Homework12

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APPM 4600

1.) 1. Consider the linear system

$$6x + 2y + 2z = -2$$
$$2x + 2/3y + 1/3z = 1$$
$$x + 2y - z = 0$$

(a) (a) Verify that $(x, y, z) = (2.6, -3.8, -5)$ is the exact solution.

$$\underbrace{\begin{bmatrix} 6 & 2 & 2 \\ 2 & \frac{2}{3} & \frac{1}{3} \\ 1 & 2 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\bar{x}} = \underbrace{\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}}_{b}$$

$$Ax = b \Rightarrow \bar{x} = A^{-1}b$$

$$\left[\begin{array}{ccc|c} 6 & 2 & 2 & -2 \\ 2 & \frac{2}{3} & \frac{1}{3} & 1 \\ 1 & 2 & -1 & 0 \end{array}\right] \begin{array}{l} \frac{1}{6}R_1 \\ R_2 - 2R_1^* \rightarrow \end{array} \left[\begin{array}{ccc|c} 1 & \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & -\frac{1}{3} & \frac{5}{3} \\ 1 & 2 & -1 & 0 \end{array}\right] \begin{array}{l} R_2 \Leftrightarrow R_3 \rightarrow \\ R_2^* - R_1 \end{array} \left[\begin{array}{ccc|c} 1 & \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{5}{3} & -\frac{4}{3} & \frac{1}{3} \\ 0 & 0 & -\frac{1}{3} & \frac{5}{3} \end{array}\right] R_2 - 4R_3 \rightarrow \left[\begin{array}{ccc|c} 1 & \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{5}{3} & 0 & -\frac{19}{3} \\ 0 & 0 & -\frac{1}{3} & \frac{5}{3} \end{array}\right] \begin{array}{l} R_1 + R_3 \\ \rightarrow \end{array} \left[\begin{array}{ccc|c} 1 & \frac{1}{3} & 0 & \frac{4}{3} \\ 0 & \frac{5}{3} & 0 & -\frac{19}{3} \\ 0 & 0 & -\frac{1}{3} & \frac{5}{3} \end{array}\right]$$

$$\begin{array}{l} R_1 - \frac{1}{5}R_2 \\ \rightarrow \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{13}{5} \\ 0 & \frac{5}{3} & 0 & -\frac{19}{3} \\ 0 & 0 & -\frac{1}{3} & \frac{5}{3} \end{array}\right] \begin{array}{l} \frac{3}{5}R_2 \rightarrow \\ -3R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{13}{5} \\ 0 & 1 & 0 & -\frac{19}{5} \\ 0 & 0 & 1 & -5 \end{array}\right] \Rightarrow (x, y, z) = (2.6, -3.8, -5) \checkmark$$

(b) (b) Using 4 digit floating point arithmetic with rounding, solve the system via Gaussian elimination without pivoting.

$$\left[\begin{array}{ccc|c} 6 & 2 & 2 & -2 \\ 2 & \frac{2}{3} & \frac{1}{3} & 1 \\ 1 & 2 & -1 & 0 \end{array}\right] \begin{array}{l} \frac{1}{6}R_1 \\ R_2 - 2R_1^* \rightarrow \\ R_3 - R_1^* \end{array} \left[\begin{array}{ccc|c} 1. & 0.3333 & 0.3333 & -0.3333 \\ 0. & 0.0001 & -0.3333 & 1.6666 \\ 0 & 1.6667 & -1.3333 & 0.3333 \end{array}\right] R_2 - \frac{1}{4}R_3 \rightarrow \left[\begin{array}{ccc|c} 1. & 0.3333 & 0.3333 & -0.3333 \\ 0 & -0.4166 & 0.0000 & 1.5833 \\ 0. & 1.6667 & -1.3333 & 0.3333 \end{array}\right] R_3 + \frac{1.6667}{0.4166}R_2$$

$$\rightarrow \left[\begin{array}{ccc|c} 1. & 0.3333 & 0.3333 & -0.3333 \\ 0. & -0.4166 & 0. & 1.5833 \\ 0. & 0. & -1.3333 & 6.6676 \end{array}\right] \begin{array}{l} R_1 + \frac{0.3333}{1.3333}R_3 \\ \rightarrow \end{array} \left[\begin{array}{ccc|c} 1. & 0.3333 & 0. & 1.3335 \\ 0. & -0.4166 & 0. & 1.5833 \\ 0. & 0. & -1.3333 & 6.6676 \end{array}\right] R_1 + \frac{0.3333}{0.4166}R_2$$

$$\rightarrow \left[\begin{array}{ccc|c} 1. & 0. & 0. & 2.6002 \\ 0. & -0.4166 & 0. & 1.5833 \\ 0. & 0. & -1.3333 & 6.6676 \end{array}\right] \begin{array}{l} \frac{1}{-0.4166}R_2 \rightarrow \\ \frac{1}{-1.3333}R_3 \end{array} \left[\begin{array}{ccc|c} 1. & 0. & 0. & 2.6002 \\ 0. & 1. & 0. & 3.8005 \\ 0. & 0. & 1. & 5.0008 \end{array}\right]$$

(c) (c) Repeat part (a) with partial pivoting. $\rightarrow$ Part (a) was verifying exact soln.? Redo (b) w/ pivoting?

$$\left[\begin{array}{ccc|c} 6 & 2 & 2 & -2 \\ 2 & \frac{2}{3} & \frac{1}{3} & 1 \\ 1 & 2 & -1 & 0 \end{array}\right] \begin{array}{l} \frac{1}{6}R_1 \\ R_2 \Leftrightarrow R_3 \rightarrow \end{array} \left[\begin{array}{ccc|c} 1. & 0.3333 & 0.3333 & -0.3333 \\ 1 & 2 & -1 & 0 \\ 2 & 0.6667 & 0.3333 & 1 \end{array}\right] \begin{array}{l} R_2 - R_1 \rightarrow \\ R_3 - 2R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 0.3333 & 0.3333 & -0.3333 \\ 0. & 1.6667 & -1.3333 & 0.3333 \\ 0. & 0.0001 & -0.3333 & 1.6666 \end{array}\right] R_2 - 4R_3 \rightarrow$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0.3333 & 0.3333 & -0.3333 \\ 0. & 1.6663 & -0.0001 & -6.3331 \\ 0. & 0.0001 & -0.3333 & 1.6666 \end{array}\right] \xrightarrow{R_1 - \frac{0.3333}{1.6663} R_2} \left[\begin{array}{ccc|c} 1. & 0. & 0.3333 & 0.9335 \\ 0. & 1.6663 & -0.0001 & -6.3331 \\ 0. & 0.0001 & -0.3333 & 1.6666 \end{array}\right] \xrightarrow{R_1 + R_3} \left[\begin{array}{ccc|c} 1. & 0. & 0. & 2.6001 \\ 0. & 1.6663 & -0.0001 & -6.3331 \\ 0 & 0.0001 & -0.3333 & 1.6666 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1. & 0. & 0. & 2.6001 \\ 0. & 1. & 0. & -3.8007 \\ 0. & 0. & 1. & -5.0003 \end{array}\right]$$

(d) (d) Which method is more accurate? i.e. stable.

From my work, I found that they have the same or similar order of stability. However, in a different example we did in class, when the size of the elements varied more greatly (i.e. > 10^4 orders of mag.), pivoting showed to be much more stable.

I imagine that I made a rounding error that helped to stabilize one of the above methods, or in cases where the elements are of similar magnitude, the stability does not rely as heavily on pivoting.