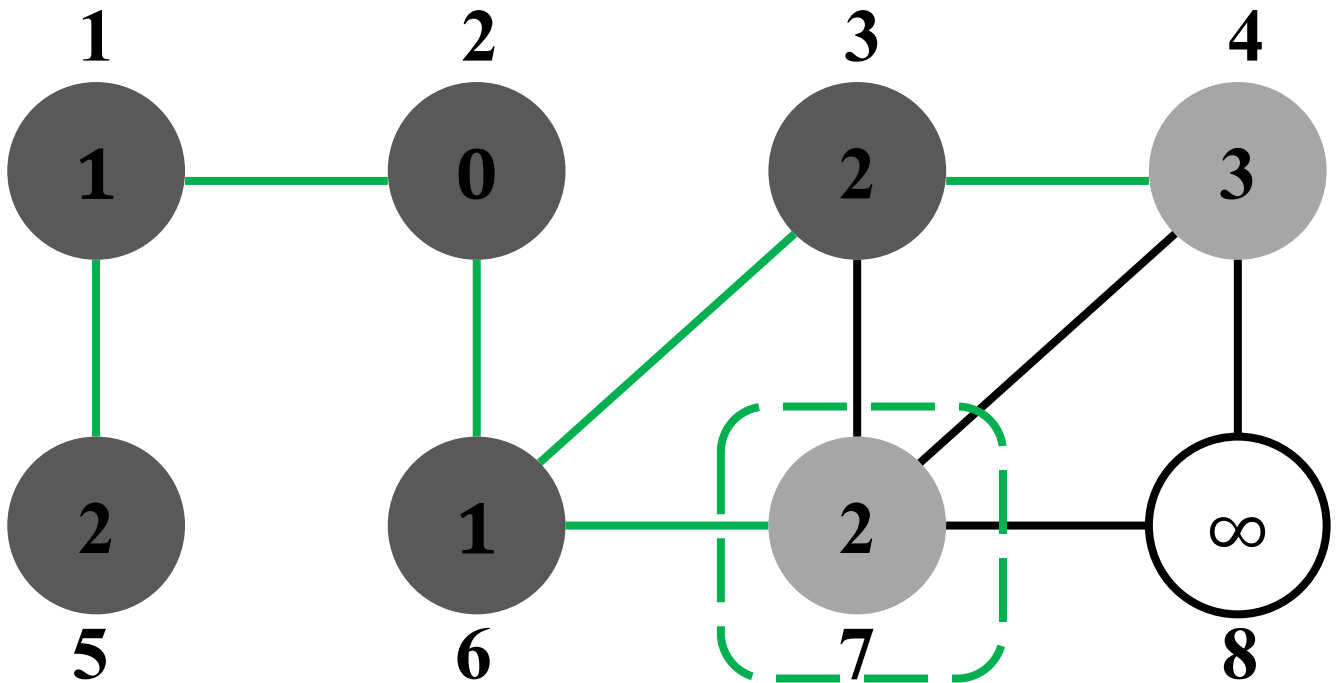
源点2



算法实例

V	1	2	3	4	5	6	7	8
$color$	B	B	B	G	B	B	G	W
$pred$	2	N	6	3	1	2	6	N
$dist$	1	0	2	3	2	1	2	∞
待处理队列	2	1	6	5	3	7	4	

源点2

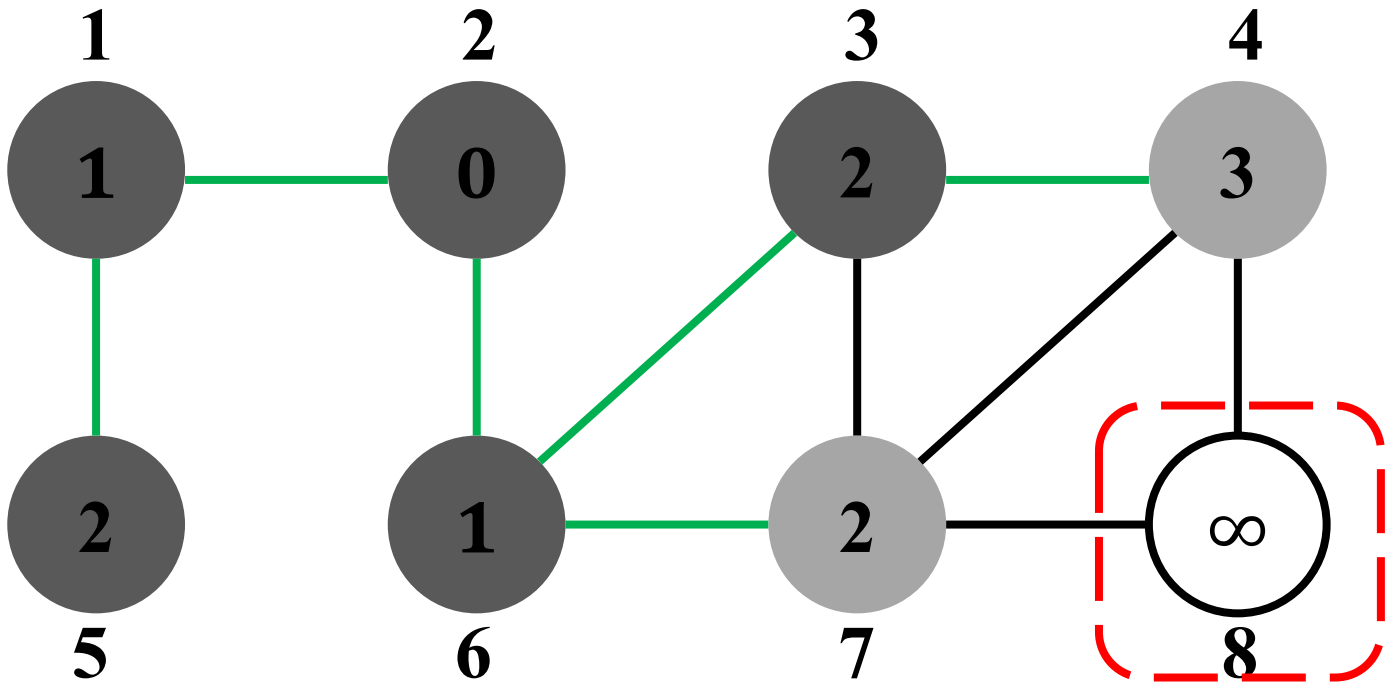


算法实例

V	1	2	3	4	5	6	7	8
$color$	B	B	B	G	B	B	G	W
$pred$	2	N	6	3	1	2	6	N
$dist$	1	0	2	3	2	1	2	∞

待处理队列	2	1	6	5	3	7	4

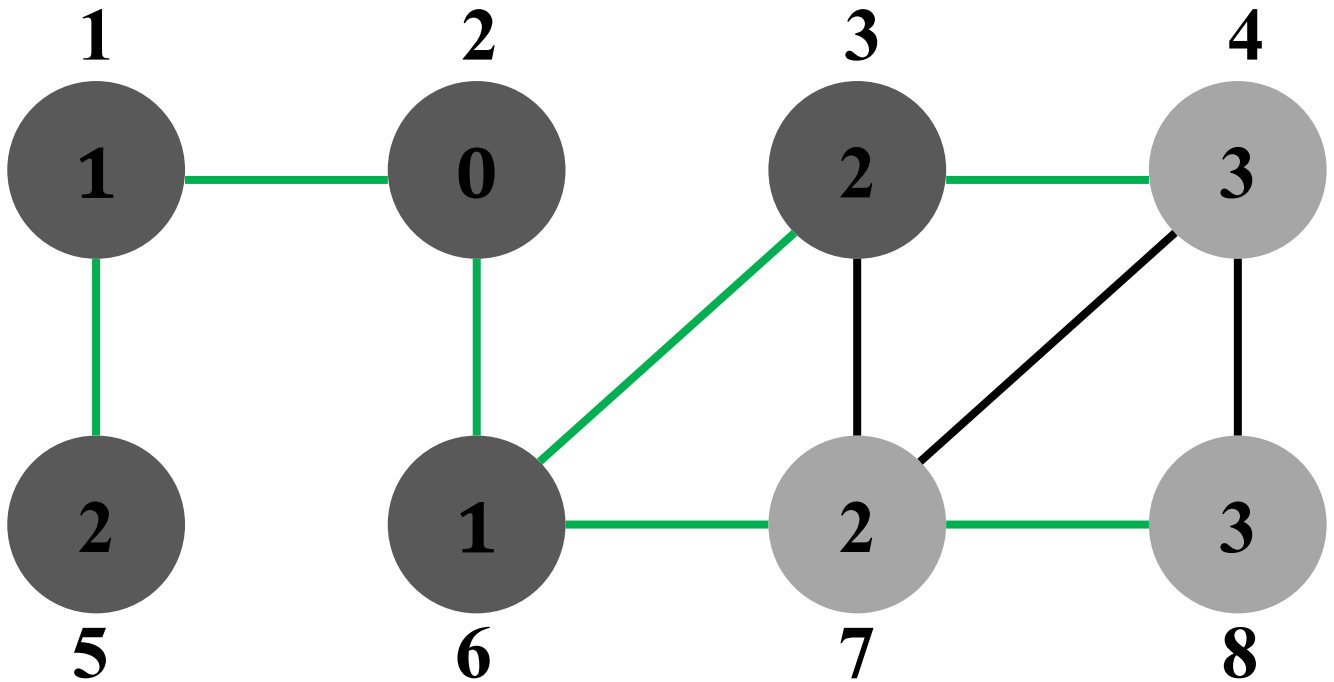
源点2



算法实例

V	1	2	3	4	5	6	7	8
$color$	B	B	B	G	B	B	G	G
$pred$	2	N	6	3	1	2	6	7
$dist$	1	0	2	3	2	1	2	3
待处理队列	2	1	6	5	3	7	4	8

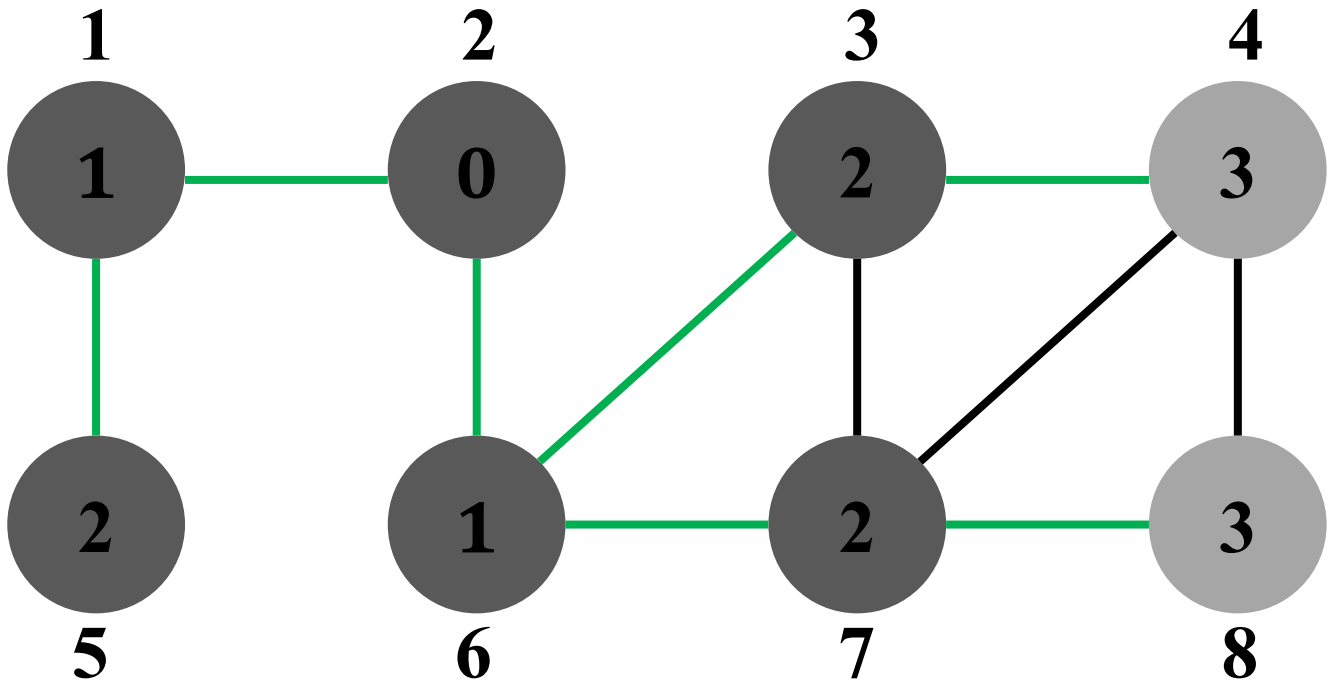
源点2



算法实例

V	1	2	3	4	5	6	7	8
$color$	B	B	B	G	B	B	B	G
$pred$	2	N	6	3	1	2	6	7
$dist$	1	0	2	3	2	1	2	3
待处理队列	2	1	6	5	3	7	4	8

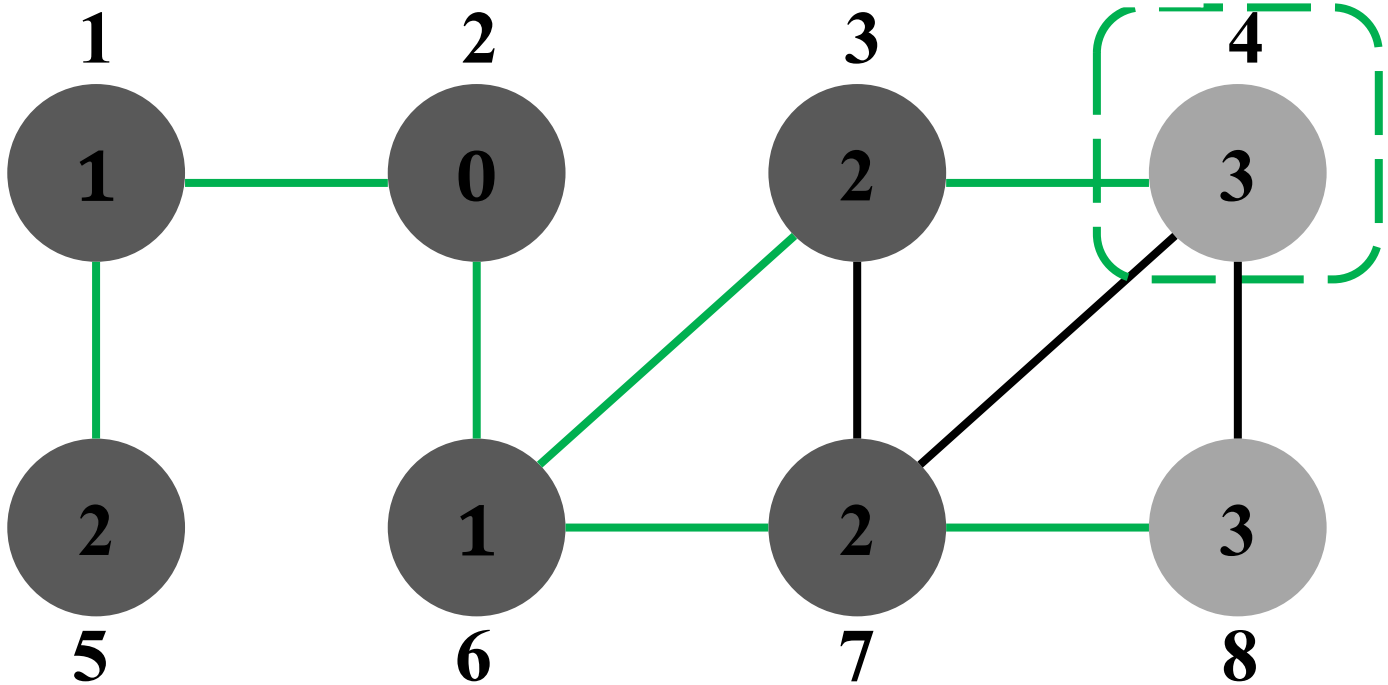
源点2



算法实例

V	1	2	3	4	5	6	7	8
$color$	B	B	B	G	B	B	B	G
$pred$	2	N	6	3	1	2	6	7
$dist$	1	0	2	3	2	1	2	3
待处理队列	2	1	6	5	3	7	4	8

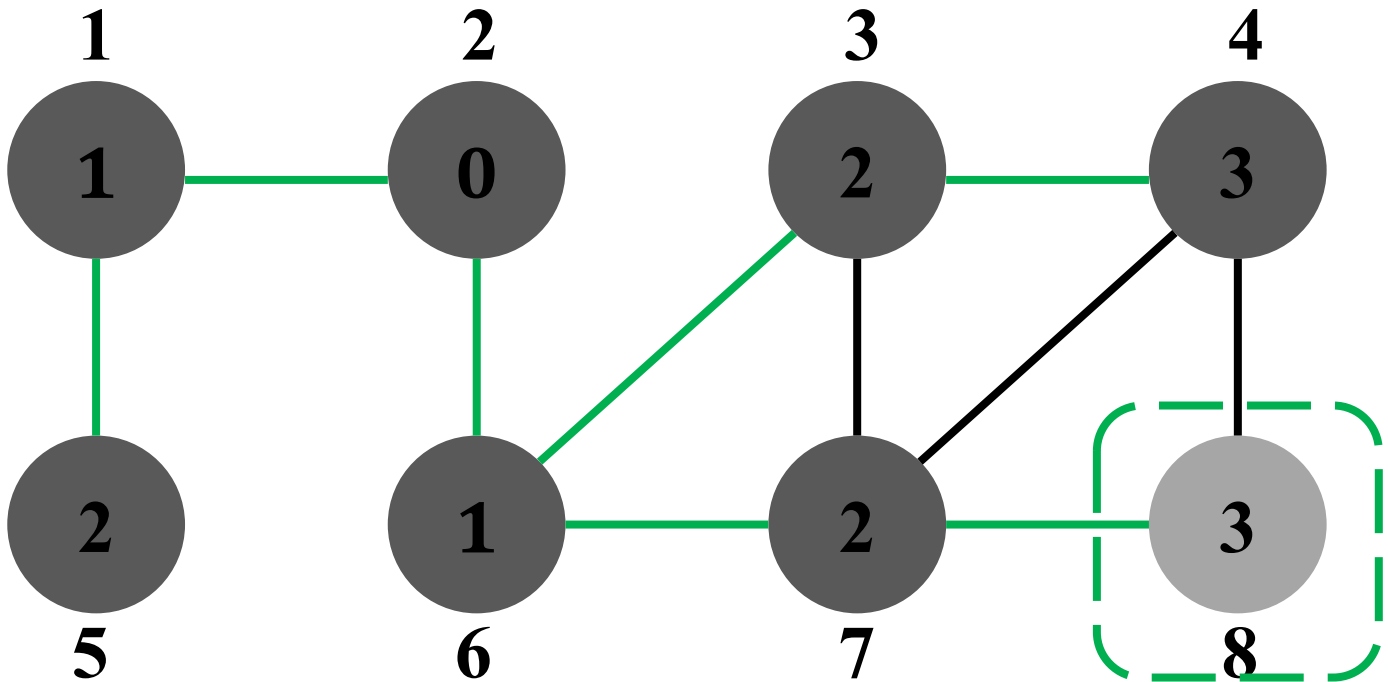
源点2



算法实例

V	1	2	3	4	5	6	7	8
$color$	B	B	B	B	B	B	B	G
$pred$	2	N	6	3	1	2	6	7
$dist$	1	0	2	3	2	1	2	3
待处理队列	2	1	6	5	3	7	4	8

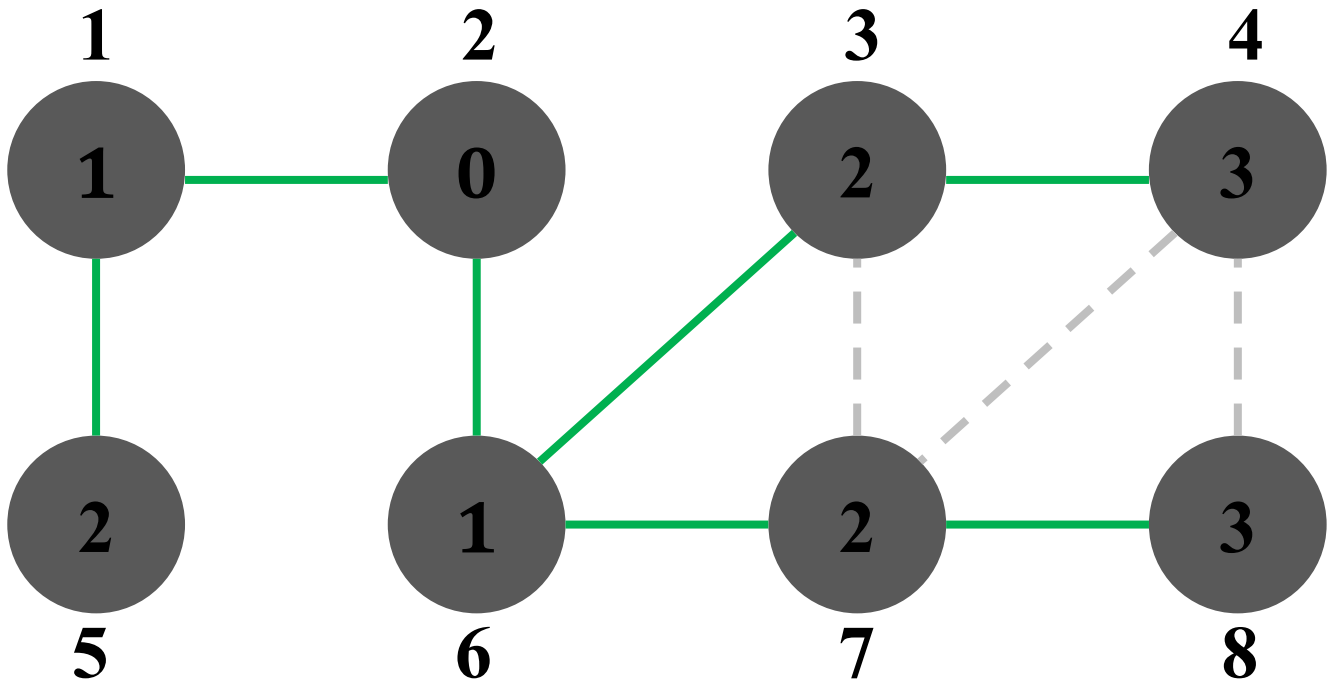
源点2



算法实例

V	1	2	3	4	5	6	7	8
$color$	B	B	B	B	B	B	B	B
$pred$	2	N	6	3	1	2	6	7
$dist$	1	0	2	3	2	1	2	3
待处理队列	2	1	6	5	3	7	4	8

源点2



搜索结束

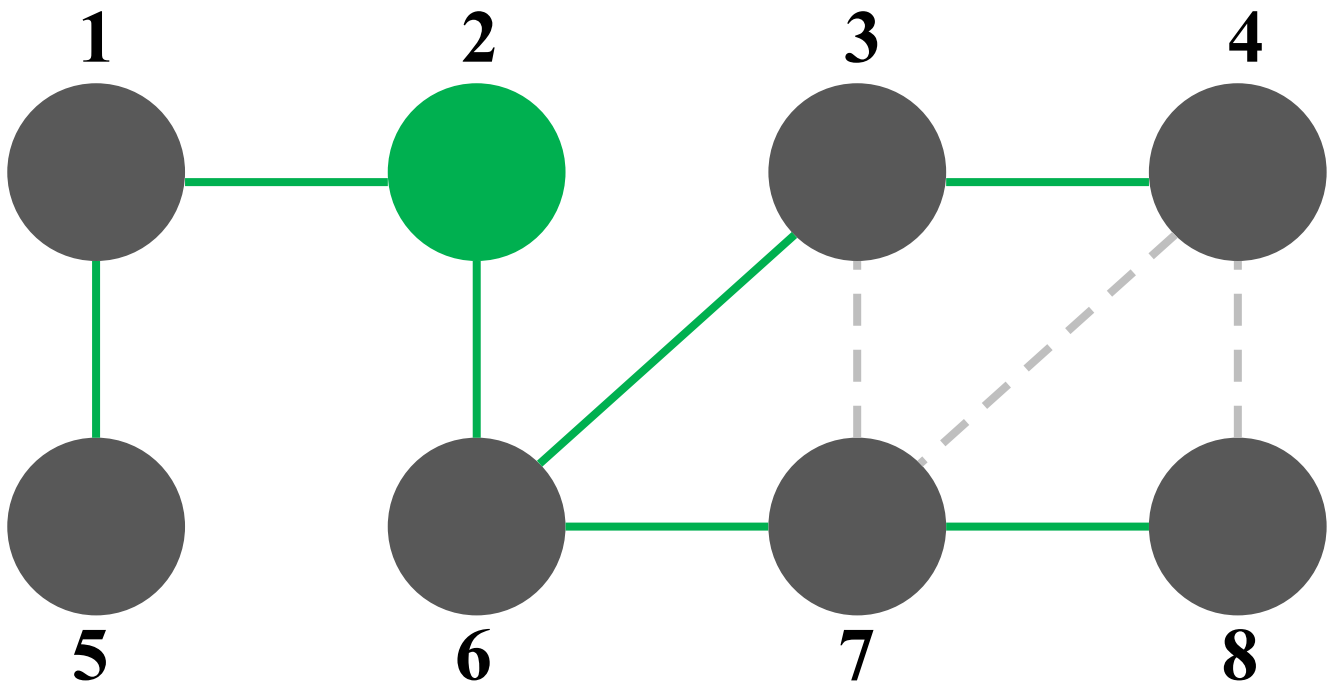
广度优先树

V	1	2	3	4	5	6	7	8
$pred$	2	N	6	3	1	2	6	7

- 辅助数组$pred[\ ]$储存了一棵树

连通、无环

$|E_T| = |V_T| - 1$



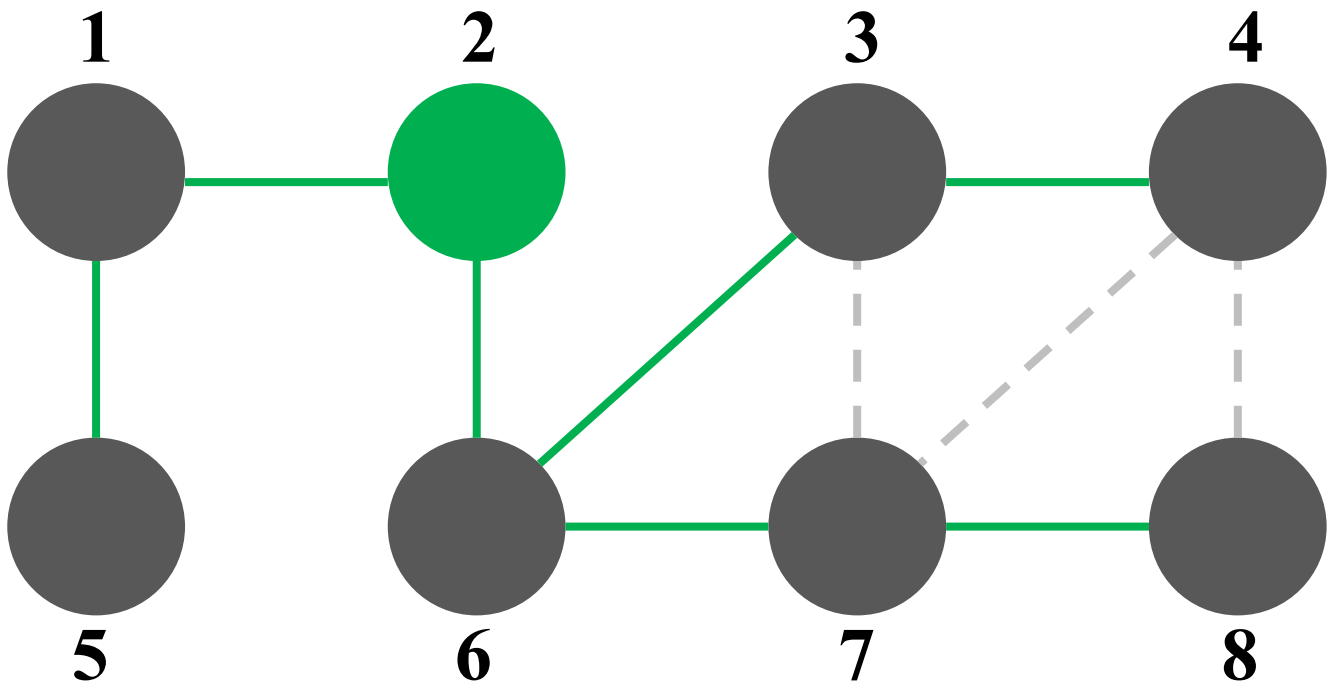
广度优先树

V	1	2	3	4	5	6	7	8
$pred$	2	N	6	3	1	2	6	7

- 辅助数组$pred[\ ]$储存了一棵树，广度优先树$T = < V_T, E_T >$

连通、无环

$|E_T| = |V_T| - 1$

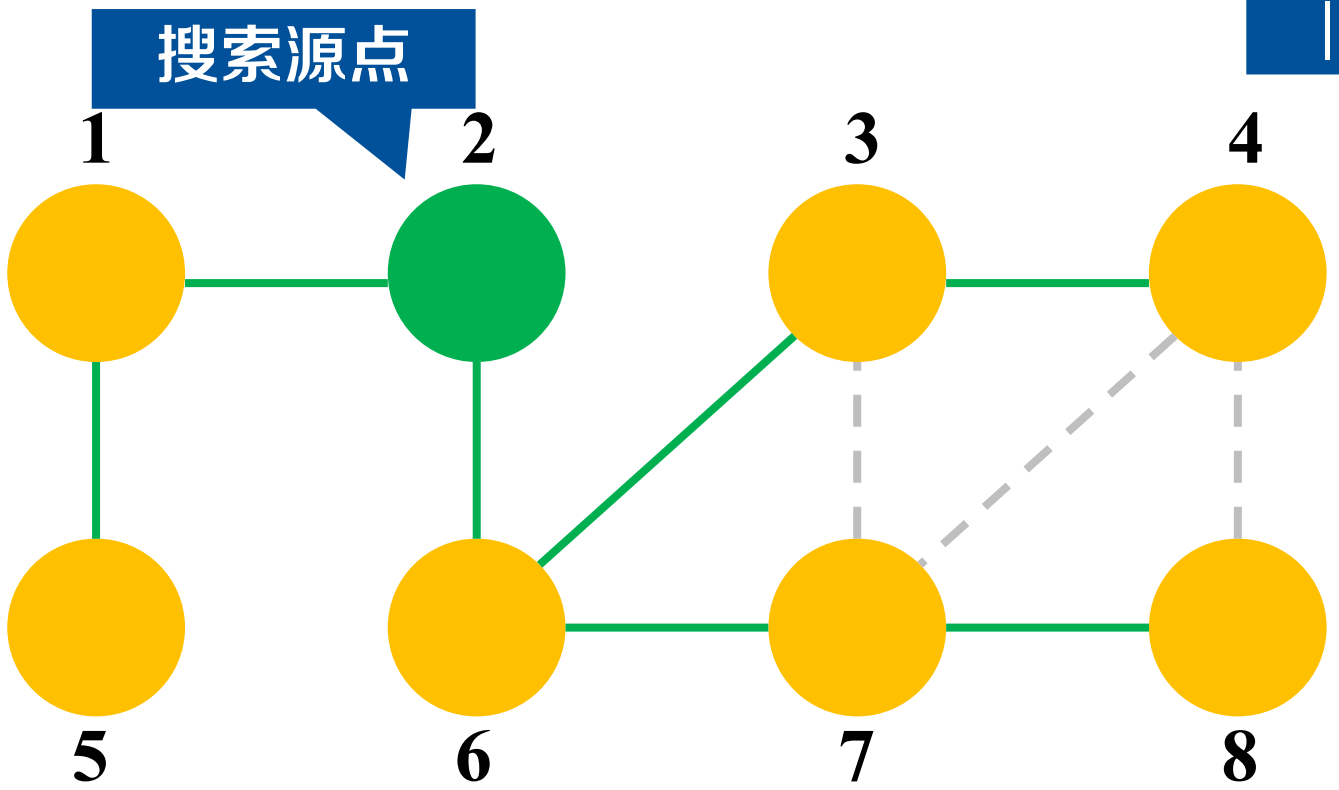


广度优先树

V	1	2	3	4	5	6	7	8
$pred$	2	N	6	3	1	2	6	7

- 辅助数组$pred[\ ]$储存了一棵树，广度优先树$T =< V_T, E_T >$
- V_T是源点s可达的顶点集合

连通、无环
$|E_T| = |V_T| - 1$



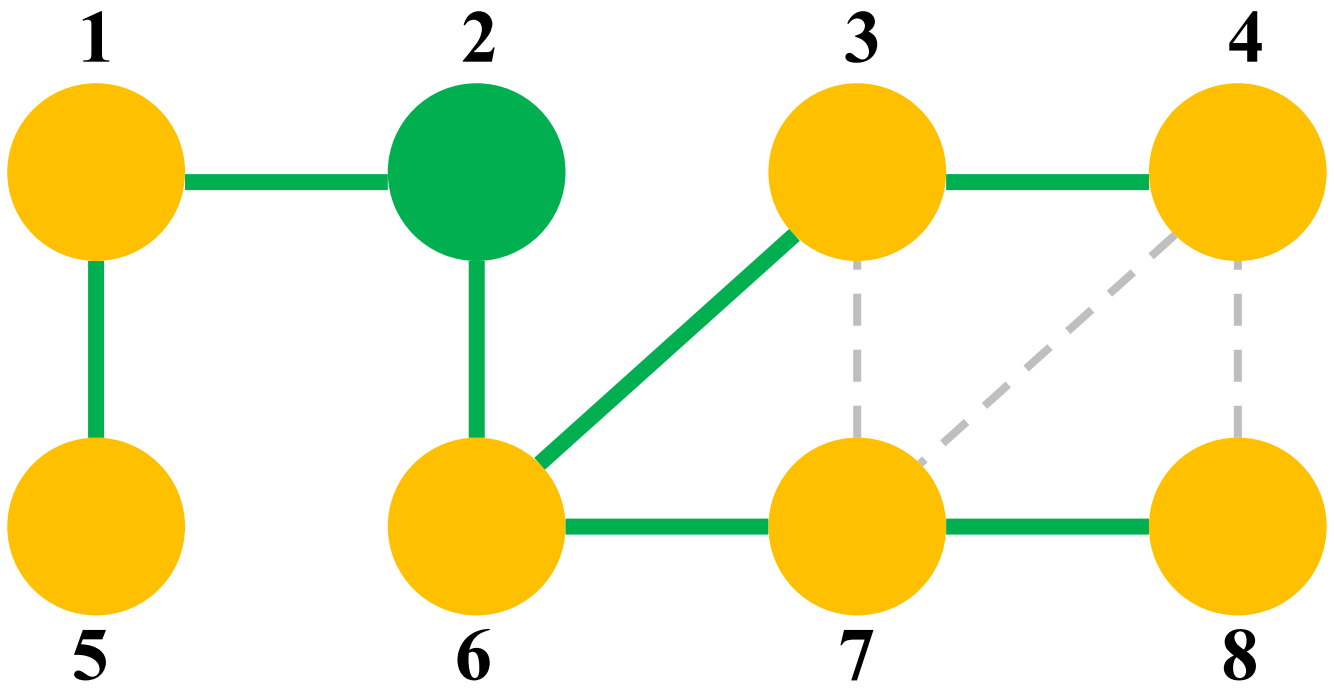
广度优先树

V	1	2	3	4	5	6	7	8
$pred$	2	N	6	3	1	2	6	7

- 辅助数组$pred[\ ]$储存了一棵树，广度优先树$T = < V_T, E_T >$
- V_T是源点s可达的顶点集合，$E_T = \{(pred[u], u): u \in V_T\}$

连通、无环

$|E_T| = |V_T| - 1$



提纲

图的搜索

算法思想

算法实例

算法分析

算法应用

伪代码

- **BFS(G, s)**

输入: 图G, 源点s
输出: 前驱数组$pred[\ ]$,距离数组$dist[\ ]$
新建一维数组$color[1..|V|]$, $pred[1..|V|]$, $dist[1..|V|]$
新建空队列Q
//初始化
for $u \in V$ do
  $color[u] \leftarrow WHITE$
  $pred[u] \leftarrow NULL$
  $dist[u] \leftarrow \infty$
end
$color[s] \leftarrow GRAY$
$dist[s] \leftarrow 0$
$Q.Enqueue(s)$

初始化各记录信息数组

伪代码

- **BFS(G, s)**

输入: 图G, 源点s
输出: 前驱数组$pred[\ ]$,距离数组$dist[\ ]$
新建一维数组$color[1..|V|]$, $pred[1..|V|]$, $dist[1..|V|]$
新建空队列Q
//初始化
for $u \in V$ **do**
    $color[u] \leftarrow WHITE$
    $pred[u] \leftarrow NULL$
    $dist[u] \leftarrow \infty$
end
$color[s] \leftarrow GRAY$
$dist[s] \leftarrow 0$
$Q.Enqueue(s)$

初始化源点状态

伪代码

- **BFS(G, s)**

输入: 图G, 源点s
输出: 前驱数组$pred[\ ]$,距离数组$dist[\ ]$
新建一维数组$color[1..|V|]$, $pred[1..|V|]$, $dist[1..|V|]$
新建空队列Q
//初始化
for $u \in V$ **do**
    $color[u] \leftarrow WHITE$
    $pred[u] \leftarrow NULL$
    $dist[u] \leftarrow \infty$
end
$color[s] \leftarrow GRAY$
$dist[s] \leftarrow 0$
$Q.Enqueue(s)$

把源点加入

伪代码

- **BFS(G, s)**

```
//广度优先搜索
while 等待队列Q非空 do
    u ← Q.Dequeue()
    for v ∈ G.Adj[u] do
        if color[v] = WHITE then
            color[v] ← GRAY
            dist[v] ← dist[u] + 1
            pred[v] ← u
            Q.Enqueue(v)
        end
    end
    color[u] ← BLACK
end
```

依次处理所有顶点

伪代码

- **BFS(G, s)**

```
//广度优先搜索
while 等待队列Q非空 do
    u ← Q.Dequeue()
    for v ∈ G.Adj[u] do
        if color[v] = WHITE then
            color[v] ← GRAY
            dist[v] ← dist[u] + 1
            pred[v] ← u
            Q.Enqueue(v)
        end
    end
    color[u] ← BLACK
end
```

从等待队列取出当前处理顶点u

伪代码

- **BFS(G, s)**

```
//广度优先搜索
while 等待队列Q非空 do
    u ← Q.Dequeue()
    for v ∈ G.Adj[u] do
        if color[v] = WHITE then
            color[v] ← GRAY
            dist[v] ← dist[u] + 1
            pred[v] ← u
            Q.Enqueue(v)
        end
    end
    color[u] ← BLACK
end
```

搜索顶点u的所有相邻顶点

伪代码

- **BFS(G, s)**

```
//广度优先搜索
while 等待队列Q非空 do
    u ← Q.Dequeue()
    for v ∈ G.Adj[u] do
        if color[v] = WHITE then
            color[v] ← GRAY
            dist[v] ← dist[u] + 1
            pred[v] ← u
            Q.Enqueue(v)
        end
    end
    color[u] ← BLACK
end
```

第一次发现顶点v

伪代码

- **BFS(G, s)**

```
//广度优先搜索
while 等待队列Q非空 do
    u ← Q.Dequeue()
    for v ∈ G.Adj[u] do
        if color[v] = WHITE then
            color[v] ← GRAY
            dist[v] ← dist[u] + 1
            pred[v] ← u
            Q.Enqueue(v)
        end
    end
    color[u] ← BLACK
end
```

标记已发现顶点v为待处理状态

伪代码

- **BFS(G, s)**

```
//广度优先搜索
while 等待队列Q非空 do
    u ← Q.Dequeue()
    for v ∈ G.Adj[u] do
        if color[v] = WHITE then
            color[v] ← GRAY
            dist[v] ← dist[u] + 1
            pred[v] ← u
            Q.Enqueue(v)
        end
    end
    color[u] ← BLACK
end
```

记录到源点的距离

伪代码

- **BFS(G, s)**

```
//广度优先搜索
while 等待队列Q非空 do
    u ← Q.Dequeue()
    for v ∈ G.Adj[u] do
        if color[v] = WHITE then
            color[v] ← GRAY
            dist[v] ← dist[u] + 1
            pred[v] ← u
            Q.Enqueue(v)
        end
    end
    color[u] ← BLACK
end
```

记录前驱顶点

伪代码

- **BFS(G, s)**

```
//广度优先搜索
while 等待队列Q非空 do
    u ← Q.Dequeue()
    for v ∈ G.Adj[u] do
        if color[v] = WHITE then
            color[v] ← GRAY
            dist[v] ← dist[u] + 1
            pred[v] ← u
            Q.Enqueue(v)
        end
    end
    color[u] ← BLACK
end
```

加入待处理队列

伪代码

- **BFS(G, s)**

```
//广度优先搜索
while 等待队列Q非空 do
    u ← Q.Dequeue()
    for v ∈ G.Adj[u] do
        if color[v] = WHITE then
            color[v] ← GRAY
            dist[v] ← dist[u] + 1
            pred[v] ← u
            Q.Enqueue(v)
        end
    end
    color[u] ← BLACK
end
```

标记当前顶点处理完成

复杂度分析

- 对于每个顶点u，搜索相邻顶点消耗时间$T_u = O(1 + deg(u))$
- 总运行时间：

$$T \leq \sum_{u \in V} T_u$$

依次搜索所有顶点

复杂度分析

- 对于每个顶点u，搜索相邻顶点消耗时间$T_u = O(1 + deg(u))$
- 总运行时间：

$$T \leq \sum_{u \in V} T_u$$

$$= \sum_{u \in V} O(1 + deg(u))$$

搜索所有相邻顶点

复杂度分析

- 对于每个顶点u，搜索相邻顶点消耗时间$T_u = O(1 + deg(u))$
- 总运行时间：

$$
\begin{aligned}
T &\leq \sum_{u \in V} T_u \\
&= \sum_{u \in V} O(1 + deg(u)) \\
&= \sum_{u \in V} O(1) + \sum_{u \in V} O(deg(u))
\end{aligned}
$$

公式化简

复杂度分析

- 对于每个顶点u，搜索相邻顶点消耗时间$T_u = O(1 + deg(u))$
- 总运行时间：

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T &\leq \sum_{u \in V} T_u \\
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\end{aligned}
$$

图的度

复杂度分析

- 对于每个顶点u，搜索相邻顶点消耗时间$T_u = O(1 + deg(u))$
- 总运行时间：

$$
\begin{aligned}
T &\leq \sum_{u \in V} T_u \\
&= \sum_{u \in V} O(1 + deg(u)) \\
&= \sum_{u \in V} O(1) + \sum_{u \in V} O(deg(u)) \\
&= O(|V| + |E|)
\end{aligned}
$$

$$deg(G) = 2 \cdot |E|$$

复杂度分析

- 对于每个顶点u，搜索相邻顶点消耗时间$T_u = O(1 + deg(u))$
- 总运行时间：

$$
\begin{aligned}
T &\leq \sum_{u \in V} T_u \\
&= \sum_{u \in V} O(1 + deg(u)) \\
&= \sum_{u \in V} O(1) + \sum_{u \in V} O(deg(u)) \\
&= O(|V| + |E|)
\end{aligned}
$$

广度优先搜索的时间复杂度为$O(|V| + |E|)$

复杂度分析

- 对于每个顶点u，搜索相邻顶点消耗时间$T_u = O(1 + deg(u))$
- 总运行时间：

$$
\begin{aligned}
T &\leq \sum_{u \in V} T_u \\
&= \sum_{u \in V} O(1 + deg(u)) \\
&= \sum_{u \in V} O(1) + \sum_{u \in V} O(deg(u)) \\
&= O(|V| + |E|)
\end{aligned}
$$

在渐近记号中，约定符号V代表$|V|$，符号E代表$|E|$

复杂度分析

- 对于每个顶点u，搜索相邻顶点消耗时间$T_u = O(1 + deg(u))$
- 总运行时间：

$$
\begin{aligned}
T &\leq \sum_{u \in V} T_u \\
&= \sum_{u \in V} O(1 + deg(u)) \\
&= \sum_{u \in V} O(1) \\
&= O(V + E)
\end{aligned}
$$

简化表示

在渐近记号中，约定符号V代表$|V|$，符号E代表$|E|$

提纲

图的搜索

算法思想

算法实例

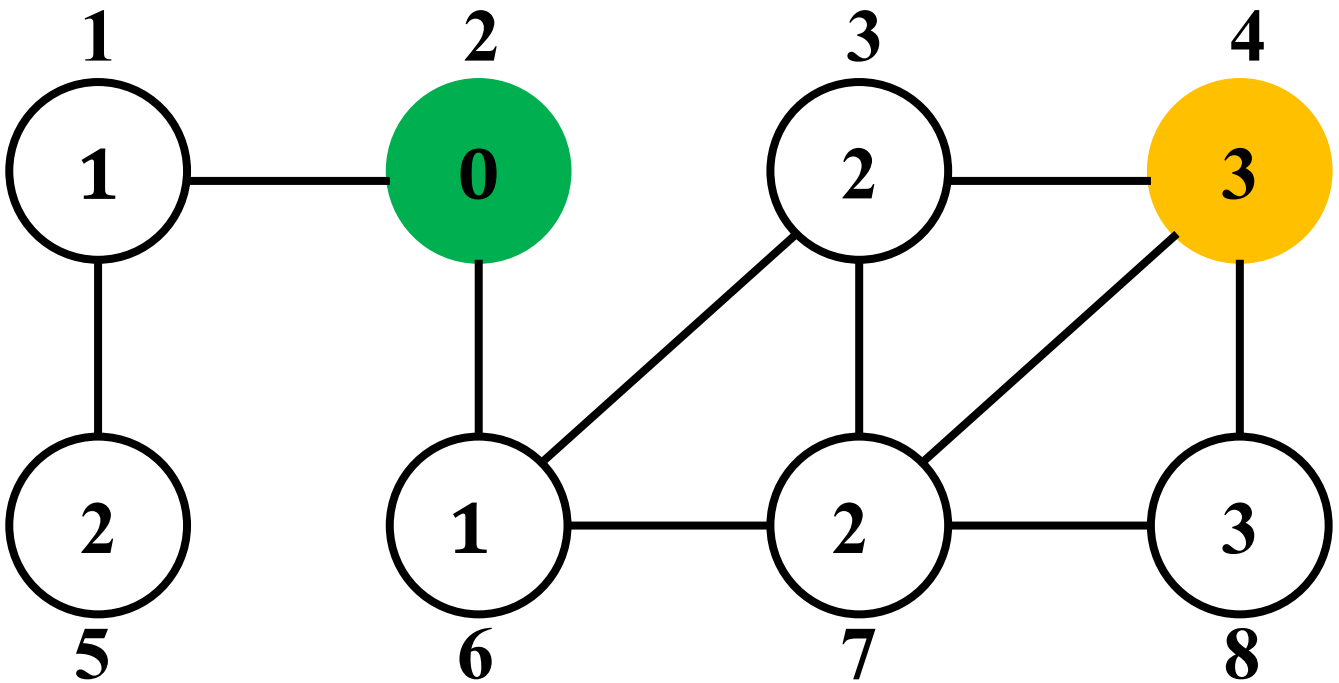
算法分析

算法应用

相关应用：最短路径

V	1	2	3	4	5	6	7	8
$pred[\,]$	2	N	6	3	1	2	6	7
$dist[\,]$	1	0	2	3	2	1	2	3

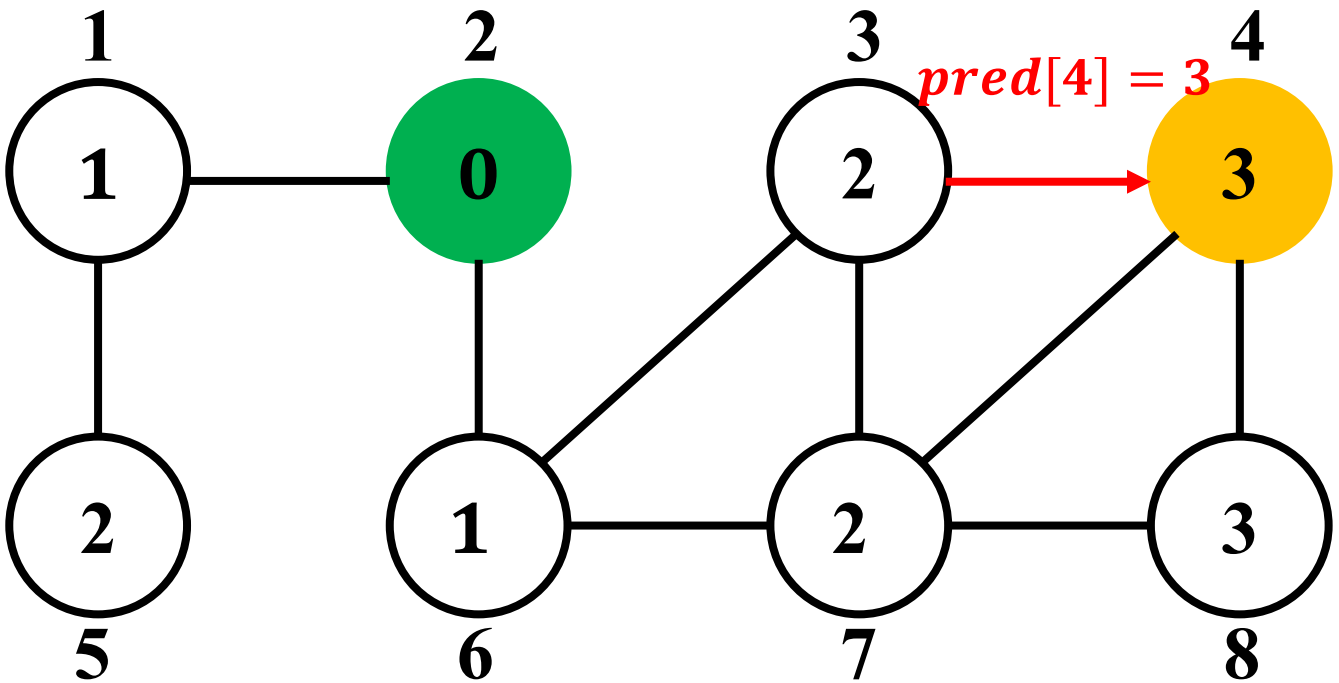
无权图G中，源点2到顶点4的最短路径



相关应用：最短路径

V	1	2	3	4	5	6	7	8
$pred[\,]$	2	N	6	3	1	2	6	7
$dist[\,]$	1	0	2	3	2	1	2	3

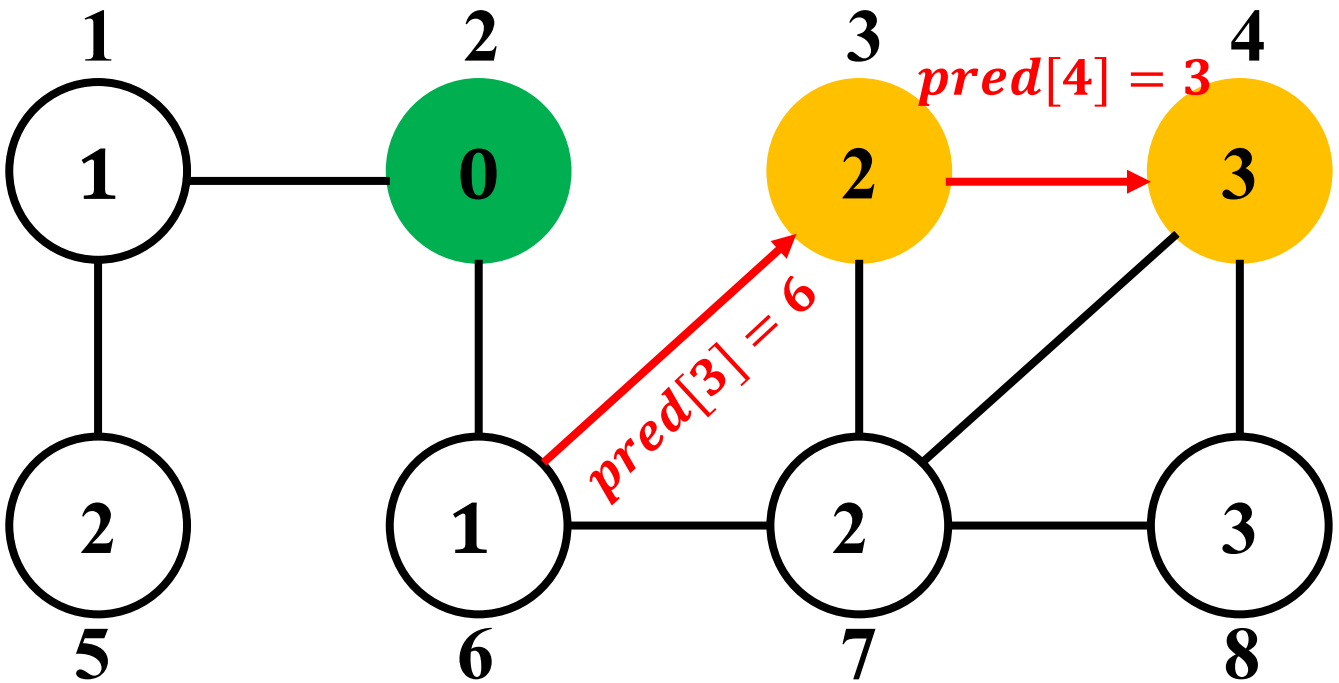
无权图G中，源点2到顶点4的最短路径



相关应用：最短路径

V	1	2	3	4	5	6	7	8
$pred[\,]$	2	N	6	3	1	2	6	7
$dist[\,]$	1	0	2	3	2	1	2	3

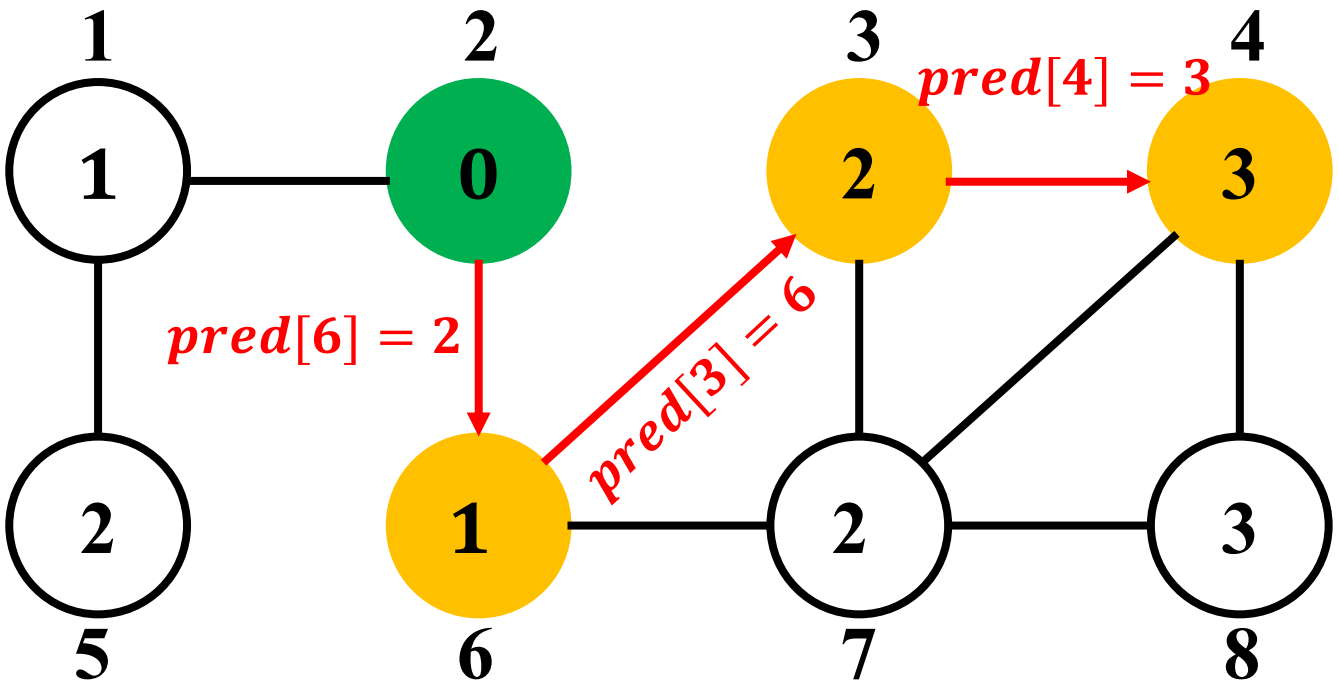
无权图G中，源点2到顶点4的最短路径



相关应用：最短路径

V	1	2	3	4	5	6	7	8
$pred[\,]$	2	N	6	3	1	2	6	7
$dist[\,]$	1	0	2	3	2	1	2	3

无权图G中，源点2到顶点4的最短路径

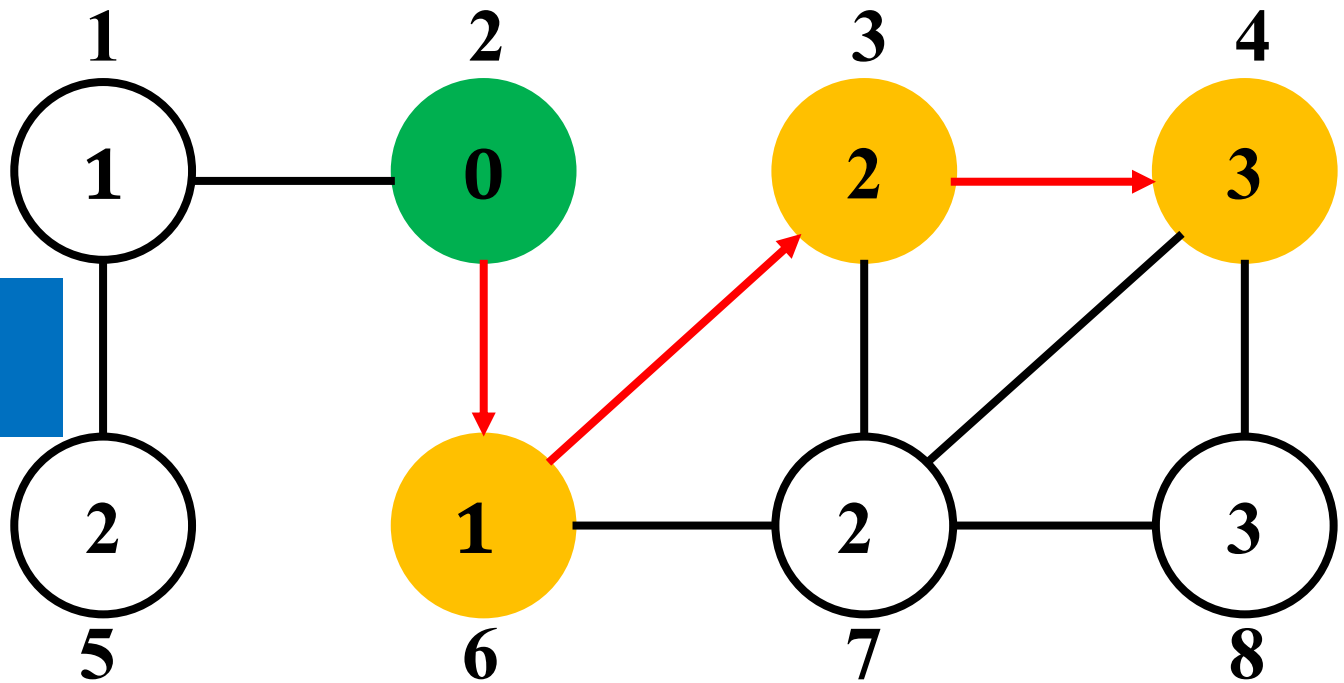


相关应用：最短路径

V	1	2	3	4	5	6	7	8
$pred[\,]$	2	N	6	3	1	2	6	7
$dist[\,]$	1	0	2	3	2	1	2	3

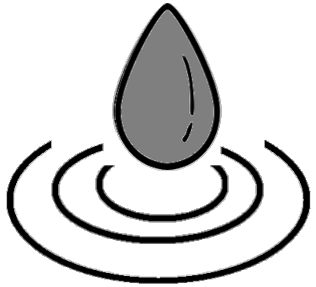
无权图G中，源点2到顶点4的最短路径

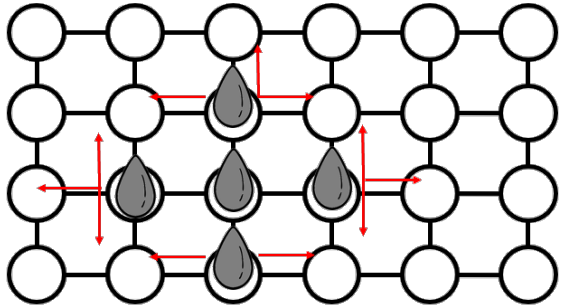
$2 \rightarrow 6 \rightarrow 3 \rightarrow 4$



小结

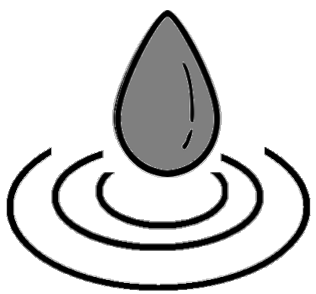
- 广度优先搜索
  - 算法核心思想：逐层扩散

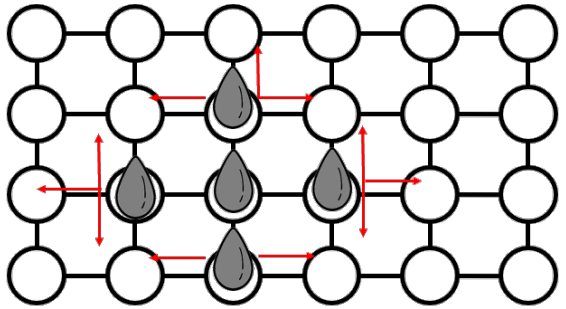




小结

- 广度优先搜索
  - 算法核心思想：逐层扩散

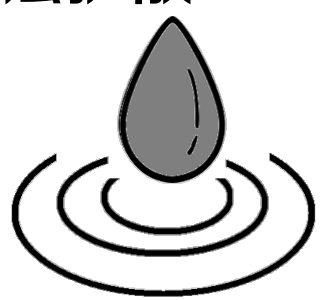


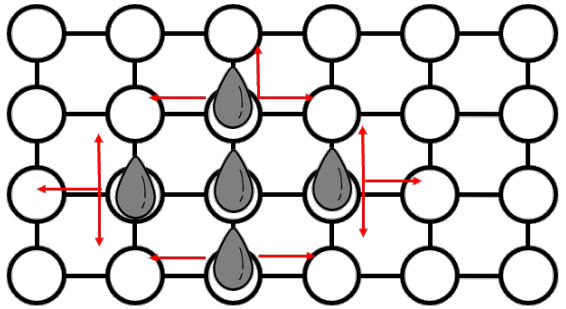


  - 算法运行时间：$O(|V|+|E|)$，简记为$O(V+E)$

小结

- 广度优先搜索
  - 算法核心思想：逐层扩散





  - 算法运行时间：$O(|V|+|E|)$，简记为$O(V+E)$
  - 算法相关应用：计算最短路径