Homework3

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APPM 4600

1.) 1. Consider the equation $2x - 1 = \sin x$.

(a) Find a closed interval $[a, b]$ on which the equation has a root r, and use the Intermediate Value Theorem to prove that r exists.

$2x - 1 = 0$ for $x = 1$, $\sin(x) = 0$ for $x = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$

Let $y = 2x - 1 - \sin(x)$ for $x = (-\frac{\pi}{2})$ $\therefore$ $y = 2(-\frac{\pi}{2}) - 1 - \sin(-\frac{\pi}{2}) = -\pi < 0$

for $x = (\frac{\pi}{2})$ $\therefore$ $y = 2(\frac{\pi}{2}) - 1 - \sin(\frac{\pi}{2}) = \pi - 2 > 0$

$\therefore$ let $a = -\frac{\pi}{2}$, $b = \frac{\pi}{2}$. Since $y = 2x - 1 - \sin(x) < 0$ for $x = -\frac{\pi}{2}$ and $y = 2x - 1 - \sin(x) > 0$ for $x = \frac{\pi}{2}$, by the Intermediate Value Theorem, there must exist some root r on $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

(b) Prove that r from (a) is the only root of the equation (on all of $\mathbb{R}$).

$2x - 1 - \sin(x) = 0$

$\frac{\partial}{\partial x}\left(2x - 1 - \sin(x)\right) = 2 - \cos(x) = 0$

Since $\cos(x)$ is bounded by $|1|$, there are no values of $x \in \mathbb{R}$ where another root exists.

(c) Use the bisection code from class (or your own) to approximate r to eight correct decimal places. Include the calling script, the resulting final approximation, and the total number of iterations used.

Code:

```
# import libraries
import numpy as np

def driver():

# use routines
    f = lambda x: 2*x - 1 - np.sin(x)
    a = -np.pi/2
    b = np.pi/2

    tol = 1e-8

    [astar,ier] = bisection(f,a,b,tol)
    print('the approximate root is',astar)
    print('the error message reads:',ier)
    print('f(astar) =', f(astar))



# define routines
def bisection(f,a,b,tol):

#    Inputs:
#     f,a,b       - function and endpoints of initial interval
#      tol  - bisection stops when interval length < tol

#    Returns:
#      astar - approximation of root
#      ier   - error message
#            - ier = 1 => Failed
```

```
#    Returns:
#      astar - approximation of root
#      ier   - error message
#            - ier = 1 => Failed
#            - ier = 0 == success

#     first verify there is a root we can find in the interval

    fa = f(a)
    fb = f(b);
    if (fa*fb>0):
      ier = 1
      astar = a
      return [astar, ier]

#   verify end points are not a root
    if (fa == 0):
      astar = a
      ier =0
      return [astar, ier]

    if (fb ==0):
      astar = b
      ier = 0
      return [astar, ier]

    count = 0
    d = 0.5*(a+b)
    while (abs(d-a)> tol):
      fd = f(d)
      if (fd ==0):
        astar = d
        ier = 0
        return [astar, ier]
      if (fa*fd<0):
         b = d
      else:
        a = d
        fa = fd
      d = 0.5*(a+b)
      count = count +1
    print('abs(d-a) = ', abs(d-a))
    print('Number of iterations: ', count)
    astar = d
    ier = 0
    return [astar, ier]

driver()
```

Output:

```
tonys@Tonys-Surface MINGW64 /c/users/tonys/Documents/APPM4600/testrep/Homework/H
omework3 (main)
$ python3 Problem1c.py
abs(d-a) =  5.851672257861651e-09
Number of iterations:  28
the approximate root is 0.8878622154822129
the error message reads: 0
f(astar) = 5.354353072029028e-09
```