Retro Fantasy AFL Optimiser Formulation

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1 Description

This model retrospectively determines the optimal AFL Fantasy team selection, captain choice, and trade sequence across a past season.

For each round $r \in R$, it chooses which players $p \in P$ are in the squad and allocates them to the required on-field and bench slots by position $k \in K$ (plus a bench utility slot). It also selects exactly one captain per round. The objective is to maximise the total realised points scored by the counted on-field players across all rounds, with the captain's counted score doubled.

The model respects:

- **Positional structure**: each round has required counts for on-field and bench positional slots by position (n_k^{on} and n_k^{bench}), plus UTILITY_BENCH_COUNT bench utility slots.
- **Position eligibility**: a player may only be assigned to a position in K (DEF/MID/RUC/FWD) if they are eligible for that position in that round.
- **Trade limits**: between consecutive rounds, at most T_r players may be traded in (and at most T_r traded out).
- **Budget / bank**: the initial bank is salary cap minus the cost of the starting squad; the bank balance is carried forward and updated each round using the round-r prices of traded-out and traded-in players; and the bank is constrained to be non-negative via its domain ($b_r \in \mathbb{R}_{\geq 0}$).
- **Scoring (bye rounds)**: in each round, exactly N_r on-field players are chosen to have their scores counted.
- **Captaincy**: exactly one of the counted on-field players is designated as captain each round, and their counted score is doubled.

This formulation is designed to be implemented as a mixed-integer linear program (MILP).

2 Sets

Let P, subscripted by p, be the set of players.

Let R, subscripted by r, be the set of rounds.

Let $K = \{\mathrm{DEF}, \mathrm{MID}, \mathrm{RUC}, \mathrm{FWD}\}$, subscripted by k, be the set of positions.

Define the eligibility-filtered index set:

$$E = \{(p, k, r) \in P \times K \times R : e_{p,k,r} = 1\}$$

This set contains exactly the (player, position, round) tuples for which the player is eligible for that position in that round.

In the implementation, positional decision variables are only created for indices in E. This reduces the number of binary variables and removes the need for explicit eligibility constraints.

3 Parameters (Vectors)

Let $s_{p,r} \in \mathbb{R}$ be the (known) number of points scored by player p in round r.

Let $c_{p,r} \in \mathbb{R}_{\geq 0}$ be the (known) price of player p in round r.

Let $e_{p,k,r} \in \{0, 1\}$ be a binary parameter indicating whether player p is eligible to be selected in position k in round r (1 if eligible, 0 otherwise).

Let $n_k^{\text{on}} \in \mathbb{Z}_{\geq 0}$ be the (known) number of on-field players required in position k.

Let $n_k^{\text{bench}} \in \mathbb{Z}_{\geq 0}$ be the (known) number of bench players required in position k.

Let $T_r \in \mathbb{Z}_{\geq 0}$ be the (known) maximum number of trades allowed in round r.

Let $N_r \in \mathbb{Z}_{\geq 0}$ be the (known) number of on-field players whose scores count toward the team total in round r (e.g. $N_r = 22$ in normal rounds and $N_r = 18$ in bye rounds).

4 Constants

Let SALARY_CAP $\in \mathbb{R}_{\geq 0}$ be the starting salary cap.

Let UTILITY_BENCH_COUNT $\in \mathbb{Z}_{\geq 0}$ be the number of bench utility slots.

5 Decision Variables

Let $x_{p,r} \in \{0,1\}$ be a binary decision variable indicating whether player p is selected in the team (in any position, on-field or bench) in round r.

Let $x^{\text{on}}_{p,k,r} \in \{0,1\}$ be a binary decision variable indicating whether player p is selected on-field in position k in round r.

Let $x^{\text{bench}}_{p,k,r} \in \{0,1\}$ be a binary decision variable indicating whether player p is selected on the bench in position k in round r.

Domain note (mirrors the implementation):

- $x^{\text{on}}_{p,k,r}$ and $x^{\text{bench}}_{p,k,r}$ only exist for $(p,k,r) \in E$.
- For $(p,k,r) \notin E$, these variables are treated as structurally fixed to 0 (i.e. they are not created at all).

Let $x^{Q,\text{bench}}_{p,r} \in \{0,1\}$ be a binary decision variable indicating whether player p is selected in the bench utility position in round r.

Let $b_r \in \mathbb{R}_{\geq 0}$ be a continuous decision variable representing the amount of cash in the bank in round r.

Let $\text{in}_{p,r} \in \{0,1\}$ be a binary decision variable indicating whether player p is traded into the team in round r.

Let $\text{out}_{p,r} \in \{0,1\}$ be a binary decision variable indicating whether player p is traded out of the team in round r.

Let $z_{p,r} \in \{0,1\}$ be a binary decision variable indicating whether player p is selected as captain in round r.

Let $y_{p,r} \in \{0,1\}$ be a binary decision variable indicating whether player p's score is counted towards the team total in round r.

6 Objective Function

Maximise the total points scored by the counted on-field players across all rounds, with the captain's counted score doubled:

$$\text{Maximise} \quad \sum_{r \in R} \sum_{p \in P} s_{p,r} \cdot (y_{p,r} + z_{p,r})$$

7 Constraints

7.1 Initial Bank Balance

Cash in the bank in round 1 is the salary cap minus the total price of the selected starting team:

$$b_1 = \text{SALARY\_CAP} - \sum_{p \in P} c_{p,1} \cdot x_{p,1}$$

7.2 Bank Balance Recurrence

The bank balance carries forward between rounds and is adjusted by the round-r prices of players traded out and traded in:

$$b_r = b_{r-1} + \sum_{p \in P} c_{p,r} \cdot \text{out}_{p,r} - \sum_{p \in P} c_{p,r} \cdot \text{in}_{p,r} \quad \forall r \in R \setminus \{1\}$$

7.3 Trade Indicator Linking

Trade-in and trade-out indicators are linked to changes in overall selection $x_{p,r}$. The following constraints linearise:

- $\text{in}_{p,r} = 1$ iff $(x_{p,r-1}, x_{p,r}) = (0, 1)$
- $\text{out}_{p,r} = 1$ iff $(x_{p,r-1}, x_{p,r}) = (1, 0)$

Lower bounds (force the indicator to switch on when a change occurs):

If the player was not selected in $r-1$ but is selected in r, then $x_{p,r} - x_{p,r-1} = 1$, which forces $\text{in}_{p,r} \geq 1$.

This is the “trigger” constraint for trade-ins.

$$\text{in}_{p,r} \geq x_{p,r} - x_{p,r-1} \quad \forall p \in P, \forall r \in R \setminus \{1\}$$

If the player was selected in $r-1$ but is not selected in r, then $x_{p,r-1} - x_{p,r} = 1$, which forces $\text{out}_{p,r} \geq 1$.

This is the “trigger” constraint for trade-outs.

$$\text{out}_{p,r} \geq x_{p,r-1} - x_{p,r} \quad \forall pinP, \forall rinR \setminus \{1\}$$

Upper bounds (prevent false positives / enforce the correct direction of change):

A trade-in can only occur if the player is selected in round r.

This prevents $\text{in}_{p,r} = 1$ when the player is not actually in the round-r team.

$$\text{in}_{p,r} \leq x_{p,r} \quad \forall p \in P, \forall r \in R \setminus \{1\}$$

A trade-in can only occur if the player was not selected in round $r-1$.

This prevents $\text{in}_{p,r} = 1$ when the player was already owned in round $r-1$ (i.e. no trade-in happened).

$$\text{in}_{p,r} \leq 1 - x_{p,r-1} \quad \forall p \in P, \forall r \in R \setminus \{1\}$$

A trade-out can only occur if the player was selected in round $r-1$.

This prevents $\text{out}_{p,r} = 1$ when the player wasn't owned in round $r-1$.

$$\text{out}_{p,r} \leq x_{p,r-1} \quad \forall p \in P, \forall r \in R \setminus \{1\}$$

A trade-out can only occur if the player is not selected in round r.

This prevents $\text{out}_{p,r} = 1$ when the player is still owned in round r.

$$\text{out}_{p,r} \leq 1 - x_{p,r} \quad \forall p \in P, \forall r \in R \setminus \{1\}$$

7.4 Linking Constraints

Overall selection must match positional selection:

$$x_{p,r} = \sum_{k \in K} x^{\text{on}}_{p,k,r} + \sum_{k \in K} x^{\text{bench}}_{p,k,r} + x^{Q,\text{bench}}_{p,r} \quad \forall p \in P, \forall r \in R$$

A player can occupy at most one lineup slot in a given round:

$$\sum_{k \in K} x^{\text{on}}_{p,k,r} + \sum_{k \in K} x^{\text{bench}}_{p,k,r} + x^{Q,\text{bench}}_{p,r} \leq 1 \quad \forall p \in P, \forall r \in R$$

7.5 Maximum Team Changes Per Round

Between consecutive rounds, at most T_r players may be traded into the team (and at most T_r traded out) between rounds $r-1$ and r.

Using the trade indicators, this is enforced by limiting the number of trade-ins (equivalently trade-outs) each round:

$$\sum_{p \in P} \mathrm{in}_{p,r} \leq T_r \quad \forall r \in R \setminus \{1\}$$

$$\sum_{p \in P} \mathrm{out}_{p,r} \leq T_r \quad \forall r \in R \setminus \{1\}$$

7.6 Positional Structure

The team must have the following positional structure in every round.

On-field positions:

$$\sum_{p \in P} x^{\text{on}}_{p,k,r} = n^{\text{on}}_k \quad \forall k \in K, \forall r \in R$$

Bench positions:

$$\sum_{p \in P} x^{\text{bench}}_{p,k,r} = n^{\text{bench}}_k \quad \forall k \in K, \forall r \in R$$

Bench utility:

$$\sum_{p \in P} x^{Q,\text{bench}}_{p,r} = \text{UTILITY\_BENCH\_COUNT} \quad \forall r \in R$$

7.7 Position Eligibility

Position eligibility is enforced structurally by the index set E.

Because $x^{\text{on}}_{p,k,r}$ and $x^{\text{bench}}_{p,k,r}$ are only defined for eligible triples $(p, k, r) \in E$, it is impossible for the model to assign a player to a position they are not eligible for in that round.

Equivalently, you can view the implementation as implicitly enforcing:

$$x^{\text{on}}_{p,k,r} = 0 \quad \forall (p, k, r) \notin E$$

$$x^{\text{bench}}_{p,k,r} = 0 \quad \forall (p, k, r) \notin E$$

and therefore explicit constraints of the form $x^{\text{on}}_{p,k,r} \leq e_{p,k,r}$ are not required.

7.8 Scoring Selection (Bye Rounds)

In each round, select which on-field players have their scores counted, up to N_r:

$$\sum_{p \in P} y_{p,r} = N_r \quad \forall r \in R$$

A player's score can only be counted if they are selected on-field in that round:

$$y_{p,r} \leq \sum_{k \in K} x^{\text{on}}_{p,k,r} \quad \forall p \in P, \forall r \in R$$

7.9 Captaincy

Exactly one captain must be selected each round:

$$\sum_{p \in P} z_{p,r} = 1 \quad \forall r \in R$$

The captain must be one of the counted on-field players in that round:

$$z_{p,r} \leq y_{p,r} \quad \forall p \in P, \forall r \in R$$