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APPM 4600

Lab 4 Derivation

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3.1 Aitken's Δ^2 acceleration technique

We will consider the acceleration technique called **Aitken's Δ^2 method**. To use this method you start with a sequence $\{p_n\}_{n=1}^{\infty}$ that converges linearly to the value p. Then you create a new sequence of approximations with iterations defined by

$$\hat{p}_n = p_n - \frac{(p_{n+1} - p_n)^2}{p_{n+2} - 2p_{n+1} + p_n}$$

Theorem 3.1 (2.14 from text). *Suppose that $\{p_n\}_{n=1}^{\infty}$ is a sequence that converges linearly to the limit p and that*

$$\lim_{n\to\infty} \frac{p_{n+1} - p}{p_n - p} < 1$$

then the Aitken's sequence $\{\hat{p}_n\}_{n=1}^{\infty}$ converges to p faster than $\{p_n\}_{n=1}^{\infty}$ in the little o sense; i.e.

$$\lim_{n\to\infty} \frac{\hat{p}_n - p}{p_n - p} = 0.$$

3.2 Exercises

- *Derivation of the method.* We know that a sequence $\{p_n\}_{n=1}^{\infty}$ converging linearly to p means that
$$\frac{p_{n+1} - p}{p_n - p} \sim \frac{p_{n+2} - p}{p_{n+1} - p}.$$
Take this equation and solve for p. The value p gives the formula for the iterates in *Aitken's Δ^2 method.*
- Write a subroutine that takes in a sequence of approximations and returns a vector of the approximations. It should also have tolerance and max number of iterations as input.
- Apply Aitken's Δ^2 method to the sequence created by the fixed point iteration in the before lab exercise set. Determine if the convergence is in fact faster than the fixed point iteration. Can you figure out the order of convergence?

$$\frac{p_{n+1} - p}{p_n - p} \approx \frac{p_{n+2} - p}{p_{n+1} - p}$$

$$(p_{n+1} - p)^2 = (p_{n+2} - p)(p_n - p)$$

$$p_{n+1}^2 - \cancel{2}p_{n+1} \cdot p + \cancel{p^2} = p_{n+2}\,p_n - \cancel{p_{n+1} \cdot p} - p_n \cdot p + \cancel{p^2}$$

$$p_{n+1}^2 - p_{n+2} \cdot p_n = p\,(p_{n+1} - p_n)$$

$$\frac{p_{n+1}^2 - p_{n+2} \cdot p_n}{p_{n+1} - p_n} = p$$