Retro Fantasy AFL Optimiser Formulation

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1 Description

This model retrospectively determines the optimal AFL Fantasy team selection, captain choice, and trade sequence across a past season.

For each round $r \in R$, it chooses which players $p \in P$ are in the squad and allocates them to the required on-field and bench slots by position $k \in K$ (plus a bench utility slot). It also selects exactly one captain per round. The objective is to maximise the total realised points scored by the counted on-field players across all rounds, with the captain's counted score doubled.

The model respects:

- **Positional structure**: each round has required counts for on-field and bench positional slots by position ($n^{\text{on}}_{k,r}$ and $n^{\text{bench}}_{k,r}$), plus UTILITY_BENCH_COUNT bench utility slots.
- **Position eligibility**: a player may only be assigned to a position in K (DEF/MID/RUC/FWD) if they are eligible for that position in that round.
- **Trade limits**: between consecutive rounds, at most T_r players may be traded in (and at most T_r traded out).
- **Budget / bank**: the initial bank is salary cap minus the cost of the starting squad; the bank balance is carried forward and updated each round using the round-r prices of traded-out and traded-in players; and the bank is constrained to be non-negative via its domain ($b_r \in \mathbb{R}_{\geq 0}$).
- **Scoring (bye rounds)**: in each round, exactly N_r on-field players are chosen to have their scores counted.
- **Captaincy**: exactly one of the counted on-field players is designated as captain each round, and their counted score is doubled.

This formulation is designed to be implemented as a mixed-integer linear program (MILP).

2 Sets

Let P, subscripted by p, be the set of players.

Let R, subscripted by r, be the set of rounds.

Let $K = \{\text{DEF}, \text{MID}, \text{RUC}, \text{FWD}\}$, subscripted by k, be the set of positions.

3 Parameters (Vectors)

Let $s_{p,r} \in \mathbb{R}$ be the (known) number of points scored by player p in round r.

Let $c_{p,r} \in \mathbb{R}_{\geq 0}$ be the (known) price of player p in round r.

Let $e_{p,k,r} \in \{0, 1\}$ be a binary parameter indicating whether player p is eligible to be selected in position k in round r (1 if eligible, 0 otherwise).

Let $n^{\text{on}}_{k,r} \in \mathbb{Z}_{\geq 0}$ be the (known) number of on-field players required in position k in round r.

Let $n^{\text{bench}}_{k,r} \in \mathbb{Z}_{\geq 0}$ be the (known) number of bench players required in position k in round r.

Let $T_r \in \mathbb{Z}_{\geq 0}$ be the (known) maximum number of trades allowed in round r.

Let $N_r \in \mathbb{Z}_{\geq 0}$ be the (known) number of on-field players whose scores count toward the team total in round r (e.g. $N_r = 22$ in normal rounds and $N_r = 18$ in bye rounds).

4 Constants

Let SALARY_CAP $\in \mathbb{R}_{\geq 0}$ be the starting salary cap.

Let UTILITY_BENCH_COUNT $\in \mathbb{Z}_{\geq 0}$ be the number of bench utility slots.

5 Decision Variables

Let $x_{p,r} \in \{0, 1\}$ be a binary decision variable indicating whether player p is selected in the team (in any position, on-field or bench) in round r.

Let $x^{\text{on}}_{p,k,r} \in \{0, 1\}$ be a binary decision variable indicating whether player p is selected on-field in position k in round r.

Let $x^{\text{bench}}_{p,k,r} \in \{0, 1\}$ be a binary decision variable indicating whether player p is selected on the bench in position k in round r.

Let $x^{Q,\text{bench}}_{p,r} \in \{0, 1\}$ be a binary decision variable indicating whether player p is selected in the bench utility position in round r.

Let $b_r \in \mathbb{R}_{\geq 0}$ be a continuous decision variable representing the amount of cash in the bank in round r.

Let $\text{in}_{p,r} \in \{0, 1\}$ be a binary decision variable indicating whether player p is traded into the team in round r.

Let $\text{out}_{p,r} \in \{0, 1\}$ be a binary decision variable indicating whether player p is traded out of the team in round r.

Let $z_{p,r} \in \{0, 1\}$ be a binary decision variable indicating whether player p is selected as captain in round r.

Let $y_{p,r} \in \{0, 1\}$ be a binary decision variable indicating whether player p's score is counted towards the team total in round r.

6 Objective Function

Maximise the total points scored by the counted on-field players across all rounds, with the captain's counted score doubled:

$$\text{Maximise} \quad \sum_{r \in R} \sum_{p \in P} s_{p,r} \cdot (y_{p,r} + z_{p,r})$$

7 Constraints

7.1 Initial Bank Balance

Cash in the bank in round 1 is the salary cap minus the total price of the selected starting team:

$$b_1 = \text{SALARY\_CAP} - \sum_{p \in P} c_{p,1} \cdot x_{p,1}$$

7.2 Bank Balance Recurrence

The bank balance carries forward between rounds and is adjusted by the round-r prices of players traded out and traded in:

$$b_r = b_{r-1} + \sum_{p \in P} c_{p,r} \cdot \text{out}_{p,r} - \sum_{p \in P} c_{p,r} \cdot \text{in}_{p,r} \quad \forall r \in R \setminus \{1\}$$

7.3 Trade Indicator Linking

Trade-in and trade-out indicators are linked to changes in overall selection $x_{p,r}$. The following constraints linearise:

- $\text{in}_{p,r} = 1$ iff $(x_{p,r-1}, x_{p,r}) = (0, 1)$
- $\text{out}_{p,r} = 1$ iff $(x_{p,r-1}, x_{p,r}) = (1, 0)$

Lower bounds (force the indicator to switch on when a change occurs):

If the player was not selected in $r - 1$ but is selected in r, then $x_{p,r} - x_{p,r-1} = 1$, which forces $\text{in}_{p,r} \geq 1$.

This is the "trigger" constraint for trade-ins.

$$\text{in}_{p,r} \geq x_{p,r} - x_{p,r-1} \quad \forall p \in P, \forall r \in R \setminus \{1\}$$

If the player was selected in $r - 1$ but is not selected in r, then $x_{p,r-1} - x_{p,r} = 1$, which forces $\text{out}_{p,r} \geq 1$.

This is the "trigger" constraint for trade-outs.

$$\text{out}_{p,r} \geq x_{p,r-1} - x_{p,r} \quad \forall p \in P, \forall r \in R \setminus \{1\}$$

Upper bounds (prevent false positives / enforce the correct direction of change):

A trade-in can only occur if the player is selected in round r.

This prevents $\text{in}_{p,r} = 1$ when the player is not actually in the round-r team.

$$\text{in}_{p,r} \leq x_{p,r} \quad \forall p \in P, \forall r \in R \setminus \{1\}$$

A trade-in can only occur if the player was not selected in round $r-1$.

This prevents $\text{in}_{p,r} = 1$ when the player was already owned in round $r-1$ (i.e. no trade-in happened).

$$\text{in}_{p,r} \leq 1 - x_{p,r-1} \quad \forall p \in P, \forall r \in R \setminus \{1\}$$

A trade-out can only occur if the player was selected in round $r-1$.

This prevents $\text{out}_{p,r} = 1$ when the player wasn't owned in round $r-1$.

$$\text{out}_{p,r} \leq x_{p,r-1} \quad \forall p \in P, \forall r \in R \setminus \{1\}$$

A trade-out can only occur if the player is not selected in round r.

This prevents $\text{out}_{p,r} = 1$ when the player is still owned in round r.

$$\text{out}_{p,r} \leq 1 - x_{p,r} \quad \forall p \in P, \forall r \in R \setminus \{1\}$$

7.4 Linking Constraints

Overall selection must match positional selection:

$$x_{p,r} = \sum_{k \in K} x^{\text{on}}_{p,k,r} + \sum_{k \in K} x^{\text{bench}}_{p,k,r} + x^{Q,\text{bench}}_{p,r} \quad \forall p \in P, \forall r \in R$$

A player can occupy at most one lineup slot in a given round:

$$\sum_{k \in K} x^{\text{on}}_{p,k,r} + \sum_{k \in K} x^{\text{bench}}_{p,k,r} + x^{Q,\text{bench}}_{p,r} \leq 1 \quad \forall p \in P, \forall r \in R$$

7.5 Maximum Team Changes Per Round

Between consecutive rounds, at most T_r players may be traded into the team (and at most T_r traded out) between rounds $r-1$ and r.

Using the trade indicators, this is enforced by limiting the number of trade-ins (equivalently trade-outs) each round:

$$\sum_{p \in P} \mathrm{in}_{p,r} \leq T_r \quad \forall r \in R \setminus \{1\}$$

$$\sum_{p \in P} \mathrm{out}_{p,r} \leq T_r \quad \forall r \in R \setminus \{1\}$$

7.6 Positional Structure

The team must have the following positional structure in every round.

On-field positions:

$$\sum_{p \in P} x^{\text{on}}_{p,k,r} = n^{\text{on}}_{k,r} \quad \forall k \in K, \forall r \in R$$

Bench positions:

$$\sum_{p \in P} x^{\text{bench}}_{p,k,r} = n^{\text{bench}}_{k,r} \quad \forall k \in K, \forall r \in R$$

Bench utility:

$$\sum_{p \in P} x^{Q,\text{bench}}_{p,r} = \text{UTILITY\_BENCH\_COUNT} \quad \forall r \in R$$

7.7 Position Eligibility

Players can only be selected into a position if they are eligible for that position (eligibility may be multi-position):

$$x^{\text{on}}_{p,k,r} \leq e_{p,k,r} \quad \forall p \in P, \forall k \in K, \forall r \in R$$

$$x^{\text{bench}}_{p,k,r} \leq e_{p,k,r} \quad \forall p \in P, \forall k \in K, \forall r \in R$$

Bench utility is unconstrained by lineup position, but still must use an eligible player in that round:

$$x^{Q,\text{bench}}_{p,r} \leq \sum_{k \in K} e_{p,k,r} \quad \forall p \in P, \forall r \in R$$

7.8 Scoring Selection (Bye Rounds)

In each round, select which on-field players have their scores counted, up to N_r:

$$\sum_{p \in P} y_{p,r} = N_r \quad \forall r \in R$$

A player's score can only be counted if they are selected on-field in that round:

$$y_{p,r} \leq \sum_{k \in K} x^{\text{on}}_{p,k,r} \quad \forall p \in P, \forall r \in R$$

7.9 Captaincy

Exactly one captain must be selected each round:

$$\sum_{p \in P} z_{p,r} = 1 \quad \forall r \in R$$

The captain must be one of the counted on-field players in that round:

$$z_{p,r} \leq y_{p,r} \quad \forall p \in P, \forall r \in R$$